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Spread Too Thin
Managing Coefficient of Variation in Monte-Carlo Based Cost Models

Stephen Koellner
Augur Consulting Inc.
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Presenter

- **Stephen Koellner** – Analyst/Technical Advisor
  - 4+ years of experience in federal project cost estimation
  - PLCCEs/POEs, IGCEs, Cost-Benefit Analysis, AoAs
  - ICEAA 2022 Team Achievement of the Year Award recipient
  - GAO Cost Guidebook contributor
  - Published author: Earth & Planetary Science Letters
  - BS in Mathematics - Penn State University

- **Augur Consulting Inc.**
  - Service-Disabled Veteran Owned Small Business (SDVOSB)
  - Supporting our government-only customer base since 2012
  - Core Competencies:
    - Cost Estimating and Analysis
    - Integrated Master Scheduling
    - Performance Management
    - Data Analytics and Visualization

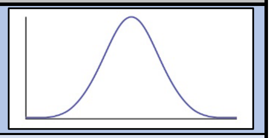
Introduction

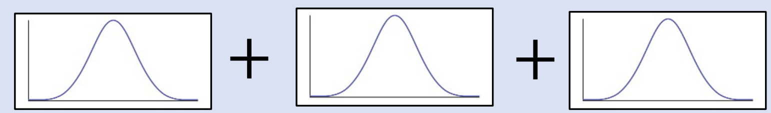
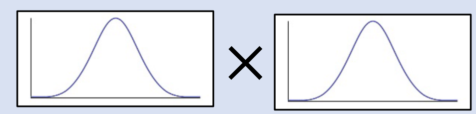
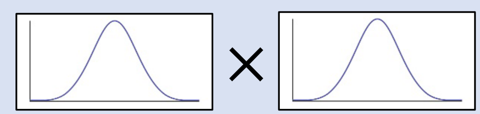
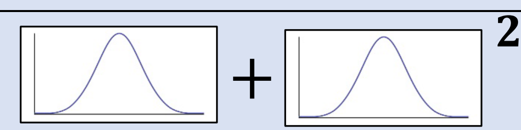
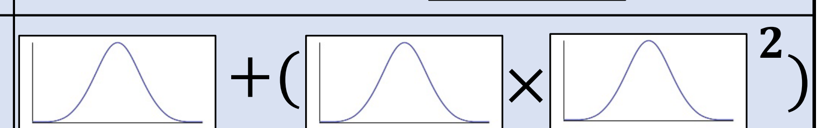
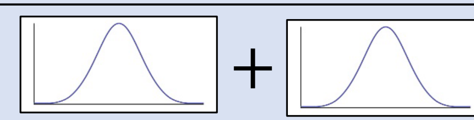
- **Problem**: Uncertainty easily underestimated in cost models
  - Inaccurate quantification of cost spread poses significant long-term risk
  - Characterized by a low Coefficient of Variation (CV)
  - Diagnosing issue often a difficult endeavor
- **Goal:** Identify modeling choices that prohibit realistic cost spread
  - Define CV as function of children elements in WBS
  - Study interactions of input level uncertainty & output level uncertainty
  - Provide modeling guidelines to cost estimators
  - Enable program managers to minimize likelihood of funding risks

Cost Uncertainty

- GAO Cost Estimating and Assessment Guide (March 2020):
  - "A credible estimate includes a risk and uncertainty analysis that quantifies the imperfectly understood risks…"
- All cost estimates should account for risk/uncertainty
  - Credible cost models produce a range (spread) of values
  - Cost modelers must primarily think of output as a distribution, not a number
  - Often brief a snapshot of distribution/spread to clients
- Analyze Results
  - Determine if cost output logically aligns with cost inputs
  - Evaluate if top-level cost uncertainty adequately matches program status
  - Identify cost drivers and quantify their impact

Application of Cost Uncertainty

Top - Level Application of Cost Uncertainty		
WBS Level	**WBS Element**	**Application of Risk**
1	**Total Contract Cost**	
2	Management	*None*
2	Development Labor	*None*
2	Prototype Materials	*None*
2	Equipment	*None*
2	Testing Labor	*None*
2	Testing Equipment	*None*

Bottom - Level Application of Cost Uncertainty		
WBS Level	**WBS Element**	**Application of Risk**
1	**Total Contract Cost**	*Composition of Children*
2	Management	
2	Development Labor	
2	Prototype Materials	
2	Equipment	
2	Testing Labor	
2	Testing Equipment	

Application of Cost Uncertainty

Comparing Applications of Uncertainty		
Application	**Pros**	**Cons**
Top - Level	• Simplifies cost modeling • Generally, more data is available to defend top level spread	• Limited ability to analyze cost drivers and quantify impact to model spread • Assumptions on spread not directly traceable to inputs Range of cost outcomes can only be viewed at top-level
Bottom - Level	• Spread of total cost directly depends on cost inputs/cost drivers • Range of cost outcomes can be viewed for any WBS element	• Complicates cost modeling/behavior of cost model • Can more easily underestimate cost uncertainty

- Both types of application have unique strengths and weaknesses
  - Choice depends on agency guidance, estimate type, estimator preference
  - E.g., ROM estimates may employ use of Top-Level application
  - Augur typically develops estimates with Bottom-Level application
- Bottom-level requires approximation methods (Monte-Carlo)

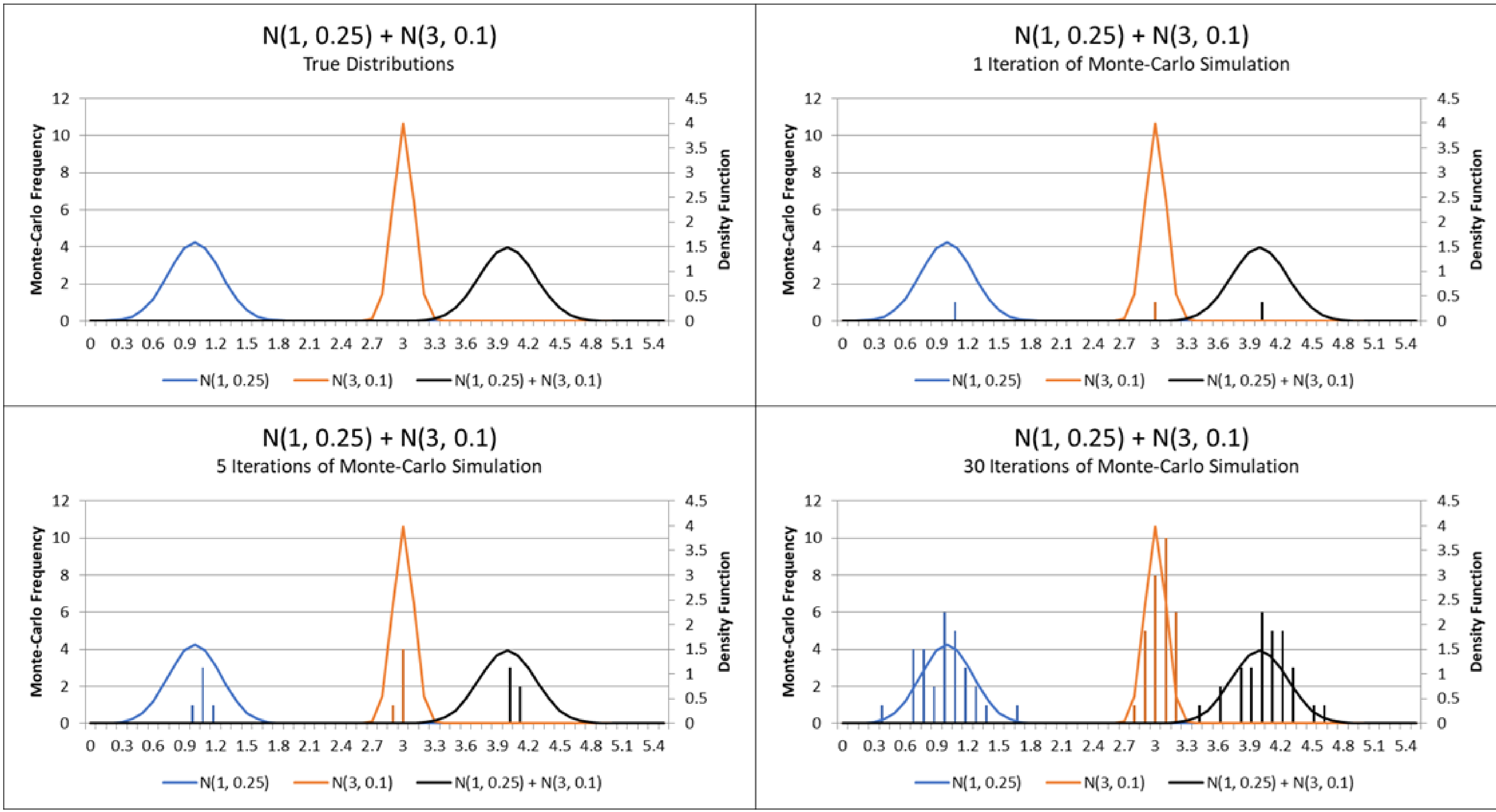
Brief evaluates behavior of Bottom-Level application of spread

Monte-Carlo Cost Modeling

- Calculate cost outputs with Monte-Carlo sampling
  - Interactions of probability distributions are incredibly complex
  - "By hand" calculations are impractical and computationally inefficient

- Monte-Carlo based cost models approximate outputs efficiently
  - Sample random values from input distributions
  - Run calculation of outputs/save results from this iteration
  - Results converge to true value as number of iterations increases

Monte-Carlo Modeling



Coefficient of Variation (CV)

- CV: standard deviation divided by mean of distribution X

$$CV_X := \frac{\sigma_X}{\mu_X}$$

- Why is CV important to cost estimators?
  - CV is a ratio that “normalizes” the spread of a distribution
  - Allows for comparison of data sets with differing means/standard deviations
  - Commonly used to check if uncertainty is appropriately captured in model
  - Higher CV indicates a wider dispersion/flatter distribution

Coefficient of Variation (CV)

- Lack of universally accepted output CV ranges for all project types
  - Top-level CV changes over time/varies with technology of program
  - Difficult for estimators: "what is the appropriate CV for this model?"
  - USAF Cost Risk Handbook provides specific guidance for top-level CV:

USAF Cost Risk Handbook	
Estimate Type	**Typical CV Range**
Space Systems/SW	0.35 - 0.45
Aircraft/Complex HW	0.25 - 0.35
Large Electronic System	0.1 - 0.2

- Common ranges of input level CVs from ACEIT*:
  - <0.15 = Optimistic level of certainty
  - 0.15-0.35 = Typical/moderate uncertainty
  - >0.35 = Higher end of uncertainty
  - Qualitative interpretations of data driven inputs
  - Can also leverage above ranges in absence of data

**Automated Cost Estimator Integrated Toolkit (SW for DoD cost estimation)*

Interpretation of a WBS as a Convolution

- Work Breakdown Structure (WBS)
  - Higher level elements (parent) are sum of lower level (children)
  - Each child element is essentially a probability distribution
  - Convolution = Linear combination of probability distributions
- Let Z be a parent-level WBS element with n children elements: X_i

WBS Level	WBS Element
1	**Total Contract Cost**
2	Management
2	Development Labor
2	Prototype Materials
2	Equipment
2	Testing Labor
2	Testing Equipment

→

WBS Level	WBS Element
k	Z
k+1	X_1
k+1	X_2
k+1	.
k+1	.
k+1	.
k+1	X_n

$$Z = \sum_{i=1}^{n} X_i$$

Convolution

- CV of Z can be defined in terms of its children
  - Approx. computationally in using Monte-Carlo simulation SW

Top Level CV Equation

$$Z = \sum_{i=1}^{n} X_i$$

- Each X_i is a distribution with parameters:
  - $r_{i,j}$ are correlation coefficients between X_i and X_j
  - μ_{Xi} is the expected value (mean) of distribution X_i
- The CV of parent level Z follows the below equation*:

$$CV_Z = \frac{\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} r_{i,j}\, CV_{X_i}\, \mu_{X_i}\, CV_{X_j}\, \mu_{X_j}}}{\sum_{i=1}^{n} \mu_{X_i}}$$

- Equation is agnostic to types of distributions

**Results can be expressed more concisely with covariance*

Verification of CV Equation

- Monte Carlo simulation approx. uncertainty distributions
  - Correlation is simplified via group strength
  - Small deviations between model & equation (depends on # of iterations)
- Formula is not useful for generating cost output
  - Cost models are more complex than simple sums
  - Equation is useful for analyzing results at WBS level

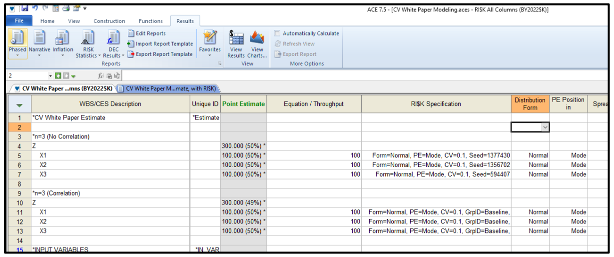
Simplified WBS (n=3)			
WBS	**Mean**	**Stan. Dev.**	**CV**
Z	**300**		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

Correlation Matrix			
	X1	**X2**	**X3**
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

$$CV_Z = \frac{\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} r_{i,j}\, CV_{X_i}\, \mu_{X_i}\, CV_{X_j}\, \mu_{X_j}}}{\sum_{i=1}^{n} \mu_{X_i}}$$

CV Calculation	
True CV (Eqn)	0.0707
Monte Carlo Sim*	0.0710
% Difference	-0.41%

** 10,000 Iterations*



Behavior of Equation

- Illustrate behavior of top-level CV
  - Use previous result as a baseline of comparison
  - Change one parameter of baseline for each scenario
  - Maintain perturbations proportionally
    - Certain results need additional normalization
  - Identify how children elements impact parent level
- Cost modeling decisions vs updates to baseline

Simplified WBS (n=3)			
WBS	**Mean**	**Stan. Dev.**	**CV**
Z	**300**		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

Correlation Matrix			
	X1	**X2**	**X3**
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Perturbation – High Child CV

- Larger spread of lower elements increases parent spread
  - Double standard deviation of one child element
  - Parent CV increase from 0.071 to 0.097
  - ~37% increase in parent CV
- Intuitive result, large impact for small WBS
  - Average CV of children elements substantially higher

Baseline WBS			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

Increased CV of WBS Element			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	20	0.2
X2	100	10	0.1
X3	100	10	0.1

CV Calculation	
Baseline	0.070711
Increased CV of WBS Element	0.097183
% Δ CV	37%

Correlation Matrix			
	X1	X2	X3
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Correlation Matrix			
	X1	X2	X3
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Perturbation – Large Mean (Normalized)

- Double mean of one element/scale others proportionally
  - Represents a ~12% increase to top level CV
  - "Grouping" spread to single element

Baseline WBS			
WBS	**Mean**	**Stan. Dev.**	**CV**
Z	**300**		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

Large Mean (Normalized)			
WBS	**Mean**	**Stan. Dev.**	**CV**
Z	**300**		
X1	200	20	0.1
X2	50	5	0.1
X3	50	5	0.1

CV Calculation	
Baseline	0.070711
Large Mean (Normalized)	0.079057
% Δ CV	**12%**

Correlation Matrix			
	X1	**X2**	**X3**
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Correlation Matrix			
	X1	**X2**	**X3**
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Perturbation – Large WBS

- Double size of WBS, maintain same total sum
- Large WBS case reduced top level CV by ~13%
  - Commonly referred to as "over-sharpening the pencil"
  - Increased fidelity dramatically reduces spread of costs
  - In depth WBS =/= more accurate estimate
    - Significant risk of underestimating cost uncertainty without sufficient correlation

Baseline WBS			
WBS	**Mean**	**Stan. Dev.**	**CV**
Z	**300**		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

Large WBS			
WBS	**Mean**	**Stan. Dev.**	**CV**
Z	**300**		
X1	50	5	0.1
X2	50	5	0.1
X3	50	5	0.1
X4	50	5	0.1
X5	50	5	0.1
X6	50	5	0.1

CV Calculation	
Baseline	0.070711
Large WBS	0.061237
% Δ CV	**-13%**

Correlation Matrix			
	X1	**X2**	**X3**
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Correlation Matrix				
	X1	**X2**	**…**	**X6**
X1	1	0.25	0.25	0.25
X2	0.25	1	0.25	0.25
…	0.25	0.25	1	0.25
X6	0.25	0.25	0.25	1

Perturbation – Strong Correlation

- Double value for single correlation coefficient
  - $r_{1,3}$ increased from 0.25 to 0.5
  - Higher dependency between X_1 and X_3
  - Top-level spread increased by ~5%
- Increasing correlation increases top-level CV

Baseline WBS			
WBS	**Mean**	**Stan. Dev.**	**CV**
Z	**300**		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

Strong Correlation			
WBS	**Mean**	**Stan. Dev.**	**CV**
Z	**300**		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

CV Calculation	
Baseline	0.070711
Strong Corr	0.074536
% Δ CV	**5%**

Correlation Matrix			
	X1	**X2**	**X3**
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Correlation Matrix			
	X1	**X2**	**X3**
X1	1	0.25	0.5
X2	0.25	1	0.25
X3	0.5	0.25	1

Perturbation – Independence (No Correlation)

- Absence of correlation
  - Dramatic reduction in top level spread: ~18%
  - Effectively independent distributions being summed
- Zero correlation is unrealistic
  - Bottom-level application of dist. transfers correlation to WBS
  - Inter-dependence of common inputs creates functional correlation
- Note: negative correlation will also reduce top level CV

Baseline WBS			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

No Correlation			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

CV Calculation	
Baseline	0.070711
No Corr	0.057735
% Δ CV	-18%

Correlation Matrix			
	X1	X2	X3
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Correlation Matrix			
	X1	X2	X3
X1	1	0	0
X2	0	1	0
X3	0	0	1

Summary of Perturbations

- Increasing spread of children elements most impactful
  - Intuitive result of CV (uncertain children -> uncertain parent)
  - Typically, this is an undesirable action for cost estimators
- WBS size second most impactful perturbation (decreases spread)
  - Higher fidelity estimates can dramatically underestimate risk
  - “Grouping” together elements will increase top-level spread
- Varying impacts of correlation to top-level CV
  - Stronger correlation off-sets impact of larger WBS’s
  - Lack of correlation will drastically underestimate top-level spread

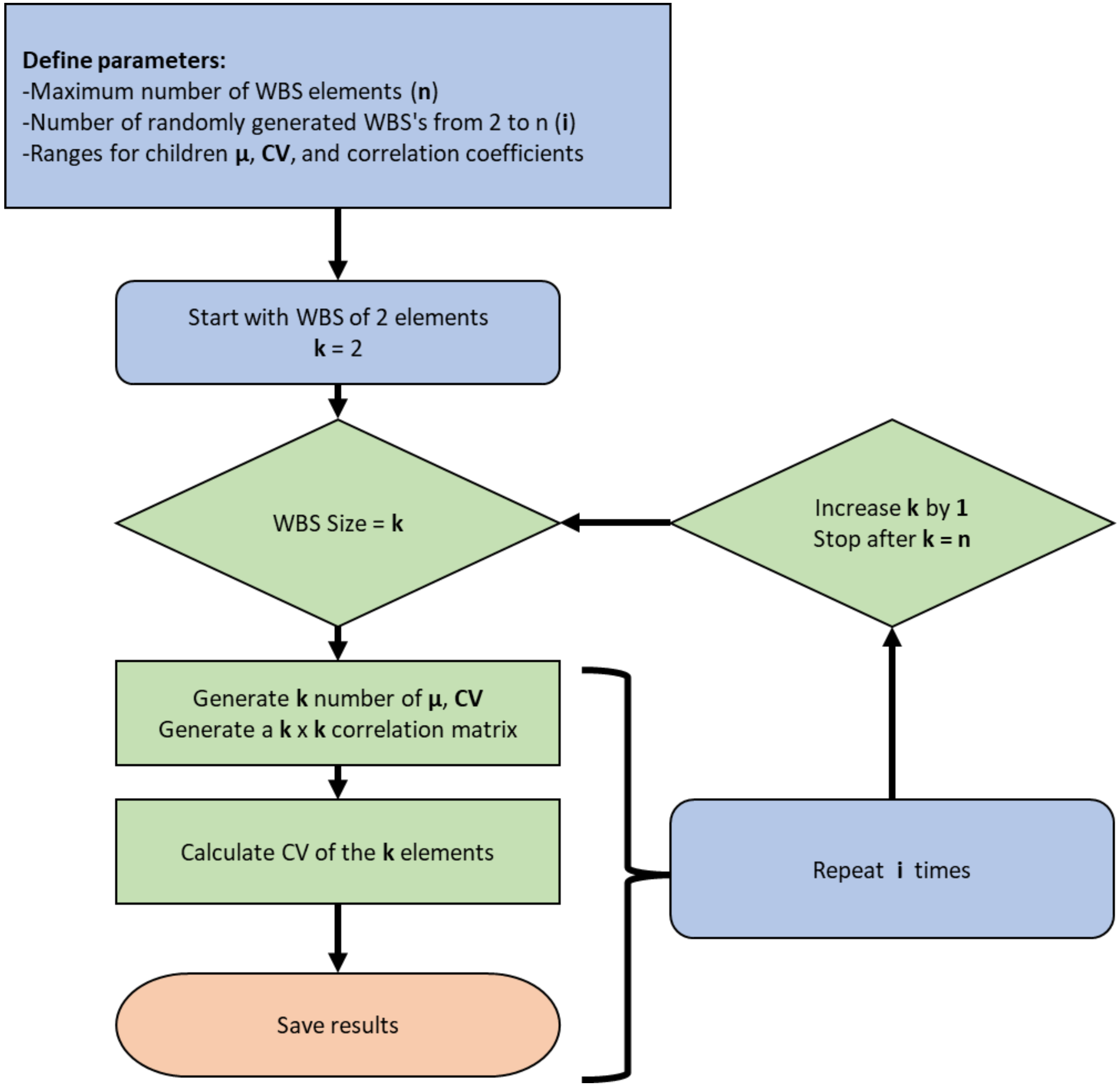
Behavior of CV			
Scenario	CV	% Δ to Baseline	Note
Baseline	**0.0707**	**0%**	***n=3, μ = 100, CV = 0.1, r = 0.25***
High Stan. Dev.	0.0972	37%	*Double one standard dev.*
Large Mean	0.0729	3%	*Double one mean*
Large Mean (Normalized)	0.0791	12%	*Double one mean, reduce mean of other elements*
Large WBS	0.0612	-13%	*Double WBS/maintain top-level mean*
Strong Correlation	0.0745	5%	*Double single correlation coefficient*
No Correlation	0.0577	-18%	*Model independent distributions*

Randomized WBS

- Formula can be used to model cost estimator behavior
  - Randomly generate WBS's and calculate top-level spread
  - Follow best practices to provide recommendations for analysts
  - Refined simulations of WBS parameters
    - More precisely model mean, children CV's, correlation, etc.
    - E.g., Children CV from uniform distribution between 0.15 – 0.6
- Randomly generate WBS
  - Common values of:
    - μ, CV, & Correlation Coefficients*
- Study CV behavior at scale
  - Simulate practices of cost estimators
  - Model impacts of correlation at the WBS level

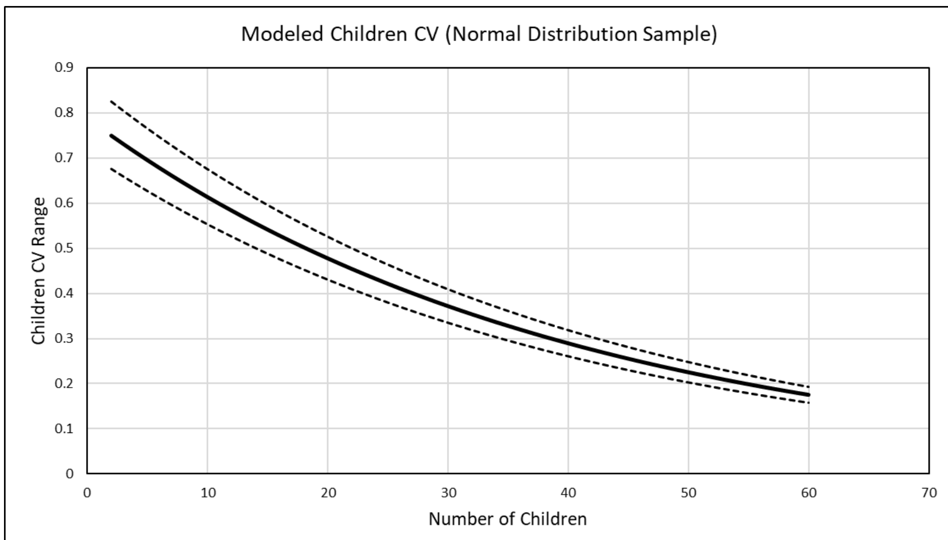
***Values scale logically as WBS size increases**

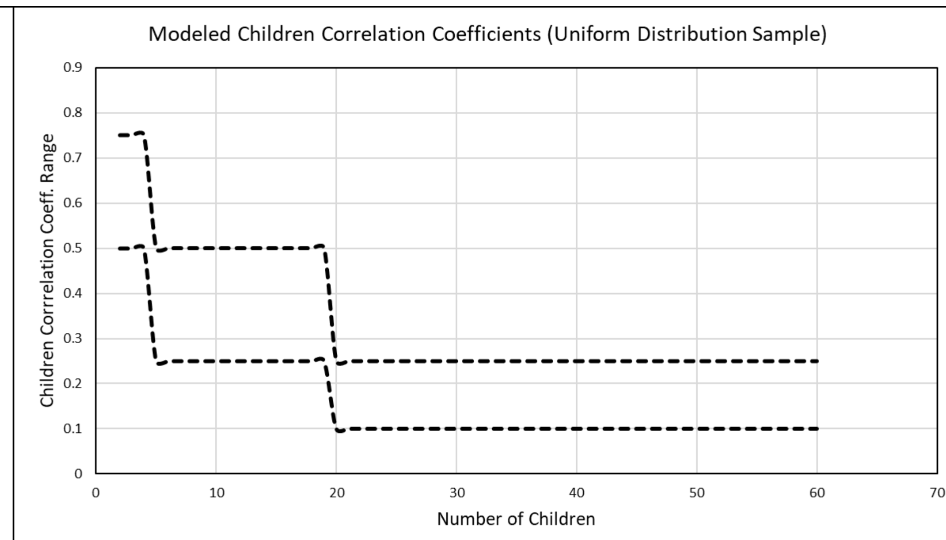
Randomized WBS



Randomized WBS

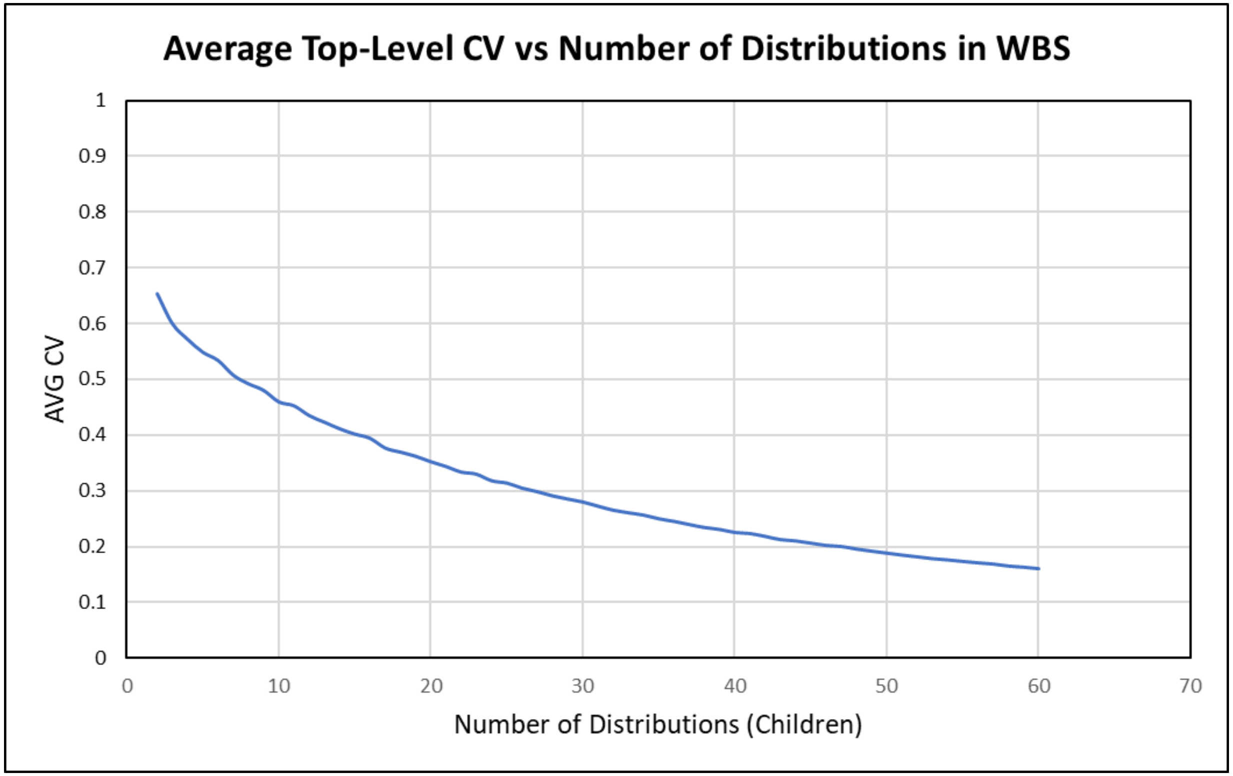
- Randomly generate statistics for children elements
  - Children CV sampled from a normal distribution
    - Mean and standard dev. decrease with more children elements
    - Higher fidelity -> less uncertainty on individual elements
  - Correlation coefficients sampled from uniform distribution
    - Step down in line with USAF guidance on correlation coefficients





Randomized WBS

- Random WBS designed to mimic ACAT I cost model
  - WBS totals between \$3.6B - \$5.8B (sum of RDTE + OP costs)
    - Choice of WBS sum is irrelevant for exercise
  - Children normalized to have sum within above range



Proposed Ranges for CV/WBS-Size

- Below are proposed ranges from USAF IT research paper
  - CV ranges by acq. milestone (based on actual cost growth)
  - Ranges are preferable since they are traceable to actual data

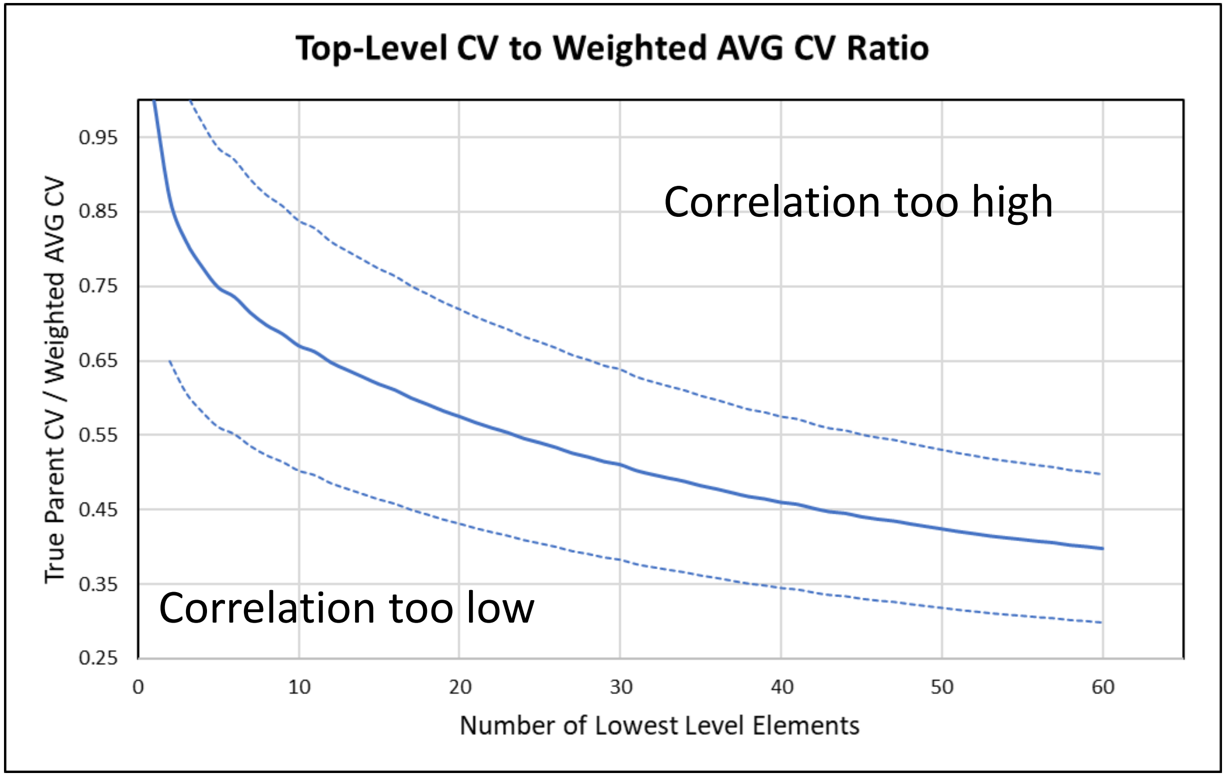
USAF IT Research Paper	
Estimate Type	**Typical CV Range**
Milestone A	0.41 - 0.74
Milestone B	0.31 - 0.54
Milestone C	0.23 - 0.32

- Ranges & randomized WBS results used for WBS size rec.
  - Compare at-scale CV behavior with ranges to make rec.
  - CVs should not be the only statistic analyzed for model health

Recommended WBS Ranges	
Acquisition Phase	**Rec. WBS Size**
Milestone A/High Uncertainty	2 - 14 Lowest Level Elements
Milestone B/Medium Uncertainty	6 - 25 Lowest Level Elements
Milestone C/Modest Uncertainty	24+ Lowest Level Elements

Ratio of True CV to Weighted Avg CV

- Ratio curve based on following modeling best practices
  - Significant deviations from this curve indicate lack of correlation
    - E.g., a WBS with 20 elements w/ratio of 0.3
    - Ratio should be ~0.55 -> correlation coefficients too low



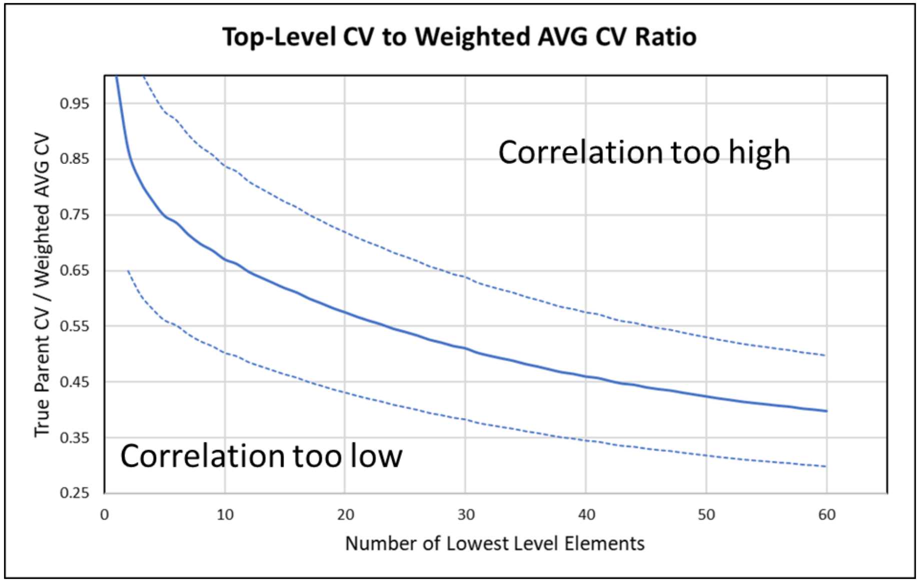
CV in Cost Estimating Applications

- Analyze output CV as a sanity check for cost model spread
  - Check ratio of top-level CV and weighted AVG CV w/WBS size
  - Check dollar value spread of outputs for reasonableness
  - CV should NOT be the only metric used for evaluation
- Observable CV behavior provides cost modeling insight
  - Early ROM estimates need small WBS/top-level risk application
  - Ensure appropriate correlation is being applied to input variables
  - WBS size should correlate with program maturity and level certainty
  - Don’t over sharpen the pencil with engineering build-ups
- **Leadership should push for higher spread in early estimates**
  - Funding requests need accurate projections of potential cost growth
  - Underestimated spread reduces MR/contingency in risk informed models

Conclusion

- Insufficient cost spread in Monte-Carlo based cost models
  - CV equation provides insight to understanding top-level CV behavior
  - Provided rules of thumb/cross-checks for diagnosing cost models
- **Top-level CV dominated by WBS size and correlation**
  - Models w/out correlation are underestimating spread
  - WBS size should fall within ranges based on lifecycle/certainty level
  - Overly detailed WBS injects overoptimism unless correlated properly

Recommended WBS Ranges	
Acquisition Phase	**Rec. WBS Size**
Milestone A/High Uncertainty	2 - 14 Lowest Level Elements
Milestone B/Medium Uncertainty	6 - 25 Lowest Level Elements
Milestone C/Modest Uncertainty	24+ Lowest Level Elements



Questions?

- Contact Presenter: skoellner@augurconsulting.net
- Contact Augur Consulting: info@augurconsulting.net

Whitepaper of this presentation is available

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BACKUP

Extreme Case

- Combine actionable positive/negative impacts to CV
  - High spread groups elements and has strong correlation
  - Low spread is a high fidelity and un-correlated WBS
  - Both have same top-level mean/avg children CV
- Wide range in top-level CV based on these choices
  - Despite both having the same top-level mean

High Spread WBS			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	200	20	0.1
X2	50	5	0.1
X3	50	5	0.1

Low Spread WBS			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	50	5	0.1
X2	50	5	0.1
X3	50	5	0.1
X4	50	5	0.1
X5	50	5	0.1
X6	50	5	0.1

CV Calculation	
High Spread	0.086603
Low Spread	0.040825
% Δ CV	-53%

Correlation Matrix			
	X1	X2	X3
X1	1	0.5	0.5
X2	0.5	1	0.5
X3	0.5	0.5	1

Correlation Matrix				
	X1	X2	...	X6
X1	1	0	0	0
X2	0	1	0	0
...	0	0	1	0
X6	0	0	0	1