Accidental Proof of Calculus for Cost Analysts

ICEAA

Professional Development & Training Workshop
May 16 -18, 2023 San Antonio, Texas

AFRL
THE AIR FORCE RESEARCH LABORATORY
LEAD | DISCOVER | DEVELOP | DELIVER

AFRL/FZC

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U.S. AIR FORCE

USSF

Presented at the ICEAA 2023 Professional Development & Training Workshop - www.iceaaonline.com/sat2023

Purpose

Background

Phasing & Problem Origin

The Math

Quick Pass & Cross Check

Calculus

Area Under the Curve

Curiosity

Next Slide

Simplify

Proof

The End

Discussion

Integrals Into Excel

Purpose

Purpose

- Calculus
  - More Than a Merrit Badge

- Tools of the Trade
  - Pencil & Paper
  - Calculator
  - MATLAB

- Cautionary Reminder
  - Cross Checks

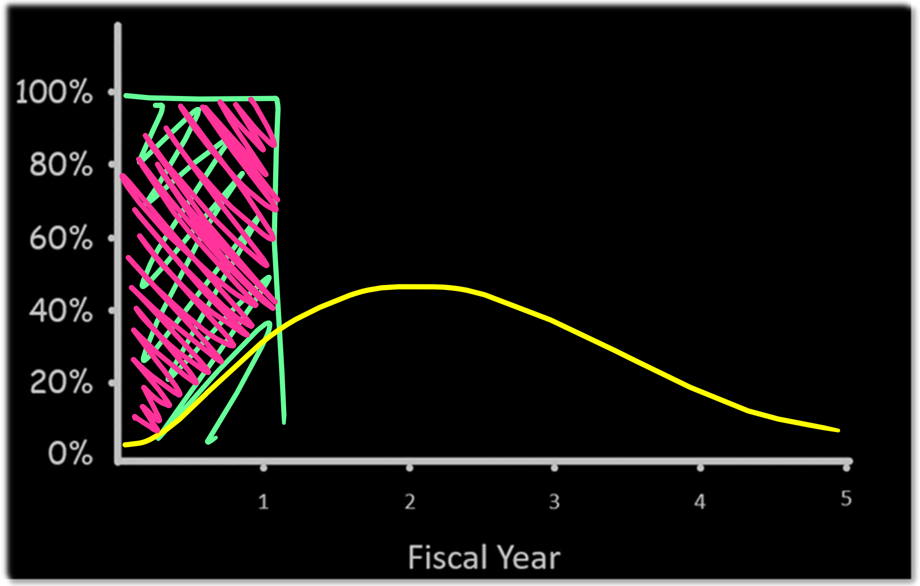
Next Slide

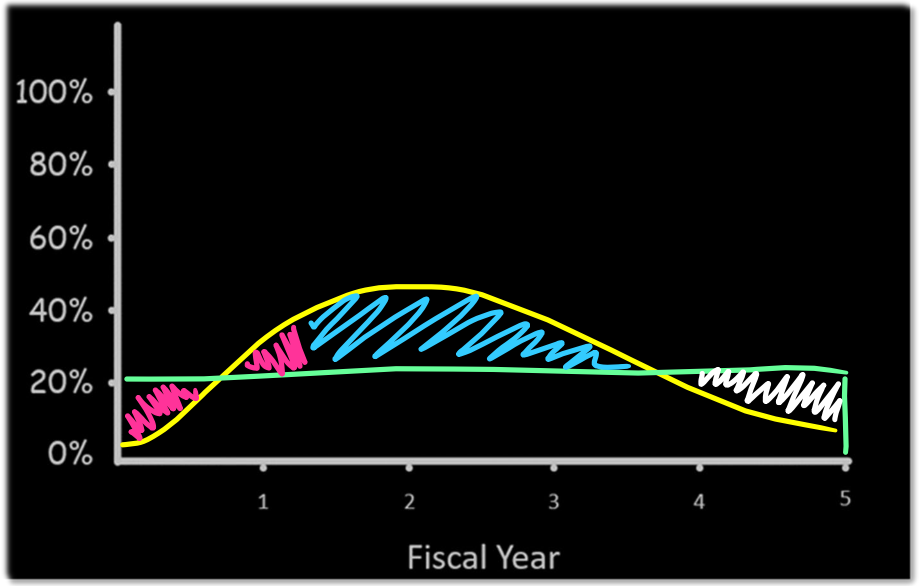
Background

Phasing & Problem Origin

Phasing

- Spread Estimate Across Time
  - Inflation & Escalation
- Align Funding with Needs
- Minimize Forward Financing





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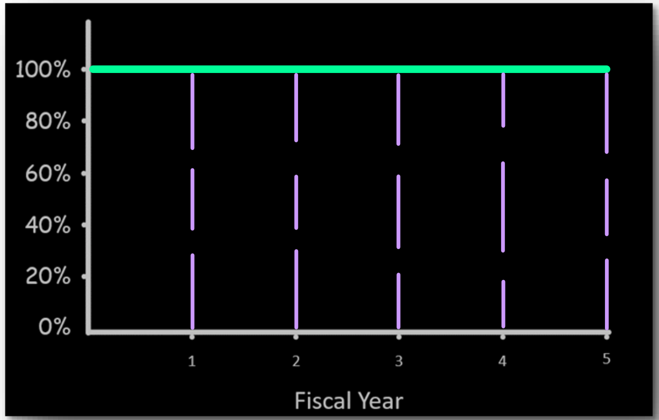
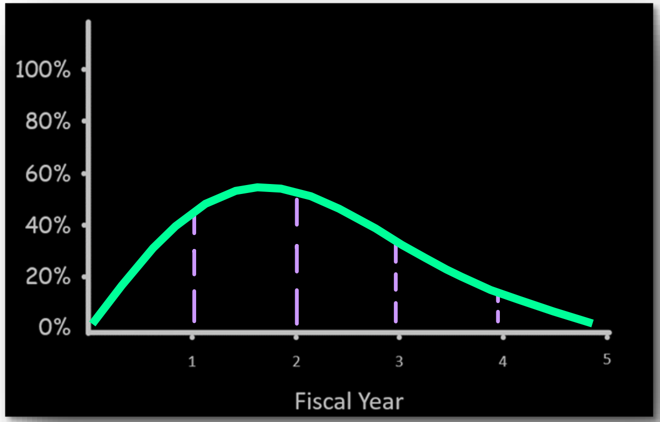
Phasing Calculator

- Flat Level of Effort (LOE)
- Front Loaded Phasing Distributions
  - Beta
  - Raleigh
  - Weibull
- Outsource the Math
  - Input
    - BY$M Input
    - Start Date (Calendar Year)
    - Duration (Months)
    - Phasing profile shape parameters
  - Output
    - TY$M *or percentages*
    - Costs Bucketed into Fiscal Years





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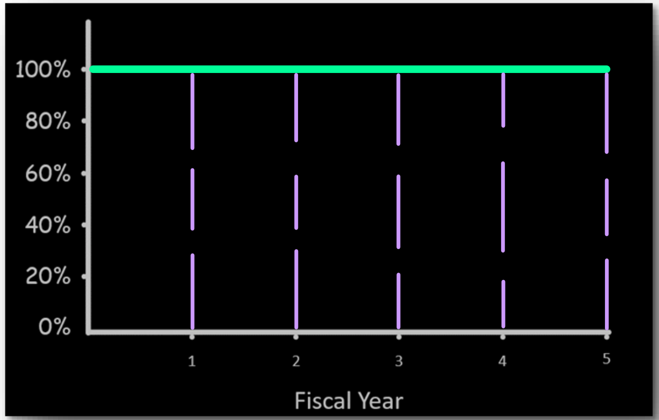
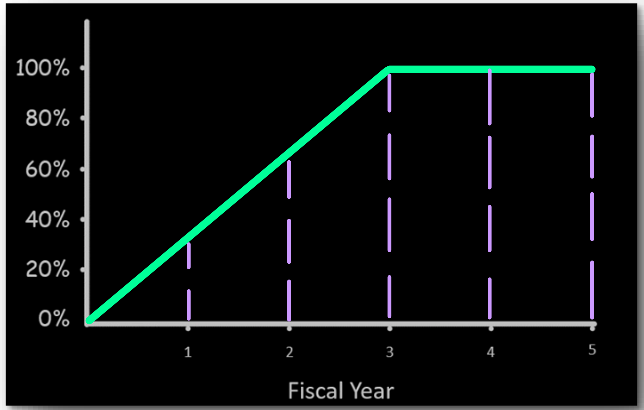
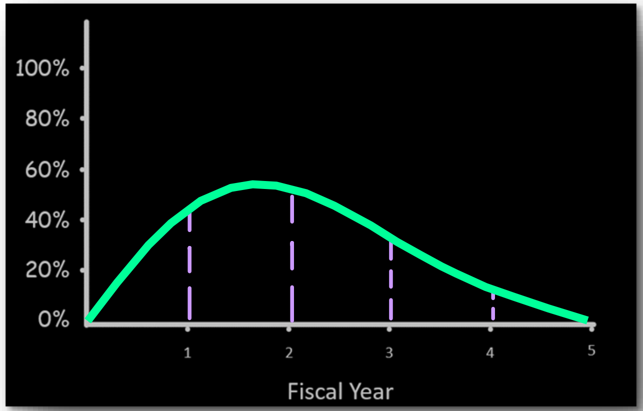
Problem Origin

- Colleague asked for a feature
- Instead of a Flat Uniform Level of Effort (LoE)
- Ramp up to a LoE across a number of years
- EZ PZ







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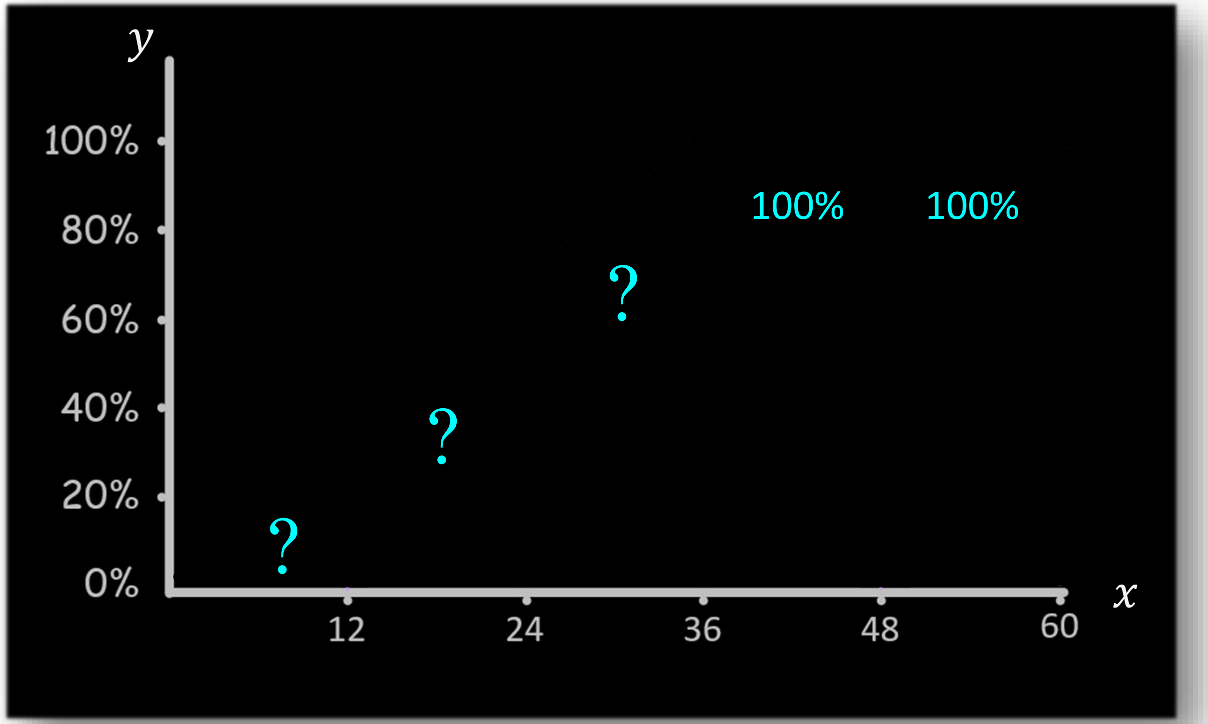
The Math

Quick Pass & Cross Check

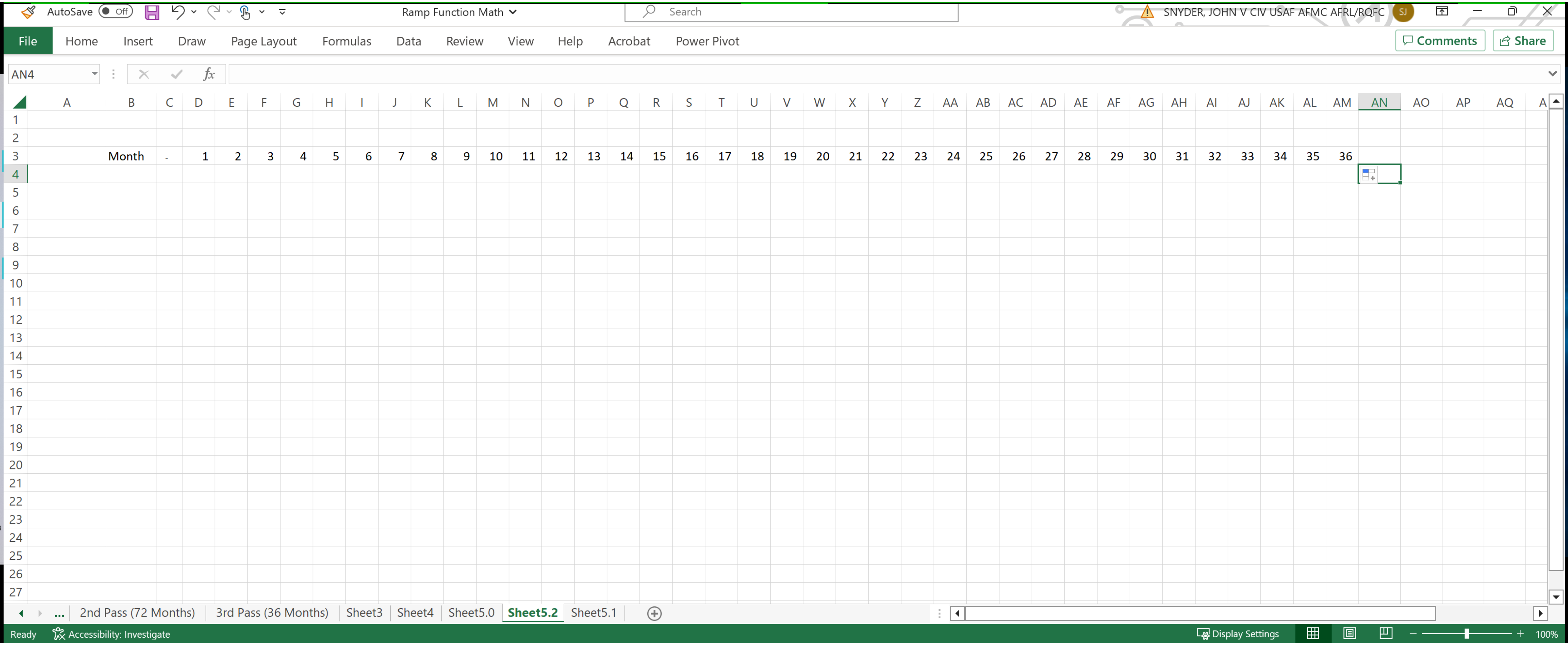
1st Step

- Sketch out the problem
  - Constraints:
    - Start month = 0
    - Ramping period = 36 months
    - Full Level of Effort @ Month 37
- Writing the equations
  - Variables
    - Independent (*y*) Months
    - Dependent (



$$\int_0^{36} \frac{1}{36} x dx = \left[\frac{1}{36} * \frac{1}{2} * x^2\right]_0^{36} \div 12 = 1.5$$

Next Slide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Month - 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36
2nd Pass (72 Months) 3rd Pass (36 Months) Sheet3 Sheet4 Sheet5.0 Sheet5.2 Sheet5.1
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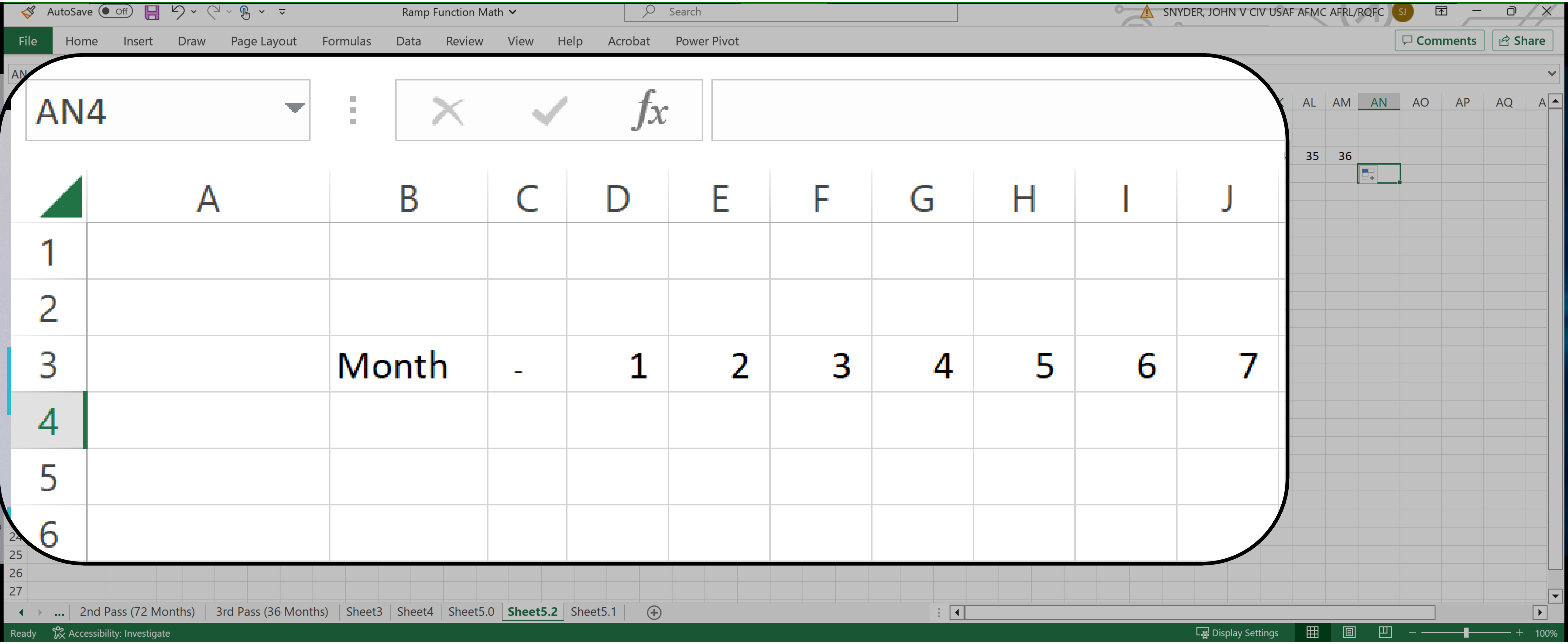
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fx
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AL AM AN AO AP AQ
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2nd Pass (72 Months)
3rd Pass (36 Months)
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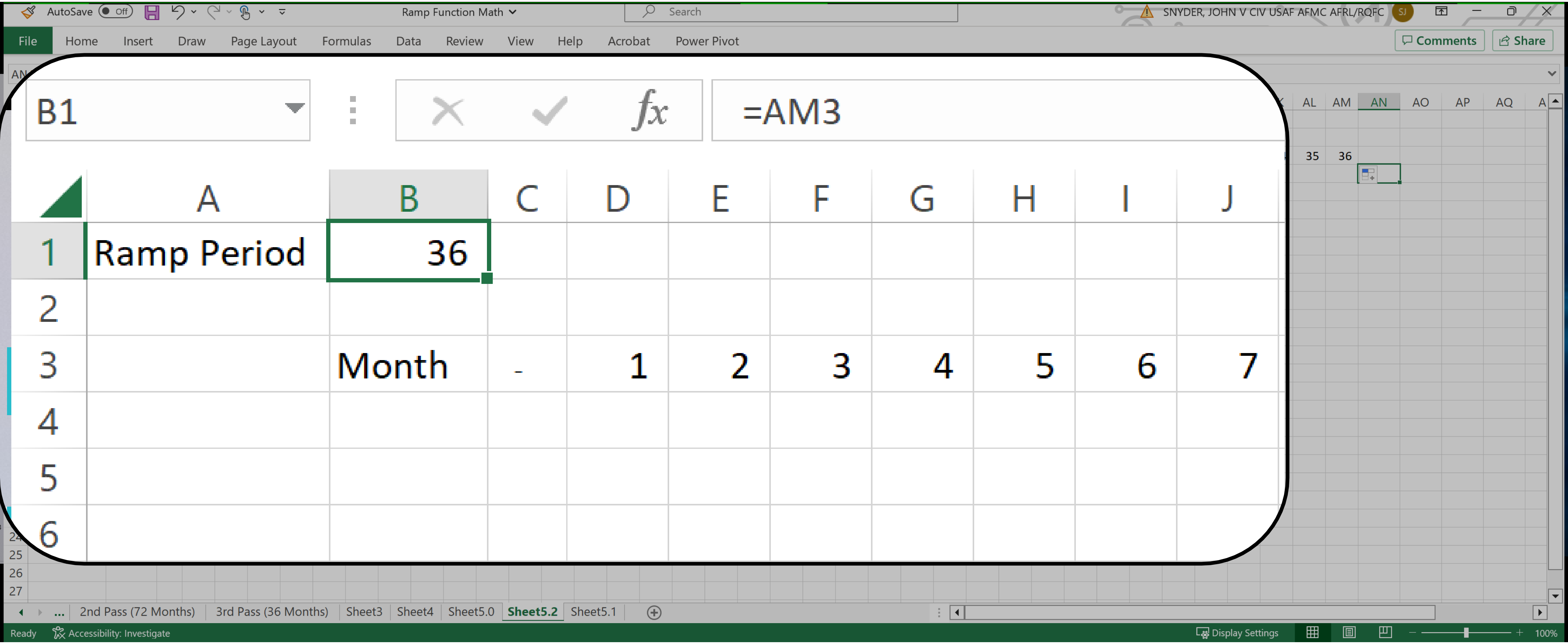
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3rd Pass (36 Months)
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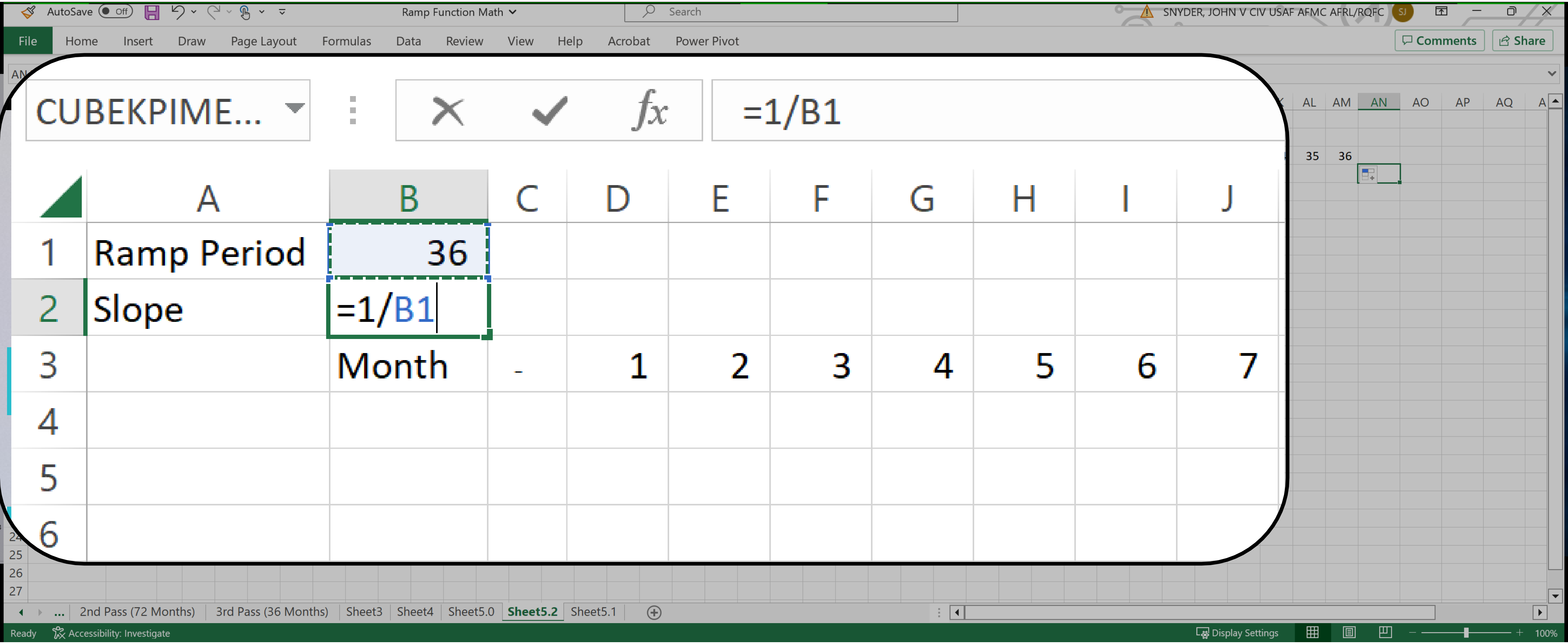
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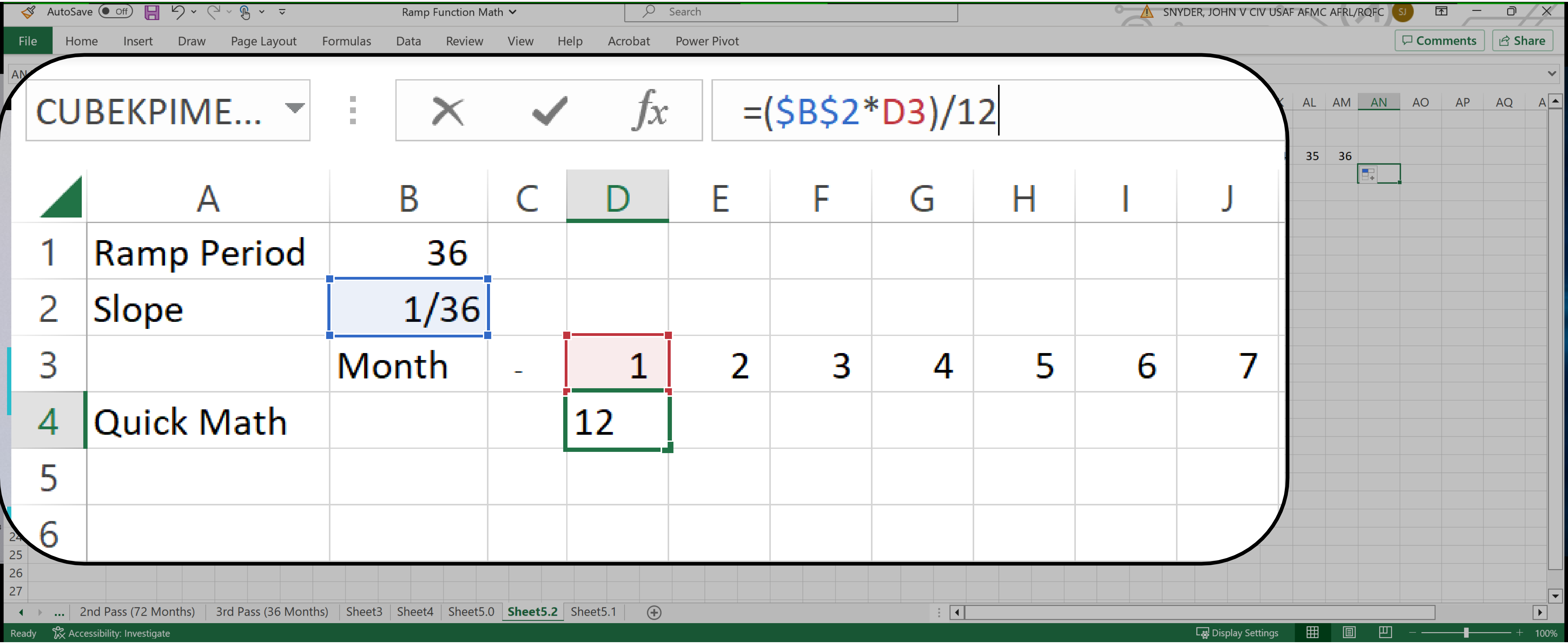
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Acrobat
Power Pivot
Comments
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CUBEKPIME...
=(B2*D3)/12
A
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C
D
E
F
G
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I
J
1 Ramp Period 36
2 Slope 1/36
3 Month - 1 2 3 4 5 6 7
4 Quick Math 12
5
6
AL AM AN AO AP AQ
35 36
2nd Pass (72 Months)
3rd Pass (36 Months)
Sheet3
Sheet4
Sheet5.0
Sheet5.2
Sheet5.1
Ready
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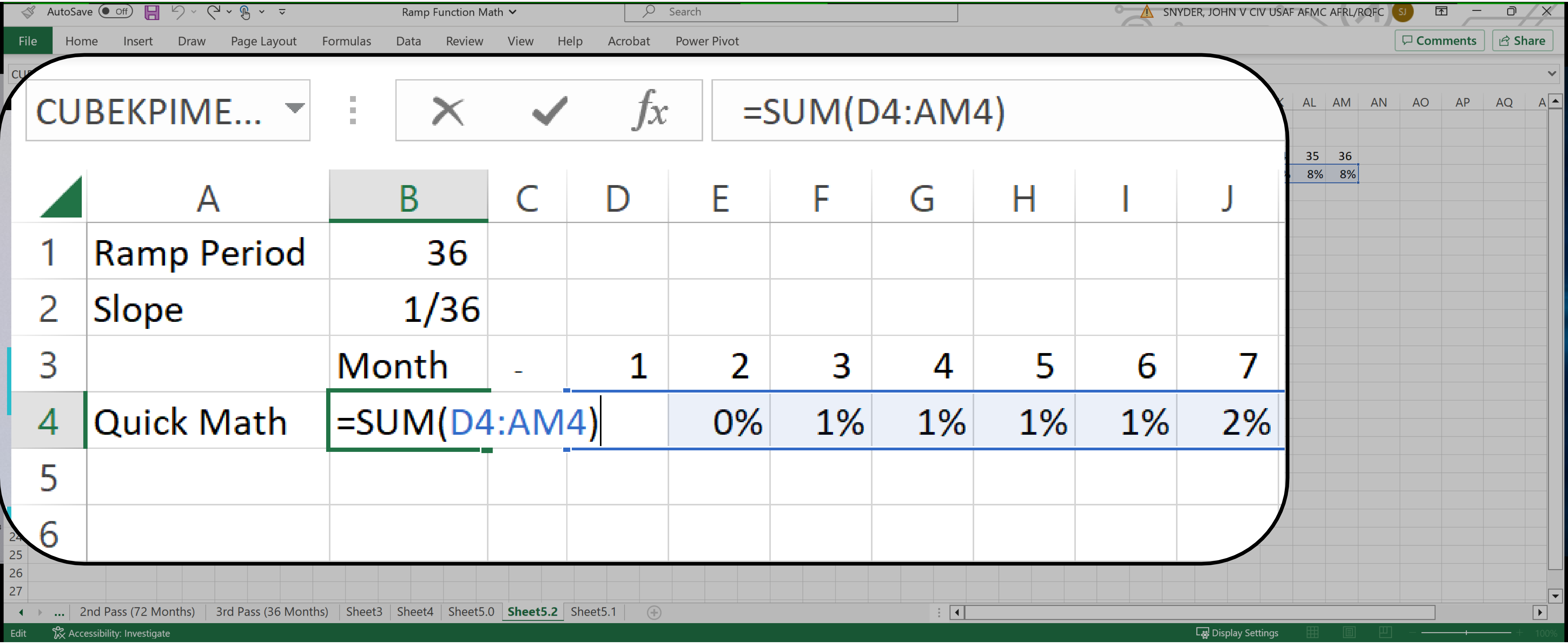
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Share
CUBEKPIME...
=SUM(D4:AM4)
A B C D E F G H I J
1 Ramp Period 36
2 Slope 1/36
3 Month - 1 2 3 4 5 6 7
4 Quick Math =SUM(D4:AM4) 0% 1% 1% 1% 1% 2%
5
6
AL AM AN AO AP AQ
35 36
8% 8%
2nd Pass (72 Months)
3rd Pass (36 Months)
Sheet3
Sheet4
Sheet5.0
Sheet5.2
Sheet5.1
Edit
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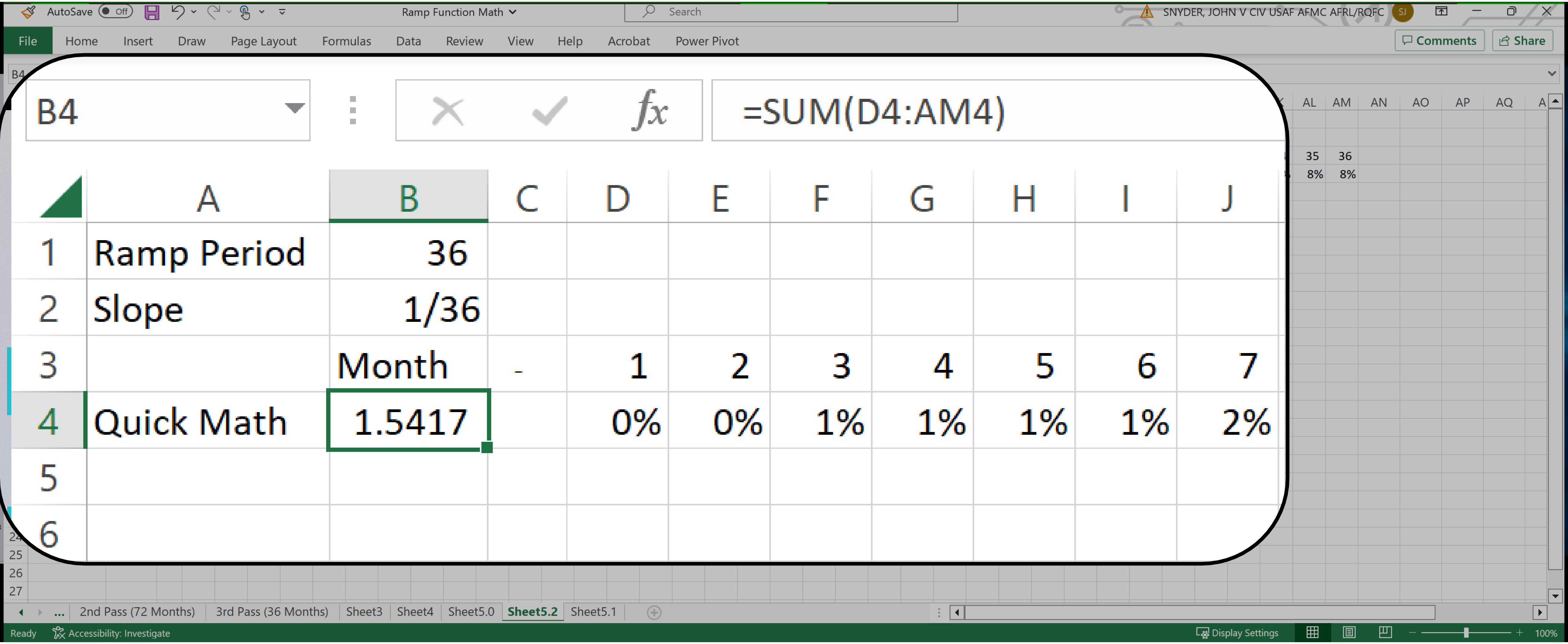
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Share
B4
=SUM(D4:AM4)
A B C D E F G H I J
1 Ramp Period 36
2 Slope 1/36
3 Month - 1 2 3 4 5 6 7
4 Quick Math 1.5417 0% 0% 1% 1% 1% 1% 2%
5
6
AL AM AN AO AP AQ
35 36
8% 8%
2nd Pass (72 Months)
3rd Pass (36 Months)
Sheet3
Sheet4
Sheet5.0
Sheet5.2
Sheet5.1
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Ramp Function Math

B4 =SUM(D4:AM4)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	Ramp Period	36																			
2	Slope	1/36																			
3		Month	-	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
4	Quick Math	1.5417		0%	0%	1%	1%	1%	1%	2%	2%	2%	2%	3%	3%	3%	3%	3%	4%	4%	4%

	A	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM
1	Ramp Period																		
2	Slope																		
3		19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
4	Quick Math	4%	5%	5%	5%	5%	6%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%

$$\sum_{n=1}^{36} \frac{1}{36} * x \div 12 = 1.5417$$

Not 1.5

2nd Pass (72 Months) | 3rd Pass (36 Months) | Sheet3 | Sheet4 | Sheet5.0 | Sheet5.2 | Sheet5.1

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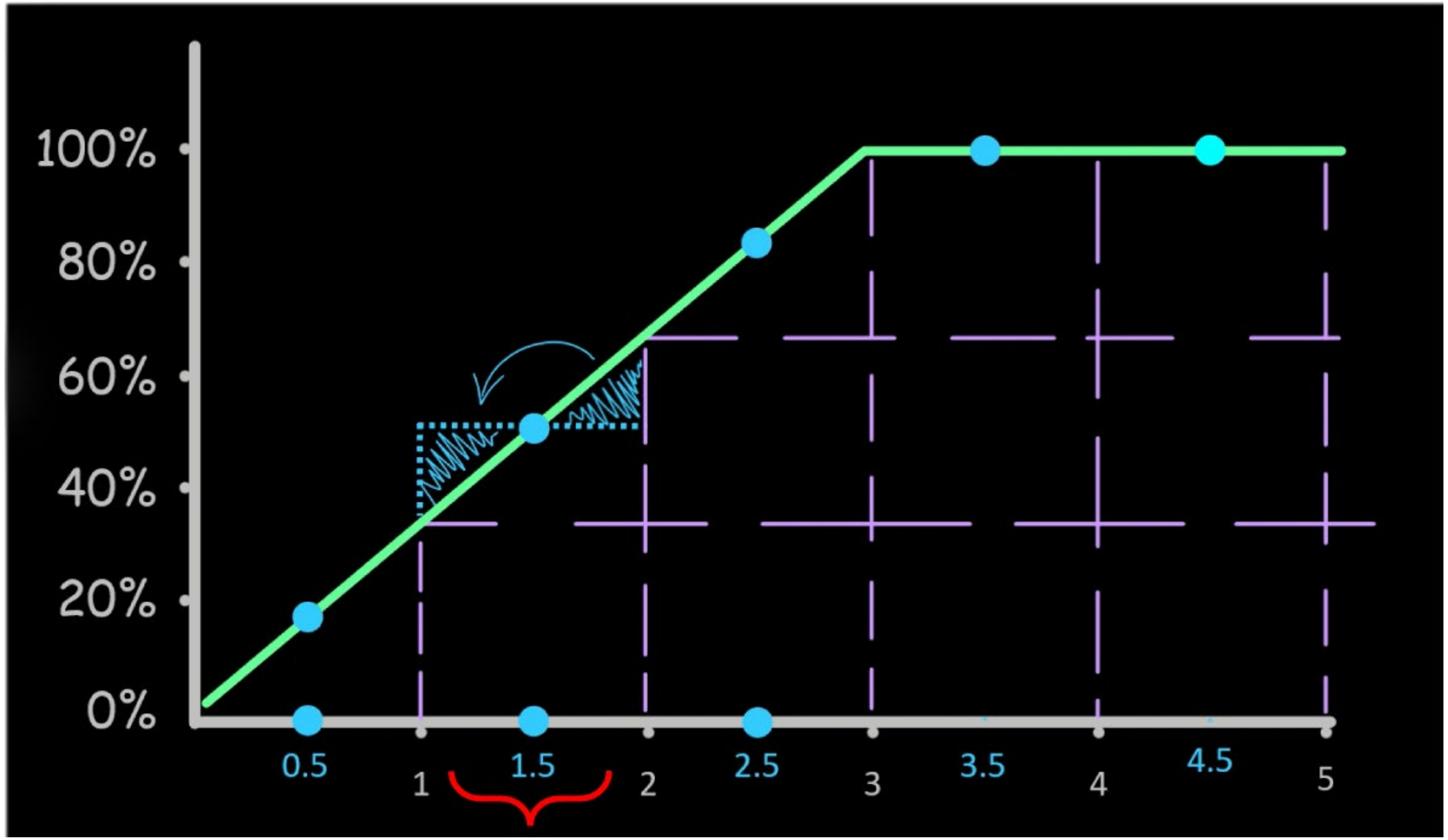
Midpoint

$$\sum_{n=1}^{36} \frac{1}{36} * (x - 0.5) \div 12$$



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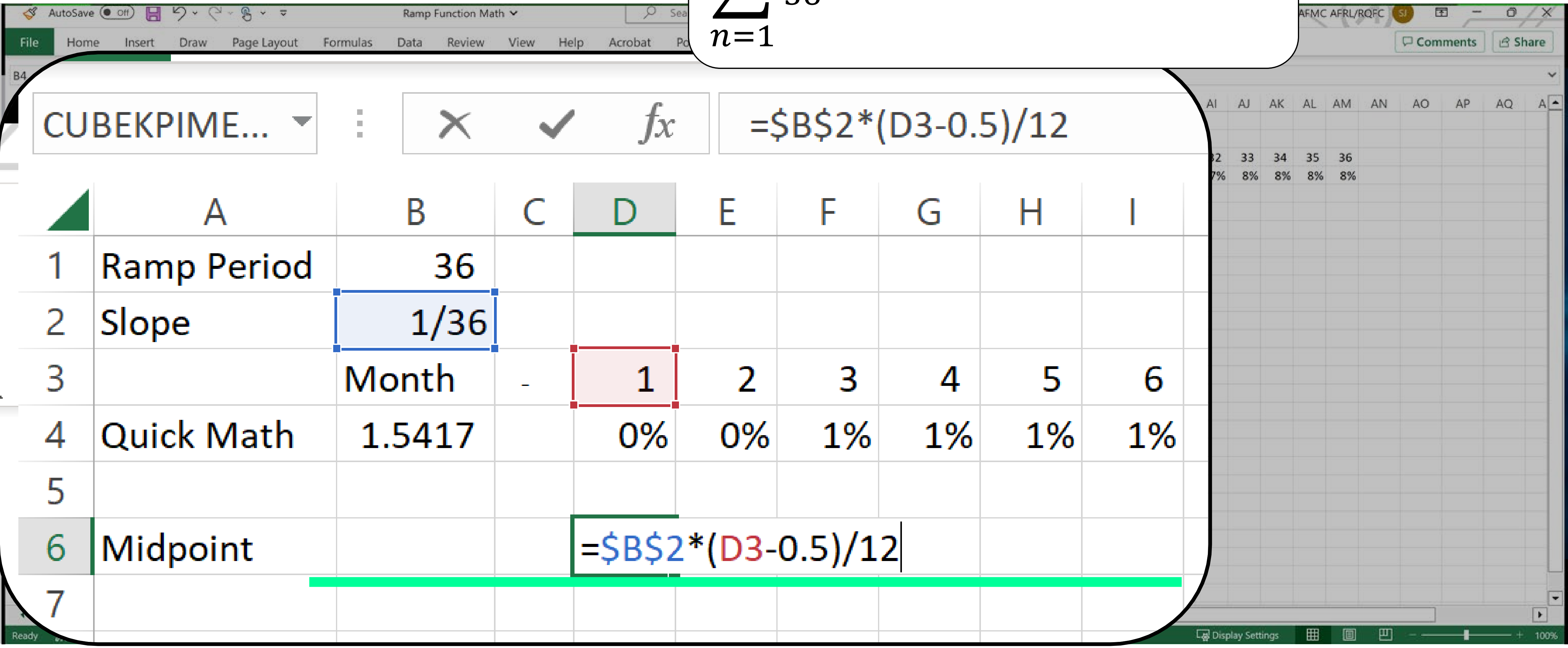
$$\sum_{n=1}^{36} \frac{1}{36} * (x - 0.5) \div 12$$



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$$\sum_{n=1}^{36} \frac{1}{36} * (x - 0.5) \div 12$$

Ramp Function Math

B7

	A	B	C	D	E	F	G	H	I
1	Ramp Period	36							
2	Slope	1/36							
3		Month	-	1	2	3	4	5	6
4	Quick Math	1.5417		0%	0%	1%	1%	1%	1%
6	Midpoint	1.5000		0%	0%	1%	1%	1%	1%
7									

Next Slide

$$\sum_{n=1}^{36} \frac{1}{36} * (x - 0.5) \div 12$$

Ramp Function Math

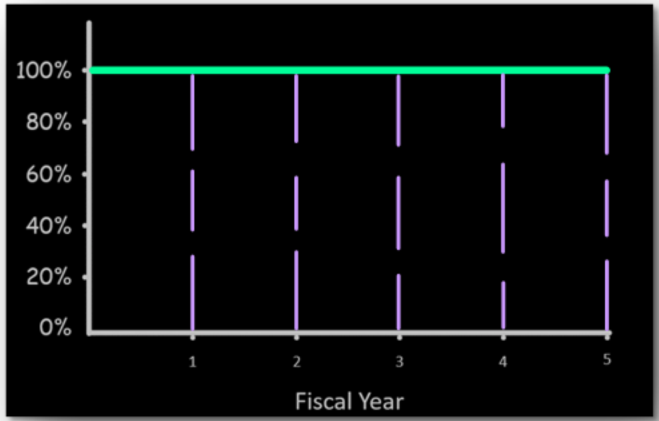
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1	Ramp Period	36																	
2	Slope	1/36																	
3		Month	-	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
4	Quick Math	1.5417		0%	0%	1%	1%	1%	1%	2%	2%	2%	2%	3%	3%	3%	3%	3%	4%
5																			
6	Midpoint	1.5000		0%	0%	1%	1%	1%	1%	2%	2%	2%	2%	2%	3%	3%	3%	3%	4%

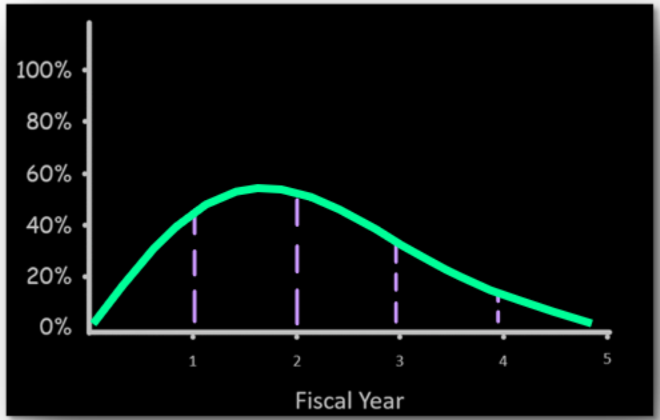
	A	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM
3		17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
4	Quick Math	4%	4%	4%	5%	5%	5%	5%	6%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%
6	Midpoint	4%	4%	4%	5%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%

2nd Pass (72 Months) | 3rd Pass (36 Months) | Sheet3 | Sheet4 | Sheet5.0 | Sheet5.2 | Sheet5.1

Phasing Calculator Tool

- Flat Level of Effort (LOE)
- Front Loaded Phasing Distributions
  - Beta
  - Raleigh
  - Weibull
- Outsource the Math
  - Input
    - BY$M Input
    - Start Date (Calendar Year)
    - Duration (Months)
    - Phasing profile shape parameters
  - Output
    - *TY$M or percentages*
    - Costs Bucketed into Fiscal Years
- Demo @ End





Phasing Calculator Tool

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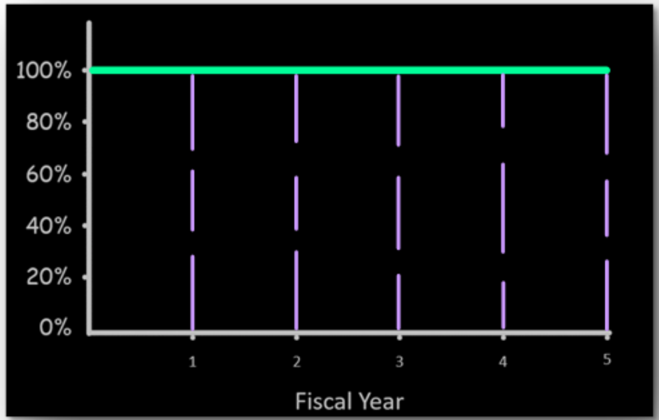
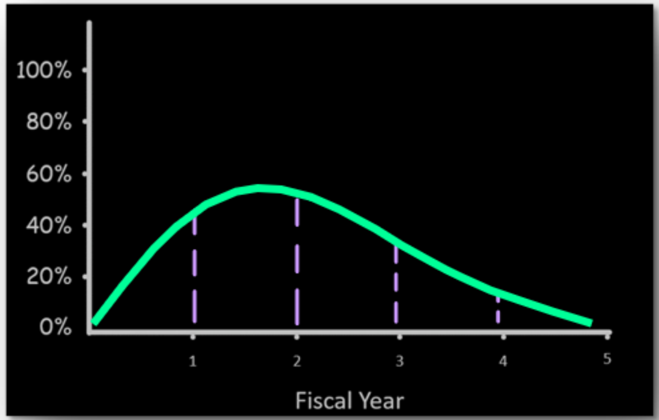
    - BY$M Input



- Output
  - TY$M *or percentages*
  - Costs Bucketed into Fiscal Years
- Demo @ End



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Ramp Function Math

Month

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	Ramp Period	36																			
2	Slope	1/36																			
3		Month	-	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
4	Quick Math	1.5417		0%	0%	1%	1%	1%	1%	2%	2%	2%	2%	3%	3%	3%	3%	3%	4%	4%	4%
5																					
6	Midpoint	1.5000		0%	0%	1%	1%	1%	1%	2%	2%	2%	2%	2%	3%	3%	3%	3%	4%	4%	4%

	A	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM
3		19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
4	Quick Math	4%	5%	5%	5%	5%	6%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%
6	Midpoint	4%	5%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%

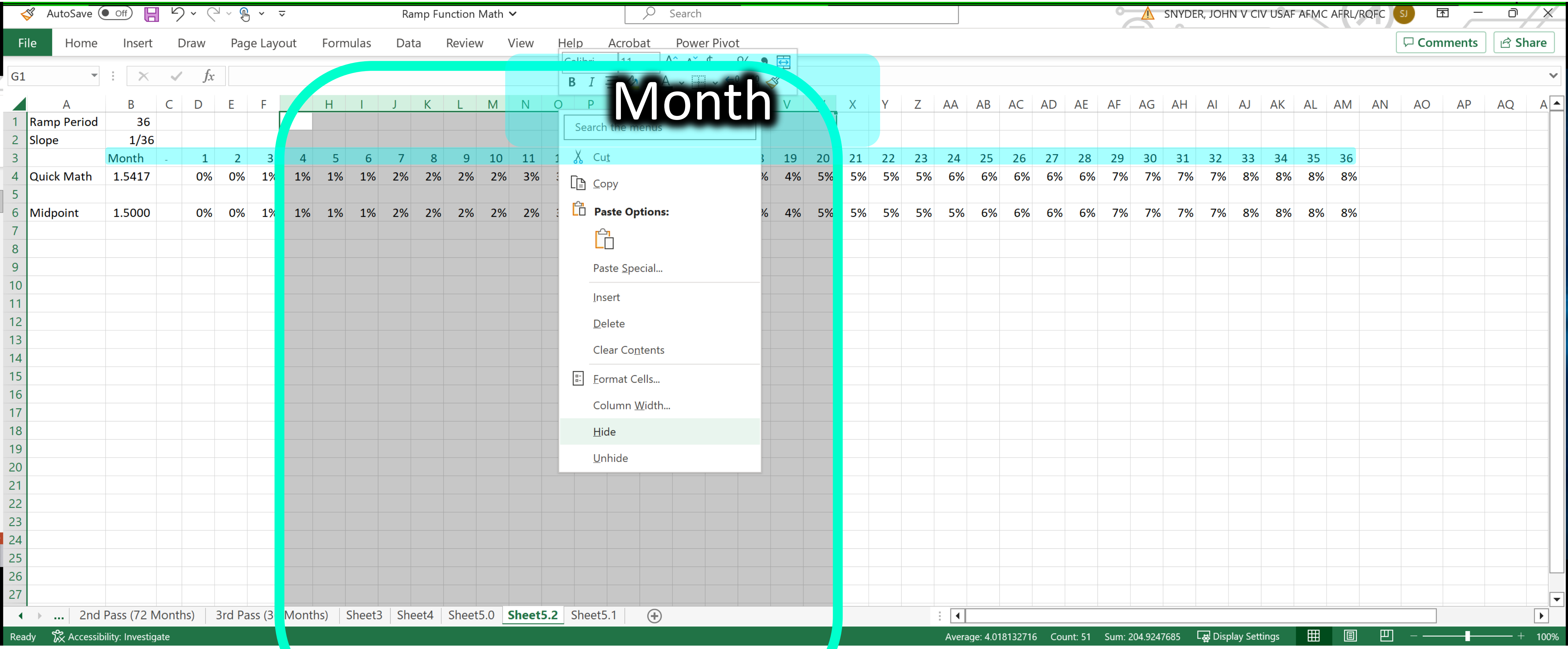
2nd Pass (72 Months) | 3rd Pass (36 Months) | Sheet3 | Sheet4 | Sheet5.0 | Sheet5.2 | Sheet5.1

Month

Ramp Function Math

Ramp Period	36																			
Slope	1/36																			
	Month	-	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Quick Math	1.5417		0%	0%	1%	1%	1%	1%	2%	2%	2%	2%	3%	3%	3%	3%	3%	4%	4%	4%
Midpoint	1.5000		0%	0%	1%	1%	1%	1%	2%	2%	2%	2%	2%	3%	3%	3%	3%	4%	4%	4%

	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Quick Math	4%	5%	5%	5%	5%	6%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%
Midpoint	4%	5%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%


Month
Ramp Function Math
Ramp Period 36
Slope 1/36
Month
Quick Math 1.5417
Midpoint 1.5000
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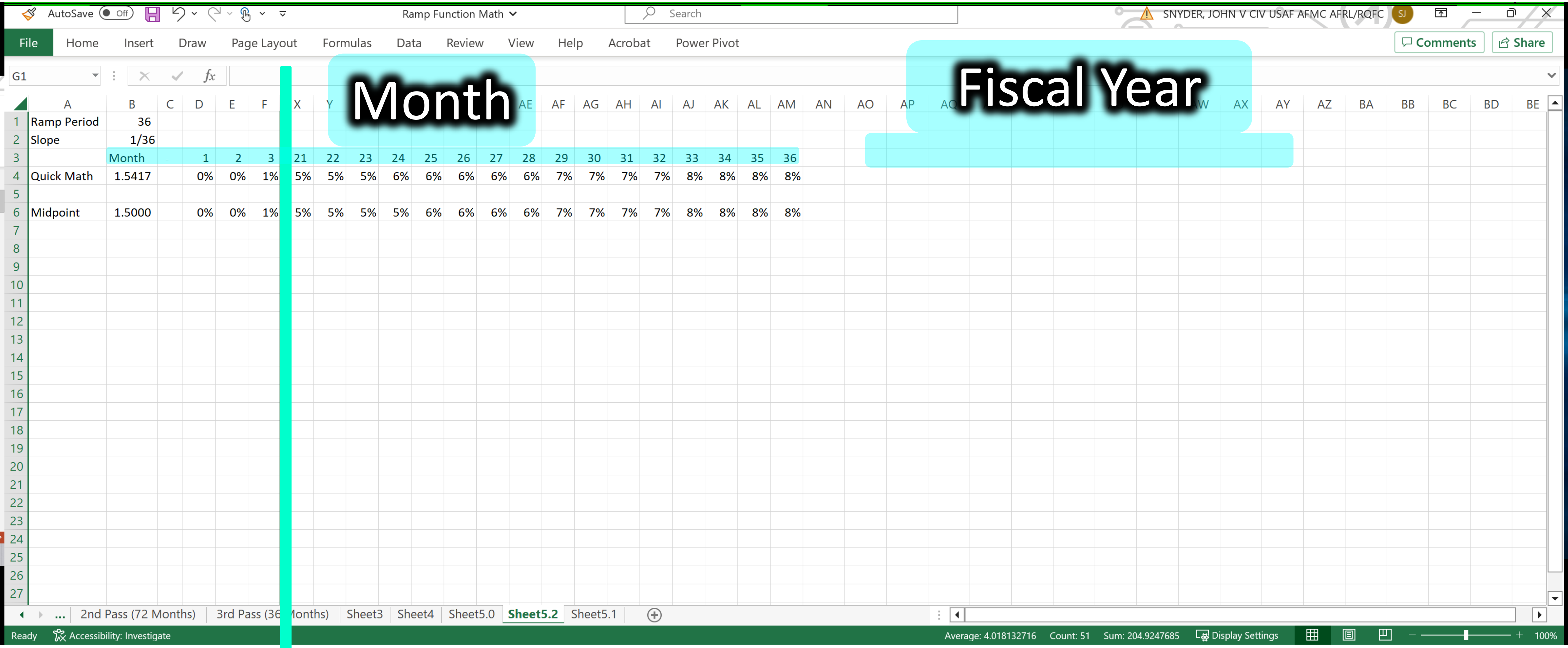
Clear Contents
Format Cells...
Column Width...
Hide
Unhide
2nd Pass (72 Months)
Sheet3
Sheet4
Sheet5.0
Sheet5.2
Sheet5.1
Ready
Average: 4.018132716
Count: 51
Sum: 204.9247685



Ramp Period	36																			
Slope	1/36																			
	Month	-	1	2	3	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Quick Math	1.5417		0%	0%	1%	5%	5%	5%	6%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%
Midpoint	1.5000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%

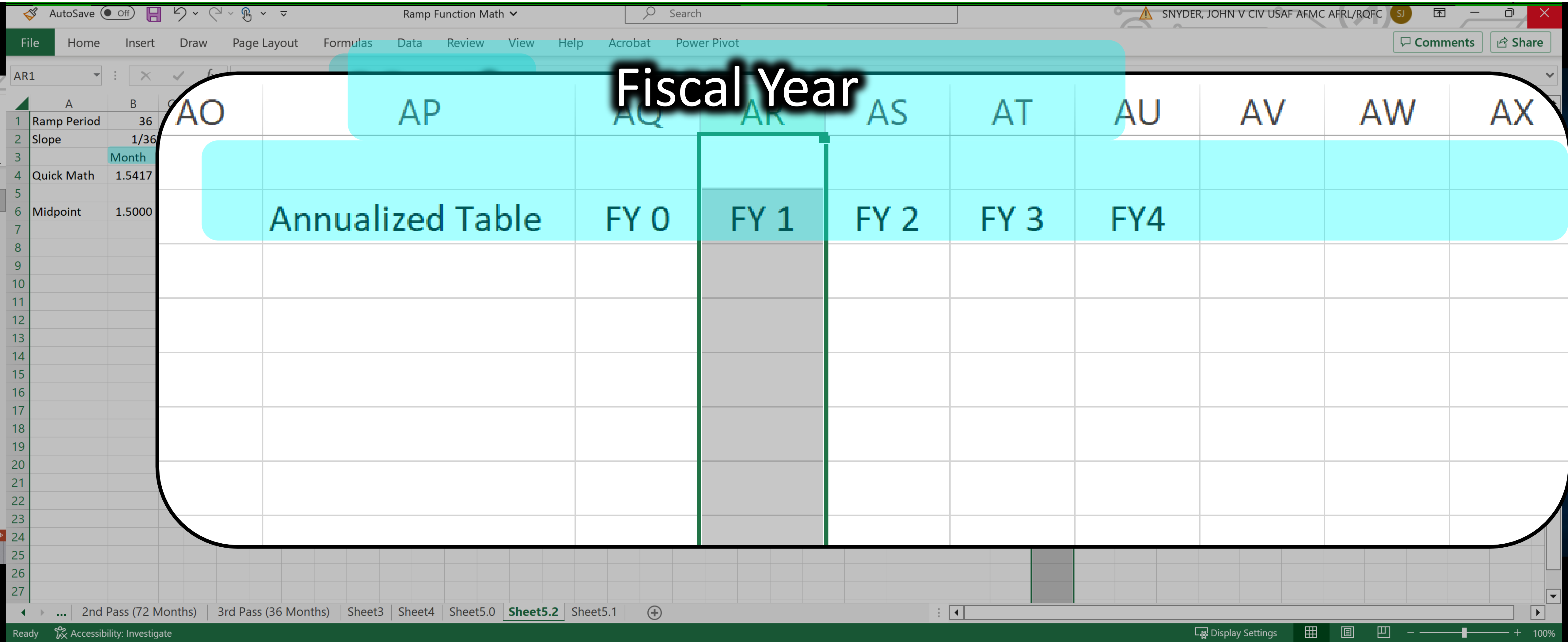
Ramp Function Math

Month

	B	C	D	E	F	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM
Ramp Period	36																				
Slope	1/36																				
	Month	-	1	2	3	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Quick Math	1.5417		0%	0%	1%	5%	5%	5%	6%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%
Midpoint	1.5000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%

Fiscal Year

Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4


Ramp Function Math
Fiscal Year
AO
AP
AQ
AR
AS
AT
AU
AV
AW
AX
Annualized Table
FY 0
FY 1
FY 2
FY 3
FY4
Ramp Period 36
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Quick Math 1.5417
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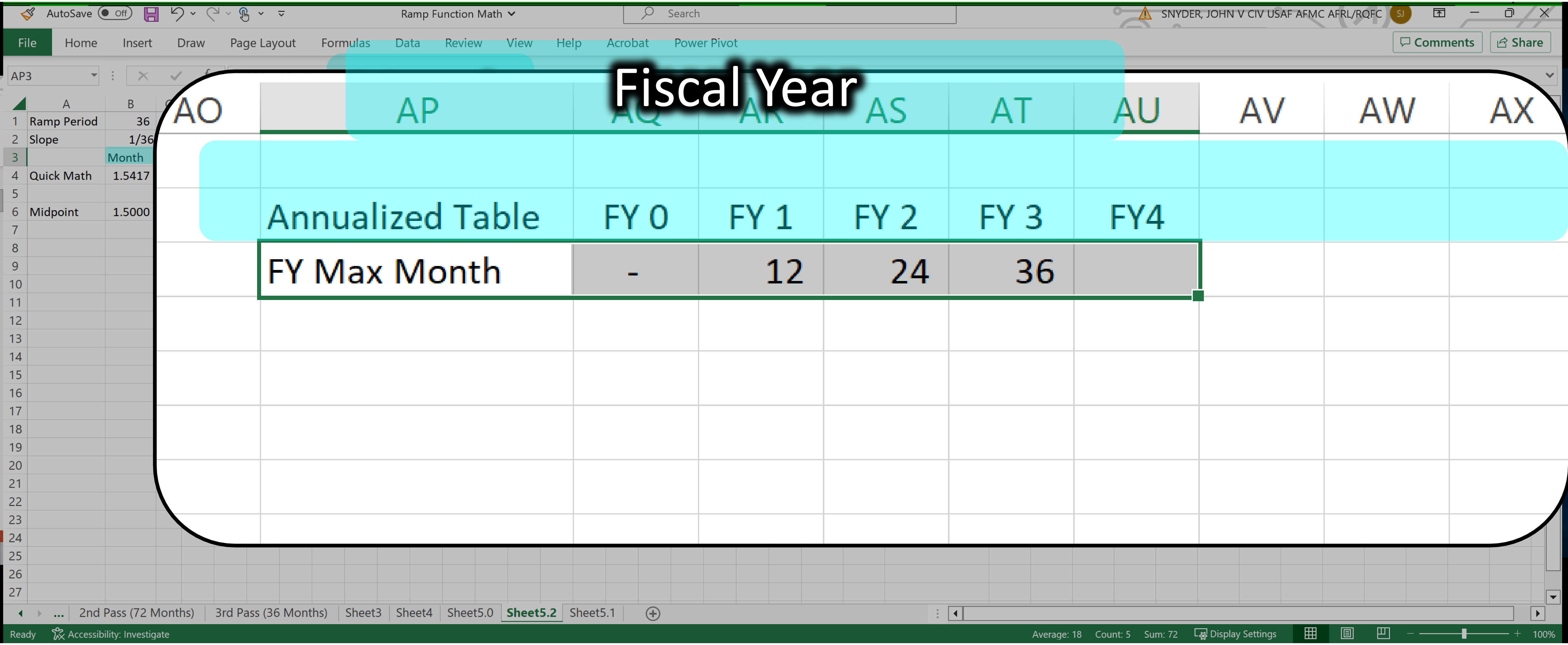

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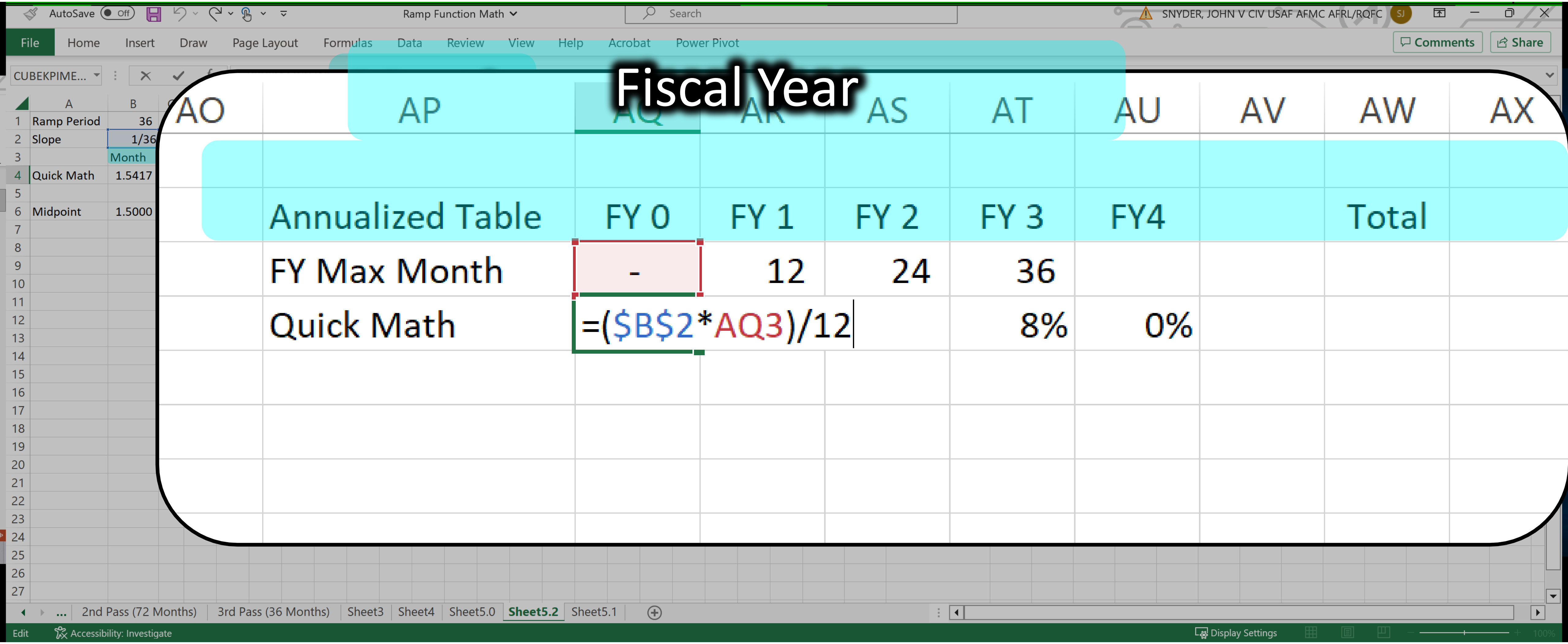
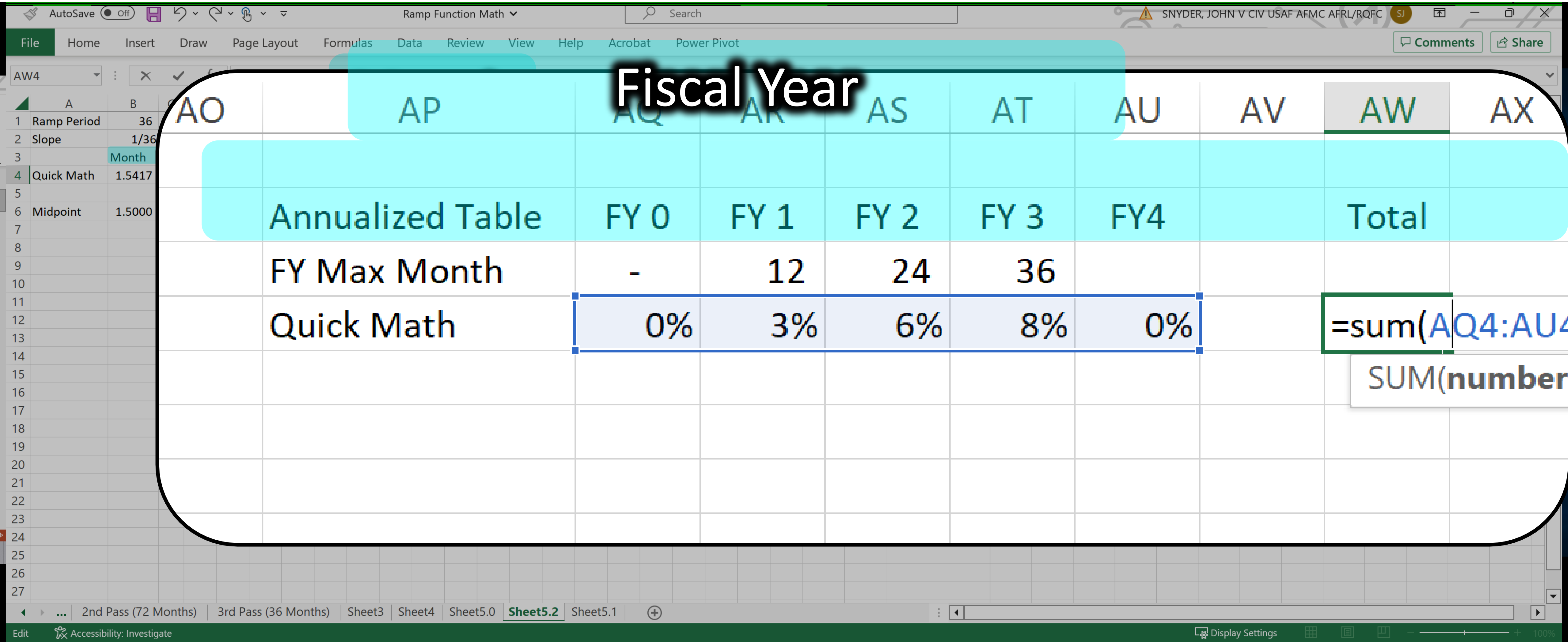
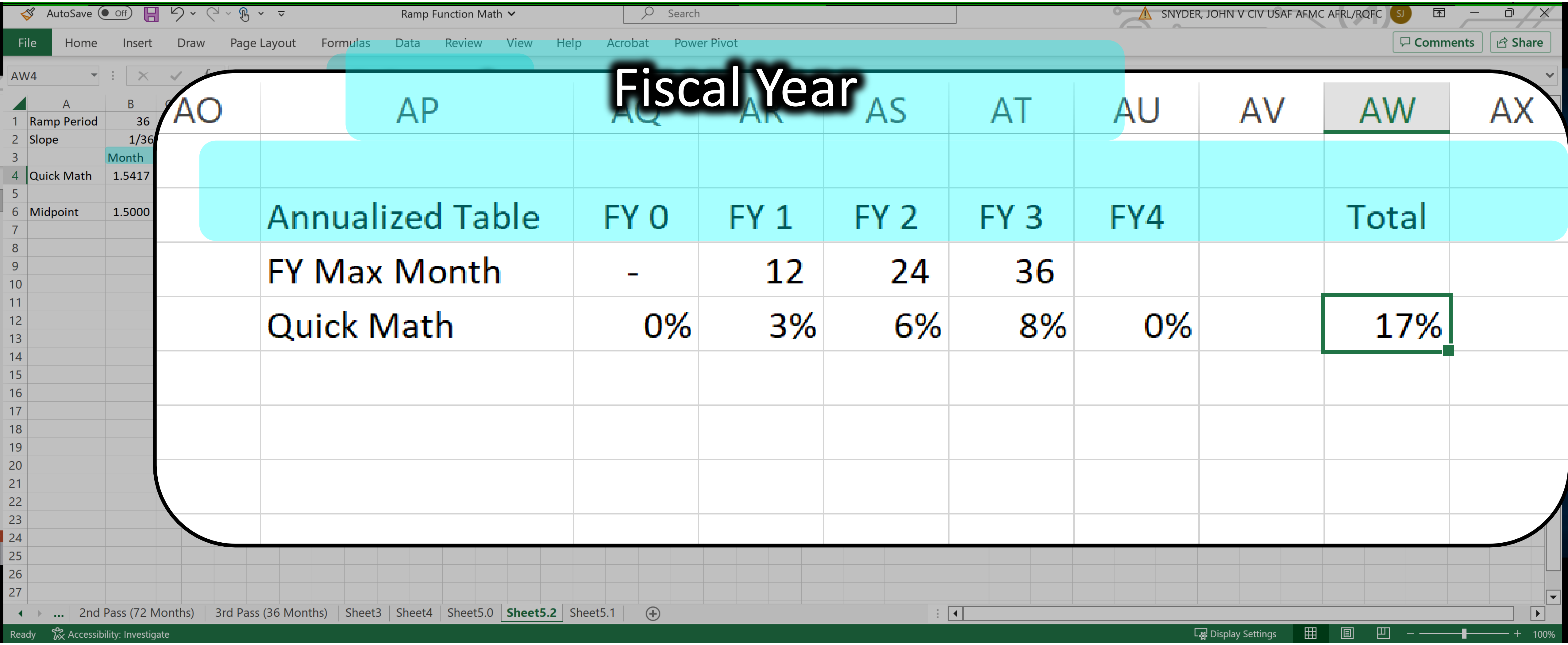

Fiscal Year
Ramp Function Math
Ramp Period 36
Slope 1/36
Month
Quick Math 1.5417
Midpoint 1.5000
AO AP AQ AR AS AT AU AV AW AX
Annualized Table
FY 0
FY 1
FY 2
FY 3
FY4
Total
FY Max Month
-
12
24
36
Quick Math
=(B2*AQ3)/12
8%
0%
2nd Pass (72 Months)
3rd Pass (36 Months)
Sheet3
Sheet4
Sheet5.0
Sheet5.2
Sheet5.1


Fiscal Year
Ramp Function Math
AW4
Ramp Period 36
Slope 1/36
Month
Quick Math 1.5417
Midpoint 1.5000
AO AP AQ AR AS AT AU AV AW AX
Annualized Table | FY 0 | FY 1 | FY 2 | FY 3 | FY4 | Total
FY Max Month | - | 12 | 24 | 36
Quick Math | 0% | 3% | 6% | 8% | 0% | =sum(AQ4:AU4
SUM(number
2nd Pass (72 Months) | 3rd Pass (36 Months) | Sheet3 | Sheet4 | Sheet5.0 | Sheet5.2 | Sheet5.1



Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4		Total
FY Max Month	-	12	24	36			
Quick Math	0%	3%	6%	8%	0%		17%

AFRL

Fiscal Year

	A	B
1	Ramp Period	36
2	Slope	1/36
3		Month
4	~~Quick Math~~	~~1.5417~~
5		
6	Midpoint	1.5000

Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4		Total
FY Max Month	-	12	24	36			
~~Quick Math~~	~~0%~~	~~3%~~	~~6%~~	~~8%~~	~~0%~~		~~17%~~

Average: 0.079457364 Count: 45 Sum: 3.416666667

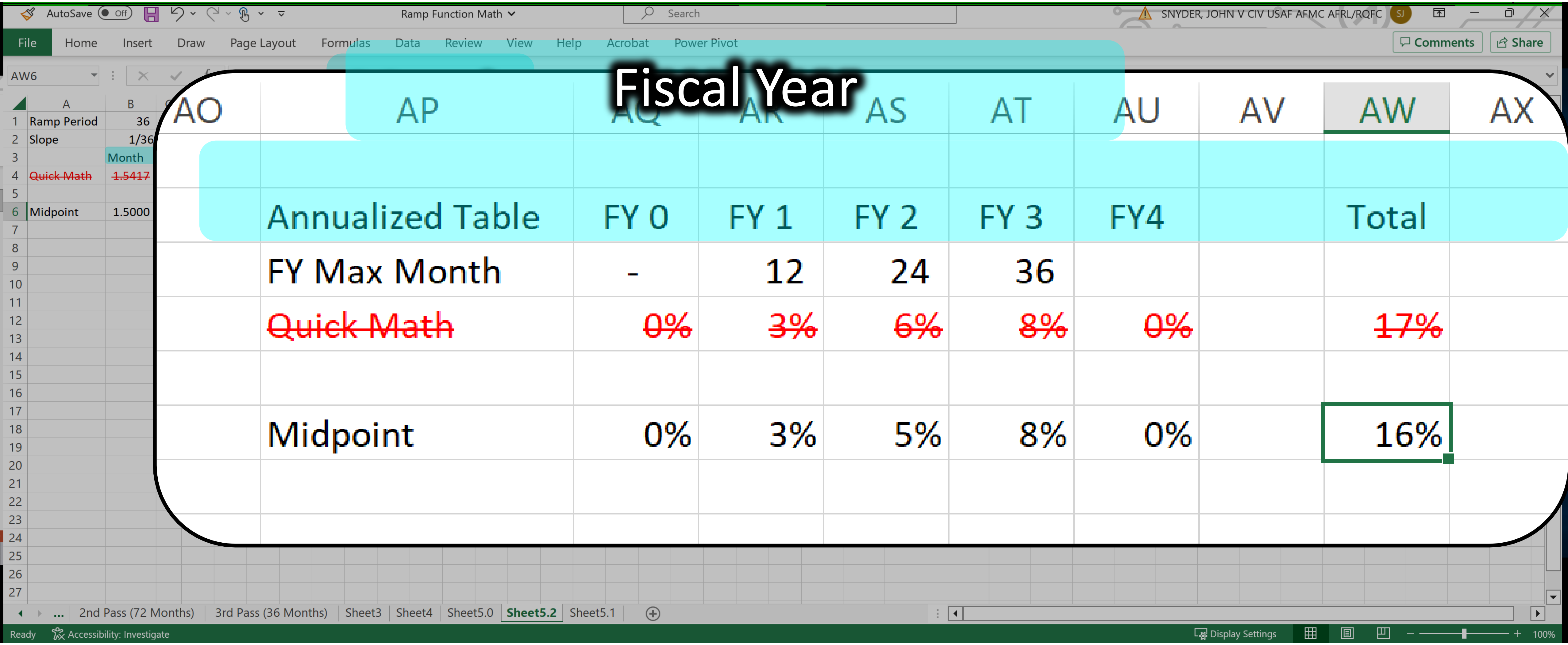
Fiscal Year

	A	B
1	Ramp Period	36
2	Slope	1/36
3		Month
4	~~Quick Math~~	~~1.5417~~
5		
6	Midpoint	1.5000

AP	AQ	AR	AS	AT	AU	AV	AW
Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4		Total
FY Max Month	-	12	24	36			
~~Quick Math~~	~~0%~~	~~3%~~	~~6%~~	~~8%~~	~~0%~~		~~17%~~
Midpoint	=B2*(AQ3-0.5)/12						

Next Slide

AFRL



AP	AQ	AR	AS	AT	AU	AV	AW
Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4		Total
FY Max Month	-	12	24	36			
~~Quick Math~~	~~0%~~	~~3%~~	~~6%~~	~~8%~~	~~0%~~		~~17%~~
Midpoint	0%	3%	5%	8%	0%		16%

Next Slide

AFRL

Fiscal Year

Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4		Total
FY Max Month	-	12	24	36			
~~Quick Math~~	~~0%~~	~~3%~~	~~6%~~	~~8%~~	~~0%~~		~~17%~~
~~Midpoint~~	~~0%~~	~~3%~~	~~5%~~	~~8%~~	~~0%~~		~~16%~~

	A	B
1	Ramp Period	36
2	Slope	1/36
3		Month
4	~~Quick Math~~	~~1.5417~~
5		
6	Midpoint	1.5000

Next Slide

AFRL

Ramp Function Math

AP6 =A6

Month | **Fiscal Year**

	A	B	C	D	E	F	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM
1	Ramp Period	36																				
2	Slope	1/36																				
3		Month	-	1	2	3	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
4	~~Quick Math~~	~~1.5417~~		~~0%~~	~~0%~~	~~1%~~	~~5%~~	~~5%~~	~~5%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~7%~~	~~7%~~	~~7%~~	~~7%~~	~~8%~~	~~8%~~	~~8%~~	~~8%~~
5																						
6	Midpoint	1.5000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%

Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4	Total
FY Max Month	-	12	24	36		
~~Quick Math~~	~~0%~~	~~3%~~	~~6%~~	~~8%~~	~~0%~~	~~17%~~
~~Midpoint~~	~~0%~~	~~3%~~	~~5%~~	~~8%~~	~~0%~~	~~16%~~

Sheets: 2nd Pass (72 Months) | 3rd Pass (36 Months) | Sheet3 | Sheet4 | Sheet5.0 | Sheet5.2 | Sheet5.1

Average: 0.053626543 Count: 7 Sum: 0.321759259

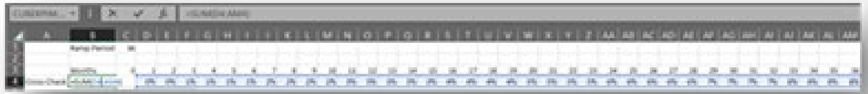
Next Slide

Calculus

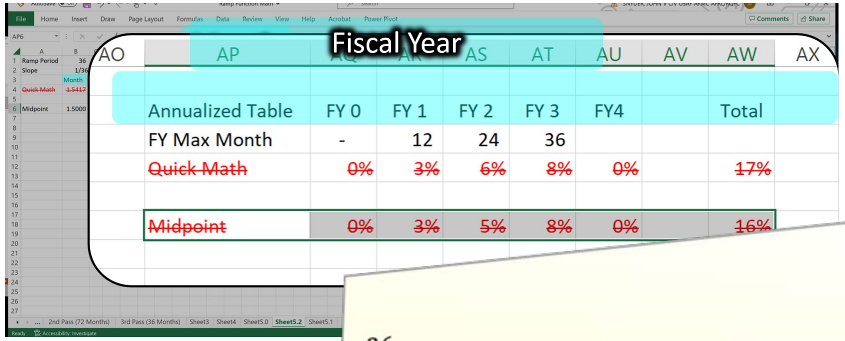
Area Under the Curve

$$\sum_{n=1}^{36} \frac{1}{36} * x \div 12 \quad = 1.5417 \; \neq 1.5$$

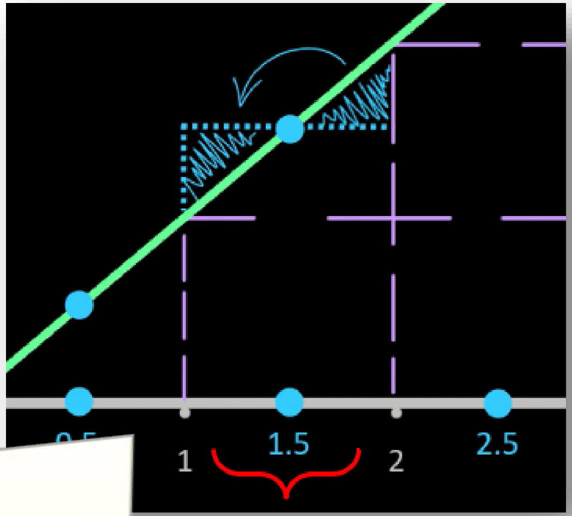
- Cross Check was Bogus



- Midpoint confirmed the calc
  - Failed to Annualize



Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4	Total
FY Max Month	-	12	24	36		
~~Quick Math~~	~~0%~~	~~3%~~	~~6%~~	~~8%~~	~~0%~~	~~17%~~
~~Midpoint~~	~~0%~~	~~3%~~	~~5%~~	~~8%~~	~~0%~~	~~16%~~



- Integrals into Excel?

$$\int_0^{36} \frac{1}{36} x dx = \left[\frac{1}{36} * \frac{1}{2} * x^2\right]_0^{36} \div 12 = 1.5$$

Next Slide

Area Under the Curve

- Tabulated Integral Reference:

$$\int x^n dx = \frac{1}{n+1} * x^{n+1} + C$$

Next Slide

Area Under the Curve

$$x^n$$

$$\int \quad dx = \frac{1}{n+1} * x^{n+1} + C$$

Next Slide

Area Under the Curve

$$\int x^n \quad dx \quad = \quad \frac{1}{n+1} \quad * \quad x^{n+1} \quad + \quad C$$

Area Under the Curve

$$\int x^n \, dx$$

$$= \frac{1}{n+1} * x^{n+1} + C$$

Next Slide

Area Under the Curve

$$\int x^n dx = x^n$$

$$\frac{1}{n+1} * x^{n+1} + C$$

Area Under the Curve

$$\int x^n dx = x^{n+1}$$

$$\frac{1}{n+1} * + C$$

Next Slide

Area Under the Curve

$$\int x^n dx = \frac{1}{n+1} * x^{n+1}$$

$+ \quad C$

Next Slide

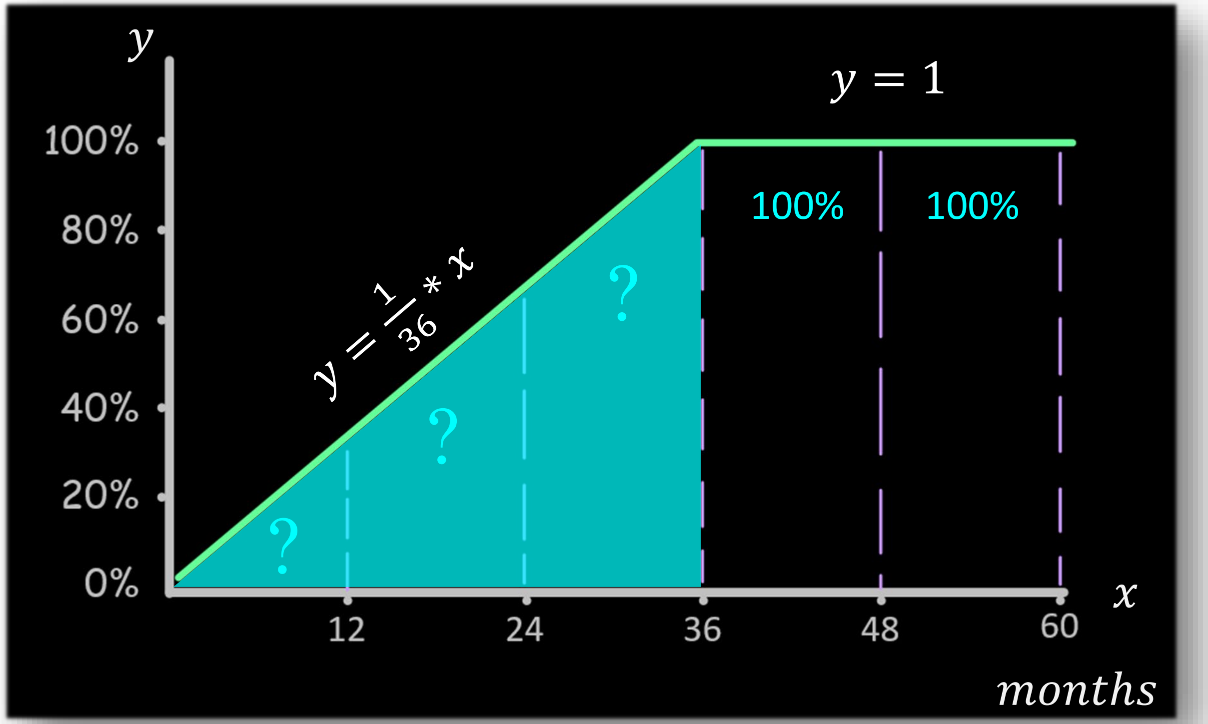
Area Under the Curve

$$\int x^n dx = \frac{1}{n+1} * x^{n+1} + C$$

Next Slide

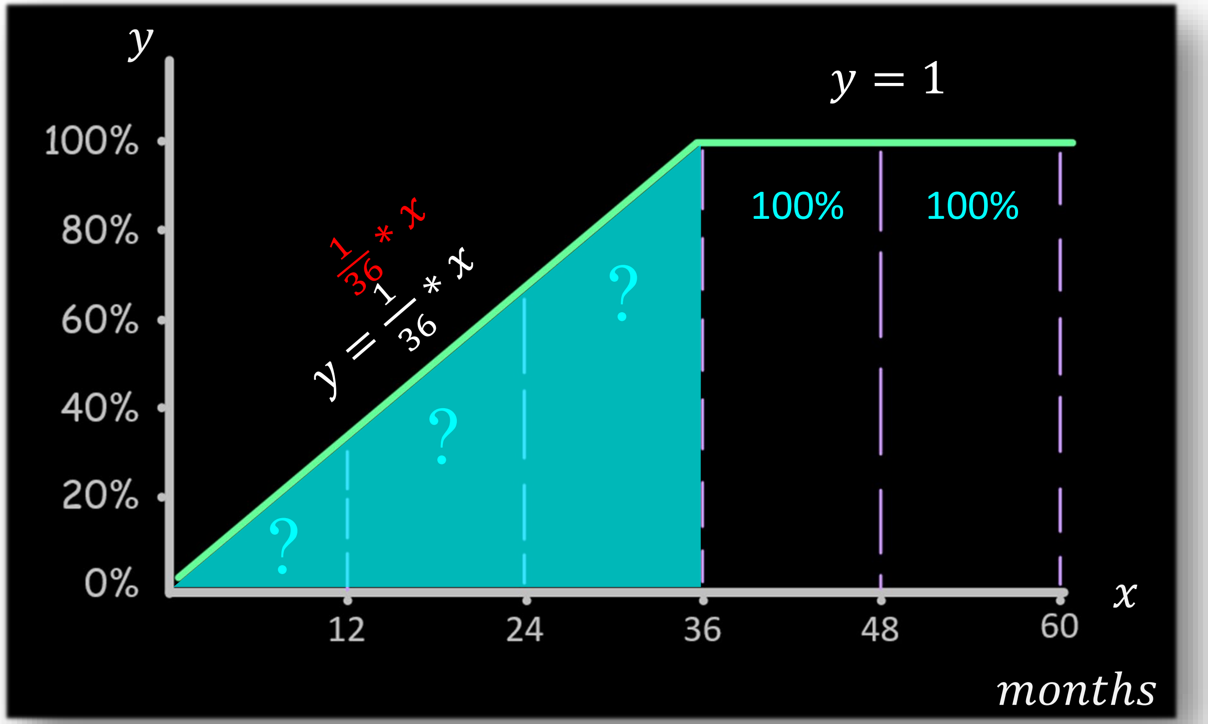
Area Under the Curve

$$\int x^n dx = \frac{1}{n+1} * x^{n+1} + C$$



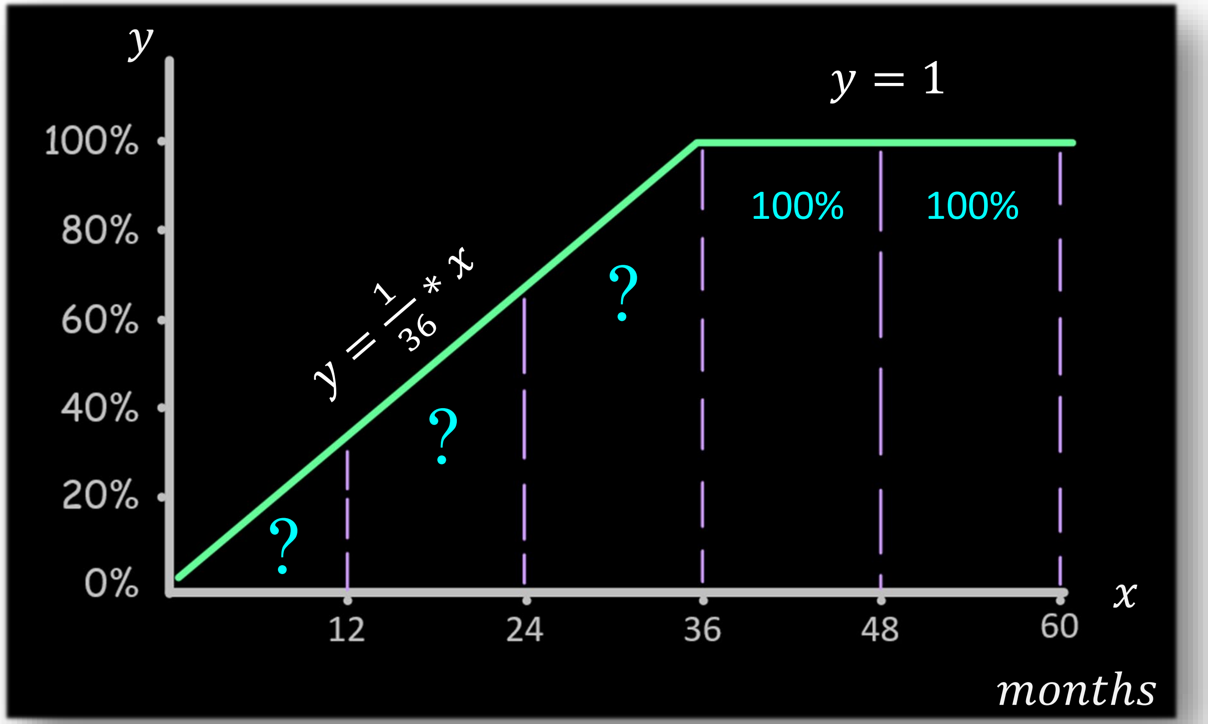
Area Under the Curve

$$\int x^n dx = \frac{1}{n+1} * x^{n+1} + C$$



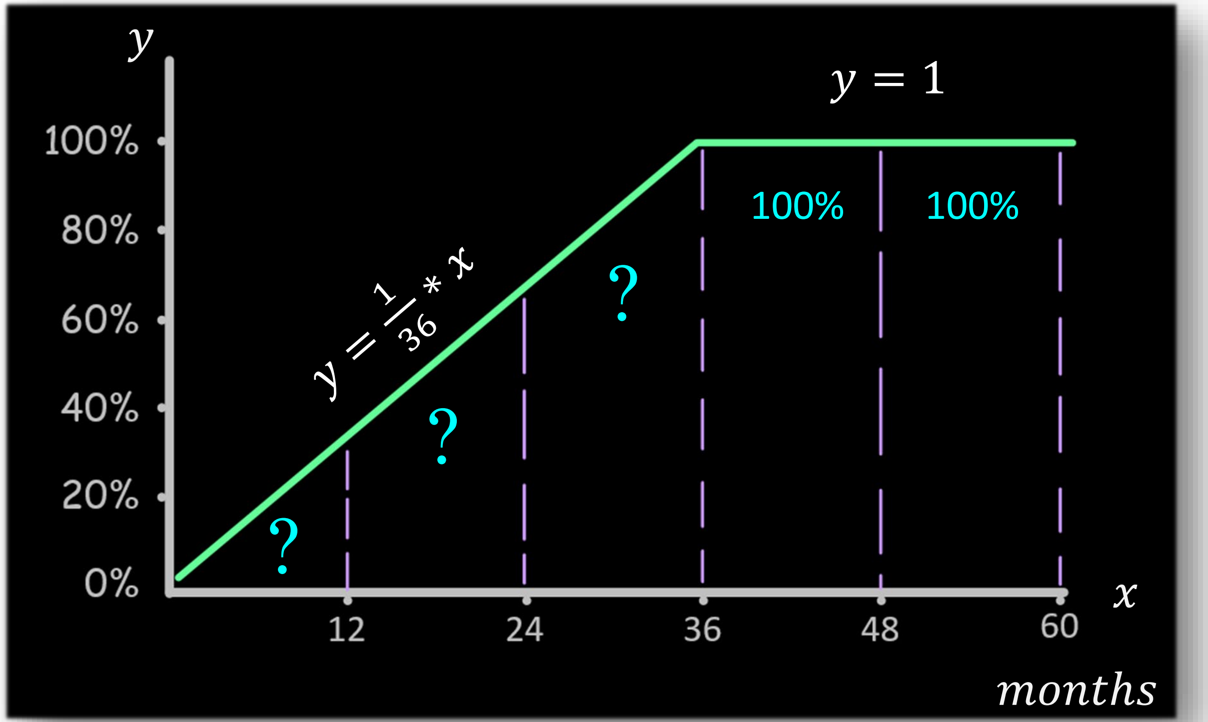
Area Under the Curve

$$\int \frac{1}{36} * x^n dx = \frac{1}{n+1} * x^{n+1} + C$$



Area Under the Curve

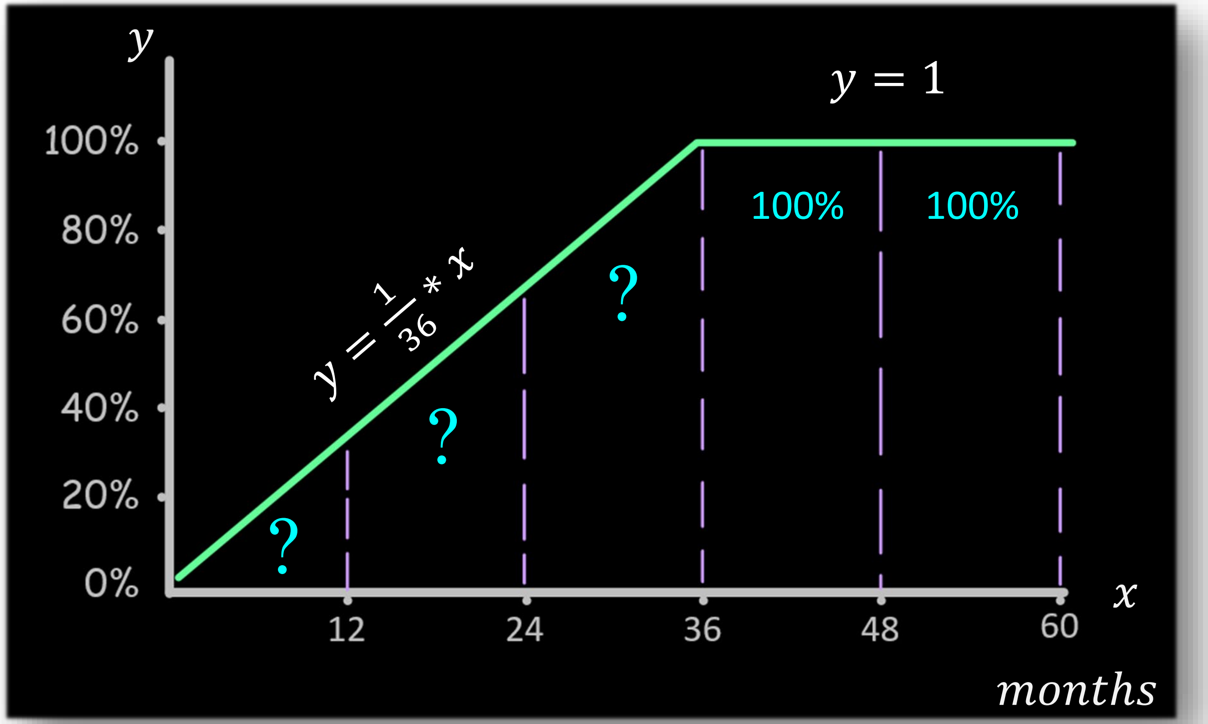
$$\frac{1}{36} * \int x^n dx = \frac{1}{n+1} * x^{n+1} + C$$



Area Under the Curve

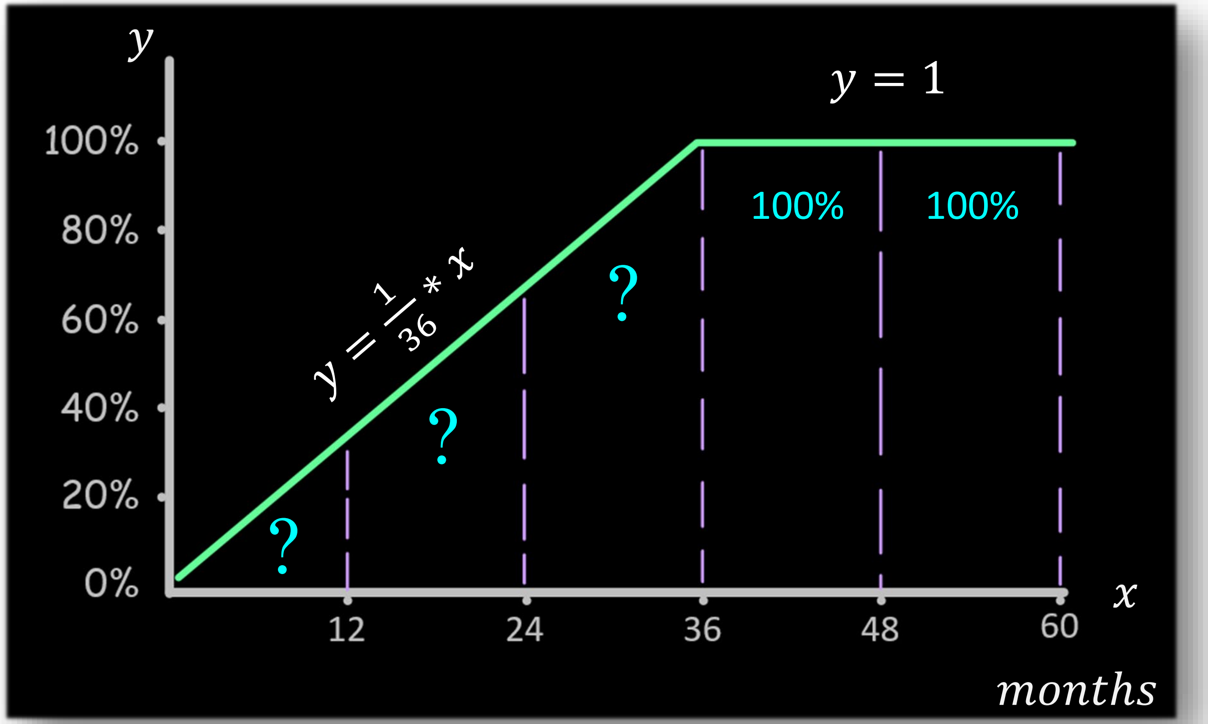
$$\frac{1}{36} * \int x^{1}\, dx = \frac{1}{n+1} * x^{n+1} + C$$

n



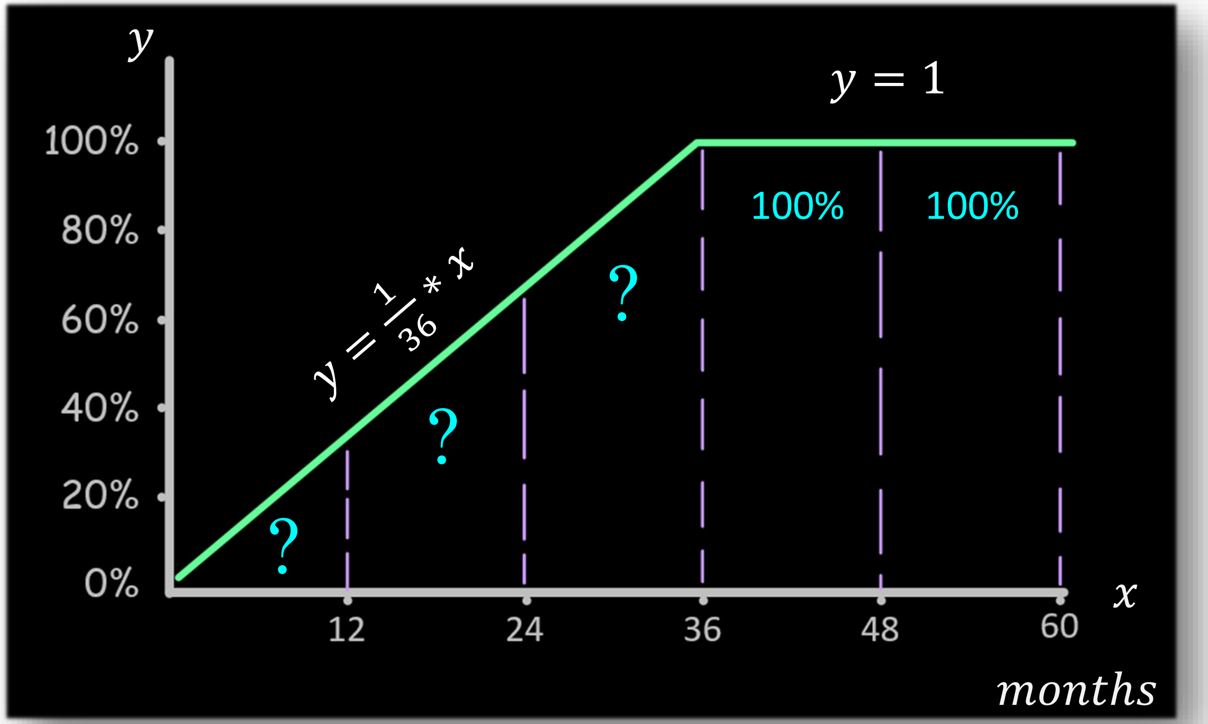
Area Under the Curve

$$\frac{1}{36} * \int x^1\,dx = \frac{1}{1+1} * x^{1+1} + C$$



Area Under the Curve

$$\frac{1}{36} * \int x^1 \, dx = \frac{1}{1+1} * x^{1+1} + C$$

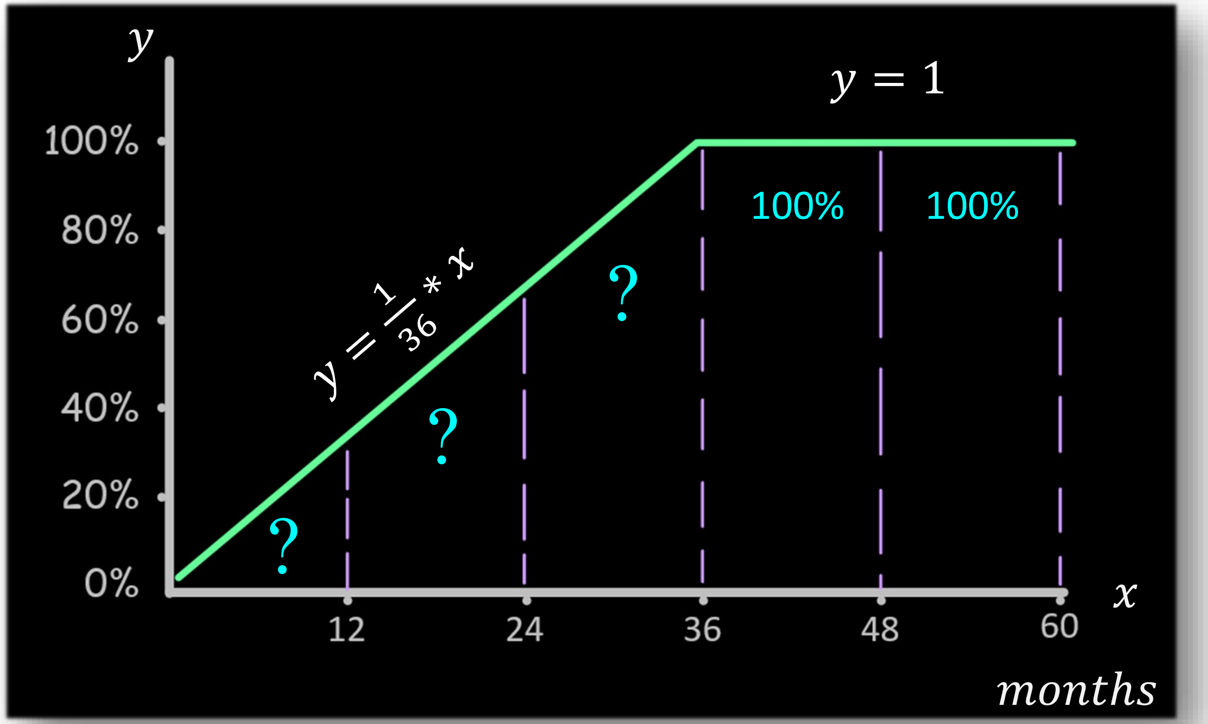


Area Under the Curve

$$\frac{1}{36} * \int x^1\, dx = \frac{1}{2} * x^2 + C$$

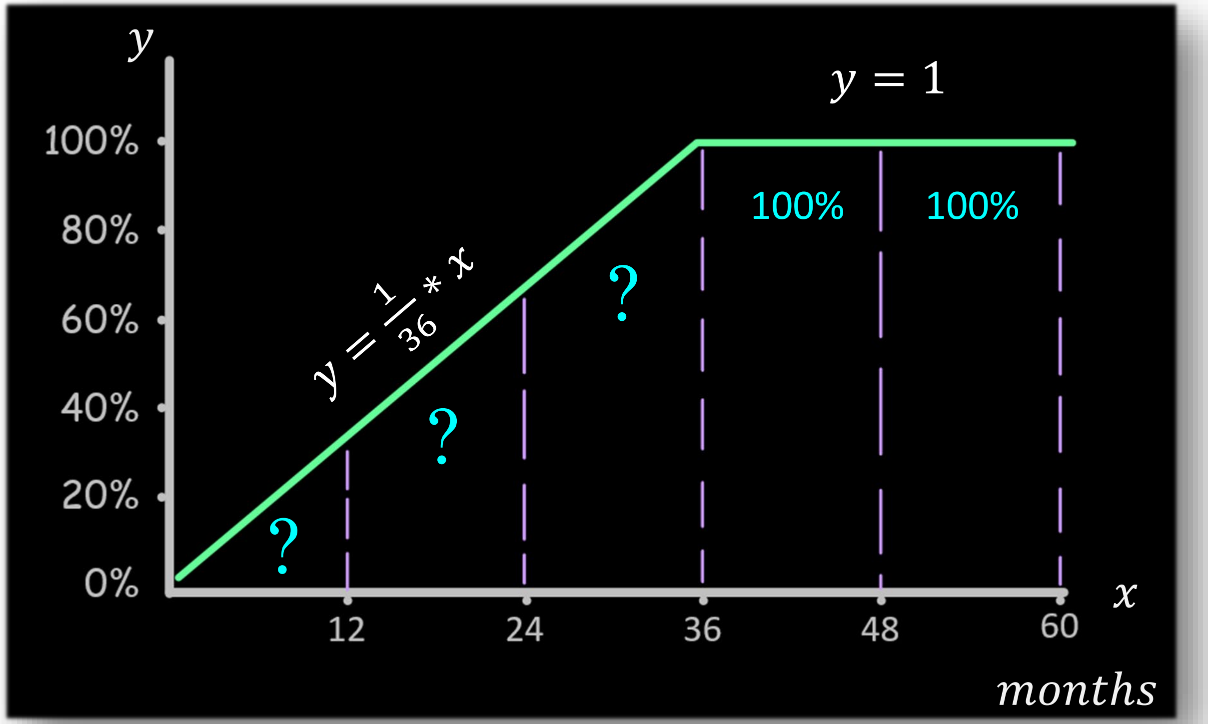
1+1

1+1



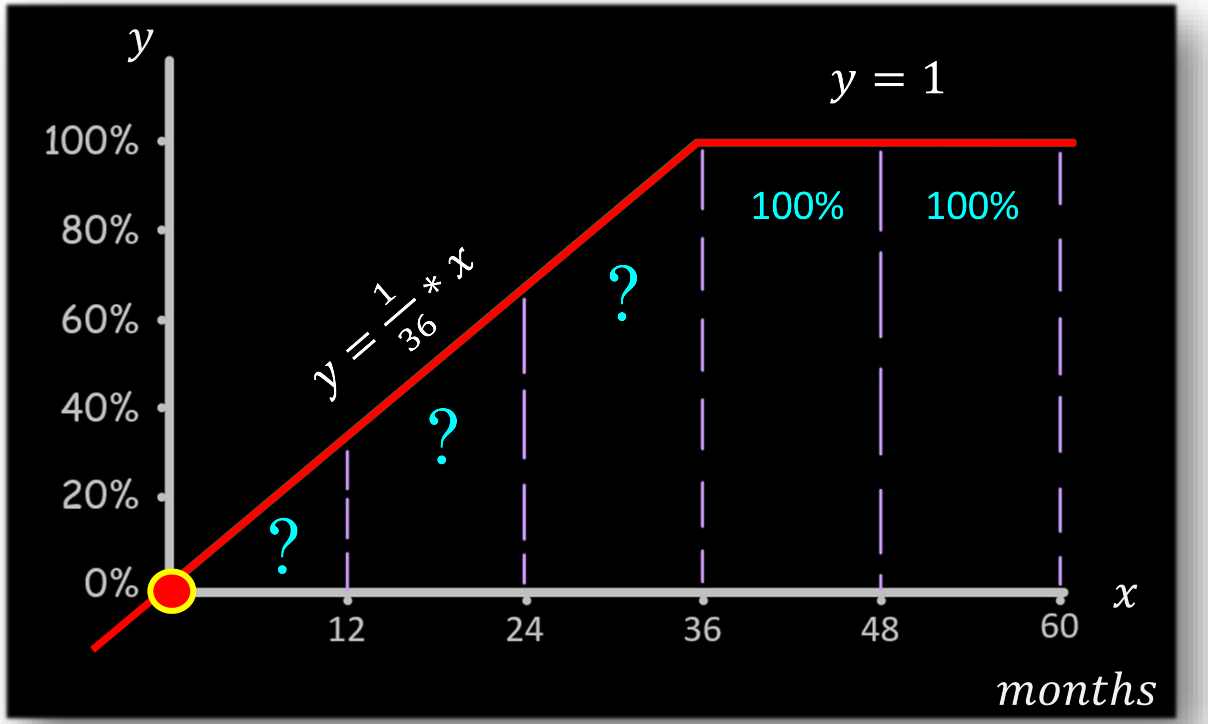
Area Under the Curve

$$\frac{1}{36} * \int x^1\,dx = \frac{1}{36} * \left[\frac{1}{2} * x^2 + C \right]$$



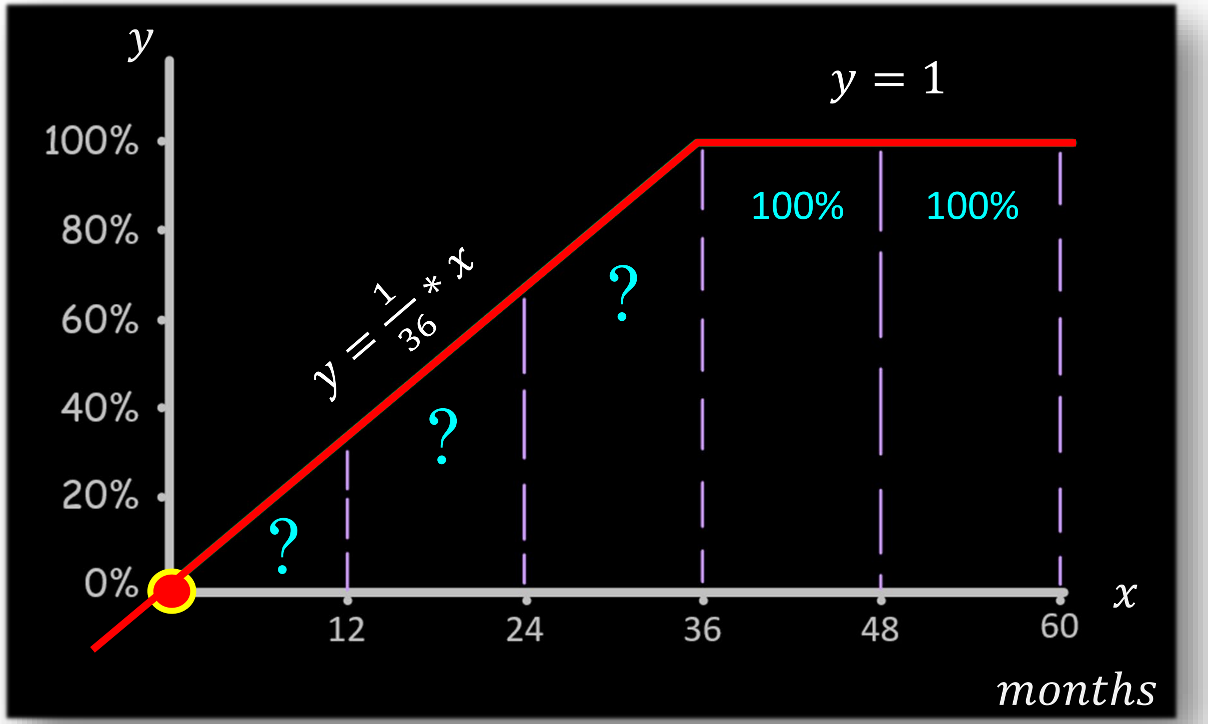
Area Under the Curve

$$\frac{1}{36} * \int x^1 \, dx = \frac{1}{36} * \left[\frac{1}{2} * x^2 + C \right]$$



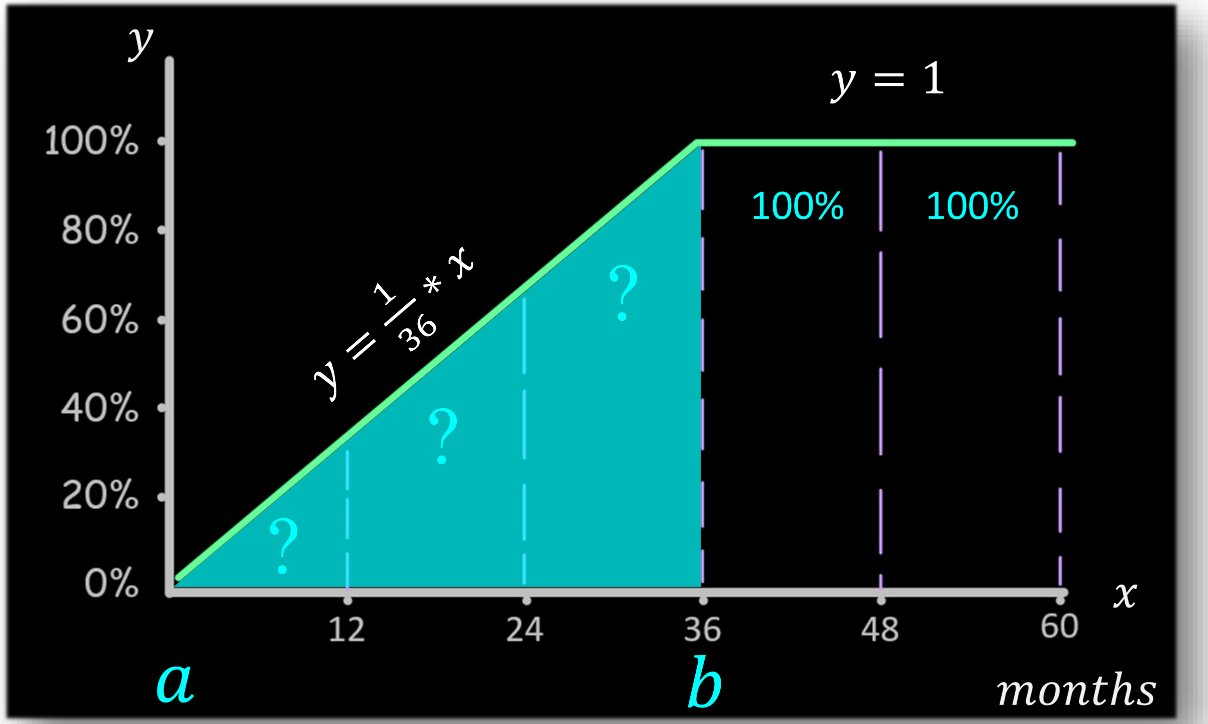
Area Under the Curve

$$\frac{1}{36} * \int x^1\, dx = \frac{1}{36} * \left[\frac{1}{2} * x^2 \right] + C$$



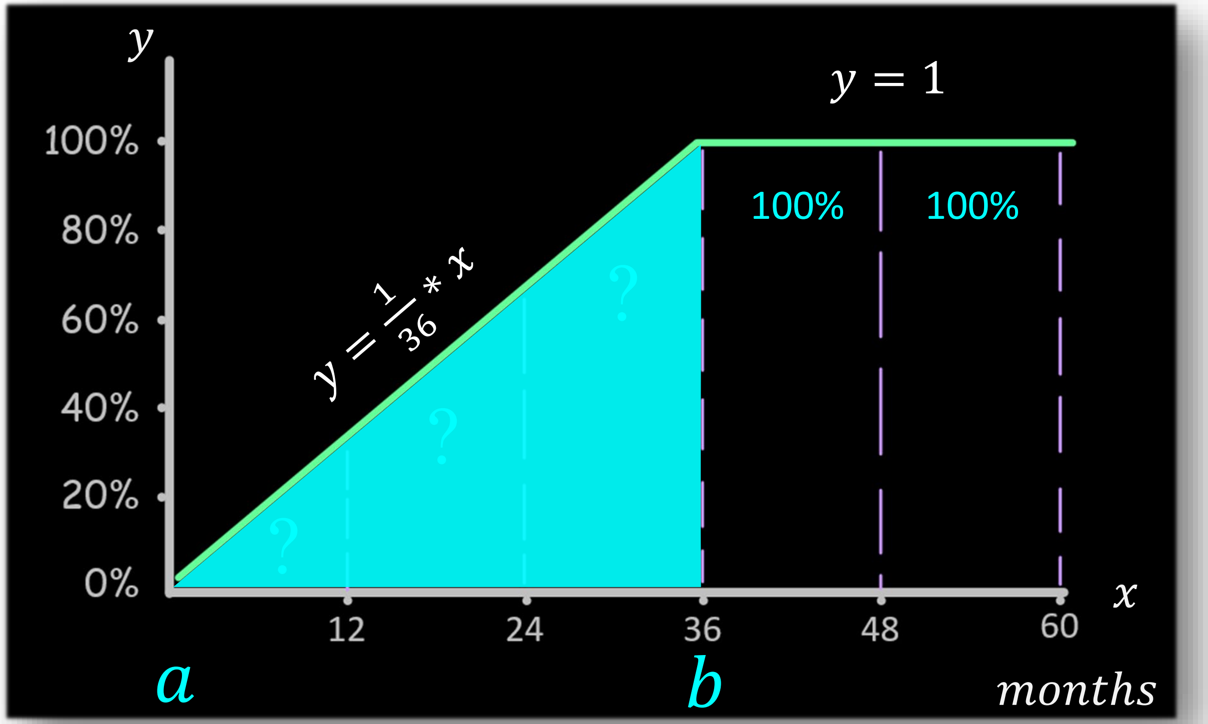
Area Under the Curve

$$\frac{1}{36} * \int x^1\, dx = \frac{1}{36} * \left[\frac{1}{2} * x^2 \right]$$



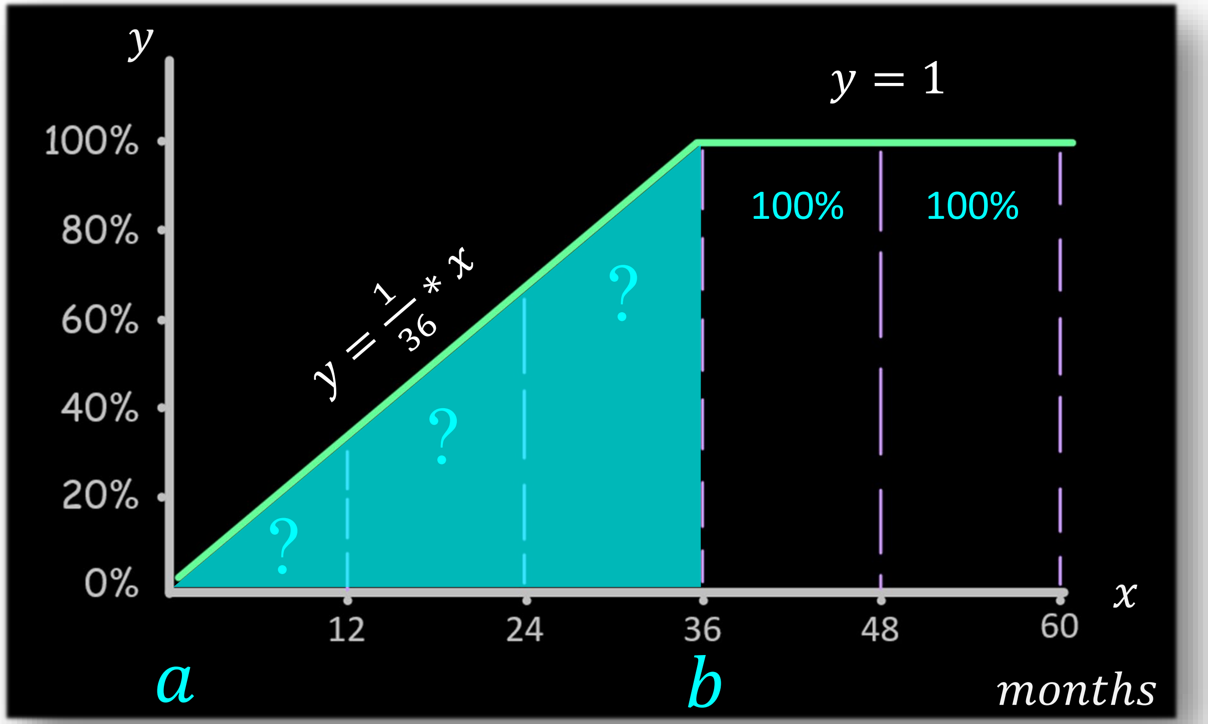
Area Under the Curve

$$\frac{1}{36} * \int_a^b x^1\,dx = \frac{1}{36} * \left[\frac{1}{2} * x^2 \right]$$



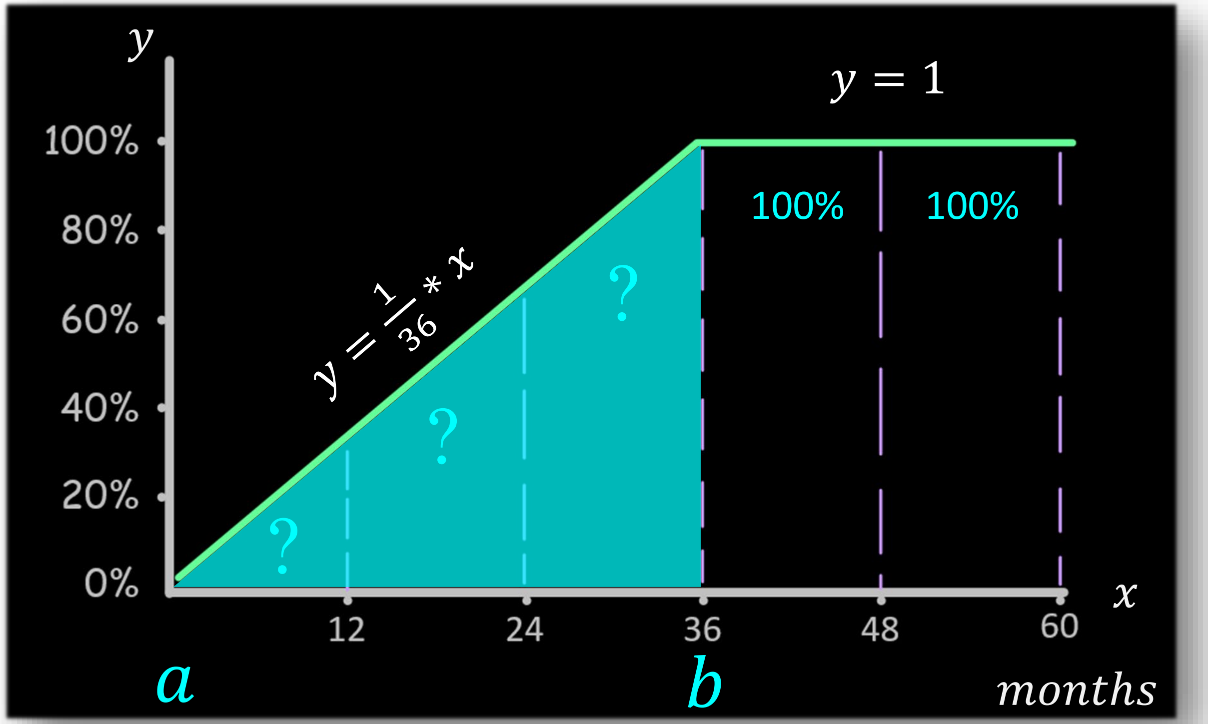
Area Under the Curve

$$\frac{1}{36} * \int_a^b x^1\,dx = \frac{1}{36} * \left[\frac{1}{2} * x^2 \right]_a^b$$



Area Under the Curve

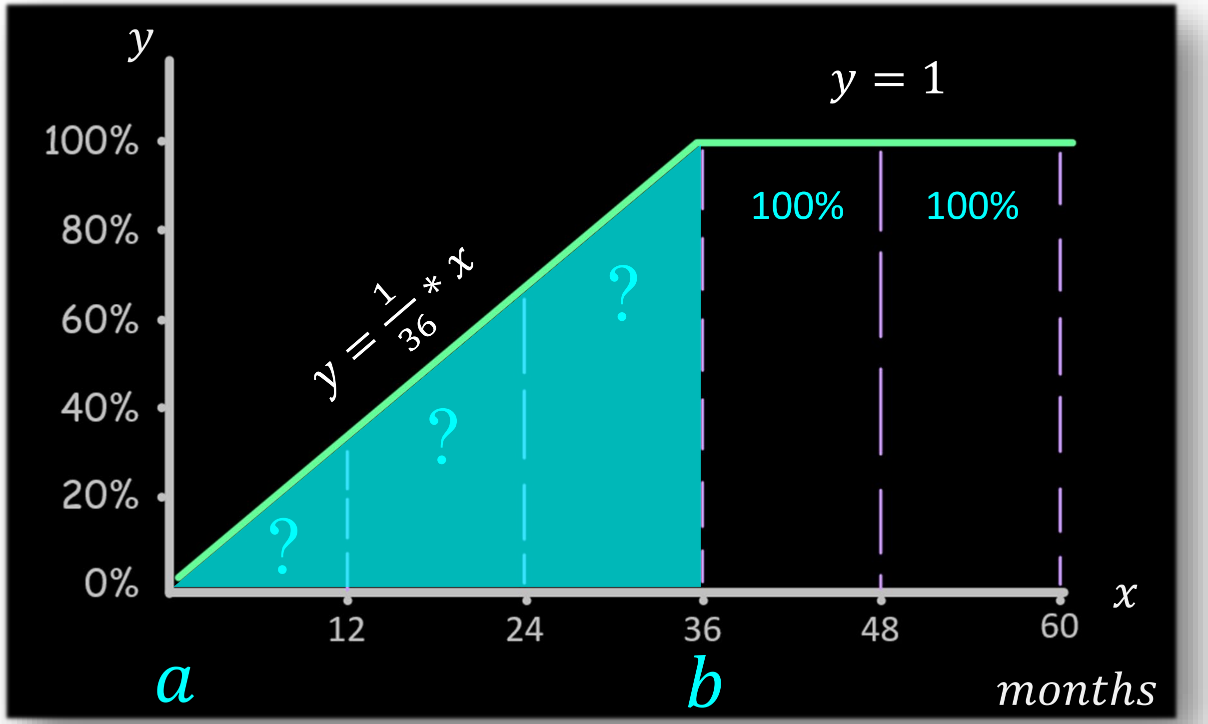
$$\frac{1}{36} * \int_{a}^{b} x^{1}\,dx = \frac{1}{36} * \left[\frac{1}{2} * b^{2} \right]_{a}$$



Area Under the Curve

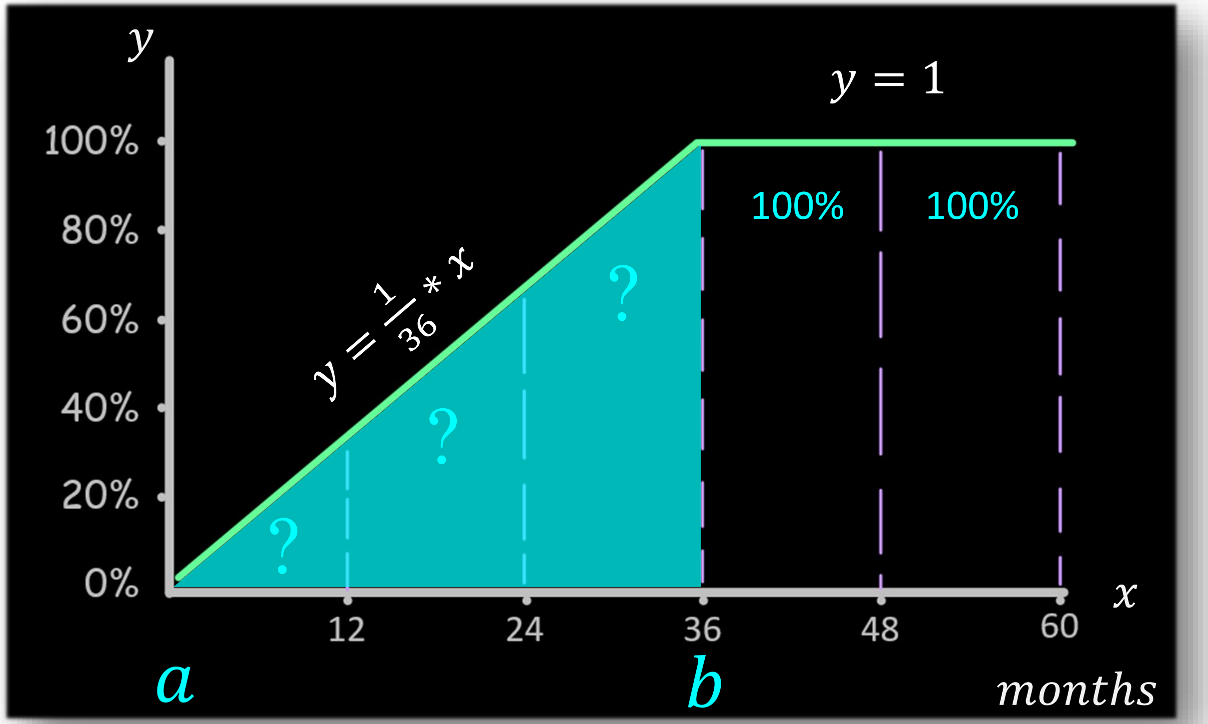
$$\frac{1}{36} * \int_a^b x^1 \, dx = \frac{1}{36} * \left[\frac{1}{2} * x^2 \right]_a$$

$$= \frac{1}{36} * \left[\frac{1}{2} * b^2 \right]$$



Area Under the Curve

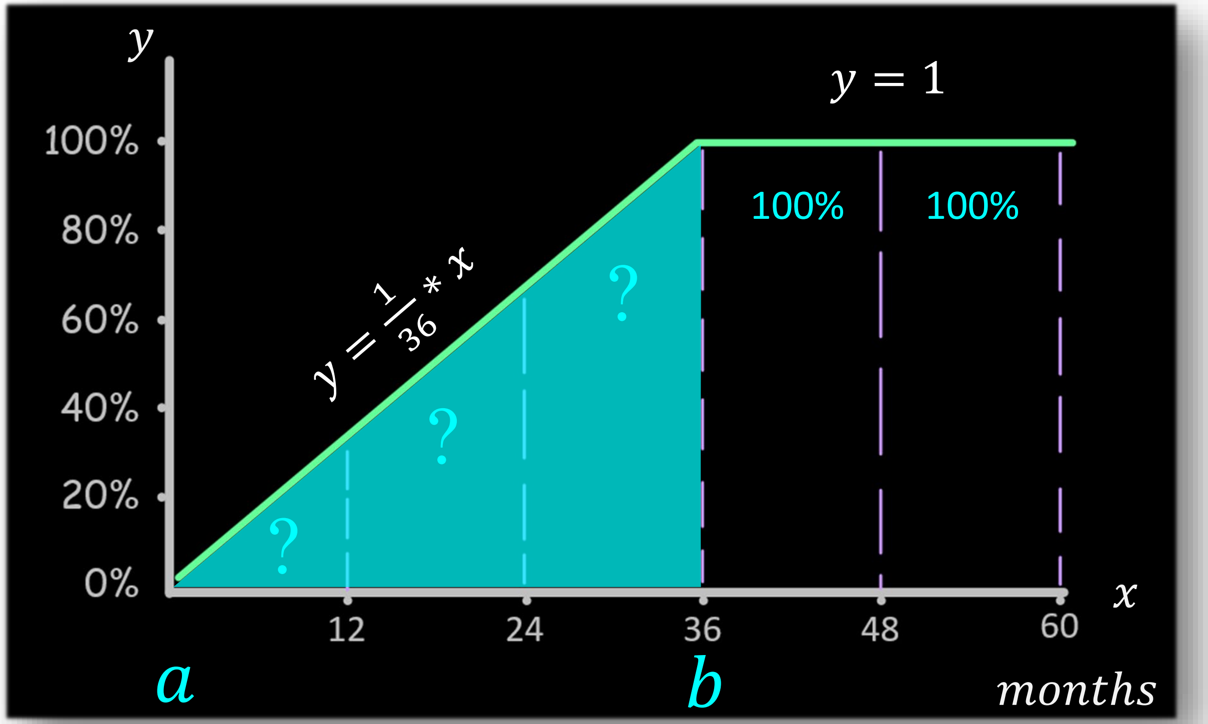
$$\frac{1}{36} * \int_a^b x^1 \, dx = \frac{1}{36} * \left[\frac{1}{2} * {\color{red}a}^2 \right]$$



$$= \frac{1}{36} * \left[\frac{1}{2} * b^2 \right]$$

Area Under the Curve

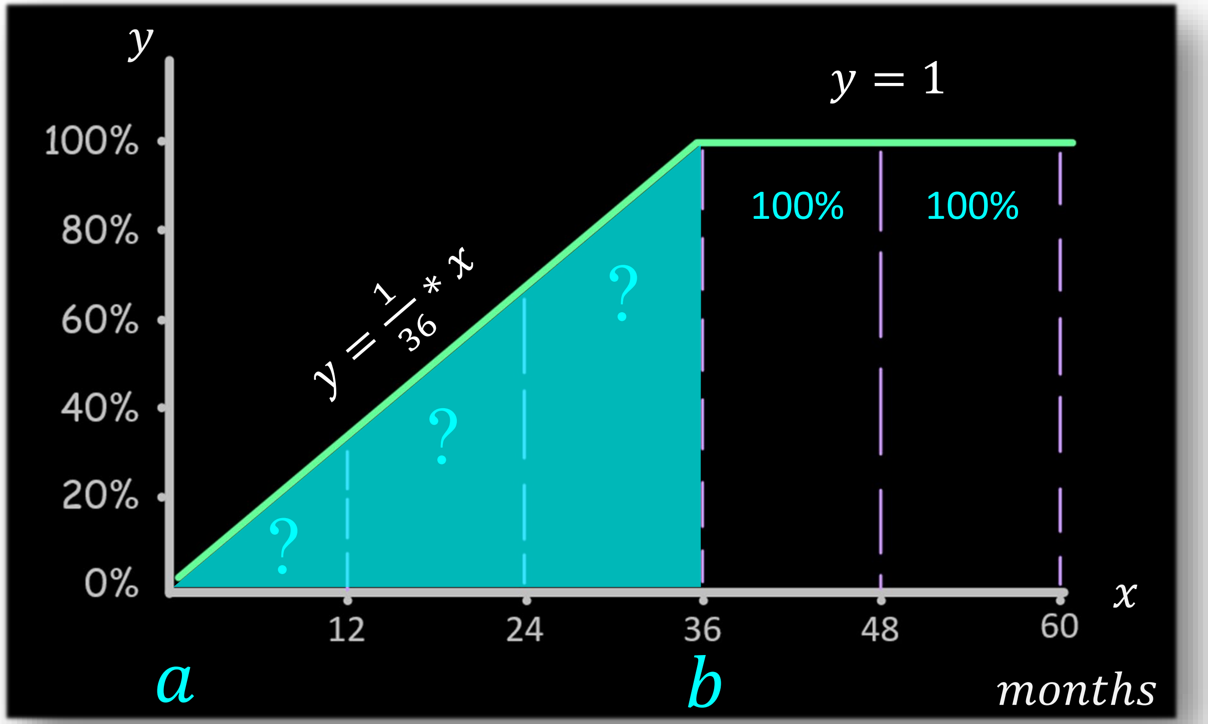
$$\frac{1}{36} * \int_a^b x^1 \, dx =$$



$$= \frac{1}{36} * \left[\frac{1}{2} * b^2 \right] - \frac{1}{36} * \left[\frac{1}{2} * a^2 \right]$$

Area Under the Curve

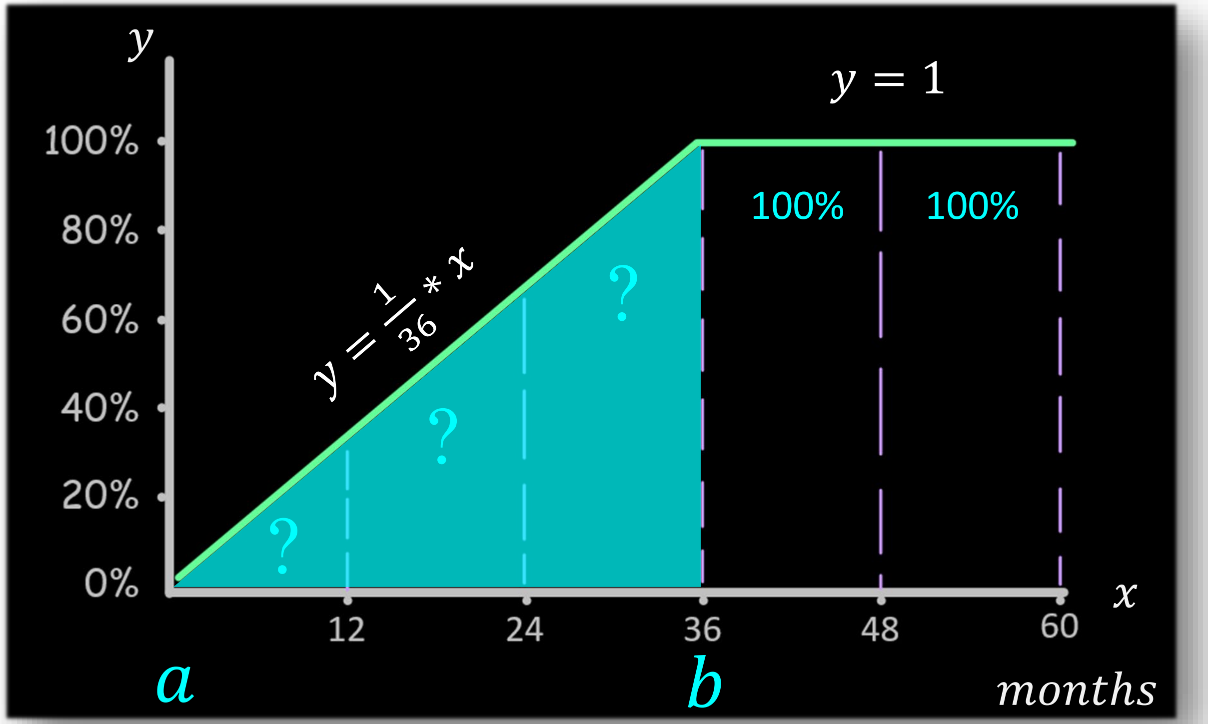
$$\frac{1}{36} * \int_a^b x^1 \, dx =$$



$$= \frac{1}{36} * \left[\frac{1}{2} * b^2 \right] - \frac{1}{36} * \left[\frac{1}{2} * a^2 \right]$$

Area Under the Curve

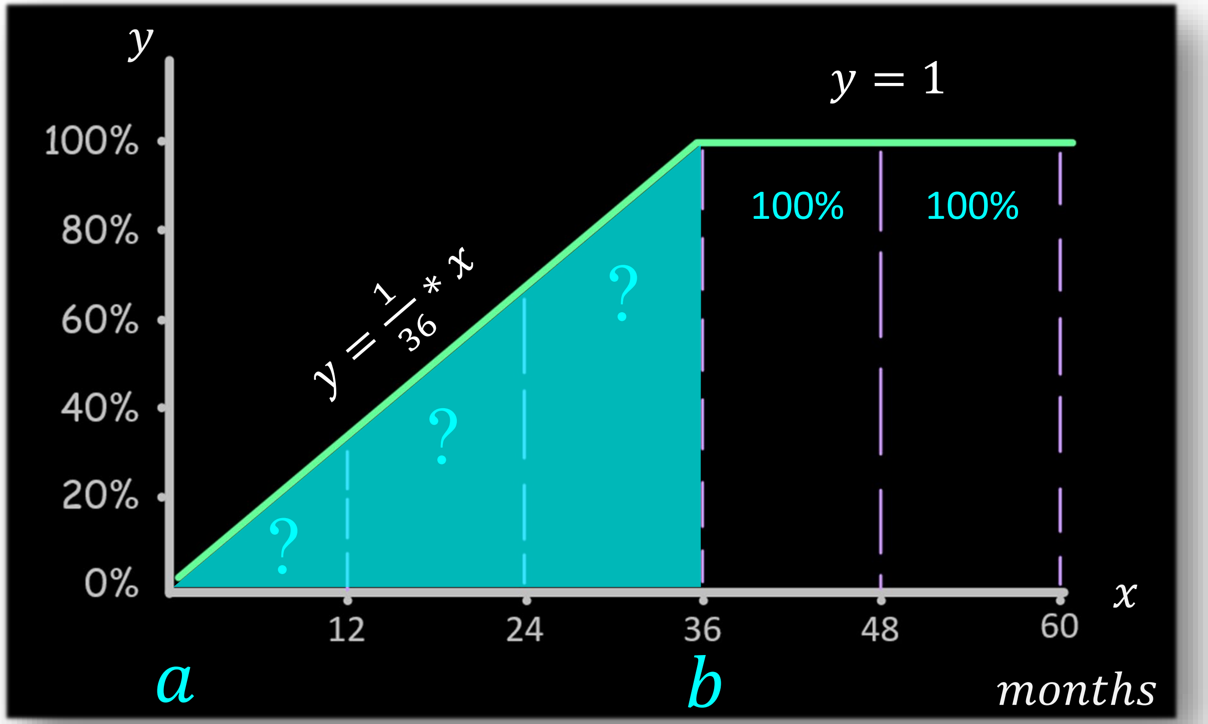
$$\frac{1}{36} * \int_a^b x^1 \, dx =$$



$$= \frac{1}{36} * \left[\left[\frac{1}{2} * b^2 \right] - \left[\frac{1}{2} * a^2 \right] \right]$$

Area Under the Curve

$$\frac{1}{36} * \int_a^b x^1\, dx =$$



$$= \frac{1}{36} * \frac{1}{2} * \left[\left[b^2 \right] - \left[a^2 \right] \right]$$

AFRL

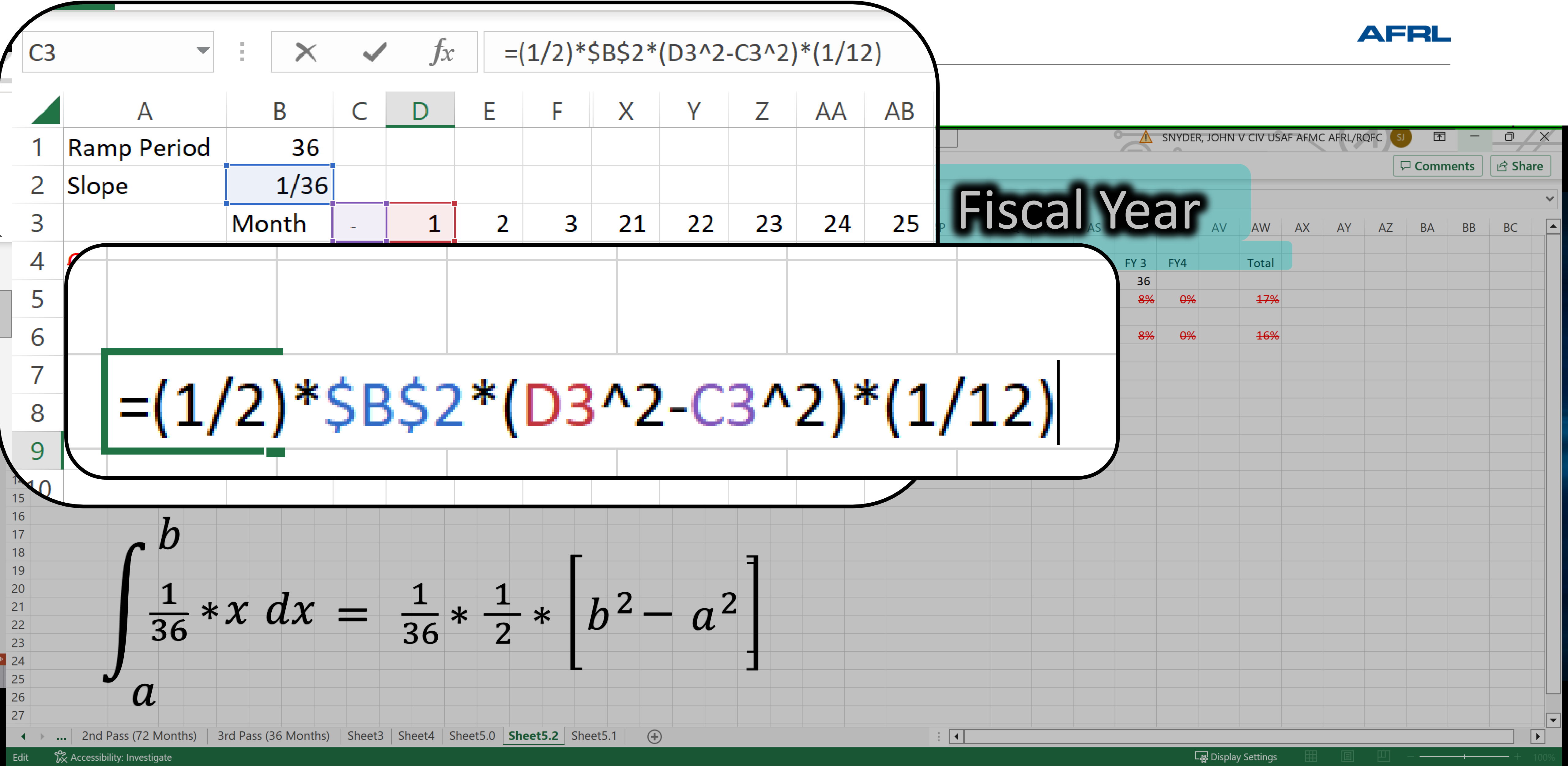
C3 =(1/2)*B2*(D3^2-C3^2)*(1/12)

	A	B	C	D	E	F	X	Y	Z	AA	AB
1	Ramp Period	36									
2	Slope	1/36									
3		Month	-	1	2	3	21	22	23	24	25
4	~~Quick Math~~	~~1.5417~~		~~0%~~	~~0%~~	~~1%~~	~~5%~~	~~5%~~	~~5%~~	~~6%~~	~~6%~~
5											
6	Midpoint	1.5000		0%	0%	1%	5%	5%	5%	5%	6%
7											
8											
9	Calculus			=(1/2)*B2*(D3^2-C3^2)*(1/12)							

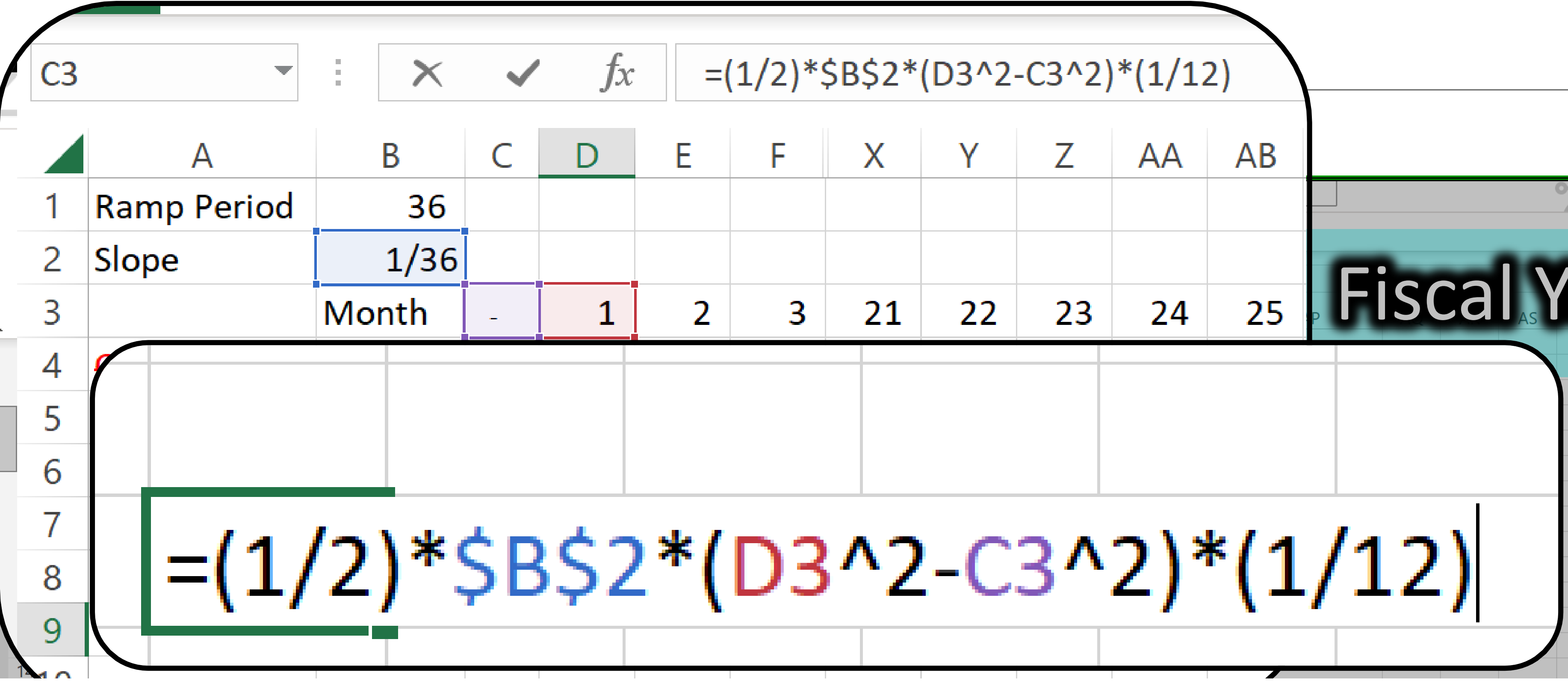
Fiscal Year

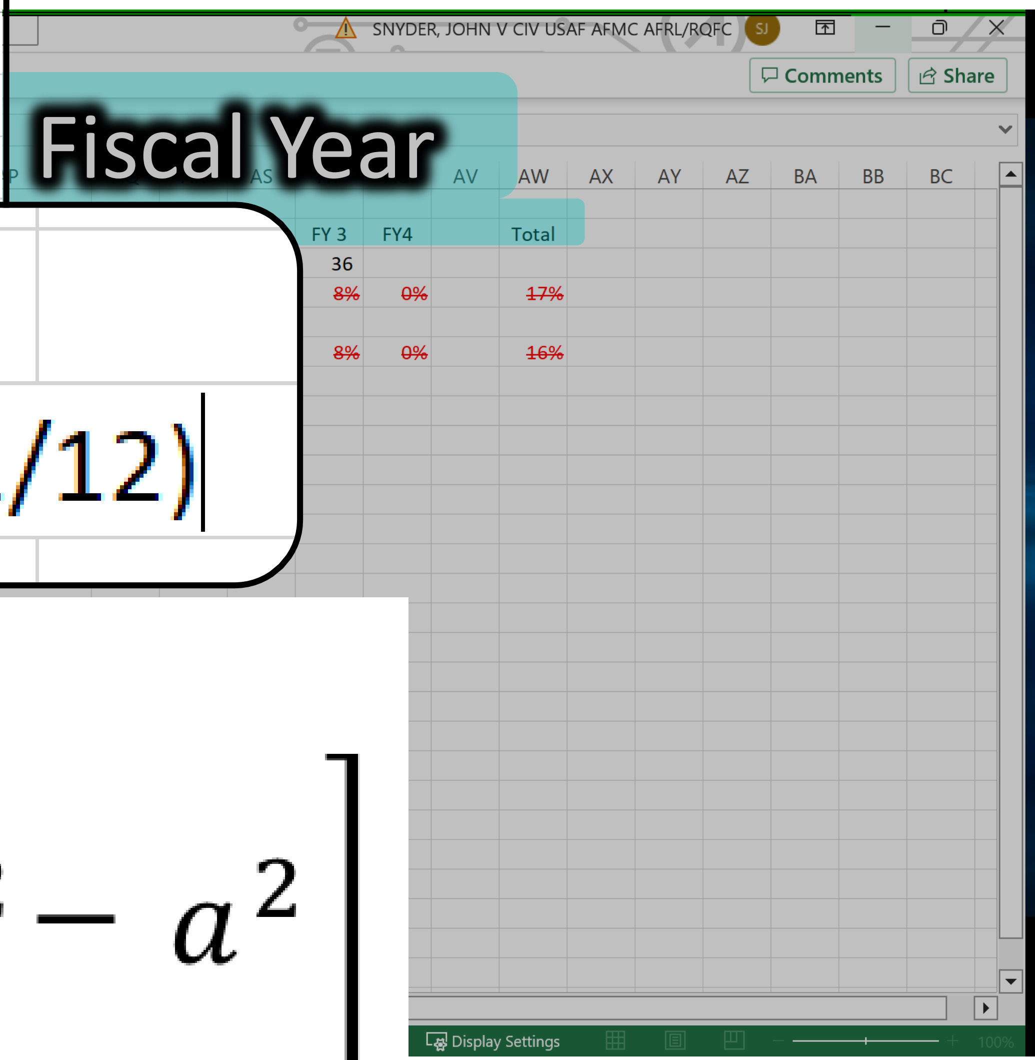
Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4	Total
FY Max Month	-	12	24	36		
~~Quick Math~~	~~0%~~	~~3%~~	~~6%~~	~~8%~~	~~0%~~	~~17%~~
~~Midpoint~~	~~0%~~	~~3%~~	~~5%~~	~~8%~~	~~0%~~	~~16%~~

$$\int_a^b \frac{1}{36} * x \, dx = \frac{1}{36} * \frac{1}{2} * \left[b^2 - a^2 \right]$$

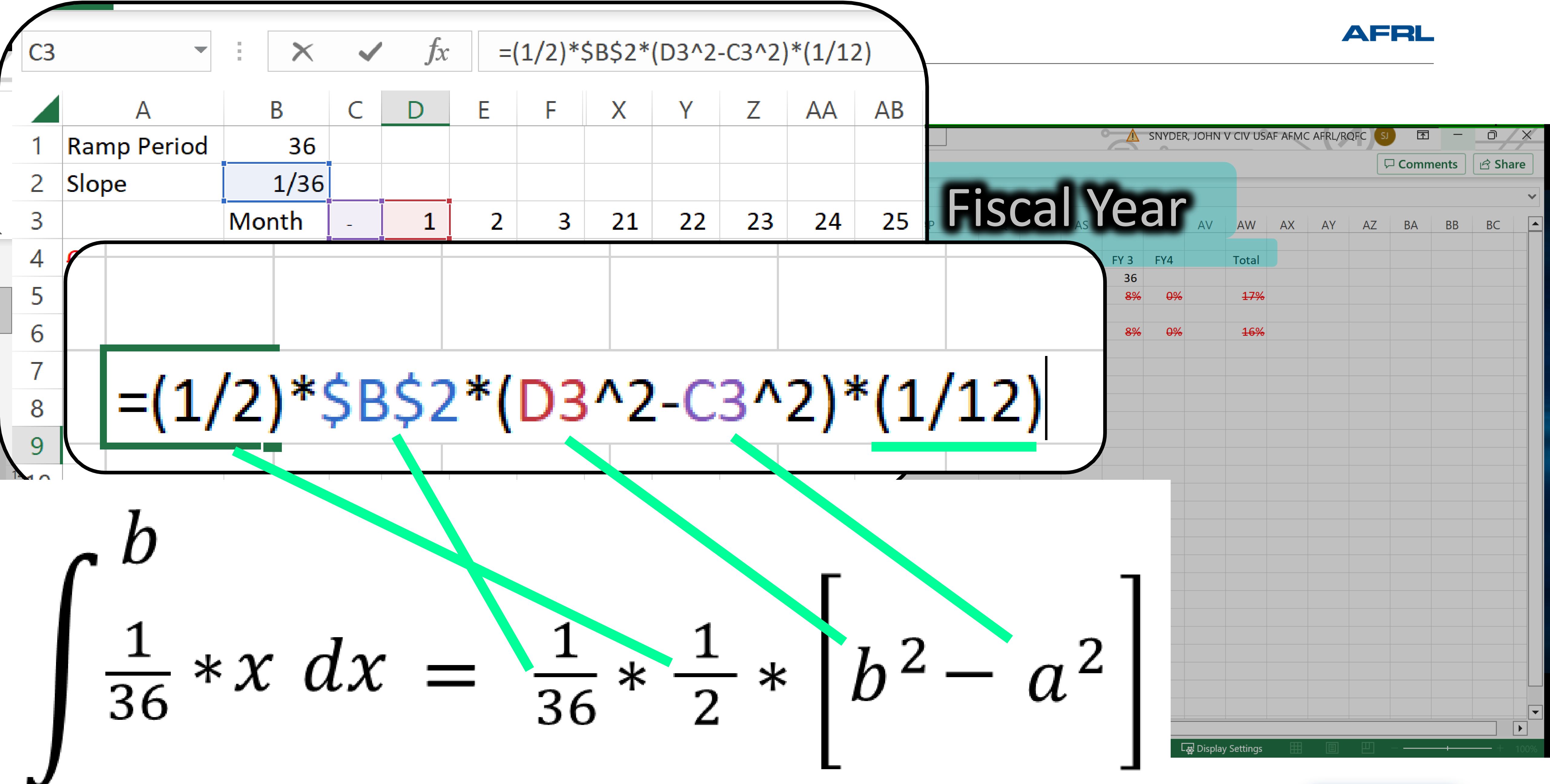


$$\int_a^b \frac{1}{36} * x \, dx = \frac{1}{36} * \frac{1}{2} * \left[b^2 - a^2 \right]$$





$$\int_a^b \frac{1}{36} * x \, dx = \frac{1}{36} * \frac{1}{2} * \left[b^2 - a^2 \right]$$



$$\int_a^b \frac{1}{36} * x \, dx = \frac{1}{36} * \frac{1}{2} * \left[b^2 - a^2 \right]$$

Next Slide

AFRL

B9 | =SUM(D9:AM9)

	A	B	C	D	E	F	X	Y	Z	AA	AB
1	Ramp Period	36									
2	Slope	1/36									
3		Month	-	1	2	3	21	22	23	24	25
4								~~%~~	~~5%~~	~~6%~~	~~6%~~
5											
6								%	5%	5%	6%
7											
8											
9	Calculus	1.5000		0%	0%	1%	5%	5%	5%	5%	6%
10											

=(1/2)*B2*(D3^2-C3^2)*(1/12)

Next Slide

AFRL

$$=(1/2)*\$B\$2*(AR3^2-AQ3^2)*(1/12)$$

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CUBEKPIME... | fx =(1/2)*B2*(AR3^2-AQ3^2)*(1/12)

Month

	A	B	C	D	E	F	X	Y						AE	AF	AG	AH	AI	AJ	AK	AL
1	Ramp Period	36																			
2	Slope	1/36																			
3		Month	-	1	2	3	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
4	~~Quick Math~~	~~1.5417~~		~~0%~~	~~0%~~	~~1%~~	~~5%~~	~~5%~~	~~5%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~7%~~	~~7%~~	~~7%~~	~~7%~~	~~8%~~	~~8%~~	~~8%~~ [illegible]
5																					
6	Midpoint	1.5000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8% [illegible]
7																					
8																					
9	Calculus	1.5000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8% [illegible]

$$\int_a^b \frac{1}{36} * x \, dx = \frac{1}{36} * \frac{1}{2} * \left[b^2 - a^2 \right]$$

AP	AQ	AR	AS	AT	AU	AV	AW
Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4		Total
FY Max Month	-	12	24	36			
~~Quick Math~~	~~0%~~	~~3%~~	~~6%~~	~~8%~~	~~0%~~		~~17%~~
~~Midpoint~~	~~0%~~	~~3%~~	~~5%~~	~~8%~~	~~0%~~		~~16%~~
Calculus		=(1/2)*B2*(AR3^2-AQ3^2)*(1/12)					

2nd Pass (72 Months) | 3rd Pass (36 Months) | Sheet3 | Sheet4 | Sheet5.0 | **Sheet5.2** | Sheet5.1

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=(1/2)*B2*(AR3^2-AQ3^2)*(1/12)

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AW9 | =sum(AQ9:AU9)

Month

	A	B	C	D	E	F	X	Y						AE	AF	AG	AH	AI	AJ	AK	AL
1	Ramp Period	36																			
2	Slope	1/36																			
3		Month	-	1	2	3	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
4	~~Quick Math~~	~~1.5417~~		~~0%~~	~~0%~~	~~1%~~	~~5%~~	~~5%~~	~~5%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~7%~~	~~7%~~	~~7%~~	~~7%~~	~~8%~~	~~8%~~	~~8%~~
5																					
6	Midpoint	1.5000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%
7																					
8																					
9	Calculus	1.5000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%

AP	AQ	AR	AS	AT	AU	AV	AW
Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4		Total
FY Max Month	-	12	24	36			
~~Quick Math~~	~~0%~~	~~3%~~	~~6%~~	~~8%~~	~~0%~~		~~17%~~
~~Midpoint~~	~~0%~~	~~3%~~	~~5%~~	~~8%~~	~~0%~~		~~16%~~
Calculus		17%	50%	83%			=sum(AQ9:A

$$\int_a^b \frac{1}{36} * x \, dx = \frac{1}{36} * \frac{1}{2} * \left[b^2 - a^2 \right]$$

2nd Pass (72 Months) | 3rd Pass (36 Months) | Sheet3 | Sheet4 | Sheet5.0 | Sheet5.2 | Sheet5.1

Enter | Accessibility: Investigate | Display Settings

=(1/2)*B2*(AR3^2-AQ3^2)*(1/12)

Ramp Function Math

AW9 =SUM(AQ9:AU9)

Month

	A	B	C	D	E	F	X	Y					AE	AF	AG	AH	AI	AJ	AK	AL
1	Ramp Period	36																		
2	Slope	1/36																		
3		Month	-	1	2	3	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
4	~~Quick Math~~	~~1.5417~~		~~0%~~	~~0%~~	~~1%~~	~~5%~~	~~5%~~	~~5%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~7%~~	~~7%~~	~~7%~~	~~7%~~	~~8%~~	~~8%~~	~~8%~~
5																				
6	Midpoint	1.5000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%
7																				
8																				
9	Calculus	1.5000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%

AP	AQ	AR	AS	AT	AU	AV	AW
Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4		Total
FY Max Month	-	12	24	36			
~~Quick Math~~	~~0%~~	~~3%~~	~~6%~~	~~8%~~	~~0%~~		~~17%~~
~~Midpoint~~	~~0%~~	~~3%~~	~~5%~~	~~8%~~	~~0%~~		~~16%~~
Calculus		17%	50%	83%			150%

$$\int_a^b \frac{1}{36} * x \, dx = \frac{1}{36} * \frac{1}{2} * \left[b^2 - a^2 \right]$$

2nd Pass (72 Months) | 3rd Pass (36 Months) | Sheet3 | Sheet4 | Sheet5.0 | **Sheet5.2** | Sheet5.1

Next Slide

AFRL

=(1/2)*B2*(AR3^2-AQ3^2)*(1/12)

AP	AQ	AR	AS	AT	AU	AV	AW
Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4		Total
FY Max Month	-	12	24	36			
~~Quick Math~~	~~0%~~	~~3%~~	~~6%~~	~~8%~~	~~0%~~		~~0.1667~~
~~Midpoint~~	~~0%~~	~~3%~~	~~5%~~	~~8%~~	~~0%~~		0.1609
Calculus		17%	50%	83%			1.5000

Next Slide

Ramp Function Math

AW9 =SUM(AQ9:AU9)

Month

Fiscal Year

	A	B	C	D	E	F	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM
1	Ramp Period	36																				
2	Slope	1/36																				
3		Month	-	1	2	3	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
4	~~Quick Math~~	~~1.5417~~		~~0%~~	~~0%~~	~~1%~~	~~5%~~	~~5%~~	~~5%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~7%~~	~~7%~~	~~7%~~	~~7%~~	~~8%~~	~~8%~~	~~8%~~	~~8%~~
6	Midpoint	1.5000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%
9	Calculus	1.5000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%

Annualized Table	FY 0	FY 1	FY 2	FY 3	FY4	Total
FY Max Month	-	12	24	36		
~~Quick Math~~	~~0%~~	~~3%~~	~~6%~~	~~8%~~	~~0%~~	~~0.1667~~
~~Midpoint~~	~~0%~~	~~3%~~	~~5%~~	~~8%~~	~~0%~~	0.1609
Calculus		17%	50%	83%		1.5000

$$\int_a^b \frac{1}{36} * x \; dx = \frac{1}{36} * \frac{1}{2} * \left[b^2 - a^2 \right]$$

Next Slide

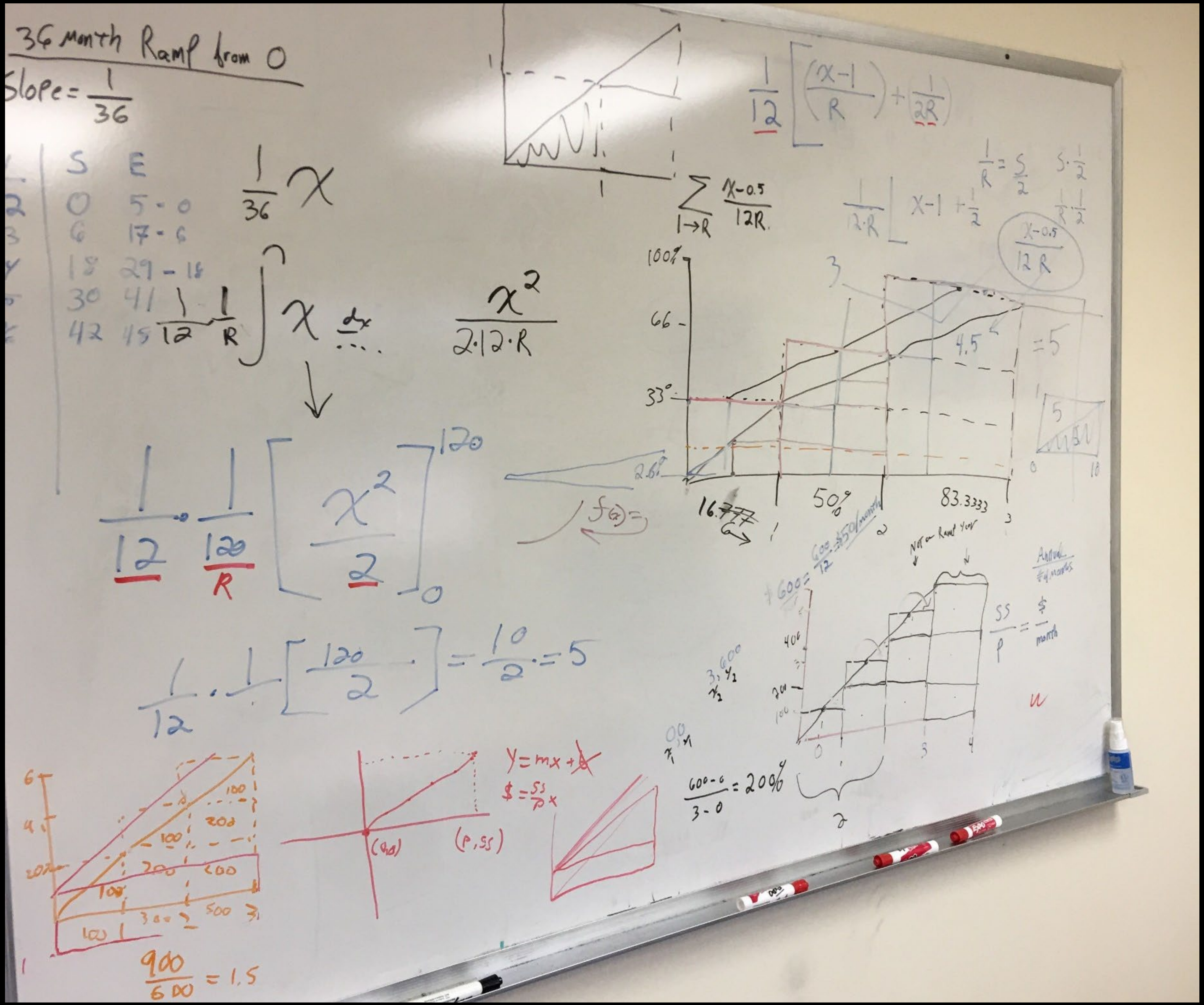
Curiosity

Next Slide

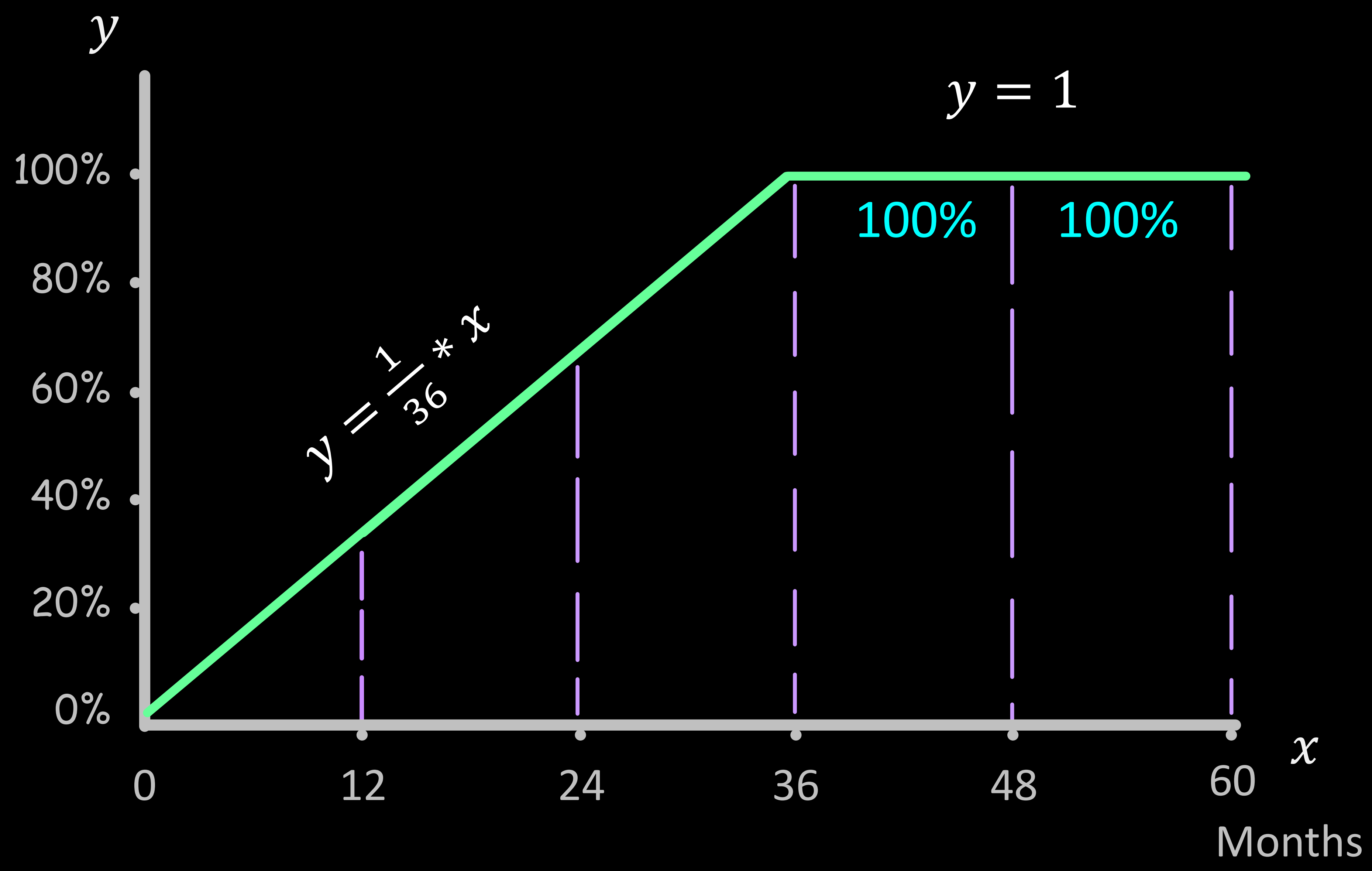
$f_x =$

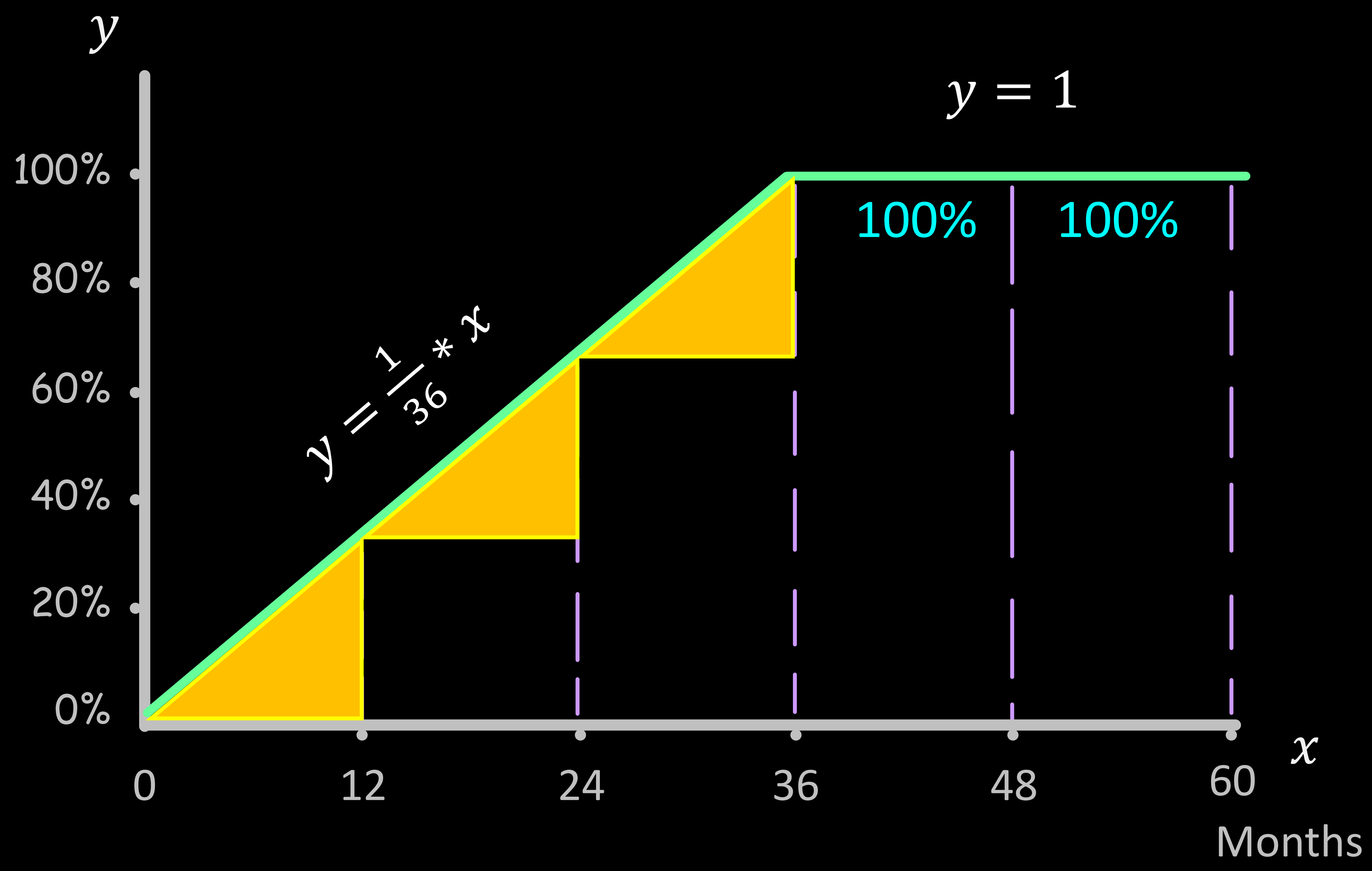
$$f_x = \left[\left(\left(S * C\right) - \left(S * P\right)\right) * \left(\frac{C - P}{12}\right)\right] * \frac{1}{2} + \left[\left(S * P\right) * \left(\frac{C - P}{12}\right)\right]$$

Next Slide

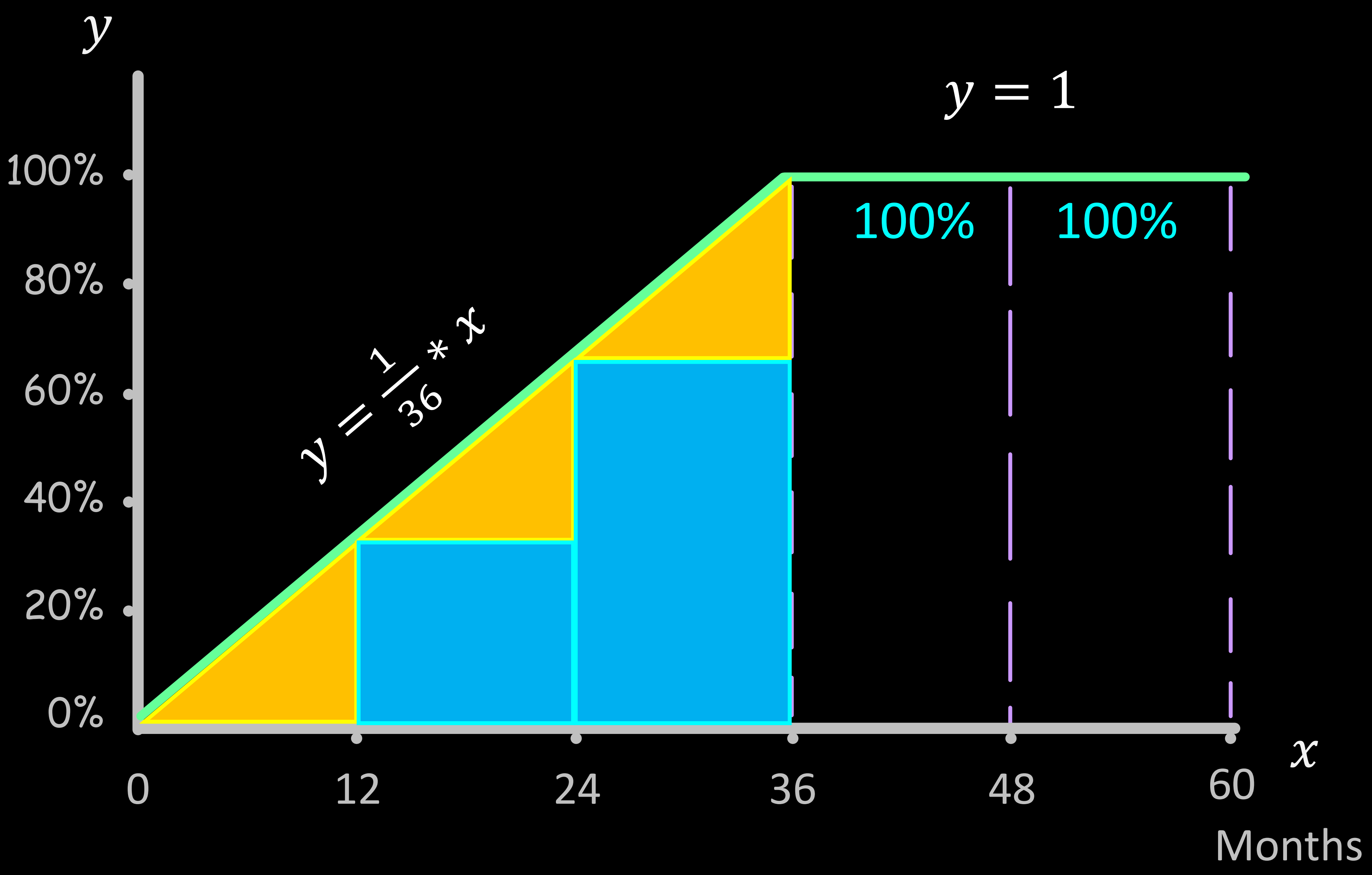

36 Month Ramp from 0
Slope = 1/36
S E
100%
66
33
50%
83.3333
y = mx + b
900/600 = 1.5
600-0/3-0 = 200%


Next Slide

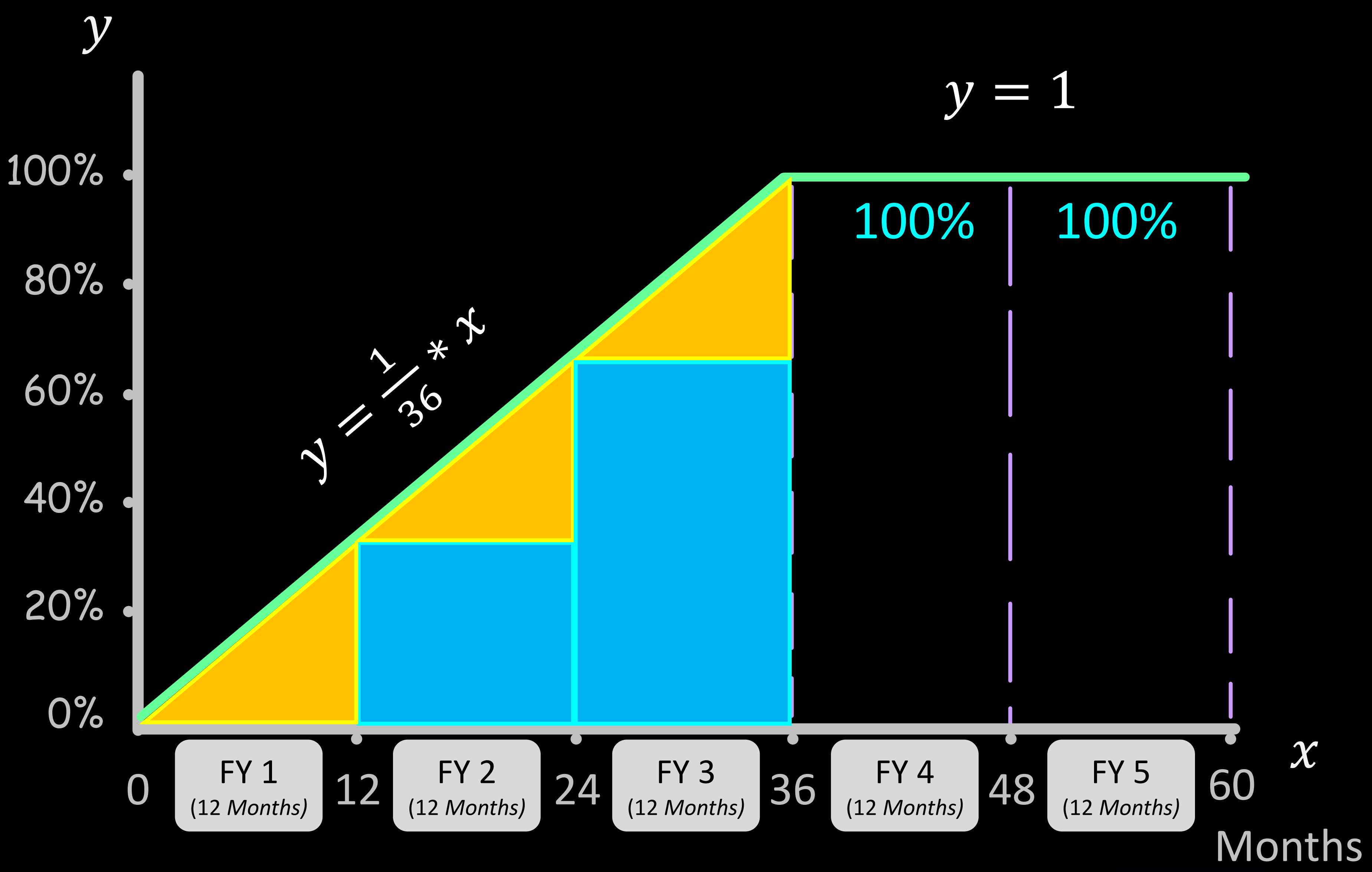

y
y = 1
100%
80%
60%
40%
20%
0%
100%
100%
y = 1/36 * x
0
12
24
36
48
60
x
Months



y
y = 1
100%
80%
60%
40%
20%
0%
y = 1/36 * x
100%
100%
0
12
24
36
48
60
x
Months



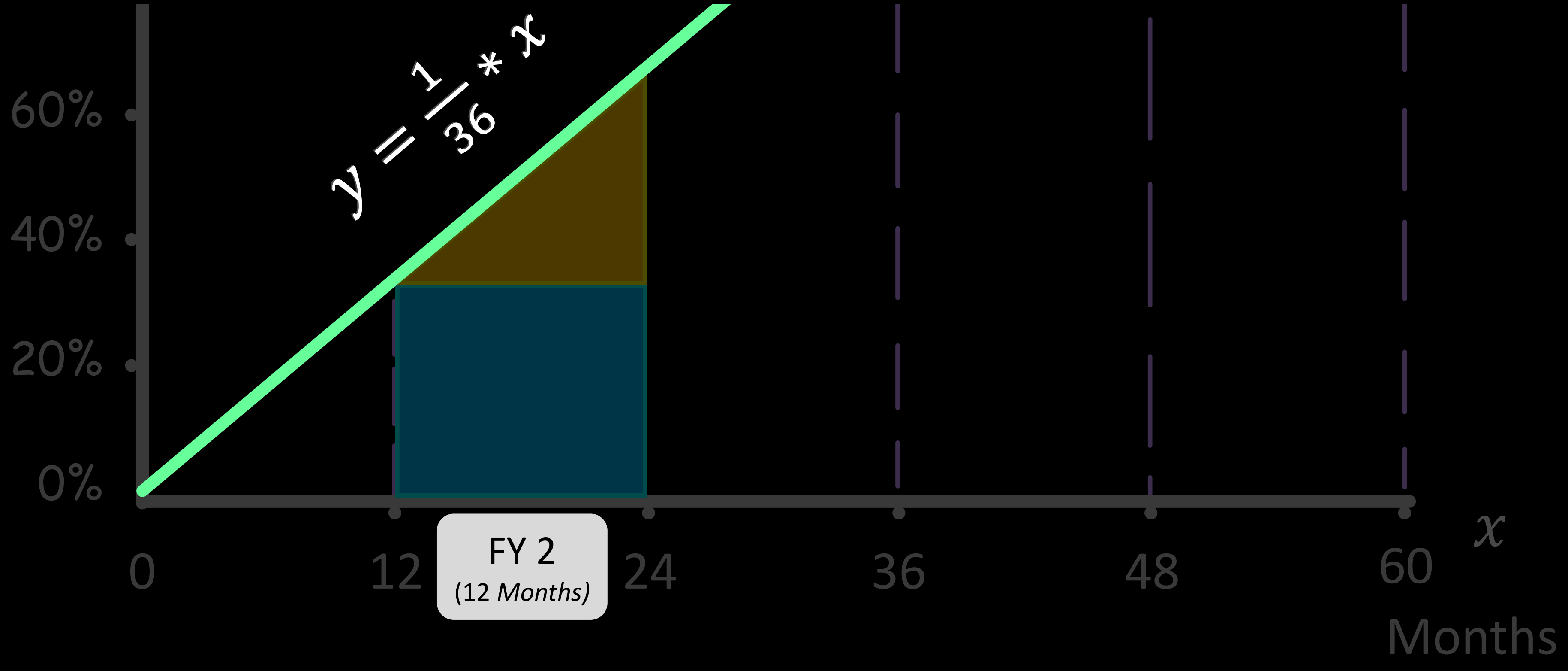
y
y = 1
100%
80%
60%
40%
20%
0%
$y = \frac{1}{36} * x$
100%
100%
0
12
24
36
48
60
x
Months



y
y = 1
100%
80%
60%
40%
20%
0%
y = 1/36 * x
100%
100%
0
FY 1 (12 Months)
12
FY 2 (12 Months)
24
FY 3 (12 Months)
36
FY 4 (12 Months)
48
FY 5 (12 Months)
60
x
Months

Next Slide

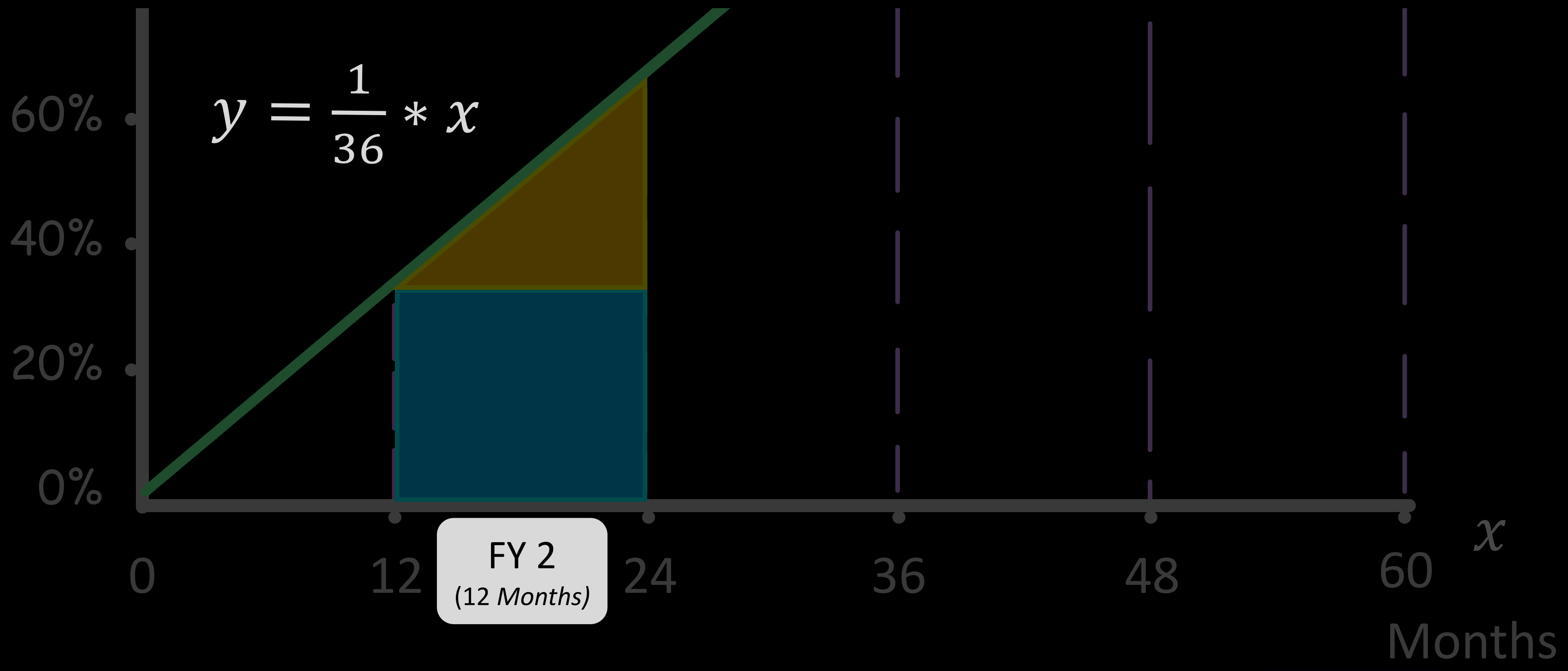
$f_x =$ Area of Triangle $+$ Area of Rectangle



$$f_x = \quad \underbrace{\qquad}_{\text{Area of Triangle}} + \underbrace{\qquad}_{\text{Area of Rectangle}}$$

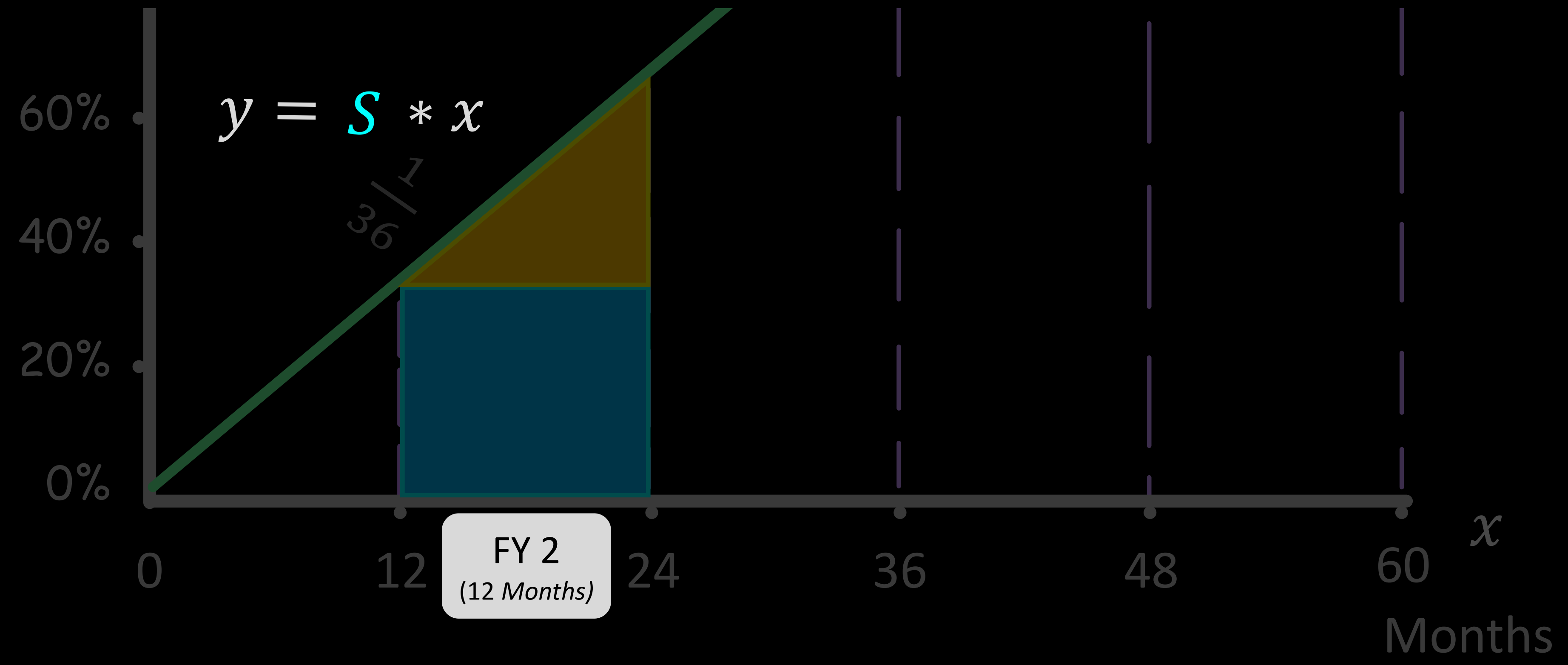
Area of Triangle

Area of Rectangle



Next Slide

$f_x =$ Area of Triangle $+$ Area of Rectangle

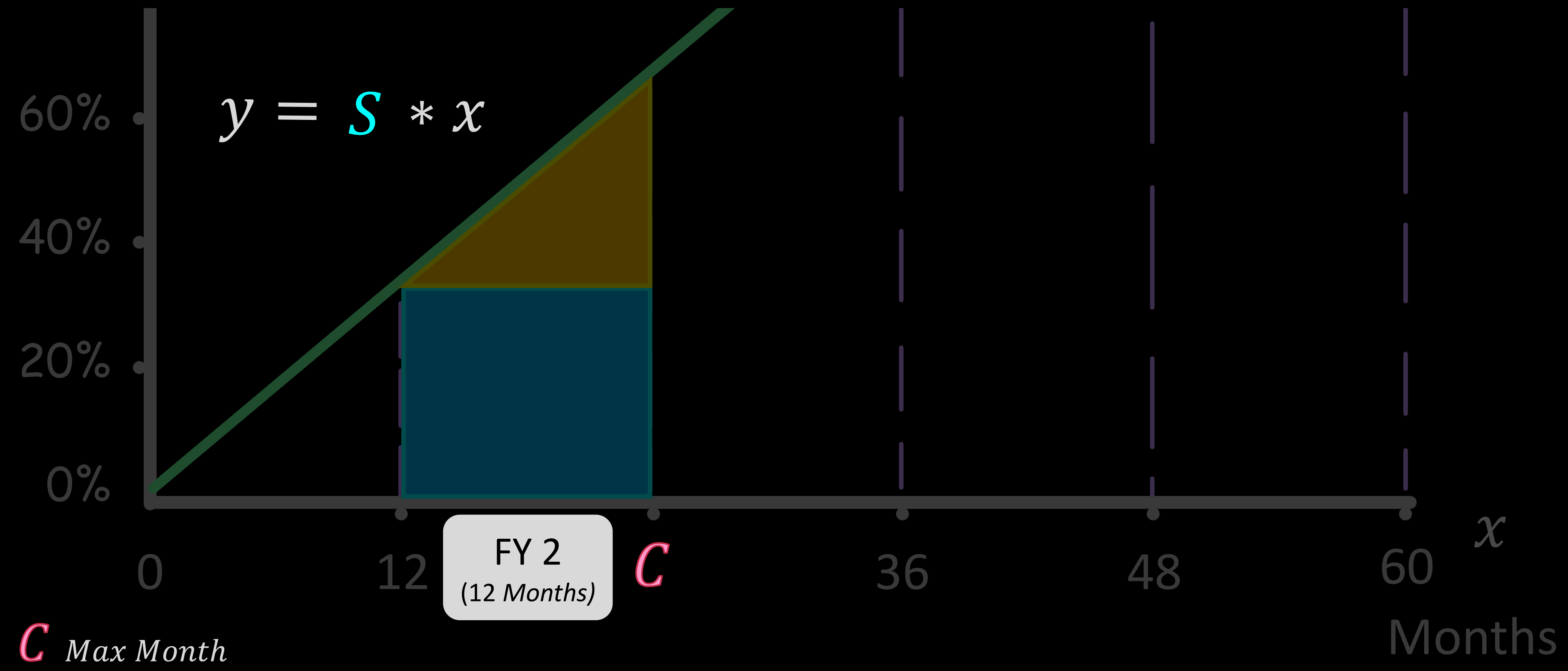


$Slope = S$

$f_x =$ [Area of Triangle] $+$ [Area of Rectangle]

Area of Triangle

Area of Rectangle



$Current\ Fiscal\ Year = C_{\ Max\ Month}$

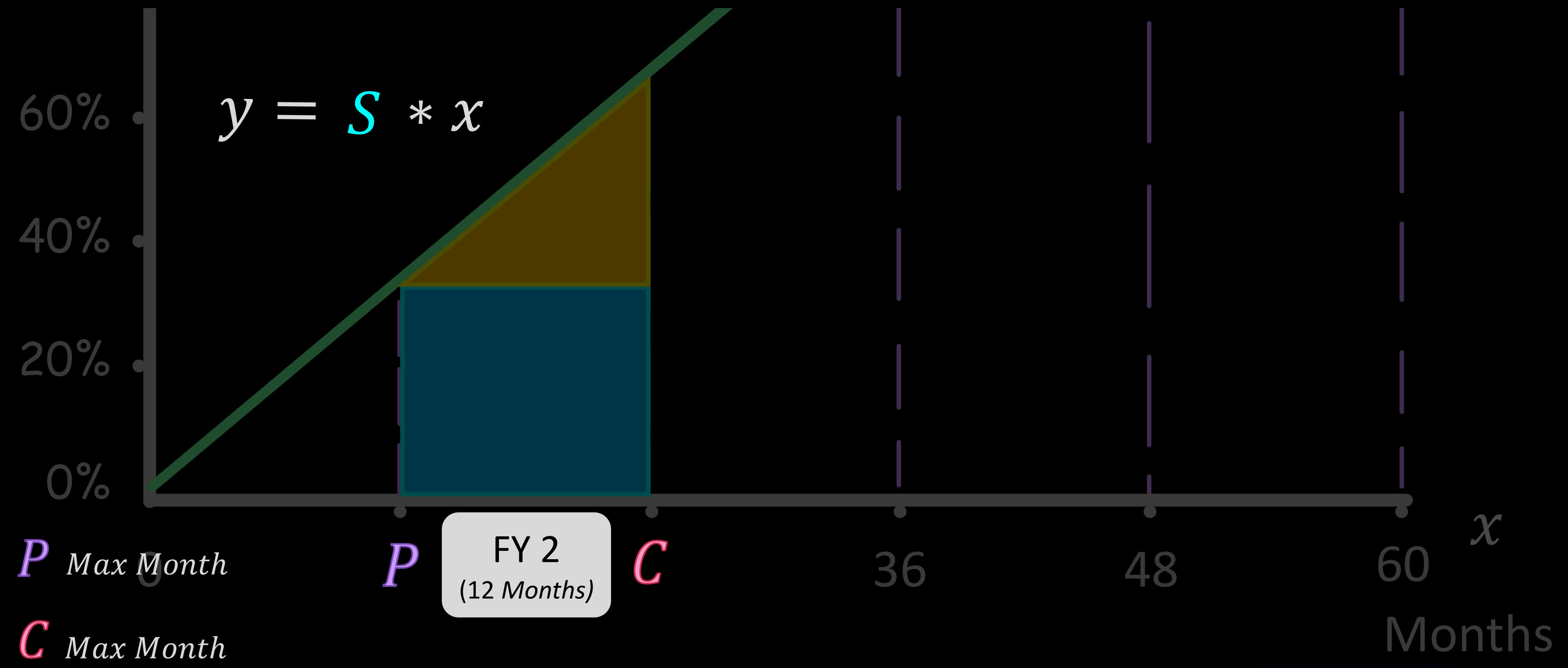
$Slope = S$

$f_x =$

Area of Triangle

$+$

Area of Rectangle



$Previous\ Fiscal\ Year = P_{\ Max\ Month}$

$Current\ Fiscal\ Year = C_{\ Max\ Month}$

$Slope = S$

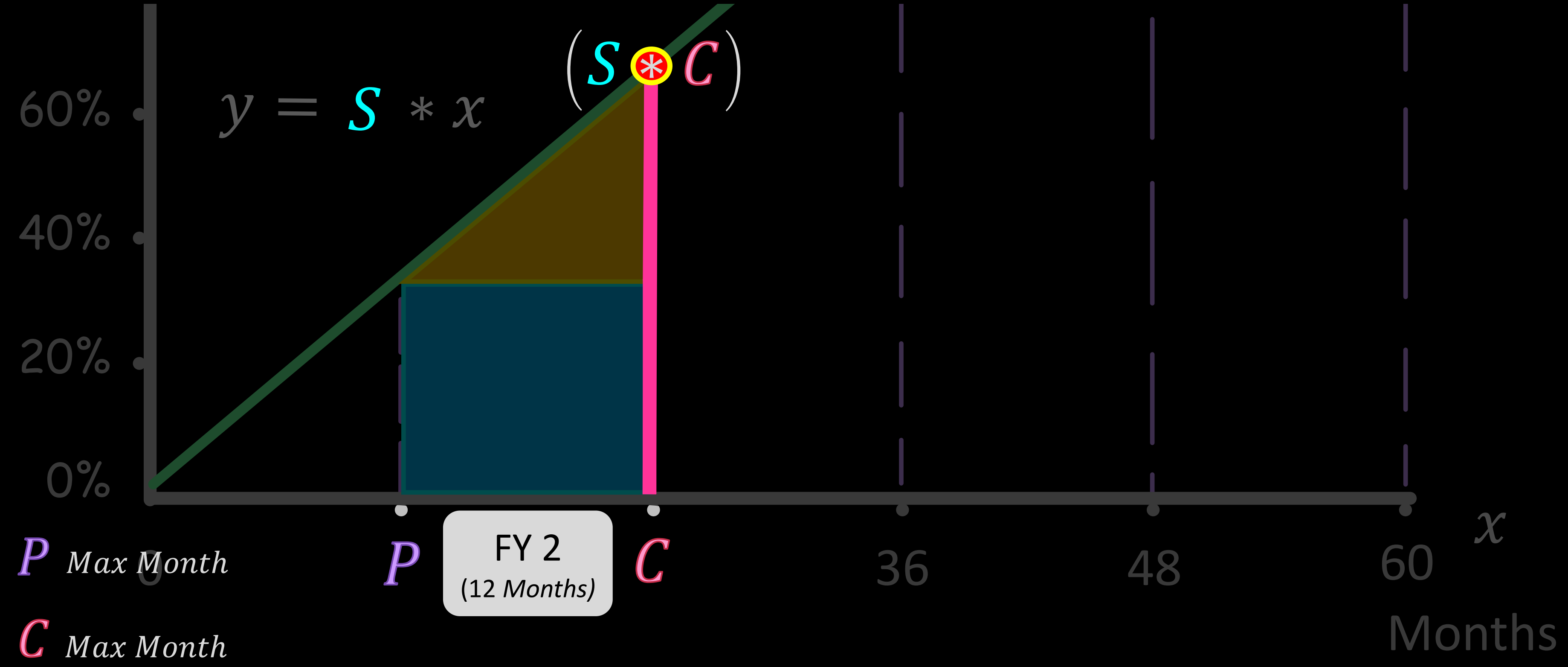
Next Slide

$f_x =$ Area of Triangle + Area of Rectangle

Height

Area of Triangle

Area of Rectangle



Previous Fiscal Year $= P$ *Max Month*

Current Fiscal Year $= C$ *Max Month*

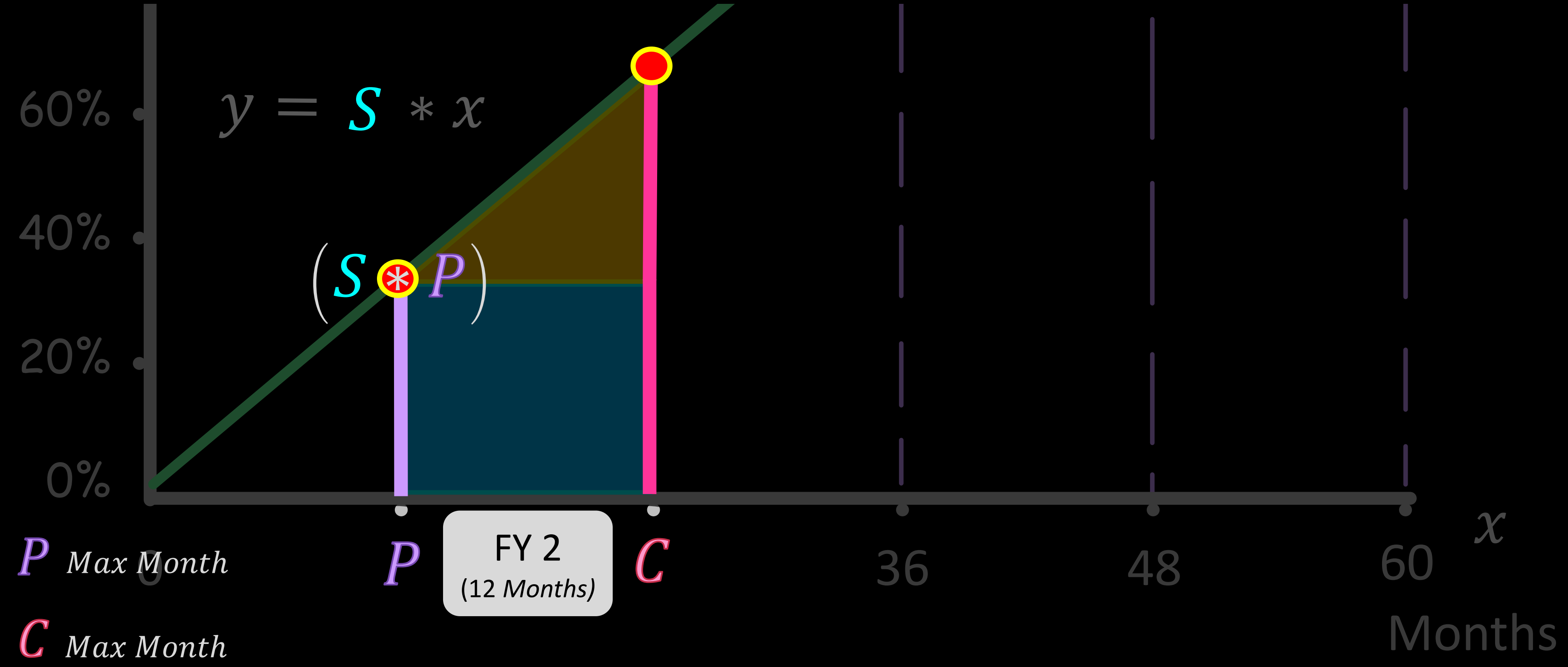
Slope $= S$

$$f_x = \left(S * C \right) + $$

Height

Area of Triangle

Area of Rectangle



$Previous\ Fiscal\ Year = P_{Max\ Month}$

$Current\ Fiscal\ Year = C_{Max\ Month}$

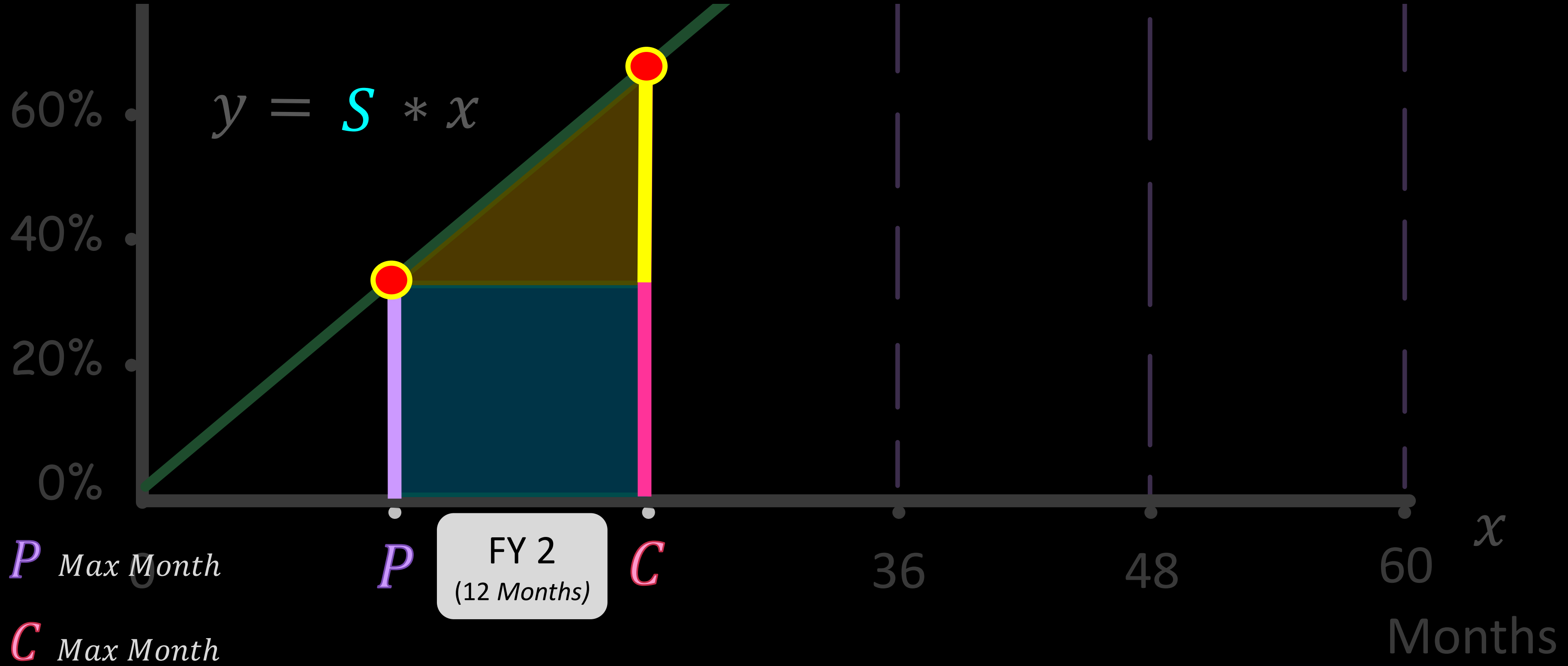
$Slope = S$

$$f_x = (S * C) - (S * P) + $$

Height

Area of Triangle

Area of Rectangle



Previous Fiscal Year = P *Max Month*

Current Fiscal Year = C *Max Month*

Slope = S

Next Slide

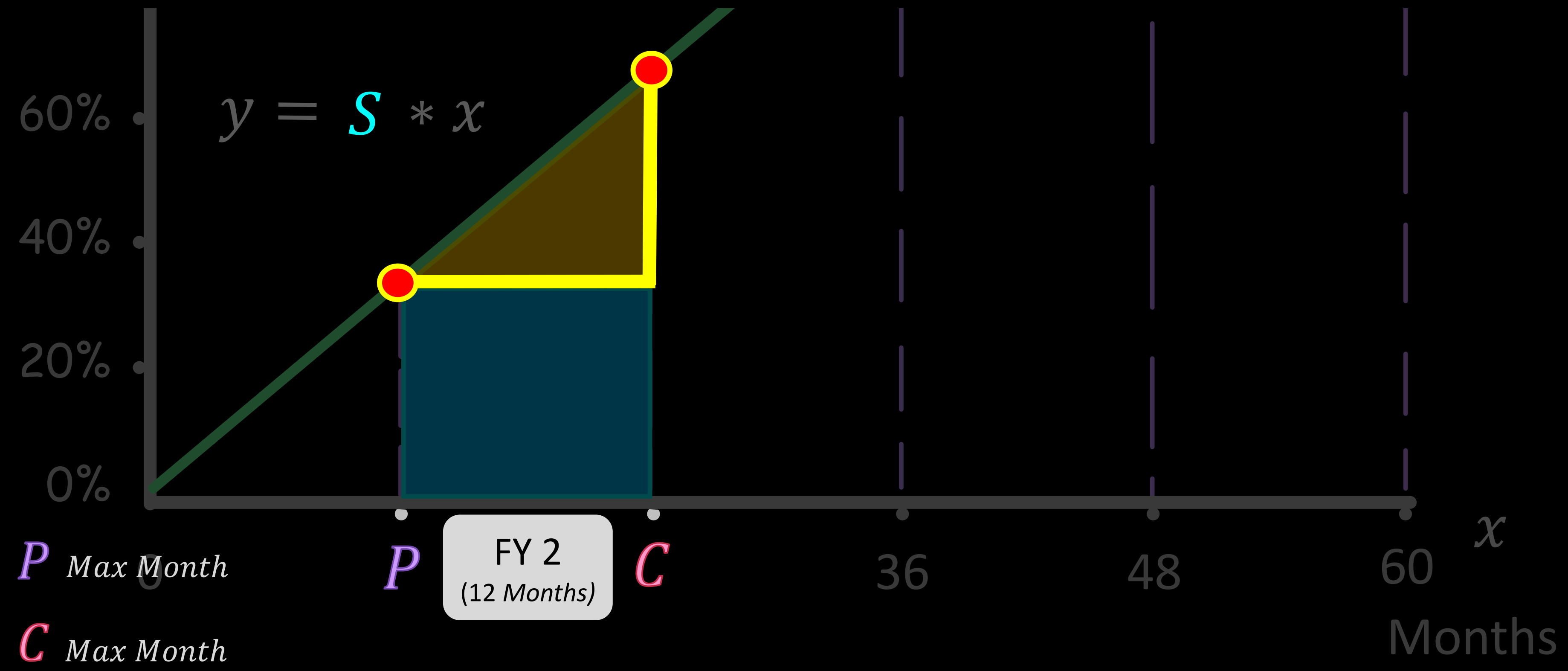
$$f_x = \Big(\big(S * C\big) - \big(S * P\big)\Big) * \big(\qquad\big) \; + $$

Height

Base

Area of Triangle

Area of Rectangle



$Previous\ Fiscal\ Year = P_{\ Max\ Month}$

$Current\ Fiscal\ Year = C_{\ Max\ Month}$

$Slope = S$

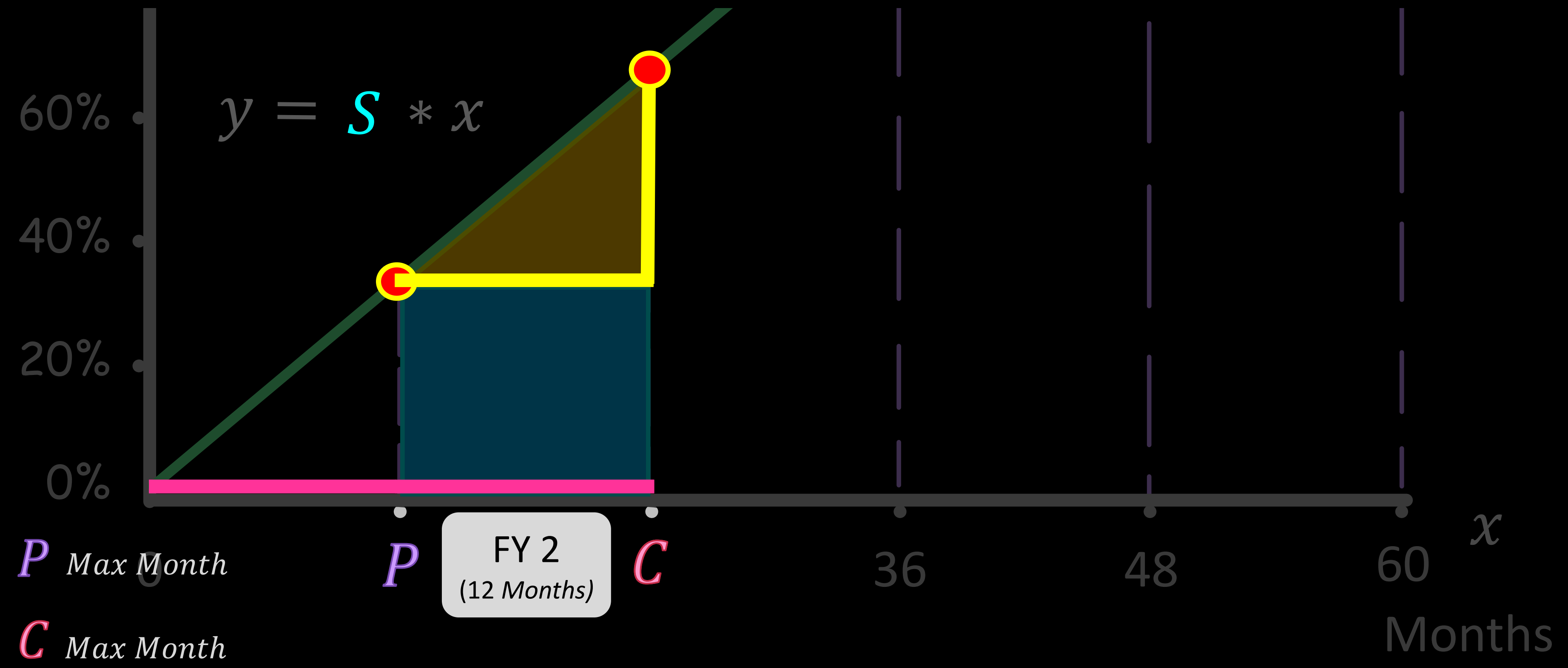
$$f_x = \big((S * C) - (S * P)\big) * (\quad) + \boxed{\quad}$$

Height

Base

Area of Triangle

Area of Rectangle



$Previous\ Fiscal\ Year = P_{\ Max\ Month}$

$Current\ Fiscal\ Year = C_{\ Max\ Month}$

$Slope = S$

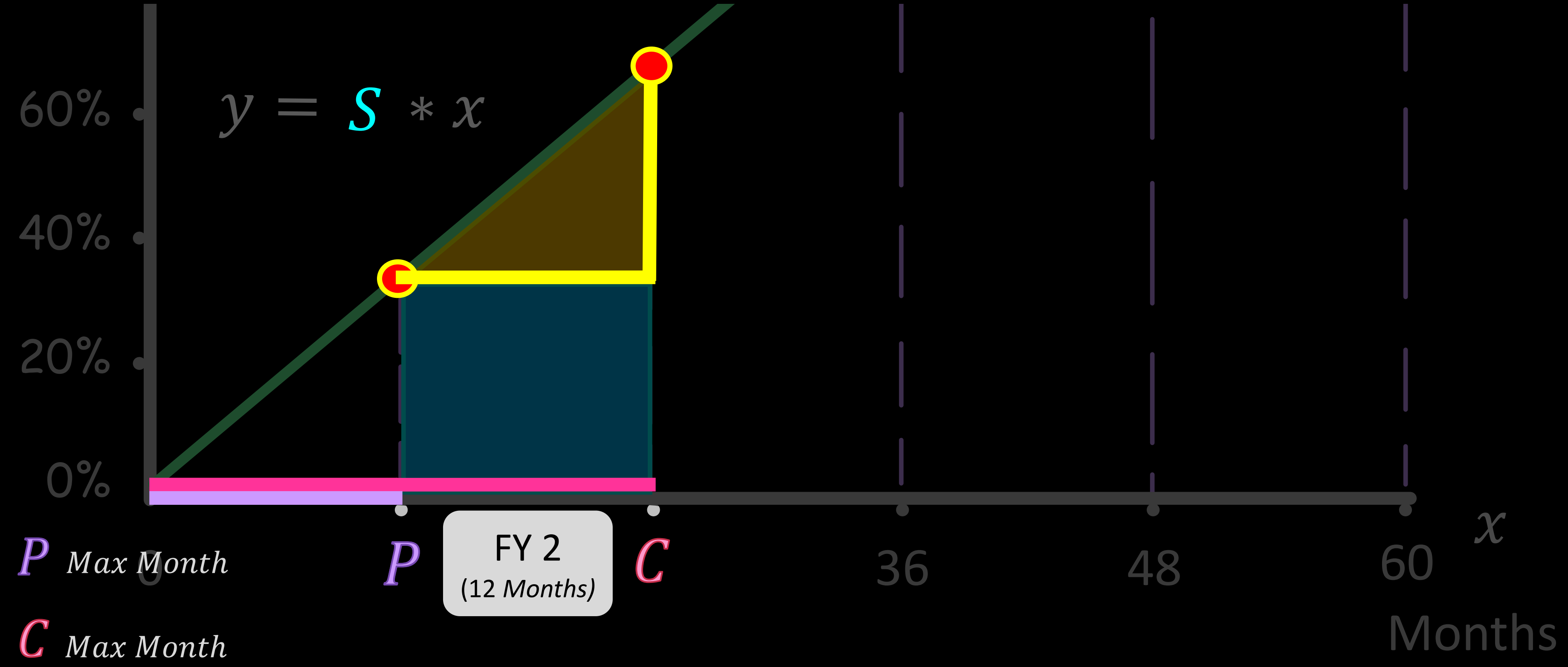
$$f_x = \big((S * C) - (S * P)\big) * (C \quad)$$

Height — Base

Area of Triangle

$+$

Area of Rectangle



$Previous\ Fiscal\ Year = P_{\ Max\ Month}$

$Current\ Fiscal\ Year = C_{\ Max\ Month}$

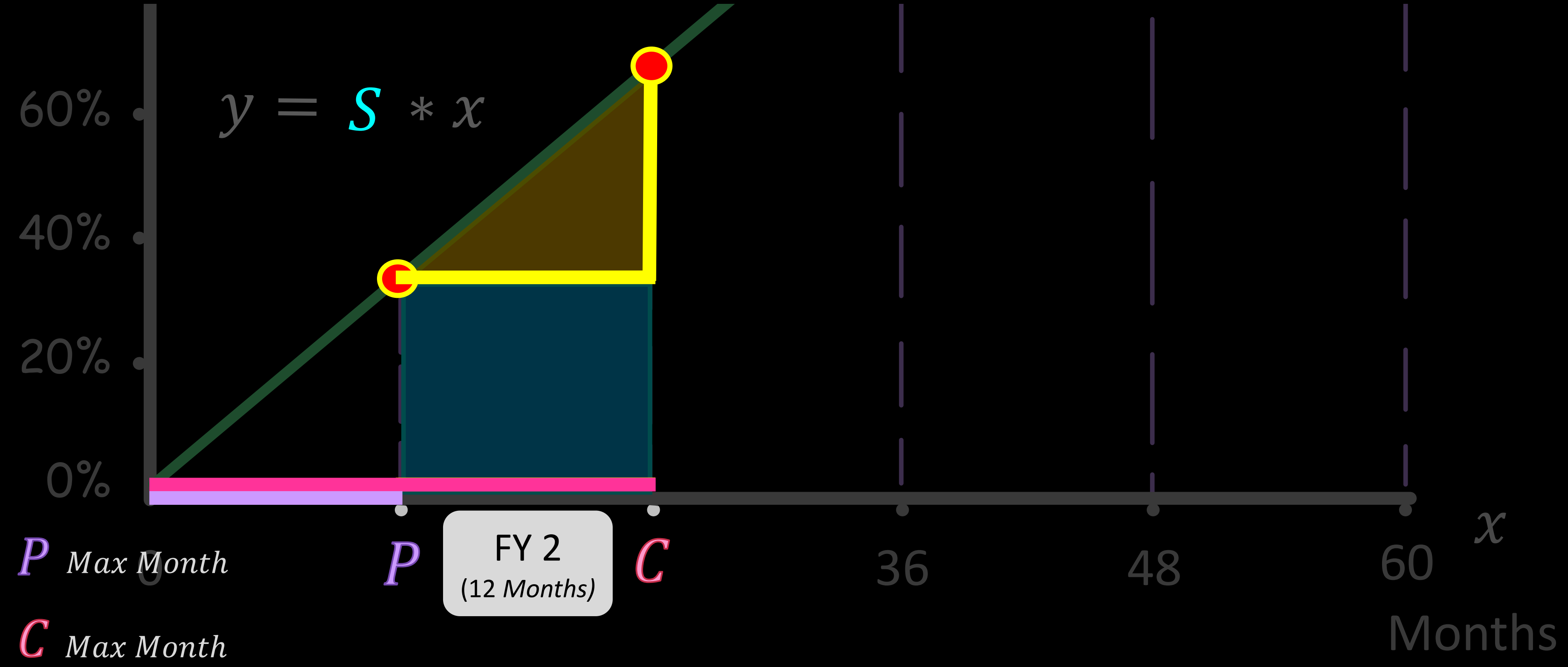
$Slope = S$

$$f_x = \Big(\big(S * C\big) - \big(S * P\big)\Big) * \big(C - P\big) + \quad$$

Height — Base

Area of Triangle

Area of Rectangle



Previous Fiscal Year = P *Max Month*

Current Fiscal Year = C *Max Month*

Slope = S

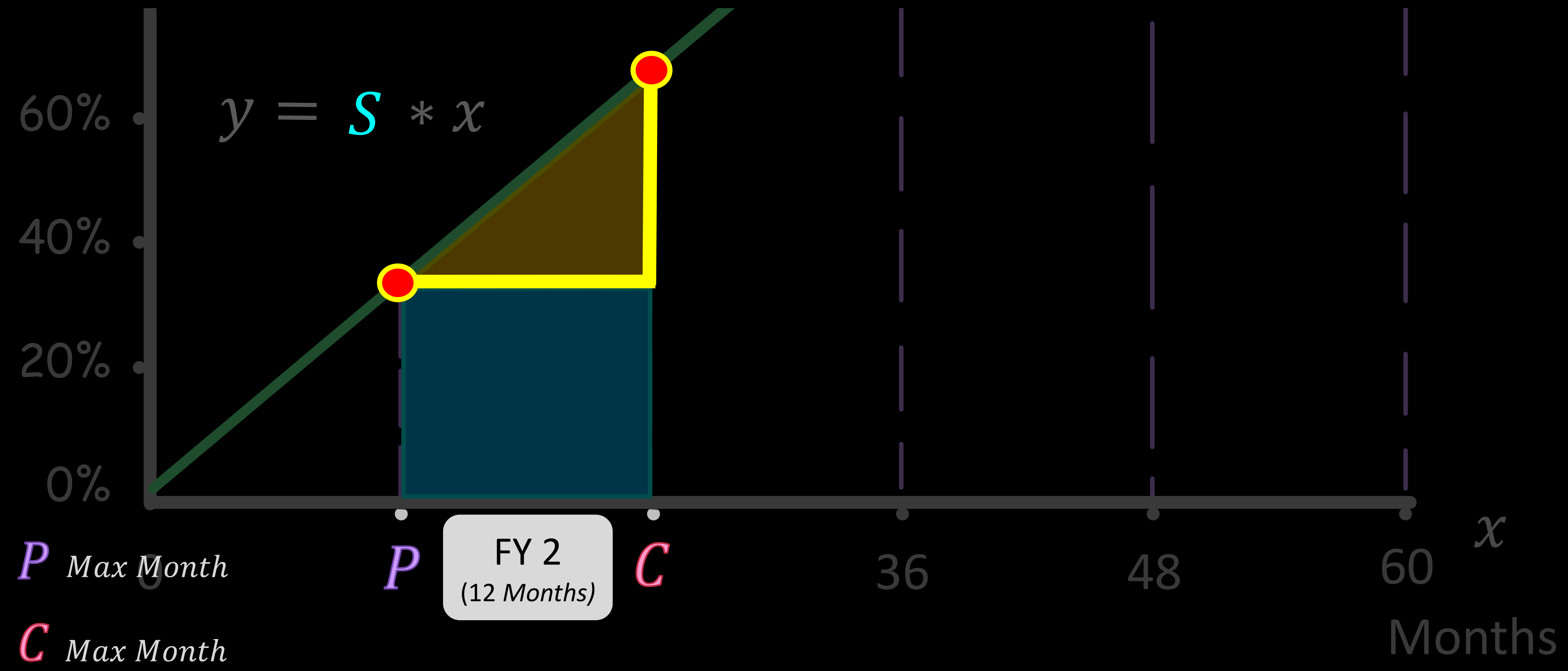
$$f_x = \left(\left(S * C\right) - \left(S * P\right)\right) * \left(C - P\right) +$$

Height

Base

Area of Triangle

Area of Rectangle



$Previous\ Fiscal\ Year = P_{Max\ Month}$

$Current\ Fiscal\ Year = C_{Max\ Month}$

$Slope = S$

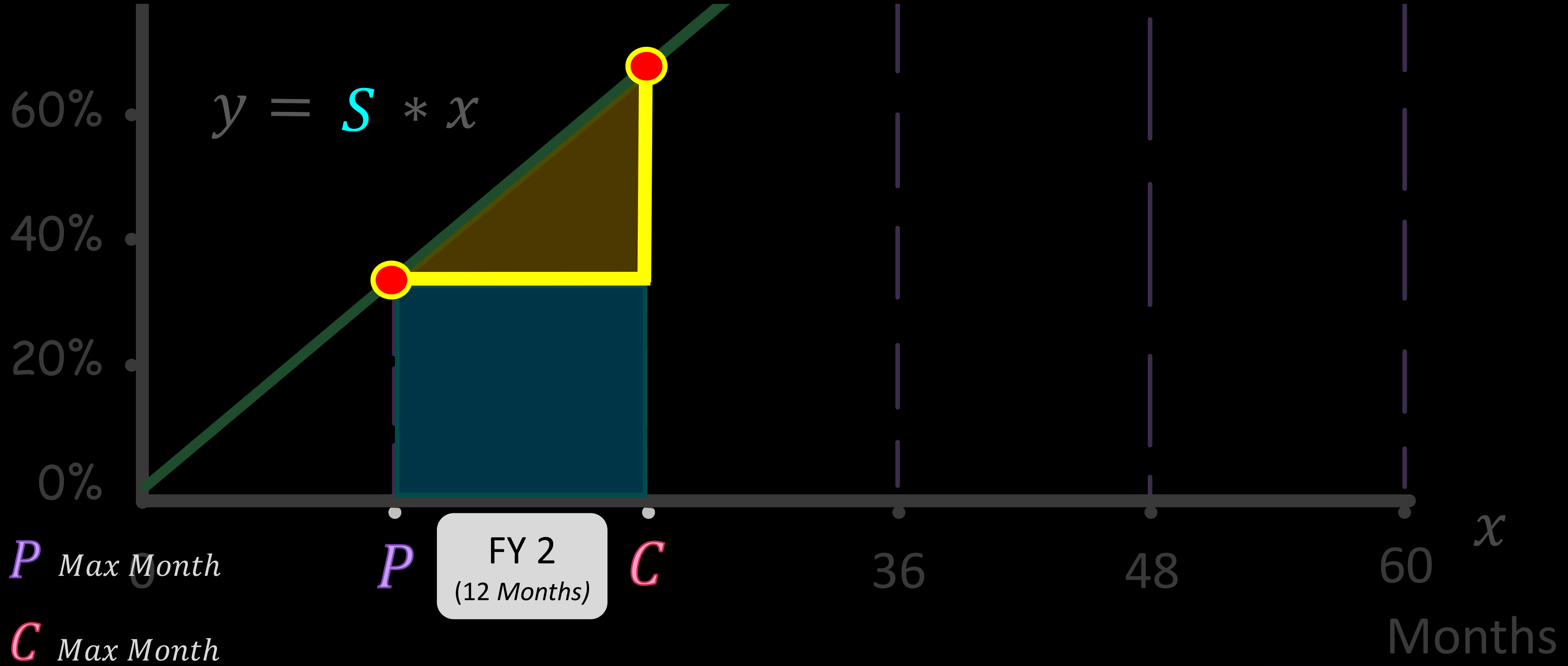
$$f_x = \big((S * C) - (S * P)\big) * (C - P) +$$

Height

Base

Area of Triangle

Area of Rectangle



$Previous\ Fiscal\ Year = P_{\ Max\ Month}$

$Current\ Fiscal\ Year = C_{\ Max\ Month}$

$Slope = S$

Next Slide

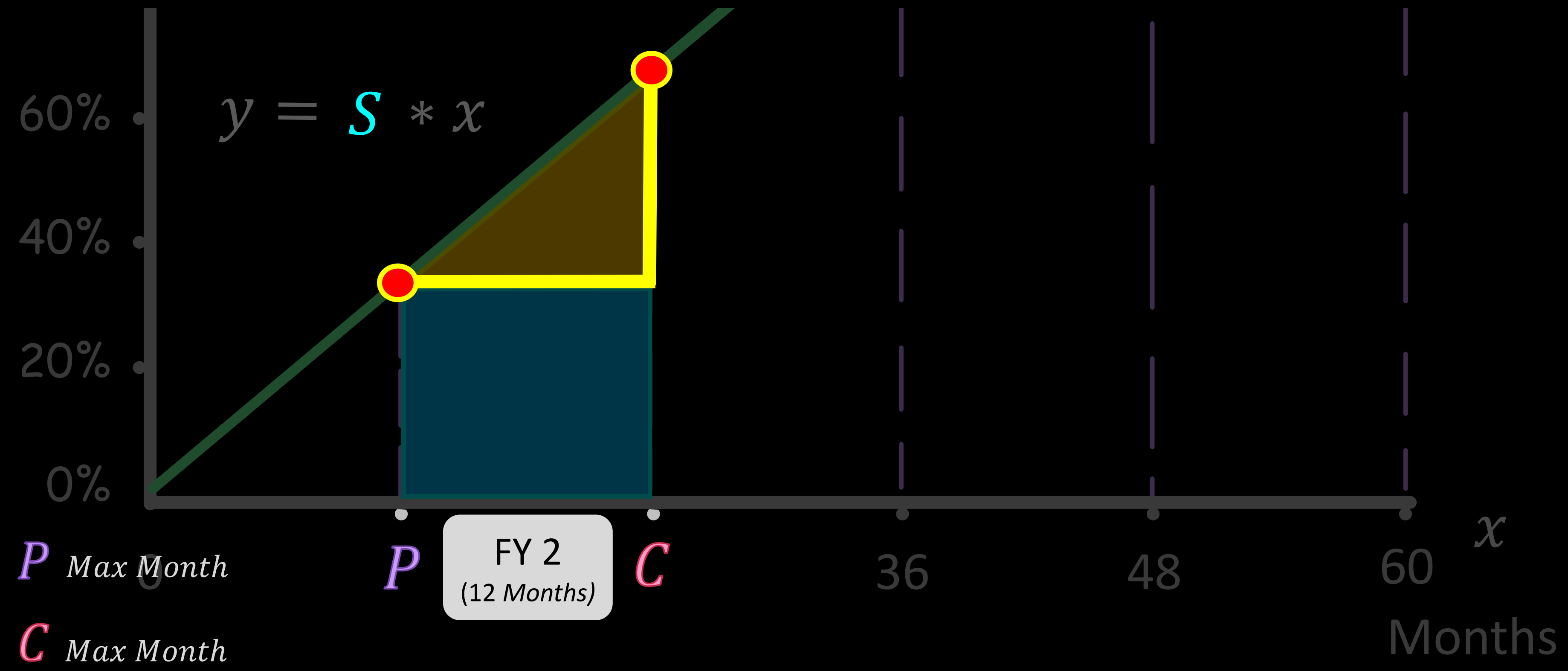
$$f_x = \Big((S * C) - (S * P)\Big) * \left(\frac{C - P}{12}\right) + $$

Height

Base

Area of Triangle

Area of Rectangle



$\textit{Previous Fiscal Year} = P_{\textit{Max Month}}$

$\textit{Current Fiscal Year} = C_{\textit{Max Month}}$

$\textit{Slope} = S$

$$f_x = \left[\left((S * C) - (S * P)\right) * \left(\frac{C - P}{12}\right)\right] * \frac{1}{2} +$$

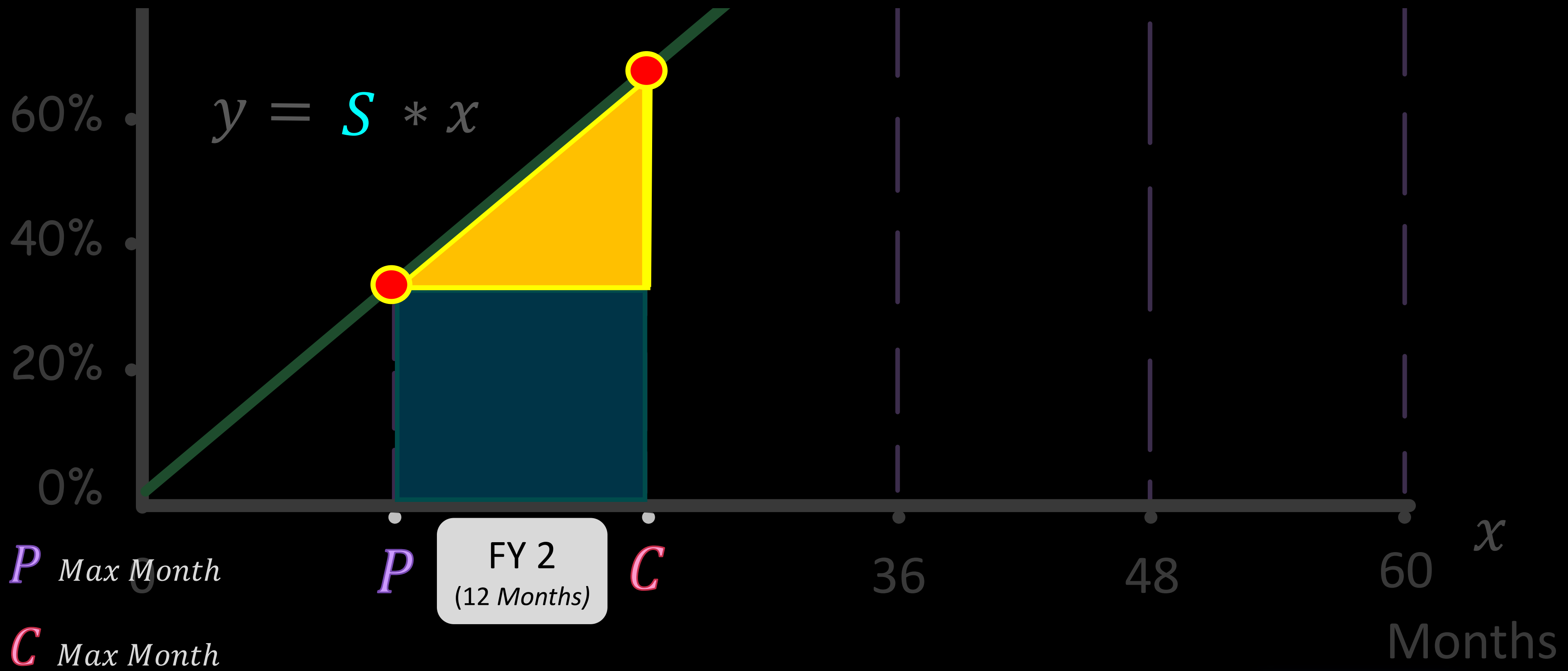
Height

Base

½

Area of Triangle

Area of Rectangle



$Previous\ Fiscal\ Year = P_{\ Max\ Month}$

$Current\ Fiscal\ Year = C_{\ Max\ Month}$

$Slope = S$

Next Slide

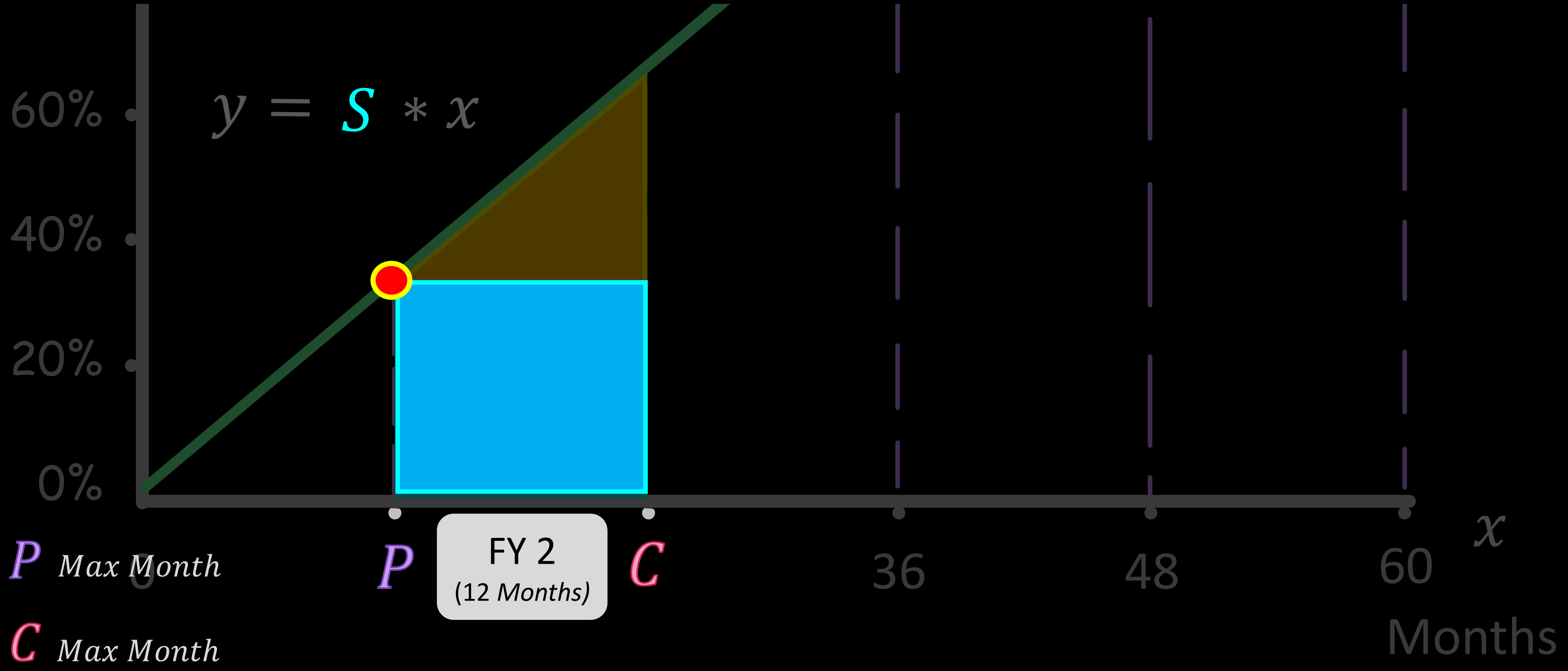
$$f_x = \left[\left(\left(S * C\right) - \left(S * P\right)\right) * \left(\frac{C - P}{12}\right)\right] * \frac{1}{2} +$$

Height | Base | ½

Area of Triangle

Height

Area of Rectangle



Previous Fiscal Year = P Max Month

Current Fiscal Year = C Max Month

Slope = S

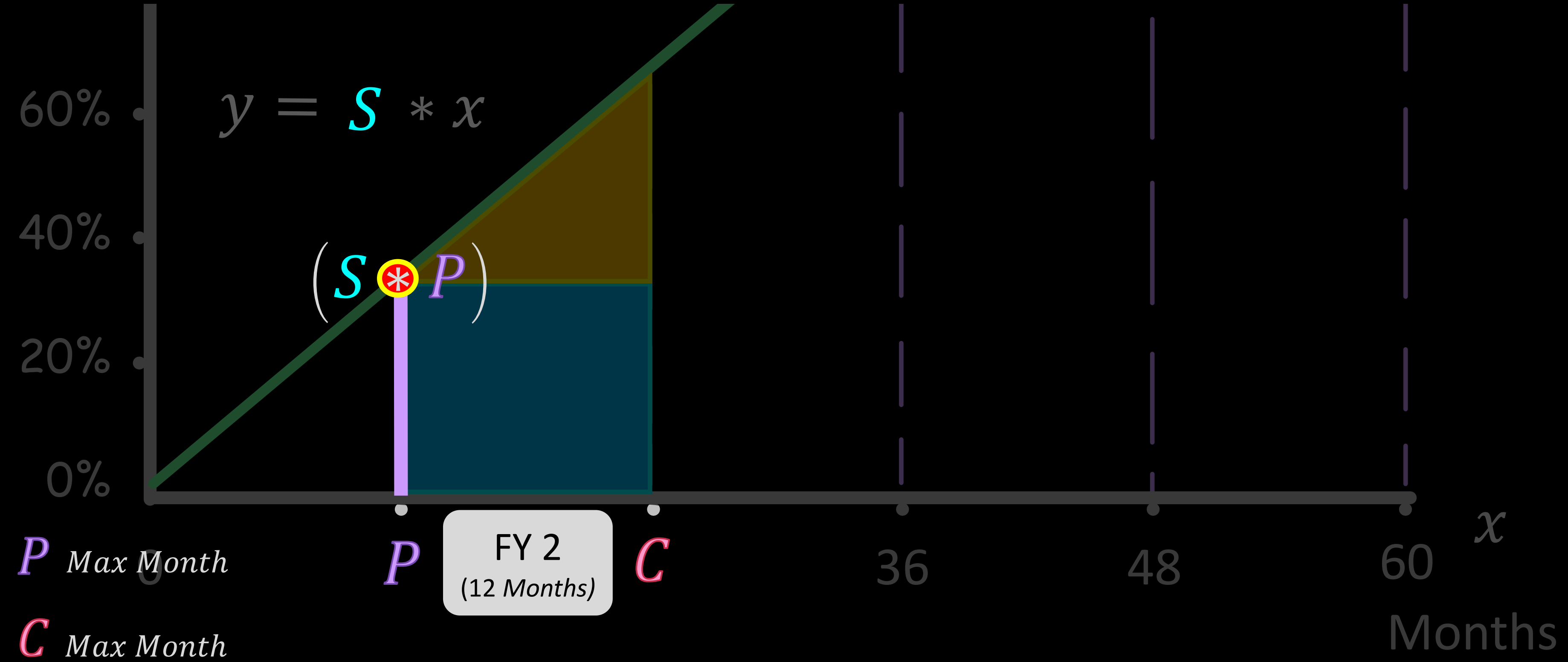
$$f_x = \left[\left((S*C) - (S*P)\right) * \left(\frac{C-P}{12}\right)\right] * \frac{1}{2} + $$

Height | Base | ½

Area of Triangle

Height

Area of Rectangle



$Previous\ Fiscal\ Year = P_{\ Max\ Month}$

$Current\ Fiscal\ Year = C_{\ Max\ Month}$

$Slope = S$

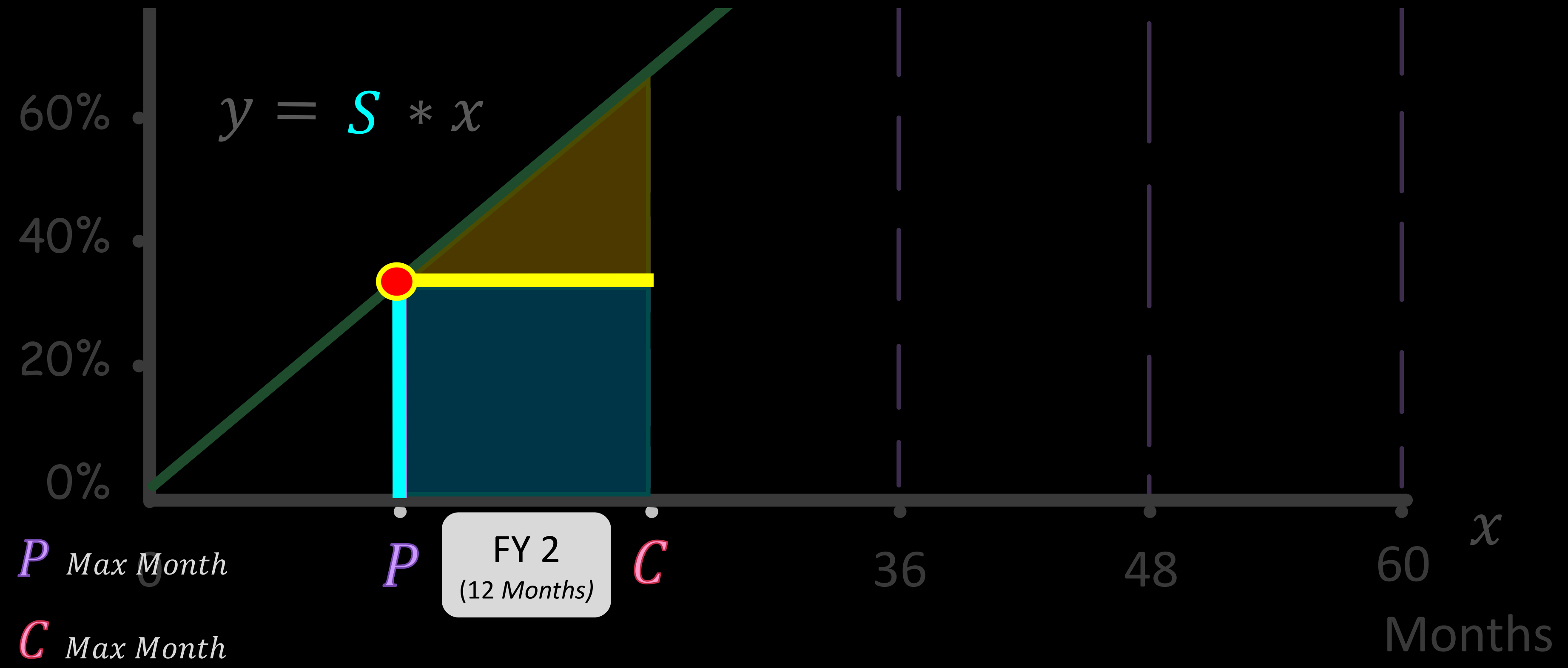
$$f_x = \left[\left((S * C) - (S * P)\right) * \left(\frac{C - P}{12}\right)\right] * \frac{1}{2} + (S * P)$$

Height | Base | ½

Area of Triangle

Height | Base

Area of Rectangle



$Previous\ Fiscal\ Year = P_{\ Max\ Month}$

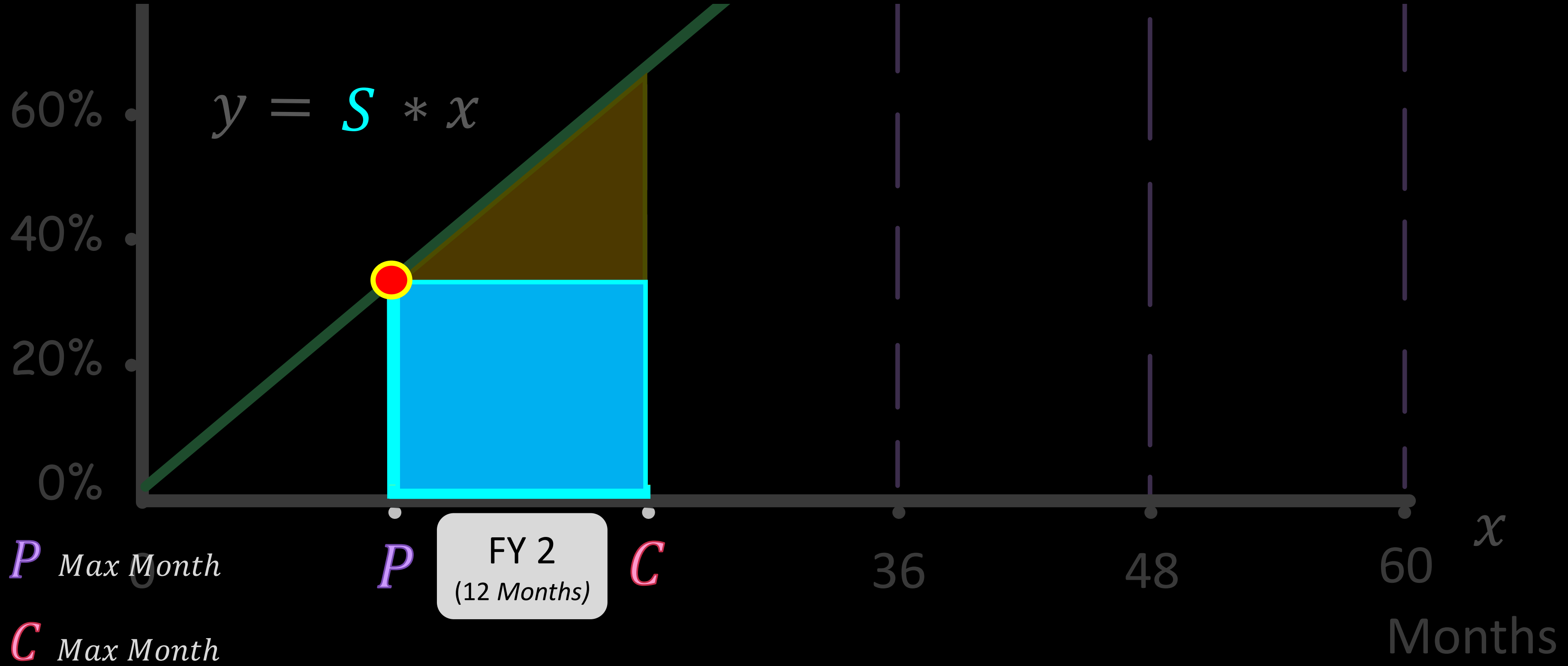
$Current\ Fiscal\ Year = C_{\ Max\ Month}$

$Slope = S$

$$f_x = \left[\left((S*C)-(S*P)\right)*\left(\frac{C-P}{12}\right)\right]*\frac{1}{2} + \left[(S*P)*\left(\frac{C-P}{12}\right)\right]$$

Area of Triangle: Height $\left((S*C)-(S*P)\right)$, Base $\left(\frac{C-P}{12}\right)$, ½

Area of Rectangle: Height $(S*P)$, Base $\left(\frac{C-P}{12}\right)$



$Previous\ Fiscal\ Year = P_{Max\ Month}$

$Current\ Fiscal\ Year = C_{Max\ Month}$

$Slope = S$

$$f_x = \left[\left(\left(S * C\right) - \left(S * P\right)\right) * \left(\frac{C - P}{12}\right)\right] * \frac{1}{2} + \left[\left(S * P\right) * \left(\frac{C - P}{12}\right)\right]$$

Height | Base | ½

Area of Triangle

Height | Base

Area of Rectangle

CUBEKPIM...

`=((($B$2*AT3)-($B$2*AS3))*(AT3-AS3)/12)*0.5+(($B$2*AS3)*(AT3-AS3)/12)`

	A	B	C	D	E	F	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM
1	Ramp Period	36																				
2	Slope *(S)*	0.02777778																				
3		Months	0	1	2	3	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
4	~~Cross Check~~	~~1.5417~~		~~0%~~	~~0%~~	~~1%~~	~~5%~~	~~5%~~	~~5%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~7%~~	~~7%~~	~~7%~~	~~7%~~	~~8%~~	~~8%~~	~~8%~~	~~8%~~
5																						
6	Midpoint	1.500000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%
7																						
8																						
9	Calculus	1.500000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%
10																						
11																						
12	Algebra	1.500000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%
13																						
14																						
15																						

	AAA	AQ	AR	AS	AT	AU	AV	AW	AX	AY
1										
2		Annualized Table	FY0	FY1	FY2	FY3	FY4		Total	
3		Max Month / FY	-	1	13	25	36			
4		~~Cross Check~~		~~0%~~	~~3%~~	~~6%~~	~~8%~~		~~17%~~	
5										
6		~~Midpoint~~		~~0%~~	~~3%~~	~~6%~~	~~8%~~		~~17%~~	
7										
8										
9		Calculus		0%	19%	53%	78%		1.500	
10										
11										
12		Algebra		0%	12)	53%	78%		1.500	
13										
14										
15										

Previous Fiscal Year = *P* *Max Month*

P | FY 2 (12 *Months*) | C | 36 | 48 | 60

Current Fiscal Year = *C* *Max Month*

Months

Slope = *S*

Next Slide

$$f_x = \left[\left(\left(S * C\right) - \left(S * P\right)\right) * \left(\frac{C - P}{12}\right)\right] * \frac{1}{2} + \left[\left(S * P\right) * \left(\frac{C - P}{12}\right)\right]$$

S C Heigh S P | C ase P | ½ | H S ght P | Ba C P

=(((B2*AT3)-(B2*AS3))*(AT3-AS3)/12)*0.5+((B2*AS3)*(AT3-AS3)/12)

CUBEKPIM... | =(((B2*AT3)-(B2*AS3))*(AT3-AS3)/12)*0.5+((B2*AS3)*(AT3-AS3)/12)

	A	B	C	D	E	F	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM
1	Ramp Period	36																				
2	Slope (S)	0.02777778																				
3		Months	0	1	2	3	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
4	~~Cross Check~~	~~-1.5417~~		~~0%~~	~~0%~~	~~1%~~	~~5%~~	~~5%~~	~~5%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~7%~~	~~7%~~	~~7%~~	~~7%~~	~~8%~~	~~8%~~	~~8%~~	~~8%~~
5																						
6	Midpoint	1.500000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%
7																						
8																						
9	Calculus	1.500000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%
10																						
11																						
12	Algebra	1.500000		0%	0%	1%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%
13																						
14																						
15																						

	AAA	AQ	AR	AS	AT	AU	AV	AW	AX	AY
1										
2		Annualized Table	FY0	FY1	FY2	FY3	FY4		Total	
3		Max Month / FY	-	1	13	25	36			
4		~~Cross Check~~		~~0%~~	~~3%~~	~~6%~~	~~8%~~		~~17%~~	
5										
6		~~Midpoint~~		~~0%~~	~~3%~~	~~6%~~	~~8%~~		~~17%~~	
7										
8										
9		Calculus		0%	19%	53%	78%		1.500	
10										
11										
12		Algebra		0%	12)	53%	78%		1.500	
13										
14										
15										

Previous Fiscal Year = *P* *Max Month*

P | FY 2 (12 Months) | C | 36 | 48 | 60 | Months

Current Fiscal Year = *C* *Max Month*

Slope = *S*

Next Slide

$$f_x = \left[\left((S*C)-(S*P)\right)*\left(\frac{C-P}{12}\right)\right]*\frac{1}{2} + \left[(S*P)*\left(\frac{C-P}{12}\right)\right]$$

S C Height S P | C ase P | ½ | H S ght P | Ba C e P

=(((B2*AT3)-(B2*AS3))*(AT3-AS3)/12)*0.5+((B2*AS3)*(AT3-AS3)/12)

CUBEKPIM... | fx | =(((B2*AT3)-(B2*AS3))*(AT3-AS3)/12)*0.5+((B2*AS3)*(AT3-AS3)/12)

	A	B	C	D	E	F	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM
1	Ramp Period	36																				
2	Slope (S)	0.02777778																				
3		Months	0	1	2	3	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
4	~~Cross Check~~	~~1.5417~~		~~0%~~	~~0%~~	~~1%~~	~~5%~~	~~5%~~	~~5%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~7%~~	~~7%~~	~~7%~~	~~7%~~	~~8%~~	~~8%~~	~~8%~~	~~8%~~
																			8%	8%	8%	8%
																			8%	8%	8%	8%
																			8%	8%	8%	8%
14																						
15																						

AQ	AR	AS	AT	AU	AV	AW	AX
Annualized Table	FY0	FY1	FY2	FY3	FY4		Total
Max Month / FY	-	1	13	25	36		
~~Cross Check~~		~~0%~~	~~3%~~	~~6%~~	~~8%~~		~~17%~~
~~Midpoint~~		~~0%~~	~~3%~~	~~6%~~	~~8%~~		~~17%~~
Calculus		0%	19%	53%	78%		1.500
Algebra		0%	12)	53%	78%		1.500

11				
12	Algebra	1.500000	0%	0%
13				

Previous Fiscal Year = *P* *Max Month*

FY 2 (12 Months)

P C 36 48 60 Months

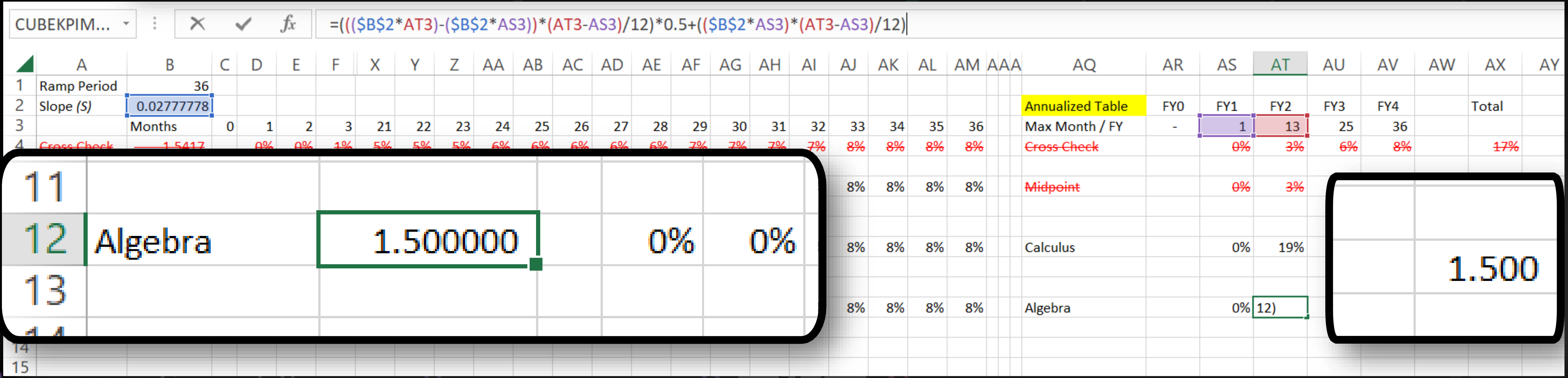
Current Fiscal Year = *C* *Max Month*

Slope = *S*

Next Slide

$$f_x = \left[\left((S * C) - (S * P)\right) * \left(\frac{C - P}{12}\right)\right] * \frac{1}{2} + \left[(S * P) * \left(\frac{C - P}{12}\right)\right]$$

=(((B2*AT3)-(B2*AS3))*(AT3-AS3)/12)*0.5+((B2*AS3)*(AT3-AS3)/12)



	A	B	C	D	E	F	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AQ	AR	AS	AT	AU	AV	AX
1	Ramp Period	36																											
2	Slope (S)	0.02777778																					Annualized Table	FY0	FY1	FY2	FY3	FY4	Total
3		Months	0	1	2	3	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	Max Month / FY	-	1	13	25	36	
4	~~Cross Check~~	~~1.5417~~		~~0%~~	~~0%~~	~~1%~~	~~5%~~	~~5%~~	~~5%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~7%~~	~~7%~~	~~7%~~	~~7%~~	~~8%~~	~~8%~~	~~8%~~	~~8%~~	~~Cross Check~~		~~0%~~	~~3%~~	~~6%~~	~~8%~~	~~17%~~
																			8%	8%	8%	8%	~~Midpoint~~		~~0%~~	~~3%~~			
																			8%	8%	8%	8%	Calculus		0%	19%			
																			8%	8%	8%	8%	Algebra		0%	12)			

11				
12	Algebra	1.500000	0%	0%
13				

1.500

Previous Fiscal Year $= P$ *Max Month*

Current Fiscal Year $= C$ *Max Month*

Slope $= S$

(12 Months)

36 48 60 Months

Next Slide

Simplify

Proof

$$f_x = \left[\left(\left(S * C\right) - \left(S * P\right)\right) * \left(\frac{C - P}{12}\right)\right] * \frac{1}{2} + \left[\left(S * P\right) * \left(\frac{C - P}{12}\right)\right]$$

$$f_x = \left[S * ((C)-(P)) * \left(\frac{C-P}{12}\right)\right] * \frac{1}{2} + \left[(S * P) * \left(\frac{C-P}{12}\right)\right]$$

$$f_x = S * \left[\left(\left(C\right) - \left(P\right)\right) * \left(\frac{C - P}{12}\right)\right] * \frac{1}{2} + S * \left[\left(P\right) * \left(\frac{C - P}{12}\right)\right]$$

$$f_x = \left[S * \left[\left(C - P \right) * \left(\frac{C-P}{12} \right) \right] * \frac{1}{2} + S * \left[P * \left(\frac{C-P}{12} \right) \right] \right]$$

$$f_x = S * \left[\left[\left(C - P \right) * \left(\frac{C - P}{12} \right) \right] * \frac{1}{2} + \left[P * \left(\frac{C - P}{12} \right) \right] \right]$$

$$f_x = S * \left[\left[(C - P) * (C - P) \right] * \frac{1}{2} + \left[P * (C - P) \right] \right]$$

$$\frac{1}{12}$$

$$\frac{1}{12}$$

$$f_x = \frac{1}{12} * S * \left[\left[(C - P) * (C - P) \right] * \frac{1}{2} + \left[P * (C - P) \right] \right]$$

$$f_x = \frac{1}{12} * S * \left[\left[(C - P) * (C - P) \right] * \frac{1}{2} + \left[(PC - P^2) \right] \right]$$

$$* \frac{1}{2}$$

$$f_x = \frac{1}{12} * S * \left[\left[\left(C - P\right) * \left(C - P\right) \right] + 2 * \left[\left(PC - P^2\right) \right] \right]$$

$$f_x = \frac{1}{2} * \frac{1}{12} * S * \left[\left[(C - P) * (C - P) \right] + 2 * \left[(PC - P^2) \right] \right]$$

$$f_x = \frac{1}{2} * \frac{1}{12} * S * \left[\left[(C - P) * (C - P) \right] + \left[(2PC - 2P^2) \right] \right]$$

C^2

$$f_x = \frac{1}{2} * \frac{1}{12} * S * \left[\left[\left(C - P\right) * \left(C - P\right) \right] + \left[\left(2PC - 2P^2\right) \right] \right]$$

$$C^2 - PC$$

$$f_x = \frac{1}{2} * \frac{1}{12} * S * \left[\left[(C - P) * (C - P) \right] + \left[(2PC - 2P^2) \right] \right]$$

$$C^2 - PC - PC$$

$$f_x = \frac{1}{2} * \frac{1}{12} * S * \left[\left[(C - P) * (C - P) \right] + \left[(2PC - 2P^2) \right] \right]$$

$$C^2 - PC - PC + P^2$$

$$f_x = \frac{1}{2} * \frac{1}{12} * S * \left[\left[(C - P) * (C - P) \right] + \left[(2PC - 2P^2) \right] \right]$$

$$f_x = \frac{1}{2} * \frac{1}{12} * S * \left[\left[C^2 \quad -2PC \quad +P^2 \right] \quad + \quad \left[\left(2PC - 2P^2\right) \right] \right]$$

$$\left(C - P\right) * \left(C - P\right)$$

$$f_x = \frac{1}{2} * \frac{1}{12} * S * \left[C^2 \quad - 2PC \quad + P^2 \qquad\qquad + 2PC - 2P^2 \right]$$

$$f_x = \frac{1}{2} * \frac{1}{12} * S * \left[C^2 + P^2 + 2PC - 2P^2 - 2PC \right]$$

$$f_x = \frac{1}{2} * \frac{1}{12} * S * \left[C^2 \qquad + P^2 \qquad - 2P^2 \right]$$

$$f_x = \frac{1}{2} * \frac{1}{12} * S * \left[\quad C^2 \qquad \begin{matrix} -2P^2 \\ +1P^2 \end{matrix} \right]$$

$$f_x = \frac{1}{2} * \frac{1}{12} * S * \left[C^2 \qquad - 1P^2 \right]$$

$$f_x = \frac{1}{2} * \frac{1}{12} * S * \left[C^2 - P^2 \right]$$

$$\int_a^b \frac{1}{36} * x \, dx = \frac{1}{36} * \frac{1}{2} * \left[b^2 - a^2 \right]$$

$$f_x = \frac{1}{2} * \frac{1}{12} * \frac{1}{36} * \left[C^2 - P^2 \right]$$

S_*

$$\int_a^b \frac{1}{36} * x \, dx = \frac{1}{36} * \frac{1}{2} * \left[b^2 - a^2 \right]$$

$$f_x = \frac{1}{36} * \frac{1}{2} * \left[C^2 - P^2 \right] * \frac{1}{12}$$

$$\int_a^b \frac{1}{36} * x \, dx = \frac{1}{36} * \frac{1}{2} * \left[b^2 - a^2 \right]$$

$$f_x = \frac{1}{36} * \frac{1}{2} * \left[C^2 - P^2 \right] * \frac{1}{12}$$

$$\int_a^b \frac{1}{36} * x \; dx = \frac{1}{36} * \frac{1}{2} * \left[b^2 - a^2 \right] * \frac{1}{12}$$

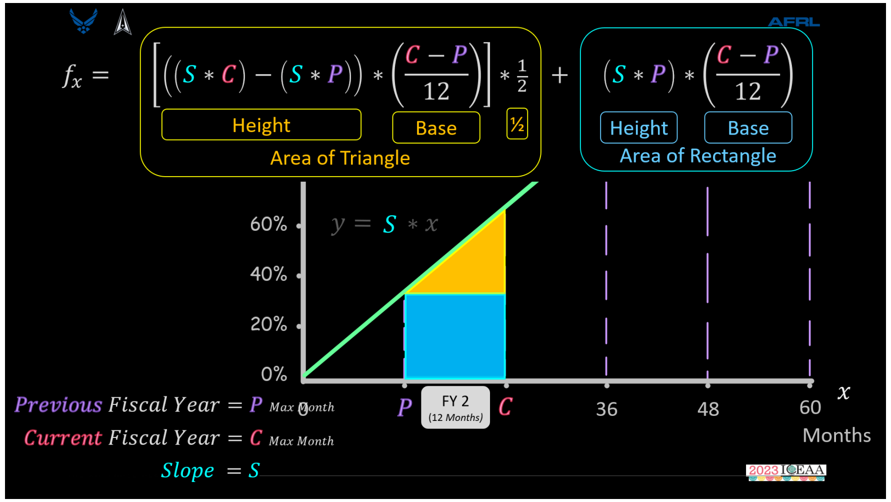
Next Slide

The Accidental Proof of Calculus for Cost Analysts

- EZ Way
  - More Than a Merrit Badge

$$\int_0^{36} \frac{1}{36} x\,dx = \left[\frac{1}{36} * \frac{1}{2} * x^2\right]_0^{36} \div 12 = 1.5$$

- The Hard Way



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The End

Purpose

- Calculus
  - More Than a Merrit Badge
- Tools of the Trade
  - Pencil & Paper
  - Calculator
  - MATLAB
- Cautionary Reminder
  - Cross Checks

Next Slide

Discussion

Quotes

Expert:

Someone who's made every mistake in a narrow field

Engineers:

Solving problems you didn't know you had

Thank you

Discussion

Quotes

Expert:

Someone who's made every mistake in a narrow field

Engineers:

Solving problems you didn't know you had

Next Slide

Thank you

Integrals Into Excel

Integrals into Excel *Reminder*

$$\int x^n\,dx = \frac{1}{n+1} * x^{n+1} + C$$

B6 =SUM(D6:AM6)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1		Ramp Period	36																
2																			
3		Months	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
4	~~Cross Check~~	~~1.5417~~		~~0%~~	~~0%~~	~~1%~~	~~1%~~	~~1%~~	~~1%~~	~~2%~~	~~2%~~	~~2%~~	~~2%~~	~~3%~~	~~3%~~	~~3%~~	~~3%~~	~~3%~~	~~4%~~
5																			
6	Midpoint	1.500000		0%	0%	1%	1%	1%	1%	2%	2%	2%	2%	2%	3%	3%	3%	3%	4%

	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM
1																				
2																				
3	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
4	~~4%~~	~~4%~~	~~4%~~	~~5%~~	~~5%~~	~~5%~~	~~5%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~6%~~	~~7%~~	~~7%~~	~~7%~~	~~7%~~	~~8%~~	~~8%~~	~~8%~~	~~8%~~
5																				
6	4%	4%	4%	5%	5%	5%	5%	5%	6%	6%	6%	6%	7%	7%	7%	7%	8%	8%	8%	8%

Next Slide

Integrals into Excel *Reminder*

$$\int x^n dx = \frac{1}{n+1} * x^{n+1} + C$$

$$\frac{1}{36} * \int x^1 dx = \frac{1}{36} * \left[\frac{1}{2} * x^2\right] = \frac{1}{36} * \left[\frac{1}{2} * b^2\right] - \frac{1}{36} * \left[\frac{1}{2} * a^2\right]$$

Next Slide