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Generating Full Monte Carlo S-Curves Without External Software

Presented at the ICEAA Professional Development & Training Workshop

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Download our paper on the Whova App!

*This presentation uses Speaker Notes as annotations. It is incomplete without the Speaker Notes.
In general, acronyms are expanded in the Speaker Notes on their first reference and on slides at the end of the presentation.
Significant acronyms are expanded in the body of the slides on their first reference.*

Agenda

- Background, Motivation, and *So What?*
- What is the IVMF?
- Importance of Risk & Uncertainty in a Comparative Analysis
- Correlation Matters (but isn't enough)
- Cholesky Decomposition & Normal to Anything (NORTA)
- Our Approach
- Our Results
- Summary / Conclusions / Next Steps

Background, Motivation, & BLUF

- MITRE is developing the Investment Value Management Framework (IVMF) to automate many commonly performed cost/benefit estimating tasks in comparative analyses
- IVMF development gives rise to a desire for "home-grown" cost risk and uncertainty analysis tool, "self-contained" in Excel, without need for external software, but with Monte Carlo functionality
- Challenge is correlating random variates in a way that preserves their original distributions
- This is simple with only two random variates, but complex when dozens (or hundreds) of cost elements are involved
- MITRE developed an innovative technique that does this!

Our technique, more or less faithfully, replicates results from *Crystal Ball*¹, with more mathematical rigor, comparable (or faster) run-times, and no software to install.

Investment Value Management Framework (IVMF)

- Excel, VBA-based tool that automates and provides structure for comparative analyses (EA, AoA, BCA, CBA, etc.) factoring in costs, non-monetizable benefits and risks
- IVMF Cost Module - somewhat akin to ACEIT¹, but focused on comparative analyses vice LCCEs
- Includes:
 - Automated population of Work Breakdown Structure (WBS) by Course Of Action²
 - Calculation of measures of merit (ROI, NPV, IRR, DPP, etc.)
 - Ongoing investment monitoring
- Provides mathematically rigorous treatment of risk and uncertainty
- Not available for external distribution

1. ACEIT is a product of Tecolote Research, Inc.

2. In other words, once the user defines a WBS, the model automatically "propagates" that structure to each COA. The user need not manually enter it.

IVMF Overview



Supports informed and evidence-based investment decisions



Considers the cost, benefit, and risk of proposed investments scenario(s) and reports results



Answers the question as to which investment scenario is the most effective and affordable relative to the status quo



Supports lifecycle management, initially using the same metrics applied to inform initial investment selection



Includes process guidelines to better manage programs and investments

CES #	Cost Element (\$K)	2025	2026
	COA 2 Cost	\$ 29,950	\$ 18,400
1	Investment	\$ 27,750	\$ 16,200
1.1	Program/Project Management	\$ 1,500	\$ 1,500
1.1.1	Planning Phase Program	\$ 1,000	\$ 1,000
1.1.2	Acquisition Phase Program	\$ 500	\$ 500
1.2	Systems Engineering	\$ 750	\$ 750
1.2.1	Planning - Systems Engineering	\$ 500	\$ 500
1.2.2	Acquisition - Systems Engineering	\$ 250	\$ 250
1.3	Business Process Re-engineering (BPR)	\$ 500	\$ 500
1.3.1	Planning Phase BPR	\$ 300	\$ 300
1.3.2	Acquisition Phase BPR	\$ 200	\$ 200
1.4	System Development	\$ 1,900	\$ 1,900
1.4.1	Prototype Investment	\$ 1,000	\$ 1,000
1.4.2	Software Development	\$ 500	\$ 500

The IVMF model is an Excel-based workbook that guides the user through calculations supporting both the initial investment and post-implementation reviews

Benefit Score				
Benefit Categories	Weight	Status Quo	Investment	Delta
Enhance Organizational Capacity	25%	3.0	3.6	▲ 19%
Improve Customer Satisfaction	11%	3.0	3.8	▲ 28%
Improve System Reliability and Information Security	11%	3.0	3.6	▲ 19%
Improve System Security and Information Security	11%	3.0	4.2	▲ 39%
Non-Monetary Benefits Subtotal	100%	3.0	3.7	▲ 22%

BENEFIT

* Quantitative normalized score is based on a scale from 1 to 5. The values of 1 to 5 represents a continuous range from 0 to 100. A score of "0" equates to absolute disagreement with the statement and a score of "100" equates to absolute agreement with the statement. A score of "50" represents neither absolute disagreement nor absolute agreement.

Legend
 Red Not acceptable
 Amber Marginally acceptable
 Green Acceptable

Project XYZ		Without	With
Method		Uncertainty	
Monetized	Incremental NPV (\$M)	\$2.4	(\$0.8)
	Incremental IRR	20.2%	4.1%
	Incremental DPP (years)	5.9	0.1
Non-Monetary Benefits	Enhance Organizational Capacity	25%	3.0
	Improve Customer Satisfaction	11%	3.0
	Improve System Reliability and Information Security	11%	3.0
	Improve System Security and Information Assurance	11%	3.0
Non-Monetary Benefits Subtotal	100%	3.0	3.7

INVESTMENT SELECT

Investment Risk Assessment				
Risk Category	Status Quo		Scenario 2	
	Rank	Score	Rank	Score
Implementation	M	53.0	ML	33.0
Performance	M	77.0	ML	30.0
Reliability	M	60.0	ML	30.0
Organizational	M	50.0	M	50.0
Max Average	MH	74.0	M	42.0

RISK

IVMF Early Warning System (EWS) - Investment Expectations				
WBS Number	Cost Element	Initial Expectation (\$K)	Current Actual (\$K)	Percent Change (%)
1.1.1.1	Leadership	\$ 1,000	\$ 1,500	50.0%
1.1.1.2	Program Control	\$ 3,000	\$ 2,500	-16.7%
1.1.1.3	Contracts	\$ 8,000	\$ 8,000	0.0%
1.1.1.4	System Development	\$ 5,000	\$ 5,000	0.0%
1.1.1.5	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.6	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.7	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.8	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.9	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.10	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.11	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.12	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.13	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.14	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.15	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.16	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.17	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.18	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.19	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.20	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.21	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.22	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.23	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.24	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
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1.1.1.26	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.27	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
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1.1.1.30	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.31	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.32	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
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1.1.1.34	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.35	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.36	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.37	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.38	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.39	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.40	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.41	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.42	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.43	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.44	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.45	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.46	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.47	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.48	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%
1.1.1.49	System Security and Information Assurance	\$ 5,000	\$ 5,000	0.0%
1.1.1.50	System Reliability and Information Security	\$ 5,000	\$ 5,000	0.0%

INVESTMENT MONITOR

- Standardization of financial and non-monetary analysis
- Automation of many commonly used AoA/BCA techniques
- Integration of cost risk and uncertainty simulation
- Identification and tracking of critical variables and metrics that drive overall value

Importance of Risk & Uncertainty in Comparative Analysis

- Comparative analyses must perform cost risk and uncertainty analysis (CRUA)—just like applied estimates!
- Consequences of failing to properly consider CRUA:
 - Risk of improper alternative selection
 - Risk of under-funding
 - Increased execution risks
 - Systemic bias toward selection of higher-risk¹, higher-reward alternatives²

PORTFOLIO OPTION	AVERAGE RETURN	STDEV OF RETURNS	Percentage of "Negative" Years	20th Percentile Return	Optimal PE COA	Optimal Risk-Adjusted COA	RAR
100% STOCKS	11.0%	15.0%	23.17%	-1.6%	X		53.3%
100% MONEY MARKET (MM)	3.5%	1.0%	0.02%	2.7%			50.0%
A % STOCKS, B % MM	8.4%	9.8%	19.53%	0.2%		X	55.1%

1. In this context, for consistency, with standard "risk-reward" terminology, we are using term "risk" to refer to variance in possible outcomes, which CEBoK defines as "uncertainty." In general, our techniques address both risk (shift in central tendency) and uncertainty (dispersion).
2. In this example, returns are assumed to be normally distributed and independent, A=65%, B=35%, and the risk-free rate of return is assumed to be 3.0%. The Risk-Adjusted Return (RAR) is calculated based on the Sharpe Ratio.

Summary of Literature Review

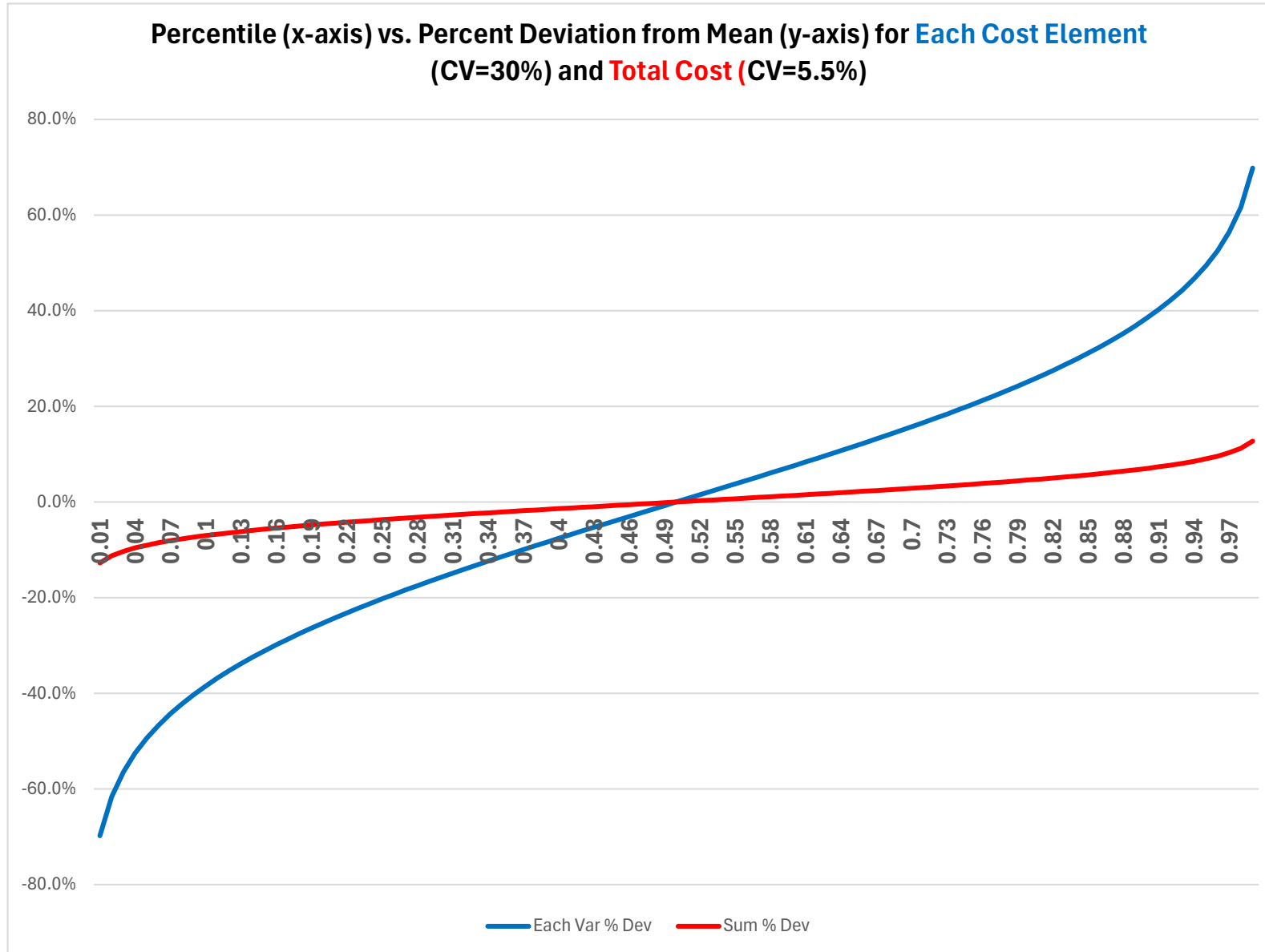
Author(s) and Year(s)	Organization and/or Journal	Paper Title	Significance & Comments
Cholesky (1918; published posthumously in 1924)	Procédé du Commandant Cholesky), Bulletin Géodésique	<i>Note sur une méthode de résolution des équations normales provenant de l'application de la méthode des moindres carrés à un système d'équations linéaires en nombre inférieur à celui des inconnues</i>	Created Cholesky decomposition
Li & Hammond (1975)	Navy Electronics Laboratories & Georgia Tech (published in IEEE)	<i>Generation of Pseudorandom Numbers with Specified Univariate Distributions and Correlation Coefficients</i>	First breakthrough, cited by NORTA authors
Cairo & Nelson (1997)	Delphi Packard & Northwestern University	<i>Modeling and Generating Random Vectors with Arbitrary Marginal Distributions and Correlation Matrix</i>	Introduced NORTA
Lurie & Goldberg (1998) (L&G)	IDA (presented at DoDCAS in 1999)	<i>An Approximate Method for Simulating Correlated Random Variables from Partially Specified Distributions</i>	Seminal work in cost community (also cited Li and Hammond but appeared unaware of NORTA)
Hu & Smith (2003)	Tecolote	<i>Impact of Correlating CER Risk Distributions</i>	Written as part of ACEIT documentation. Adaptation of L&G using "Group Strength" heuristic.
Braxton, Coleman, Druker, Hughes (2008)	Northrup Grumman (presented at SCEA)	<i>A Non-Simulation-Based Method for Inducing Pearson's Correlation Between Input Random Variables</i>	Refers to L&G as "landmark" and "the only method for inducing correlation the authors are aware of." Uses custom VBA Cholesky package from "The Foxes Team in Italy." Inputs only. Uses non-linear optimization to adjust L matrix. No full implementation.
Smart (2015)	MDA (presented at ICEAA)	<i>Beyond Correlation: Don't Use the Formula that Broke Wall Street</i>	Refers to copulas as "beyond correlation" and writes equations to define non-linear copulas for normal and lognormal distributions. Limited Implementation in joint cost/schedule risk analysis tool.

Correlation Matters (but isn't enough)

- In the prior example, we assumed independent investments
- But costs within an estimate (and their inputs) are typically positively correlated with each other
- Most cost estimates contain insufficient correlation
- Various default correlation coefficients have been proposed, generally ranging from 20% to 60%¹
- The "correct" amount depends upon the context of the estimate, and the amount of functional correlation already in the model
- But even extremely high correlation by itself is often not enough—copulas are needed² because (untransformed) correlation captures only *linear* dependence (discussion in Speaker Notes)
- Our method doesn't use (non-linear) copulas directly but can be built upon to include them³

1. Prominent recommendations include 0.2 (Steve Book), 0.3 (DoD Joint Cost and Schedule Risk and Uncertainty Handbook) and 0.6 (Christian Smart).
2. Smart, Christian. *Beyond Correlation: Don't Use the Formula That Killed Wall Street* (ICEAA 2015). In this context, copulas are multivariate CDFs in the interval (0,1). They are used to model conditional dependence—for example, with bivariate normal or bivariate lognormal distributions.
3. Wang, Tianyang and Dyer, James. *A Copula-Based Approach to Modeling Dependence in Decision Trees*. *Operations Research*, January-February 2012, Vol. 60, No. 1, pp. 225-242. The use of standard injected correlation, strictly speaking, also involves copulas. We are using the term as Smart did in footnote 2, to refer to non-linear copulas. Non-linear copulas correspond to **relational correlation** in CEBoK.

Correlation Matters: The "Portfolio Effect"



Correlation Matters: Creating Correlated Random Variates

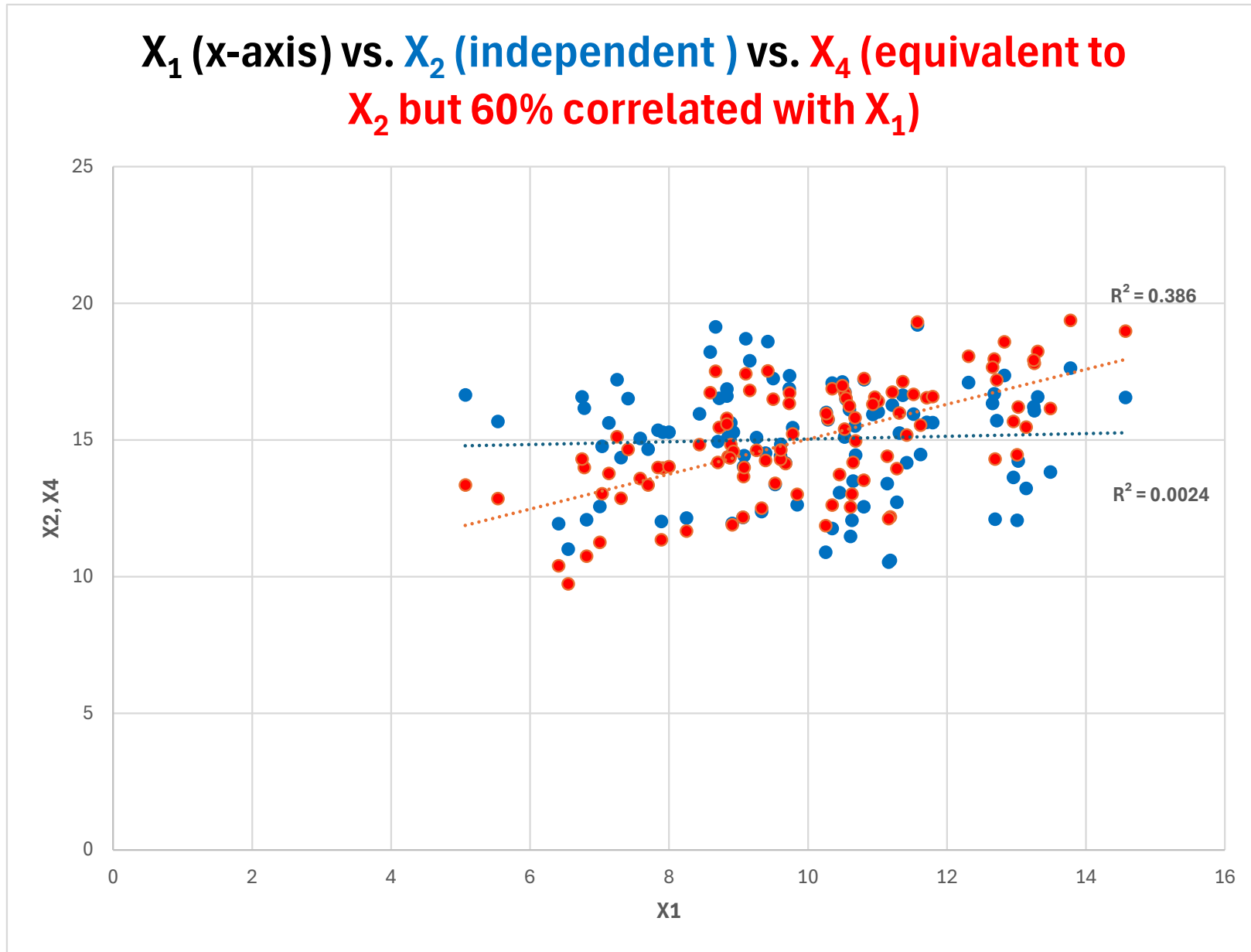
- It's easy to generate a pair of correlated random variables...with some assumptions
- Let X_1 & X_2 be independent, normally distributed random variates with a constant variance: $X_1 \sim N(\mu_1, \sigma)$ & $X_2 \sim N(\mu_2, \sigma)$
- We want to preserve the distribution of X_2 , but correlate it to degree ρ with X_1
- Set X_3 to be a linear combination of X_1 & X_2 , with "weights" influenced by ρ :
$$X_3 = \rho X_1 + \sqrt{1 - \rho^2} X_2$$
- Then X_3 will also be normally distributed with standard deviation σ and will be correlated with X_1 to degree ρ
 - But its mean will be $\mu_3 = \rho\mu_1 + \sqrt{1 - \rho^2} \mu_2$, which could be greater (or less) than μ_2
 - Let $X_4 = X_3 - \mu_3 + \mu_2$, to correct the mean. Then $X_4 \sim N(\mu_2, \sigma)$ and has correlation ρ with X_1

Our paper generalizes this to the three-variable case. But if any variable is non-normal or has a non-constant variance, or there are more than three of them---matrix algebra is needed!

Correlation Matters: Example Graph

Presented at the ICEAA 2025 Professional Development & Training Workshop - www.iceaonline.com/at12025

X_1 (x-axis) vs. X_2 (independent) vs. X_4 (equivalent to X_2 but 60% correlated with X_1)



Correlation Matters:

A Subjective Rank Ordering of Correlation Techniques

1. Functionally specify all correlation (generally impossible due to external dependencies)
2. Functional & injected correlation, with (non-linear) copulas (future envisioned IVMF)
3. Functional & injected Pearson¹ correlation (IVMF)
4. Injected Pearson correlation ("Outputs Only" tool)
5. Functional & injected "Pearson-like" correlation (ACEIT)
6. Functional & injected rank-order (Spearman²) correlation (CB, @Risk, Argo)³
7. Functional correlation only (as typically implemented in cost models)
8. Ignoring correlation altogether
9. Artificially injecting *negative* correlation & unwitting correlation due to "convenience"

Pearson formula:
$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}$$

1. Pearson (or Pearson Product Moment) correlation measures the strength of a linear relationship between two variables and is given by the formula above
2. Spearman (or rank-order) correlation measures only the strength of monotonicity and is the Pearson correlation of the *ranks* of the variables.
3. @Risk is a product of the Palisades Corporation. Argo is an open-source tool developed by Booz Allen Hamilton.

What is Cholesky Decomposition?

- Technique¹ that decomposes a positive definite matrix into the product of a lower triangular matrix and its transpose
- Used to generate correlated random variates based on fundamental equation:

$$[U] * [Ch] = [Corr]$$

- Where [U] is the matrix of uncorrelated, normally distributed numbers, [Ch] is the Cholesky decomposition² of the matrix of *target correlations*, and [Corr] is a matrix of correlated normal random numbers whose pairwise correlations reasonably approximate the target correlations

1. Invented by French mathematician André-Louis Cholesky (1875-1918). Cholesky was killed in action in World War I; the work was published posthumously.
2. We can use either the lower triangle or its transpose (an upper triangle), depending on the dimensionality of the original matrix of uncorrelated numbers.

What is Normal to Anything (NORTA)?

- Technique¹ used to generate correlated random numbers that follow any specified distribution(s)
- Starts with correlated, normal random numbers
- Uses CDFs and their inverses to transform the normally distributed numbers to target distributions, preserving correlations
- Why does it work?
 - Mathematical foundation of simulation in general: Transformation Theory² (an inverse exists for any CDF, and CDF is a map into $[0, 1]$)
 - NORTA: If I can create correlated $[0, 1]$ variables, I can map them back to any $f(x) = F^{-1}(g(x))$
 - When mapping them back, the correlations are preserved. In other words, the correlations of the CDF values (percentiles) approximately match the correlations of the underlying numbers

1. First published by Cario and Nelson (1997).
2. "Probability", Alan F. Karr, p.60

Our Approach: Overview

1. Use Cholesky decomposition to transform uncorrelated normal variates into correlated normal variates
2. Use NORTA to transform correlated normal variates into the desired distributions, preserving their correlations
3. When applying "outputs only" risk/uncertainty (injected correlation only), store the starting points ("seeds") so that results are generated instantaneously
4. "Budget" each cost element at the percentile such that total cost matches the user-desired percentile
5. Graph total cost at each whole number percentile to instantaneously generate the S-Curve and related statistics
6. Time-phase risk-adjusted estimate using same proportions as point estimate (*not shown in example*)

Our Approach: Ingredients (Three-Element, Four-Trial Example)

WBS #	WBS NAME	ESTIMATED COST
1.1	Program Mgmt	\$100
1.2	Prime Mission Product	\$1,000
1.3	Integ, Assembly, & Test	\$500

Point Estimates (3x1)

	1.1	1.2	1.3
1	-0.989	-1.353	0.327
2	1.404	1.668	0.652
3	0.608	0.818	-0.461
4	-2.716	-2.456	0.420

Matrix of independent standard normal "random seeds" (4x3)

	1.1	1.2	1.3
1.1	1.000	0.9	0.5
1.2	0.9	1.000	0.6
1.3	0.5	0.6	1.000

Matrix of target correlations (3x3)

WBS #	WBS NAME	ASSUMED DISTRIBUTION	PE Position	Mean, Median, or LB	StDev (or ML)	UB
1.1	Program Mgmt	Normal	Mean	\$100	\$30	
1.2	Prime Mission Product	Triangular	Most Likely	\$900	\$1,000	\$1,500
1.3	Integ, Assembly, & Test	Lognormal	Median	\$500	\$135	

Assigned distributions to each cost element. The order of columns is (μ, σ) for Normal; (Lower Bound, Most Likely, Upper Bound) for Triangular; and (unit-space) (median, σ) for Lognormal. The IVMF makes use of benchmark CVs found in the JCSRUH to automate this process.

Our Approach: Intermediate Calculations

	1.1	1.2	1.3
1	-0.989	-1.063	1.494
2	1.404	0.929	-0.465
3	0.608	0.621	-1.231
4	-2.716	-0.026	2.249



	1.1	1.2	1.3
1.1	1.000	0.900	0.500
1.2	0.000	0.436	0.344
1.3	0.000	0.000	0.795



	1.1	1.2	1.3
1	-0.989	-1.353	0.327
2	1.404	1.668	0.652
3	0.608	0.818	-0.461
4	-2.716	-2.456	0.420

Matrix of independent standard normal "random seeds" (4x3)
[U]

Cholesky decomposition of matrix of target correlations (3x3)
[Ch]

Matrix of correlated standard normals (4x3) [Corr]

END OF CHOLESKY DECOMPOSITION ; BEGINNING OF NORTA

	1.1	1.2	1.3
1	0.161	0.088	0.628
2	0.920	0.952	0.743
3	0.728	0.793	0.323
4	0.003	0.007	0.663

CDFs (Percentiles) of [Corr] (4x3)

	1.1	1.2	1.3
1	\$70.34	\$972.67	\$545.29
2	\$142.11	\$1,380.46	\$594.46
3	\$118.23	\$1,250.91	\$442.49
4	\$18.53	\$920.54	\$558.94

Inverse CDFs, evaluated at these percentiles, for our specified distributions. These are our Monte Carlo "runs." (4x3)

\$558.94 is the 66.3% percentile of a lognormal distribution with median \$500 and standard deviation \$135

1.3	Integ, Assembly, & Test	Lognormal	Median	\$500	\$135
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Our Results: Example Case

	1.1	1.2	1.3
1	\$70.34	\$972.67	\$545.29
2	\$142.11	\$1,380.46	\$594.46
3	\$118.23	\$1,250.91	\$442.49
4	\$18.53	\$920.54	\$558.94

	1.1	1.2	1.3
1.1	1.000	0.9	0.5
1.2	0.9	1.000	0.6
1.3	0.5	0.6	1.000

Our results

Target correlations

- Our results have all the desired properties of a Monte Carlo run, with no need for simulation or external software—and instantaneous results
- When run with many trials (n=1,000), all target distributions and pairwise correlations are accurately replicated
- In this example, $\text{Correl}(\text{WBS } 1.1, \text{WBS } 1.2) = 0.954$ (target = 0.900)

Our Results: Screenshot from "Outputs Only" Tool

Original distribution specifications

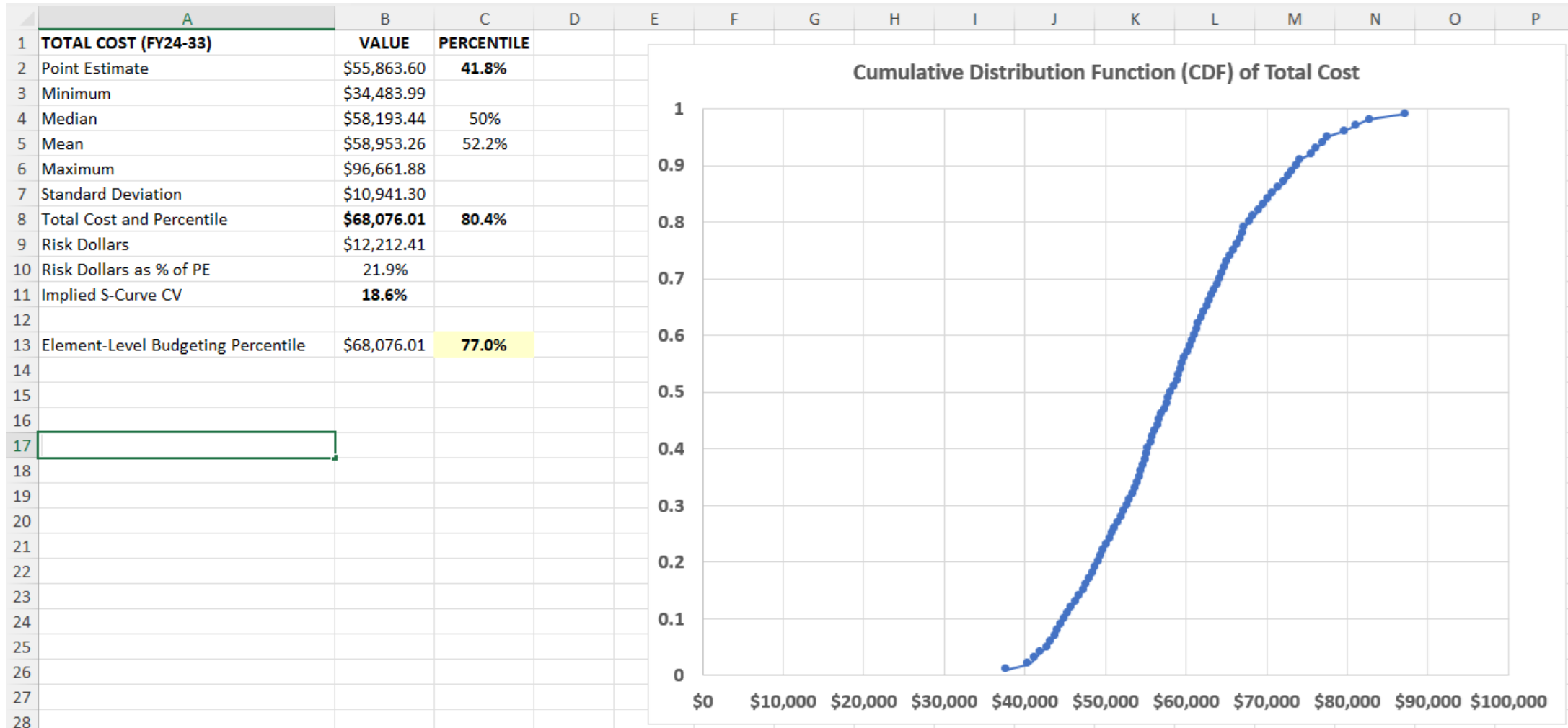
Our results

Targets implied by original distributions

ITERATION (DOWN); VARIABLE (ACROSS)	47	48	61	62
	Uniform	Triangular	Normal	Lognormal
Lower Bound or Mean	\$2,704.52	\$21,375.00	\$7,000.00	\$9.62
Most Likely or StDev		\$30,000.00	\$1,916.37	\$0.27
Upper Bound	\$5,022.68	\$48,825.00		
Dist Mean	\$3,863.60	\$33,400.00	\$7,000.00	\$15,551.96
Dist StDev	\$669.20	\$5,730.70	\$1,916.37	\$4,257.61
Median:	\$3,796.82	\$32,432.32	\$6,909.93	\$14,821.48
Mean:	\$3,810.68	\$32,998.37	\$6,896.69	\$15,247.51
StDev:	\$662.45	\$5,701.54	\$1,920.69	\$4,193.08
Correlation with Variable 47:	1.000	0.632	0.640	0.625
CV:	17.4%	17.3%	27.8%	27.5%
Target CVs:	17.3%	17.2%	27.4%	27.4%
Target Correlations:	1.000	0.630	0.630	0.630

Screenshot from our tool showing selected variables. Means, standard deviations, CVs, and target correlations are faithfully replicated over 1,000 trials.

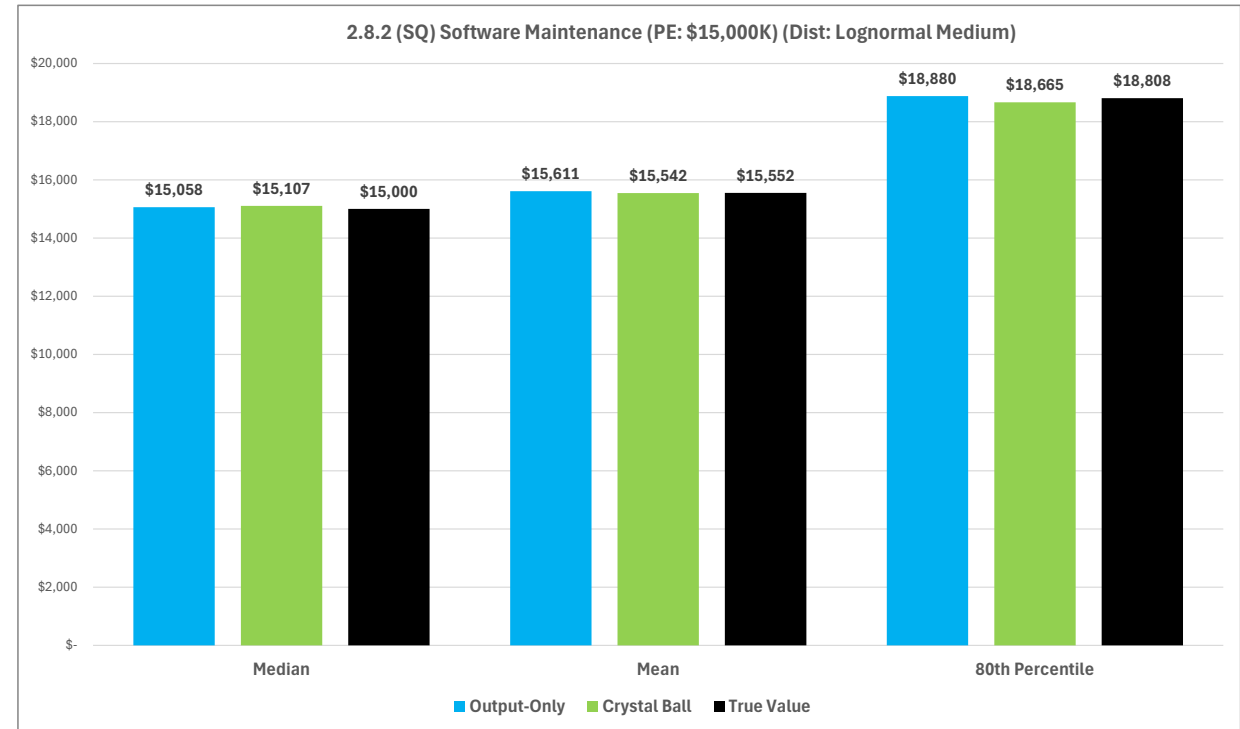
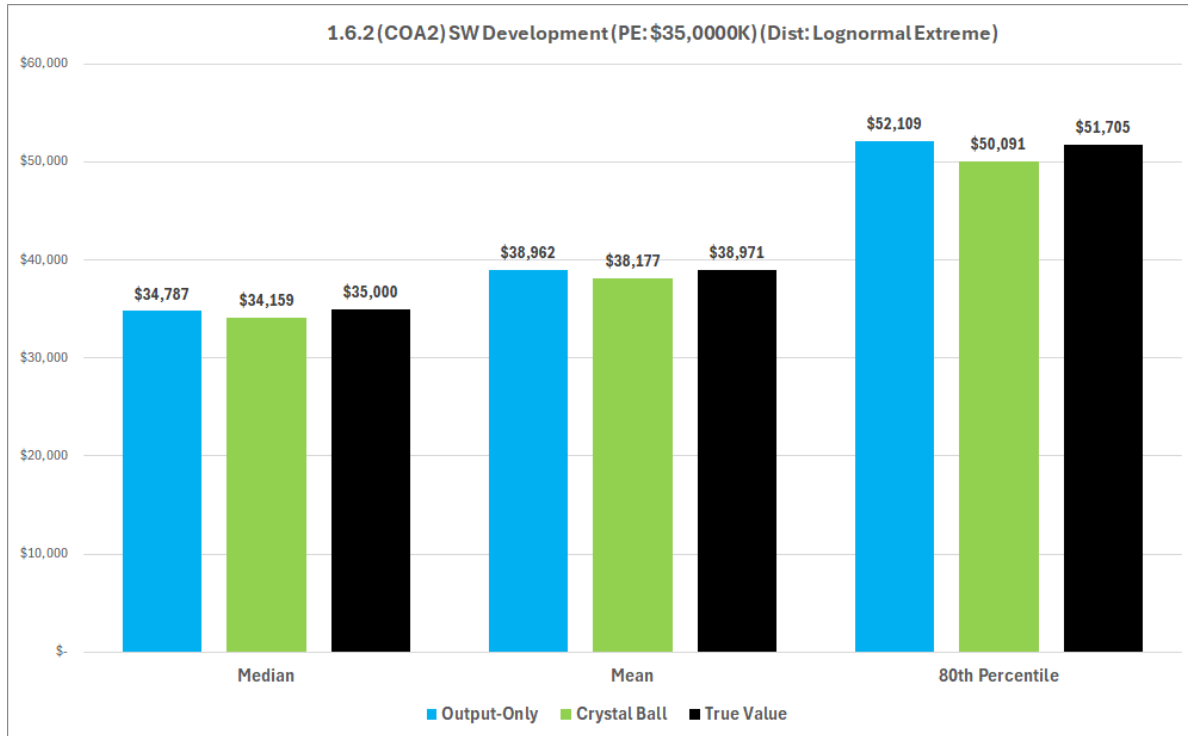
Our Results: S-Curve from "Outputs Only" Tool



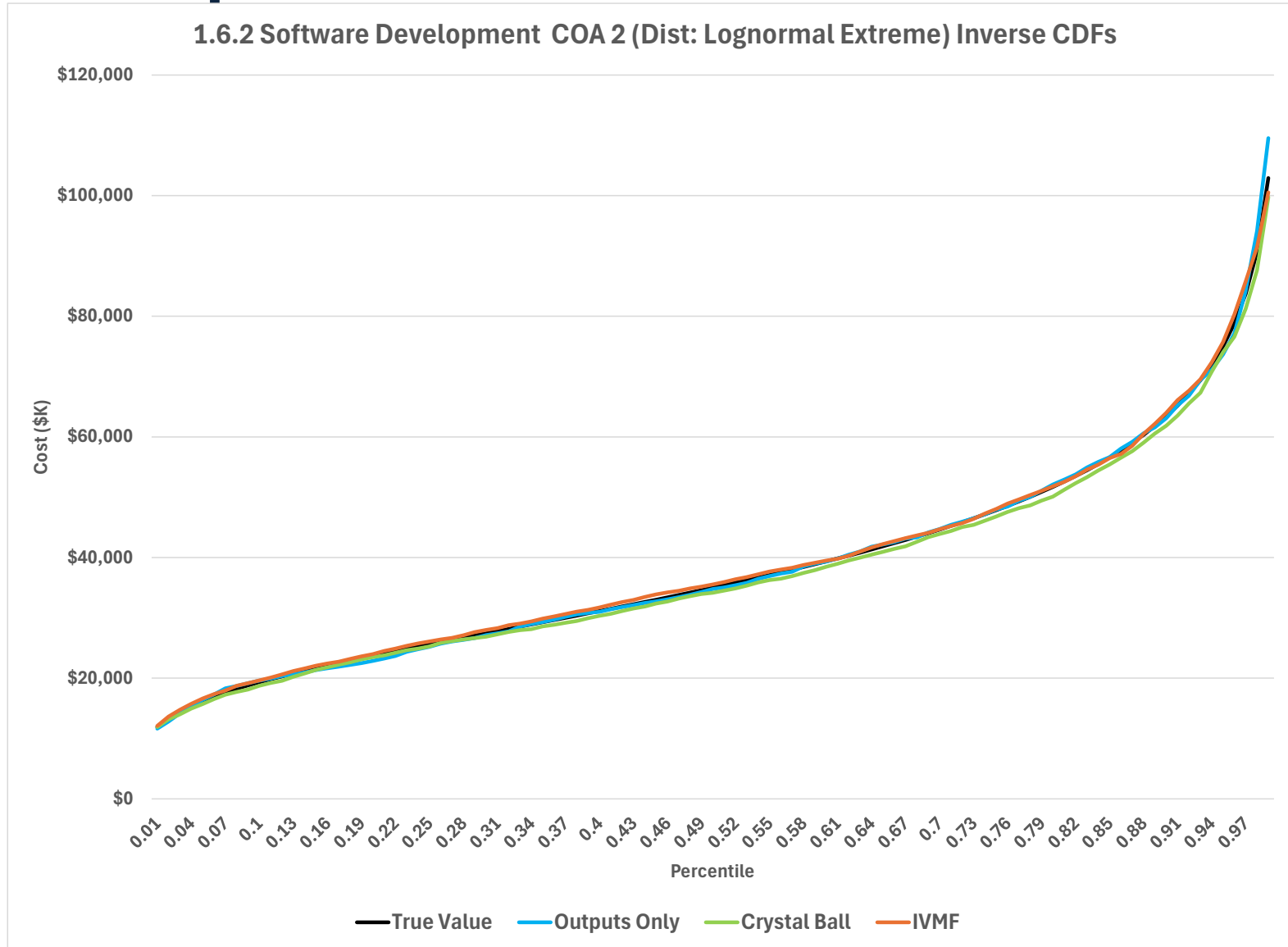
It's easy to generate S-Curves and other cost risk statistics from our trials. Moreover, risk dollars can be automatically allocated based on a desired percentile. Here, each element is budgeted to the 77th percentile, so that total cost lies at the 80th percentile.

Our Results: Numerical Experiments

- We saw *some* evidence that our method captures the tails (in tail-heavy distributions) better than Crystal Ball
- This makes sense because it's using actual (not rank-order) correlation
- The below results are from a representative test case (notional data)
- Target correlations were also better captured: 1.87% vs. 3.43% SEE (discussion in Notes)



Numerical Experiments: Element-Level Inverse CDFs



Numerical Experiments: NORTA vs. Rank-Order Correlation

Test	Variables	Rank-Order Time (s)	Rank-Order Error	NORTA Time (s)	NORTA Error	Linearly Dependent?
1	2	2.51	0.043	0.19	0.008	N
2	10	4.48	0.131	0.95	0.134	N
3	10	4.51	0.147	0.92	0.160	N
4	10	4.67	0.304	0.93	0.844	Y
5	50	5.11	0.822	5.28	0.734	N
6	50	4.71	13.549	5.27	17.765	Y
7	50	4.52	8.575	4.98	23.461	Y
8	18	3.67	0.379	1.82	0.264	Y

Rank-order correlation and NORTA correlation performed comparably. Generally, NORTA outperformed only for linearly independent matrices. However, only the final test contained real-world data. In that (linearly dependent) test, NORTA also outperformed.

Numerical Experiments: Percentile Optimization vs. Book's¹ Method

Cost Element	Point Estimate	Book's Budget	Percentile	Expected Delta	Optimization Budget	Percentile	Expected Delta
Investment	\$2,337,843	\$2,600,244	80.0%	-\$186,651	\$2,600,244	80.0%	-\$186,651
Program/Project Management	\$30,000	\$31,669	55.9%	-\$5,568	\$35,141	70.1%	-\$3,920
Planning Phase Program/Project Management	\$15,000	\$15,835	55.9%	-\$2,781	\$17,561	70.1%	-\$1,970
Acquisition Phase Program/Project Management	\$15,000	\$15,835	55.8%	-\$2,777	\$17,574	70.1%	-\$1,957
Systems Engineering (or Systems Analysis)	\$134,043	\$151,208	63.5%	-\$42,507	\$164,896	71.4%	-\$38,846
Planning Phase Systems Engineering	\$104,703	\$119,223	63.1%	-\$39,976	\$128,954	70.1%	-\$38,447
Acquisition Phase Systems Engineering	\$29,340	\$31,985	59.2%	-\$10,285	\$35,730	70.1%	-\$9,637
Business Process Re-engineering (BPR) / Change Management	\$599,100	\$666,071	69.8%	-\$83,129	\$677,371	72.7%	-\$80,113
Planning Phase BPR/Change Management	\$104,355	\$111,446	59.9%	-\$19,022	\$119,042	70.1%	-\$16,732
Acquisition Phase BPR/Change Management	\$494,745	\$554,625	69.2%	-\$80,858	\$558,271	70.1%	-\$79,578
System Development	\$300,000	\$317,521	64.7%	-\$33,163	\$339,148	79.9%	-\$28,703
Software Development	\$30,000	\$31,214	60.3%	-\$4,251	\$32,723	70.1%	-\$3,902
Data Development & Transition Planning	\$30,000	\$31,229	60.2%	-\$4,218	\$32,757	70.1%	-\$3,854
Data Base Standards/Dictionary	\$30,000	\$31,896	59.8%	-\$6,362	\$34,185	70.1%	-\$5,868
Training Development	\$30,000	\$31,901	62.2%	-\$6,667	\$33,941	70.1%	-\$6,114
Test and Evaluation	\$30,000	\$32,631	59.7%	-\$8,452	\$35,945	70.1%	-\$7,548
Logistics Support Development	\$30,000	\$31,861	59.2%	-\$6,271	\$34,168	70.1%	-\$5,841
Development/Prototype Facilities	\$30,000	\$31,184	60.2%	-\$3,977	\$32,546	70.1%	-\$3,724
Environmental Studies	\$30,000	\$31,863	61.8%	-\$6,245	\$33,847	70.1%	-\$5,726
Other Development	\$30,000	\$31,835	58.4%	-\$6,065	\$34,534	70.1%	-\$5,236
System Procurement	\$1,274,700	\$1,433,774	77.3%	-\$136,207	\$1,383,675	70.1%	-\$146,926

Our method implicitly includes a method of allocating risk dollars. Comparing it to Steve Book's seminal method reveals fewer likely overruns, due to a wider/more proportionate spread of risk dollars.

S-Curve Generation from IVMF Model

Application of Our Method

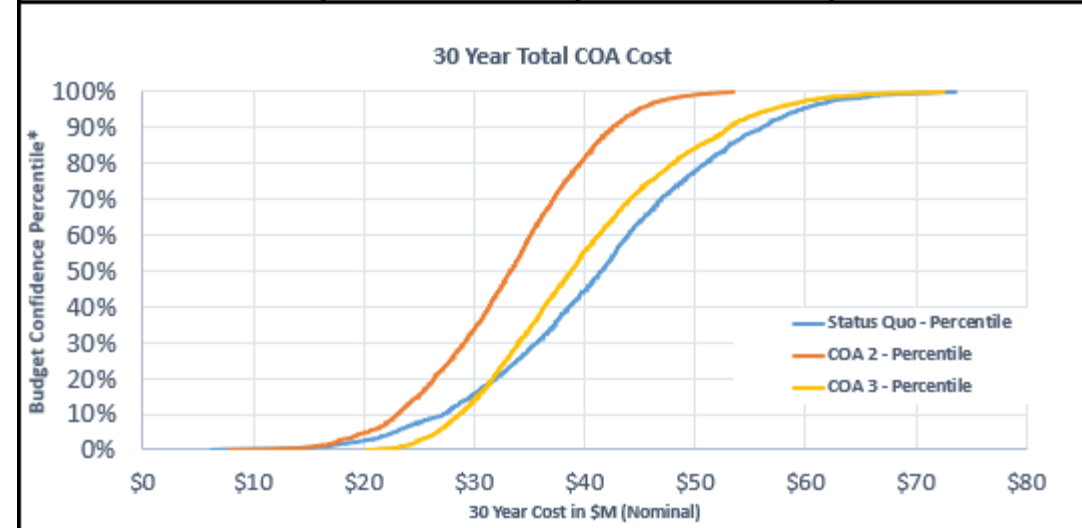
Output-Only Version

- Perform Monte Carlo simulation on cost model outputs
- User builds cost model within the file and specifies distribution parameters and correlations between cost elements
- Model maintains desired cost element distributions and correlations instantaneously produces S-curves and cost model distribution statics

IVMF (All In One) Version

- Allows the user to perform cost risk uncertainty analysis on both inputs and/or outputs
- User builds cost model within the file and specifies distribution parameters and correlations between cost inputs and/or cost elements
- Allows for comparative analysis between different COAs (factors in non-monetary assessments)
- IVMF is highly automated to simplify customization model setup and analysis

Comparison and Precision Data			
Comparison Percentile at the Cost Element Level or Total Cost Level			Total
Comparison Percentile			80%
Step Size			0.001
Cost Element to Plot:	Total Cost - \$K Nominal		
Cost Element Name	Status Quo	COA 2	COA 3
Point Estimate (PE):	\$32,500	\$32,771	\$36,162
Percentile of PE:	21.1%	47.1%	39.0%
Minimum:	\$3,241	\$5,886	\$19,596
Median:	\$41,525	\$33,314	\$38,825
Mean:	\$41,342	\$33,004	\$39,938
Percentile of Mean:	49.6%	48.6%	55.4%
Maximum:	\$79,210	\$55,064	\$74,235
Standard Deviation:	\$11,156	\$7,595	\$9,357
CV:	0.27	0.23	0.23
Risk-Adjusted Cost:	\$50,940	\$39,611	\$48,084
Risk Adjustment Percentile:	80.1%	80.1%	80.1%
Percent increase to PE:	56.7%	20.9%	33.0%
Risk Dollars:	\$18,440	\$6,840	\$11,922



*Note - Official Budget numbers require consideration of funding source and may need to be converted to Then Year dollars.

Summary / Conclusions / Next Steps

- MITRE's IVMF automates many commonly performed comparative analysis cost estimating tasks
- Failure to perform CRUA systematically favors high risk alternatives
- CRUAs must consider correlation
- The "right" way to capture correlation is via non-linear copulas—but our Cholesky/NORTA method is a step in the right direction!
- Method is more mathematically rigorous—and appears slightly better "at the tails"—than Crystal Ball
- In Output-Only mode, generates S-Curves and all related statistics instantaneously
- Anyone can do this—using only Excel, with no external software to "run"!
- Potential next steps:
 - Conduct more robust numerical experiments
 - Add non-linear copula-based & enhanced risk dollar allocation capabilities
 - Create standalone IVMF Excel product/Add-In

Questions?

Acronyms

ACRONYM	EXPANSION
ACEIT	Automated Cost Estimating Integrated Tools
AoA	Analysis of Alternatives
BCA	Business Case Analysis
CB	Crystal Ball
CBA	Cost-Benefit Analysis
CDF	Cumulative Distribution Function
CEBoK	Cost Estimating Body of Knowledge
COA	Course of Action
Correl	Correlation
CRUA	Cost Risk and Uncertainty Analysis
CV	Coefficient of Variation
DoD	Department of Defense
DoDCAS	Department of Defense Cost Analysis Symposium
DPP	Discounted Payback Period
EA	Economic Analysis
ESBM	Enhanced Scenario-Based Method
ICEAA	International Cost Estimating and Analysis Association
IDA	Institute for Defense Analyses

ACRONYM	EXPANSION
IRR	Internal Rate of Return
IVMF	Investment Value Management Framework
JCSRUH	Joint Cost and Schedule Risk and Uncertainty Handbook
L&G	Lurie and Goldberg
LB	Lower Bound
LCCE	Life-Cycle-Cost Estimate
MDA	Missile Defense Agency
ML	Most Likely
NORTA	Normal to Anything
NPV	Net Present Value
PE	Point Estimate
ROI	Return on Investment
SCEA	Society for Cost Estimating and Analysis
SEE	Standard Error of the Estimate
StDev	Standard Deviation
UB	Upper Bound
VBA	Visual Basic for Applications
WBS	Work Breakdown Structure