# Minding Your P's and Q's as Prices Rise

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# Abstract

Inflation is running at its highest levels in decades in the U.S. and abroad. The U.S. dollar today is worth about 93 cents compared to a year ago, or 93 cents of buying power in the marketplace. The same situation of diminished value holds for a government budget, a salary, a pension, a dividend, or a company's cash balances. In other words, 93 cents would buy as much 12 months ago as 100 cents will buy now.

Many firms and government organizations are attempting to develop custom escalation indices for labor, material, and construction in the face of high inflation.[1] But questions remain as to the consistency of any such indices with local labor-market conditions, their technical quality and accuracy, the scope of their coverage, and the degree to which self-fulfilling prophecies are at play.

In short, choices and challenges abound in combining individual prices (P's) and quantities (Q's) of labor and material and equipment into one measure of overall escalation for a project. This research illuminates the issue of *which* type of index to use *when* in the relentless fight against *Money Illusion* – or the cognitive tendency of people and organizations to view their wealth, income, and expenses in nominal-dollar rather than real-dollar terms.

Further, in building budgets in the public sector of the economy, and in establishing collective bargaining agreements, projections of inflation are necessary. As a bonus, this paper also addresses the estimation of probability distributions of inflation rate forecasts, using financial instruments actively traded on Wall Street, such as Treasurys and, more recently, inflation derivatives. As economists note, focus sharpens and credibility rises when prices are set by market agents who bear financial risk, such as pension, insurance, and hedge-fund managers.



**Keywords**: *Inflation, Escalation, Nominal-dollar, Real-dollar*


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[1] Importantly, government-prescribed indices for programming and budgeting, such as the U.S. Gross Domestic Product (GDP) Implicit Price Deflator, measure average inflation rates across all sectors of the economy, though 70% of GDP is personal expenditures (housing/utilities, food/beverages, health care, financial/insurance services, etc.). Consumer spending trends are not comparable to technical activities such as Navy shipbuilding.

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# Introduction

## Current Trends

The U.S. dollar always seems to be the same. But in reality, it's always changing. It's unstable like all other currencies of the world such as the euro, the British pound, the Turkish lira, the French franc, and the Canadian dollar. Important problems grow out of the fact that units of money are not stable in buying power. A government's budget, a worker's salary, a pension, a dividend, or a company's cash flow or market capitalization aren't worth what they used to be – because of inflation. Inflation, in short, renders false the aphorism that "a dollar is a dollar" and "a euro is a euro."

The U.S. Federal Reserve's "transitory inflation" of last summer has proven anything but transitory. Inflation today at 7% per annum is running at its highest level in 40 years.[2] The U.S. dollar is worth about 93 cents compared to a year ago, or 93 cents of buying power in the marketplace. That is, 93 cents a year ago would have bought the same amount of goods and services as 100 cents today. The dollar is not the dollar that existed before. And compared to 40 years ago? The dollar today buys only 35 cents of what it bought then.

Similar trends hold internationally. For example, the U.K.'s Consumer Price Index with Housing (CPIH) increased over 5% in 2021, the highest level in 30 years, and their Retail Price Index (RPI) increased over 7%.[3] Similarly, Canada's Consumer Price Index (CPI) and the European Union's Harmonized Index (HCPI) are currently running at roughly 5% per annum, the highest in three decades.[4]

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[2] "Inflation Hits Fastest Clip Since '82," The Wall Street Journal, 13 January 2022. "The Consumer Price Index rose 7% in December [2021] from the same month a year earlier," as reported by the U.S. Department of Labor on 12 January 2022.

[3] "UK Inflation Highest in 30 Years," The Wall Street Journal, 20 January 2022.

[4] Data from the U.K.'s Office of National Statistics, Statistics Canada, and the European Union Central Bank. The HCPI is "harmonized" in that all the countries in the European Union follow the same estimation methodology.

## Challenges

Given the recent spike in prices, firms and government organizations globally are renewing efforts to develop custom-built escalation[5] indices for labor, material, and construction projects.[6] Labor unions are increasingly demanding the insertion of Cost of Living Adjustment (COLA) provisions in new contracts.[7] But questions remain regarding the composition of a market basket of goods that make up the indices, their consistency with local labor-market conditions, and their technical quality and accuracy.

In short, choices and challenges abound in collecting and combining hundreds and sometimes thousands of individual prices (P's) and quantities (Q's) into one measure of overall escalation, dubbed the "Index Number Problem." The research described in this paper illuminates the trade space for index-number creation and recommends best practices – all in an effort to combat *Money Illusion* - or the psychological or cognitive tendency of people and organizations to view their wealth and income in nominal-dollar rather than real-dollar terms.

## Estimation Imperative

The requirement to generate cost estimates *over the life cycle* in then-year or out-turned dollars, or any currency, confounds matters greatly, regardless of the inflation indices employed. Differences, or deltas, in annual inflation rates can generate a large financial impact ten or twenty years in the future due to compounding. For example, a 100 basis-point delta in rates from 2.0% per annum to a custom-built rate of 3.0% per annum yields a multiplier of 1.22 (22% increase) after 20 years.[8] In today's economy, where inflation has accelerated sharply and departed significantly from previous government

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[5] Some authorities use the terms "inflation" and "escalation" indices interchangeably. Others make a distinction. Generally, "inflation" is applied to the macro economy and "escalation" to the micro economy, or to individual agents in the marketplace. For example, see "Inflation and Escalation Best Practices for Cost Analysis," Office of the Secretary of Defense, Cost Assessment and Program Evaluation, December 2021. On the other hand, the Naval Sea Systems Command uses the term "Shipbuilding Inflation." Further, defense planners use the term "defense inflation" rather than "escalation."

[6] Escalation on construction projects can often-times be attributed to factors that go beyond the erosion of the value of the currency over time. Programmatic hiccups such as schedule delays due to unforeseen project risks contribute to increases in project costs.

[7] "As Inflation Soars, More Workers Demand Cost-of-Living Protection," The Wall Street Journal, David Harrison and Sarah Chaney Cambon, 22 December 2021.

[8] $\text{Yearly Multiplier} = (1 + x)^n \div (1 + y)^n, where$ *x* and *y* are alternative yearly inflation rates. A basis point or "beep" is 1/100 of a percentage point.

estimates of 2% per annum, a 300 basis-point delta is more in tune with reality, yielding a multiplier of 1.97 (79% increase)

The stakes are extremely high. For example, a sample of Major Defense Acquisition Programs (MDAPs) in the U.S. reveals that inflation accounts for 17% of the total nominal cost of the programs, on average.[9] For *Virginia Class* submarines, $29B is required to fund escalation through the end of production, or 32% of total procurement cost.[10] And as inflation rises, escalation's share of total cost rises too. If the inflation estimate is off, then so will be the nominal cost estimate.[11]

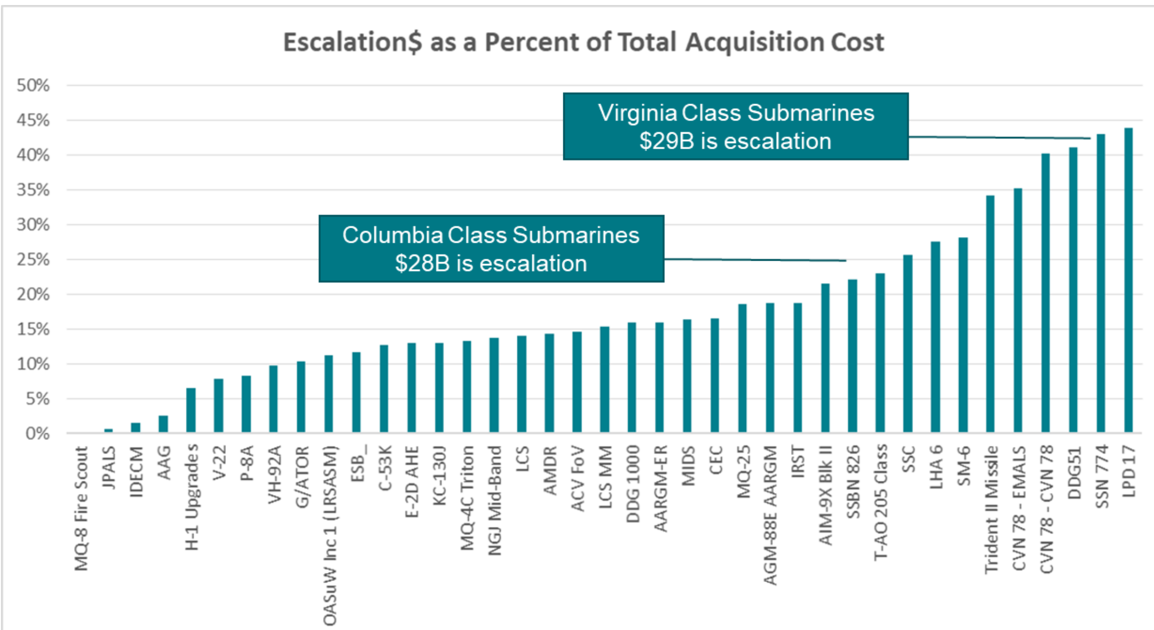


*Figure 1: Escalation Dollars for Navy Programs*

To close the loop, our research presents a methodology for forecasting rates of inflation based on the probability distribution functions (PDFs) inherent in the put-call spreads associated with inflation-derived securities. The probabilistic forecasts, in turn, enable tradeoffs between risk and rates in building budgets for programs and projects.

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[9] Data from Selected Acquisition Reports (SAR) Summary Tables, December 2019.
[10] Selected Acquisition Report for *Virginia Class* submarines, December 2019.
[11] Some organizations, such as Australia's Department of Defense, generate cost estimates for their project in constant dollars. Escalation is tacked onto the estimates by Treasury, but, the basic problem persists, albeit at a higher level of authority in government.

# Money Illusion

## Explanation

Many of us in the western world have been anesthetized to weakening currencies by decades of low inflation.  However, a COVID-impacted economy, supply-chain disruptions on a global scale, and an unprecedented increase in the money supply (at least in the U.S.) brought about fiscal stimulus that has led to a significant reduction in purchasing power. Inflation is now a world-wide phenomenon, with sharply rising prices in developed countries such as the U.K., Canada, and the U.S., and in emerging economies such as Brazil, Peru, and Turkey.[12]  And, to cap off the bad news, the European Union's statistics agency just reported that inflation in January 2022, in the Eurozone, is running at the fastest pace *ever*.[13]

High rates of inflation impact nearly every aspect of commercial and consumer finance. Incomes, investments, debt, and spending are not what they used to be in terms of purchasing power of a currency.  As the award-winning author Ray Dalio posits, "*If you are like most people, you are not nearly as aware of your currency risk* as *you need to be,"* since

*"Of the roughly 750 currencies that have existed since 1700, only about 20 percent remain, and all of them have been devalued."*[14]

During recent times of relatively low inflation, such as the 1990's and 2000's, the purchasing power of a currency did not experience much change from one year to the next.  Governments, companies, and workers tended to ignore the phenomenon that inflation slowly but surely erodes their income and wealth.  The difficulty in grasping the instability of a currency was captured by John Maynard Keynes when he coined the term "Money Illusion."  Irving Fisher put it best:

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[12] "Inflation is a global phenomenon.  What we have seen in recent months, we see pretty much everywhere."  World Bank Chief Economist, Ayhan Kose, as cited in yahoo!news, 12 January 2022.
[13] "Eurozone Inflation Hits a Record," The Wall Street Journal, 3 February 2022.
[14] The Changing World Order, Avid Reader Press, Ray Dalio, 2021, page 131.

*"Why this oversight? Why is it that we have been so slow to take up these fundamental problems which are of vital concern to all people? It is because of the "Money Illusion"; that is, the failure to perceive that the dollar, or any other unit of money, expands or shrinks in value."*[15]

## Examples

Money illusion is commonplace. It's likely to remain a problem as long as inflation persists. Individuals and organizations might believe they're doing better financially than they really are because they're paying attention to nominal values instead of what money can buy.

Real-world examples include:[16]

- The Dow first achieved 36,000 in November 2021 but it is equivalent to only 22,600 in terms of the purchasing power of the U.S. dollar that prevailed in 1999 when James Glassman made his famous prediction.[17]
- Average hourly earnings in the U.S. increased 4% last year. In terms of purchasing power, this represents a 3% loss.[18]
- A cost analyst's salary has increased 1.5% per annum over the last ten years. They make 16% more now than when they started. Are they better off? They *lost money* in terms of buying power, with inflation running at 1.96% during the same time span.
- The U.K.'s defense budget has almost doubled from 2000 to 2021, from £28B to £53B. But in terms of constant purchasing power, it's increased only 7%, to £30B.

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[15] The Money Illusion, Irving Fisher, page 4.
[16] Adjustments are made using the consumer price indices in the U.S. and the U.K.
[17] Journalist James Glassman and economist Kevin Hassett wrote in late 1999 that the Dow Jones Industrial Average would hit 36,000 as soon as 2005; it took another 16 years to reach that level.
[18] "Inflation Is Cutting into Pay Raises for Workers," The Washington Post, 23 January 2022.

- Gas prices in the U.S. today average $3.30 per gallon (regular grade), thought to be high and making the headlines. But, in constant purchasing power, they're unchanged from 40 years ago.
- When the Pentagon operates under a continuing resolution, the U.S. Congress hasn't passed a defense bill. Nominal defense spending is consequently flat year over year. But in *real terms* – or the ability to buy items such as tanks, ships, missiles and ammunition, spending is 7% lower.

## Solution

A long-standing challenge in overcoming *Money Illusion* is the development and application of a metric upon which to measure the changing nature of the dollar. That is, when has the value of a currency changed, and by how much? What's the means or mechanism by which to measure the change in terms of real value?

Index numbers, the subject of this research, provide the answer. They measure the relative change in price, quantity, value, or other items of interest from one time period to another (time series) or, alternatively, at a single point in time across geographical locations or some other dimension of analysis (cross-sectional). As such, index numbers are expressed in terms of percent deltas from a base year, period, or value, usually denoted as 1.00 or 100.

Inflation indices (which are a subset of the larger domain of index numbers) measure the average percentage change in the prices of a collection of representative goods and services, or a *market basket*, from one point of time to another. If such indices are turned upside down, so to speak, there emerges an index of the *buying power of the currency*. As the economist and mathematician, Irving Fisher noted,

*"The two indexes play see-saw with each other, one going up or down as the other goes down or up. So there are always these two indexes, one of prices and the other of the buying power of the dollar. Both tell us the same story but in opposite ways."*[19]

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[19] Ibid.

Through the development and application of a robust set of indices, then, the cognitive bias of money illusion can be lessened, and decisions strengthened – based upon *real* rather than *nominal* buying power.

# The Index Number Problem

## Definition

Practical problems arise in developing robust and accurate inflation or escalation indices for labor, material, and acquisition programs and projects in the U.S. and abroad. In short, how exactly should microeconomic information involving hundreds or even thousands of prices and quantities be aggregated into a smaller number of price and quantity variables?[20]

To illustrate, Table 1 presents a subset of wage rates (unit prices) and headcounts (quantities) for a major North American ship-construction project in two consecutive years.

**Year 1**

| Item | Unit Price | Quantity |
|---|---|---|
| Laborer | $16 | 175 |
| Welder | $27 | 300 |
| Machinist | $31 | 250 |
| Engineer | $49 | 200 |

**Year 2**

| Item | Unit Price | Quantity |
|---|---|---|
| Laborer | $17 | 150 |
| Welder | $29 | 350 |
| Machinist | $32 | 250 |
| Engineer | $49 | 75 |

*Table 1: Raw Data for Wage Rates for a Ship-Construction Project*

All of the wage rates, with the exception of engineering, increased based on local labor market conditions or demand and supply. The headcounts changed based on needs of the shipyard as the project progressed from design to construction. How should the price changes be separated from the quantity changes to construct either a pure labor escalation index or a pure quantity-change index?

This is a classic case of the "Index Number Problem," officially stated by the Organization of Economic Cooperation and Development (OECD) as:

[20] The U.S. Consumer Price Index – Urban Consumers (CPI-U) is based on a market basket of 80,000 goods and services, thought to represent price changes of literally millions of items bought and sold in the marketplace.

*"How to combine the relative changes in the prices and quantities of various products into*

- *A single measure of the relative change of the overall* price level*, and*
- *A single measure of the relative change of the overall* quantity level*.*

*Or, conversely, how a value ratio pertaining to two periods of time can be decomposed into a component that measures the overall change in prices between the two periods – that is the price index – and a component that measures the overall change in quantities between the two periods – that is the quantity index."*[21]

## Mathematical Expression

Mathematically, the product of price (*p*) and quantity (*q*) for an item is denoted by "*value*," in the parlance of index-number theory, or

1) $v_i^t = p_i^t q_i^t, where\ i = the\ i^{th}\ item\ in\ the\ market\ basket\ in\ time\ period\ t.$

For example, *value per hour* for all the *Welders* in the shipyard in the first year = $27 x 300 = $8,100.  Depending upon the application, *value* might be material expense, labor cost (as in the table), consumption expenditure, and so on.

Summing across all items in the first year, denoted by *t = 0*, or the base year, and likewise in the second year, denoted by *t*, and taking the ratio of the two yields the total change in value (OECD's "value ratio"), in index form, from one year to another

2) $V^{0,t} = \frac{V^t}{V^0} = \frac{\sum_{i=1}^{n} p_i^t q_i^t}{\sum_{i=1}^{n} p_i^0 q_i^0} = \frac{\$24375}{\$28450} = 0.86\ in\ the\ example.$

Changes in value, then, are a function of changes in *both* price and quantity.  Herein lies the index problem, and the challenge – to develop pure price and quantity indices which isolate or hold constant the influence of the other variable in the value function, or

3) $V^{0,t} = f(P^{0,t}, Q^{0,t}), where\ P^{0,t} = price\ index\ and\ Q^{0,t} = quantity\ index.$

[21] OECD Glossary of Statistical Terms, 2008, p. 382.

In the example, total labor expense decreases in year two because far fewer high-wage-rate engineers are employed during ship construction than in design. But this says *nothing* about pure labor escalation.

## A Cautionary Note

Price indices produced and published by national statistical agencies are sometimes based on simplistic calculations of average wages over time using only two values: *total employee expenses* and *total headcount*, with one term divided by the other, due to limitations in data collection.[22] Changes in the mix of labor across occupations are consequently overlooked, with the Average Hourly Earnings series from the U.S. Bureau of Labor Statistics (BLS) a good example.[23]

Using such indices as measures of *pure price change* can lead to meaningless numbers, as demonstrated in Table 2, where electricians and janitors uniformly receive a 10% pay raise.[24]

Headcount, importantly, is *not* constant. The number of low-wage janitors goes from 10 to 5. Ignoring the change yields a labor escalation rate of 25.7%, computed as the ratio of total expenses to total number of workers. The rate goes way up since the high-priced electricians now have a bigger share in the computation of expenses.

But the true escalation rate is 10% across the board, established beforehand ($50 to $55 and $20 to $22).

The right-side section of the table provides a solution. Only by holding headcount constant (10 electricians and 10 janitors) is the true value of labor escalation computed.[25]

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[22] Ideally, the indices are based on more granular information, e.g., salary and benefits across multiple occupation titles and headcounts by occupation and grade.

[23] “Escalation in Employer Costs for Employee Compensation: A Guide for Contracting Parties,” Albert Schwenk, Bureau of Labor Statistics. By contrast, BLS’s Employment Cost Index is a fixed-employment weighted index which tracks changes in wages free from employment shifts in occupations and industries.

[24] Ibid.

[25] An alternative in this case would be to hold headcount constant using the end values (10 and 5 for electricians and janitors), and to re-compute initial expenses and the average wage rate based on these headcounts.

| Baseline | | | |
|---|---|---|---|
| Job | Hourly Wage | Headcount | Total Expense |
| Electricians | $50.00 | 10 | $500.00 |
| Janitors | $20.00 | 10 | $200.00 |
| | | | $700.00 |
| Average Wage Rate = $700/20 | | | $35.00 |
| Baseline Wages and Headcount | | | |

| Workers Receive 10% Pay Raise but Number of Janitors Cut | | | |
|---|---|---|---|
| Job | Hourly Wage | New Headcount | Total Expense |
| Electricians | $55.00 | 10 | $550.00 |
| Janitors | $22.00 | 5 | $110.00 |
| | | | $660.00 |
| Average Wage Rate = $660/15 | | | $44.00 |
| Wage Escalation = 25.7% (wrong) | | | |

| Computation with Fixed Headcount | | | |
|---|---|---|---|
| Job | Hourly Wage | Original Headcount | Total Expense |
| Electricians | $55.00 | 10 | $550.00 |
| Janitors | $22.00 | 10 | $220.00 |
| | | | $770.00 |
| Average Wage Rate = $770/20 | | | $38.50 |
| Equals 10% Escalation (right) | | | |


*Table 2: Separation of Prices and Quantities*

To recap, it's essential to understand the formulaic underpinnings of all published indices, whether the source is government agencies or private-sector management, construction, and engineering firms.

## Index Formulae

The economist and mathematician Irving Fisher investigated the properties of over 200 formulaic constructs for index numbers, ranging from simplistic averages of price relatives to his "ideal index." Table 3 presents a subset of these using the raw data from the shipyard construction project, with inclusion of a particular index based on either its foundational importance or its statistical qualities.

| Index | Formula | Shipyard Example | Explanation |
|---|---|---|---|
| **Unweighted Index Numbers** | | | |
| Dutot | $P_{Dutot}^{0,t} = \frac{(\sum_{i=1}^{n} p_{ti})/n}{(\sum_{i=1}^{n} p_{0i})/n}$ | $P_{Dutot}^{0,t} = 3.25\%$ | Ratio of average prices. Equivalent to the ratio of the sum of prices in the end period and the sum in the base period |
| Carli | $P_{Carli}^{0,t} = \frac{1}{n}\sum_{i=1}^{n}\frac{p_{ti}}{p_{0i}}$ | $P_{Carli}^{0,t} = 4.22\%$ | Arithmetic average of price relatives |
| Jevons | $P_{Jevons}^{0,t} = (\prod_{i=1}^{n}\frac{p_{ti}}{p_{0i}})^{(\frac{1}{n})}$ | $P_{Jevons}^{0,t} = 4.18\%$ | Geometric mean of price relatives |
| Harmonic | $P_{Harmonic}^{0,t} = \frac{n}{\sum_{i=1}^{n}\left(\frac{1}{\frac{p_{ti}}{p_{0i}}}\right)}$ | $P_{Harmonic}^{0,t} = 4.14\%$ | Harmonic mean of price relatives |
| **Weighted Index Numbers** | | | |
| Laspeyres | $P_{L}^{0,t} = \frac{\sum_{i=1}^{n} p_{ti}q_{0i}}{\sum_{i=1}^{n} p_{0i}q_{0i}}$ | $P_{L}^{0,t} = 3.60\%$ | Weighted by base-period quantities. Popular in EPA clauses and national statistics |
| Paasche | $P_{P}^{0,t} = \frac{\sum_{i=1}^{n} p_{ti}q_{ti}}{\sum_{i=1}^{n} p_{0i}q_{ti}}$ | $P_{P}^{0,t} = 4.73\%$ | Weighted by end-period quantities. Less intuitive than the Laspeyres index. |
| Fisher | $P_{F}^{0,t} = \sqrt{P_L P_P}$ | $P_{F}^{0,t} = 4.16\%$ | An “ideal” index which captures the phenomenon of weights changing over time |

*Table 3: Weighted and Unweighted Indices*

Indeed, several of the unweighted indices can be said to support the construction of their weighted counterparts. For example, a Laspeyres index is a weighted Carli index since both use arithmetic means. Similarly, the Paasche index is a weighted Harmonic index. Stretching slightly, the Jevons index becomes a Fisher index since both use geometric means.

## Unweighted Index Numbers

The first class of indices are unweighted. They make no attempt to adjust for changes in quantity, nor do they consider the relative importance of an item in a market basket. The Dutot Index, for example, is a *simple aggregate* which is obtained by dividing the sum of current prices in a market basket by their corresponding values in the base period.

An alternative method of comparing two sets of prices would be to calculate first a separate index or ratio for each item and then to average these. The ratios are *price relatives*, computed as a current price divided by a base-period price, or $p_t/p_0$. The averages of the price relatives, in turn, are tallied by means of any one of several measures of central tendency, such as the arithmetic mean (Carli index), the geometric mean (Jevons index), and the harmonic mean. Many, if not most, economic price adjustment clauses in contracts use price relatives, albeit *with* weights.

## Weighted Index Numbers

The second set of indices attempts to separate the influence of changes in quantity from changes in prices. A generalization of Dutot's *simple aggregate index* forms the basis for the *weighted aggregate index*, equation (4), with the weights ($w_i$*'s*) chosen such that they express the relative importance of the roles they play in a market basket of wage rates or material prices

4) $$P^{0,t}_{Dutot,\ weighted} = \frac{(\sum_{i=1}^{n} p_{ti} w_i)}{(\sum_{i=1}^{n} p_{0i} w_i)}.$$

The weights are often expressed as *quantities*, such as headcounts for a job classification or number of units of a material item for a construction project. Quantities, in turn, might be those of the based period, denoted $q_0$, or those of a given year, denoted $q_t$. The choice of either is fundamentally arbitrary.

The Laspeyres Price Index, denoted $P_L$, uses base-period weights (*time 0*) and the Paasche Price Index, denoted $P_P$, current-period weights (*time t*) in the market basket of 1 to *n* items, as shown in the table.

In terms of the value function of equation (2), the denominator of the Laspeyres Index represents total expenditures in the base period, such as material expenses or labor cost. The numerator represents what these expenses would have been in time period *t* if the same quantities had been purchased as in the base period.

The Paasche Index, on the other hand, uses current total expenses in the numerator and, in the denominator, what these expenses would have been in the base period had the same quantities prevailed.

The Laspeyres and Paasche indices have their advantages and disadvantages. The Laspeyres index is arguably more intuitive than the Paasche. It's less expensive to produce, as quantity information is difficult for statistical agencies to obtain, more so than prices. On the other hand, the Laspeyres index runs the risk of becoming less relevant if purchasing habits (the quantity weights) change over time, with base periods ranging from a few years old to even a few decades.[26]

The Fisher Price Index, denoted $P_F$, captures the notion that quantities usually don't remain constant at either end of the time span (base year and current period) but change over time. Fisher's index is expressed in expanded form in equation (5)

$$5)\quad F_{Fisher}^{0,t} = \sqrt{\frac{\sum_{i=1}^{n} p_{ti} q_{0i}}{\sum_{i=1}^{n} p_{0i} q_{0i}} \frac{\sum_{i=1}^{n} p_{ti} q_{ti}}{\sum_{i=1}^{n} p_{0i} q_{ti}}}.$$

The Fisher index is in many ways better than either the Laspeyres or Paasche. First, it does not express any preference for either of the two sets of weights. More importantly,

[26] The base period for the CPI-U, for example, is the three-year average 1982 to 1984, inclusive.

only the Fisher index, of the three, satisfies highly important criteria such as the *time reversal test* and the *factor reversal test*.

- *Time reversal test*
    - If Joe is twice as tall as John, then it stands to reason that John is half as tall as Joe.
    - Applying this simple logic to index numbers, if an index for 2030 with a base year of 2020 is equal to 200, then the same index for 2020 with a base year of 2030 should be equal to 50. That is, prices have doubled in both instances: from 100 to 200 and from 50 to 100. The index should reflect this.
- *Factor reversal test*
    - If the price of cheese doubles and the amount of cheese sold triples, then total revenue should increase six-fold, or 2 x 3.
    - In the context of index numbers, the product of a price index and a quantity index should present the value index of equation (2).[27]

*To emphasize, surprisingly enough, both the Laspeyres and the Paasche indices* ***fail*** *to fulfill these intuitively necessary and desirable properties.*

For these reasons, the authors recommend use of Fisher's Ideal Index as best-in-class in practical applications.

## Practical Considerations

Conversion of weighted inflation indices to price relatives, and vice versa, is of great practical importance in crafting Economic Price Adjustment (EPA) clauses in contracts and in developing labor and material escalation indices.

[27] Mathematical derivation is available upon request from the authors.

## Price Relatives

For example, an EPA clause for labor in a major ship-construction contract uses four international price indices, according to where the labor cost is incurred, as shown in Table 4.[28]

| Nation | Index | Weight | Source |
|---|---|---|---|
| United States | Mean wage of electrical engineers | $W_{USA}$ | Bureau of Labor Statistics |
| France | Mean labor cost – electrical and mechanical industries | $W_{France}$ | National Institute of Economic and Statistical Studies |
| Canada | Average hourly earnings for machinery manufacturing | $W_{Canada}$ | Statistics Canada |
| Germany | Industrial employees in the metal industry | $W_{Germany}$ | Statistisches Bundesamt |

*Table 4: Labor Escalation Indices in an EPA Clause*

Mathematically, the aggregate labor escalation index is represented by equation (6), a base-period weighted arithmetic mean of prices relatives.

$$6)\quad EPA_{Labor}^{0,t} = \left(\frac{p_t^{USA}}{p_0^{USA}}\right) w_{USA} + \left(\frac{p_t^{France}}{p_0^{France}}\right) w_{France} + \cdots + \left(\frac{p_t^{Germany}}{p_0^{Germany}}\right) w_{Germany}.$$

The price relatives are the ratios of the indices in a country from one time period to another; that is, current to base. The weights are each country's percent of total expenditures in the base period. For example, a weight of 5% for the USA means that 5% of total labor cost is incurred by American labor. The weights sum to 100%.

Similarly, a material EPA clause in the same contract uses five international indices, also weighted by base-period relative expenditures.

[28] Details of the indices and their weights are omitted due to the business-sensitive nature of the data.

## Conversions

Interestingly, the specification of equation (6) is a Laspeyres index in disguise. Equations (7) and (8) provide the proof.

First, in equation (7), a unitary term is inserted in the numerator. The terms are then re-arranged in equation (8) to yield an expression for weighted price relatives.

7) $$P_L^{0,t} \equiv \frac{\sum_{i=1}^{n} p_{ti} q_{0i}}{\sum_{i=1}^{n} p_{0i} q_{0i}} = \frac{\sum_{i=1}^{n} p_{ti} q_{0i} (p_{0i}/p_{0i})}{\sum_{i=1}^{n} p_{0i} q_{0i}}$$

8) $$P_L^{0,t} = \frac{\sum_{i=1}^{n} p_{0i} q_{0i} (p_{ti}/p_{0i})}{\sum_{i=1}^{n} p_{0i} q_{0i}} = \sum_{i=1}^{n} w_{0i} \left(\frac{p_{ti}}{p_{0i}}\right), where$$

$$w_{0i} = \frac{v_{0i}}{\sum_{i=1}^{n} v_{0i}} = \frac{p_{0i} q_{0i}}{\sum_{i=1}^{n} p_{0i} q_{0i}}, and\ where$$

$$v_{0i} = the\ value\ (price\ x\ quantity = expenditure)\ of\ an\ item\ in\ the\ base\ period.$$

These weights, then, are exactly those used in the EPA clause, or an item's expenditure relative to total expenditures for all items.

Alternatively, to close the loop, the EPA clauses could have used current-period rather than base-period weights. The Paasche Price Index is a current-period harmonic mean of price relatives, as shown in equation (9), where $w_{ti}$ is the expenditure weight in the current (or end) period for the $i^{th}$ ratio of price relatives.[29]

9) $$P_P^{0,t} \equiv \frac{\sum_{i=1}^{n} p_{ti} q_{ti}}{\sum_{i=1}^{n} p_{0i} q_{ti}} = \frac{1}{\sum_{i=1}^{n} w_{ti} \left(\frac{p_{ti}}{p_{0i}}\right)^{-1}}.$$

[29] Proof is available upon request from the authors.

## Implications

Many indices, in practical applications, are built according to equation (6); that is, as weighted price relatives. The construct is common not only in contracting and collective bargaining agreements, but in cost estimating and analysis as well.

For example, equation (10) shows a material escalation index for Class Standard Equipment (CSE) developed recently to support an Independent Cost Estimate (ICE) for construction of U.S. *Arleigh Burke Class* destroyers. The market basket of CSE consists of 61 distinct items ranging from high-cost gas turbine engines and controllable pitch propellers to lower-cost pumps and valves.

$$10) Material_{CSE}^{0,t} = \left(\frac{p_t^{Engines}}{p_0^{Engines}}\right) w_1 + \left(\frac{p_t^{Propellers}}{p_0^{Propellers}}\right) w_2 + \cdots + \left(\frac{p_t^{Item\#61}}{p_0^{Item\#61}}\right) w_{61}.$$

For material indices or construction projects, the quantities purchased might change little, sometimes not at all, if the time span between the base and current periods is short, say less than half a dozen years. The *Burke* destroyers *always* require four gas turbine engines per ship set, at both ends of the time span and at all points in between. *But*, if the engines become relatively more expense than other items, then their end-period weight will change, as will the weights of all other items (since the weights sum to 100%). And, for longer periods, it’s unlikely that constant quantities will prevail due to technological innovation in materials, processes, or both.

For labor indices, on the other hand, weights and quantities are very seldom constant, even for short time spans. Headcounts can and do change significantly. Therefore, Laspeyres, Paasche, and Fisher weighted indices yield different values.

Great care must be taken, then, in the interpretation of indices built as weighted price relatives, as in equation (6), since they’re Laspeyres indices in disguise. While better than indices that make no attempt to hold quantity constant, they’ll lack some of the desirable properties of a Fisher index.

# Escalation Measurement

## Overview

Several steps are used to measure escalation for labor, material, select components of overhead, and projects in general, and to generate probabilistic forecasts, including:

- Develop unique indices for a basket of items based on an extensive data collection effort
- Leverage the linkage between core inflation in the macro economy and unique escalation, based on an evaluation of historical data, and using metrics such as Compound Annual Growth Rates (CAGR's)
- Deflate historical escalation indices using the CPI-U, the most widely used measure of inflation in the economy
- Compute deltas between unique escalation and macro inflation
- Leverage market-implied forecasts of macro inflation to generate point and probabilistic estimates of escalation up to thirty years out.

A key step in the process, then, is to accurately measure escalation in the first place and assess its historical linkage to the CPI-U.[30] But practical problems arise. These include: which commodities or items to include in the index, how to determine prices, how to determine weights, and which formulaic construct to employ.

[30] Forecast methodology and issues are discussed in the next section.

## Data Collection

Many organizations such as the U.S. Department of Defense have traditionally allocated scant resources to the daunting task of developing concrete, repeatable, *sampling processes* and *index formulae* which summarize accurately and parsimoniously the changes in prices and quantities of tens if not hundreds of thousands of items bought and sold in particular sectors of the economy, such as shipbuilding or aircraft production.

In attempting to fill this capability gap in several practical applications across federal departments and international boundaries, the authors sought to build escalation indices and to generate forecasts using the general taxonomy of Figure 2.

Given the theoretical issues discussed previously in the paper, an emphasis was placed on collecting as much detailed data as possible to develop Fisher indices.

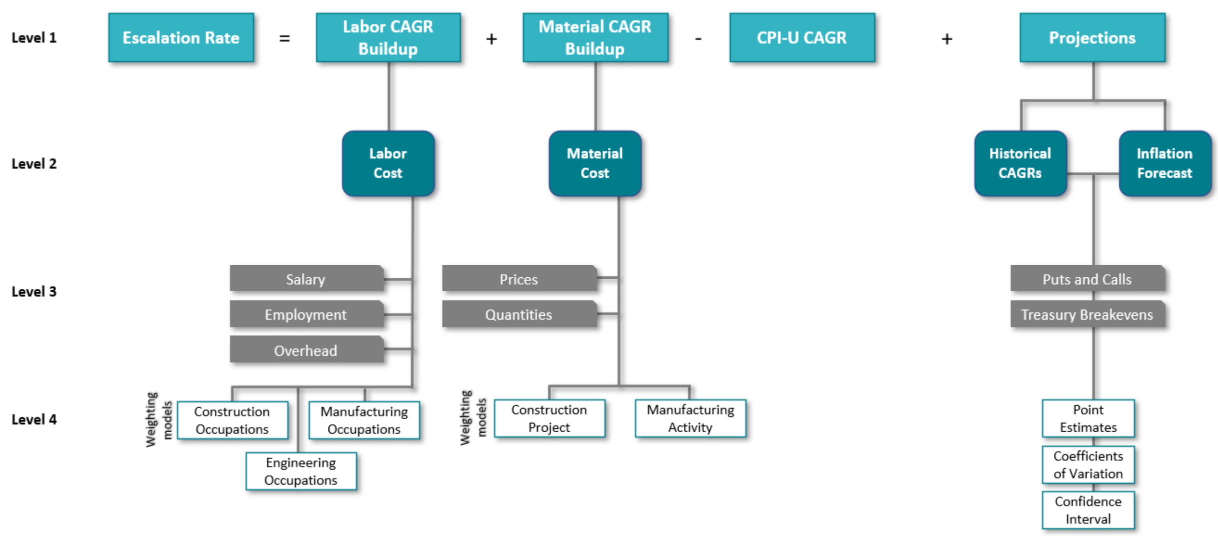


*Figure 2: Taxonomy for Data Collection*

Actual prices, headcounts, and quantities were obtained whenever possible for various categories of labor and material. The result in these instances: high-quality escalation metrics.

In other cases, however, data was difficult or impossible to obtain given resource constraints or shortcomings in accounting systems.  Reliance on escalation measurements from government organizations such as the U.S. Bureau of Labor Statistics or Statistics Canada was a next-best solution.

Data was also collected for the CPI-U to (1) de-escalate unique indices and to (2) support forecasting.

The CPI-U is arguably the most widely known and used measure of inflation in the U.S. The index directly affects the income of almost 80 million Americans based on statutory requirements: Social Security beneficiaries, military and Federal Civil Service retirees and survivors, and food stamp recipients.[31]  Further, over two million workers who are covered by collective bargaining agreements receive wage-rate increases tied to the CPI-U.[32]

Importantly, the CPI-U is *the* metric used in trillion-dollar markets to adjust inflation swaps and options, which, in turn, enable the estimation of PDFs for forecasts.

## Data Aggregation

The domain for which an escalation index is calculated needs to be defined carefully.  It could be, for instance, labor escalation for an aerospace company or material escalation for a hydro-electric dam.  Figure 3 shows the labor and material components of a construction project.  It's important to group like items together into separate buckets, such as engineering labor or manufacturing labor, with many job codes within each. Then, the buckets are weighted according to their proportional expense.

But herein lies an issue, referred to as "consistency in aggregation."  Ideally, the sub-indices for a bucket yield the same value, when summed, as if the domain aggregate had been calculated directly in a single operation.  Consistency, in turn, depends upon the mathematical form of an index.

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[31] U.S. Bureau of Labor Statistics.
[32] Ibid.

In Figure 3, for example, let *D* represent the total domain of all goods and services in the construction project, split into *K* domains, *d* = 1, 2, …, *K*.

Then, a Laspeyres index for the domain, *D*, can be calculated as

- $P^{0,t}_{Laspeyres,D} = \sum_1^K P^{0,t}_{Laspeyres,d}\, s^0_d, where\ s^0_d$ = the value share of domain d within the total market basket D.

The Laspeyres index, then, is consistent in aggregation. So is the Paasche, and by implication the Fisher since it's the geometric mean of the two. But, all this said, in practice it's unlikely that all sub-indices are always created using the same formulaic construction. Some may be Laspeyres indices, others Paasche indices, and yet others Fisher indices. And yet others may be none of these because of limitations in data collection. The bottom line: in common practice aggregate domain indices are sometimes approximations to a pure, theoretically end state.

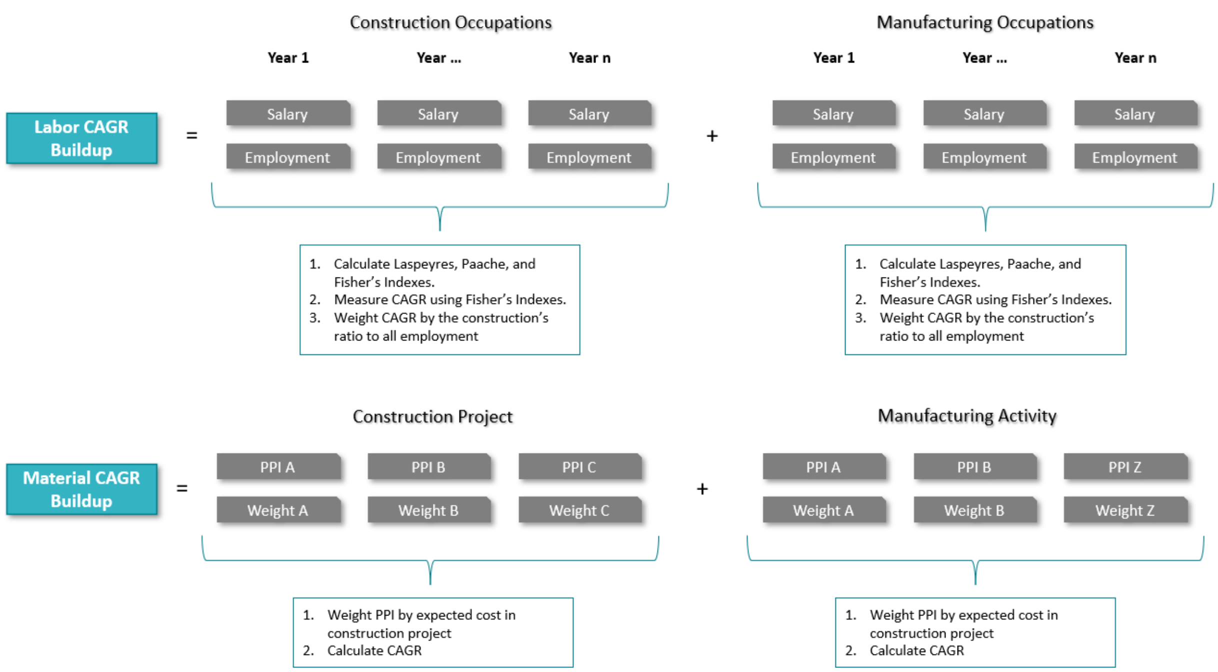


*Figure 3: Example of Data Aggregation*

### Linkage to the CPI-U

Based on recent research by the authors, here are some examples of escalation-rate deltas, above and beyond the annual rate of increase in the CPI-U, measured in basis points.

- Health Care. 100 to 150 basis points over the last decade, using BLS's "Employer Cost of Employee Compensation – Heath Care" as a gauge. The index, using a Fisher formulaic construct, measures a firm's cost-per-employee hour worked for employee healthcare compensation.
- Labor Rates. Defense wage-rate increases for selected occupations can sometimes run roughly 50 to 100 basis points above core inflation, based on data from the prime contractors. In other cases, however, the rates roughly match the CPI-U. Much, then, seems to depend on company need and the local job market.
- Material Items. High-end construction projects can run up to 100 basis points above the CPI-U, on average.

## Forecasts

### Risk and Uncertainty

For *whichever* indices are employed in life-cycle cost estimates, the forecasts of their out-year (or out-turned) values remain problematic and a major issue in generating realistic S-curves in nominal terms. Simply employing a triangular distribution, with endpoints taken as plus and minus a percentage-point or two from the mode, seems wholly inadequate. A common practice is to take +/- 10% of mean values based on historical trends.

This paper offers survey- and market-based measures for the estimation of future probability distributions of inflation, with the underlying framework pictured in Figure 4. The financial instruments in the U.S. marketplace use the Consumer Price Index - All

Urban Consumers (CPI-U), which covers 93% of the population, as their metric for inflation adjustments.[33]

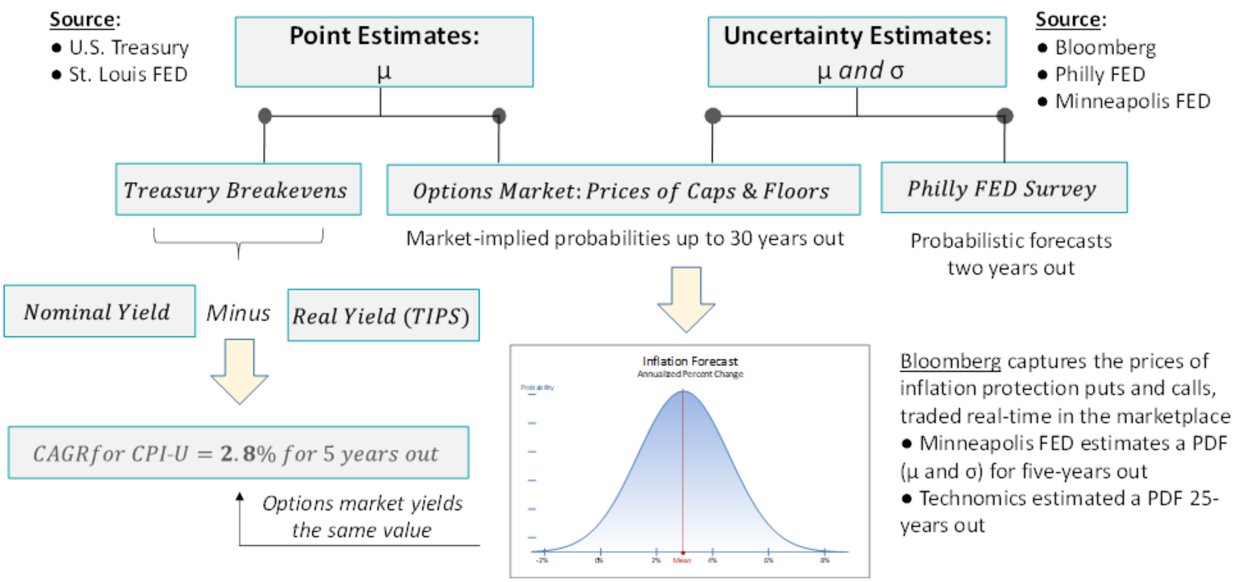


*Figure 4: Framework for Inflation Forecasts*

Inferences are drawn from analyses of *survey-* and *market-based* measures of inflation expectations. *Survey-based* measures examine projections of inflation made by government officials and professional forecasters. *Market-based* measures, on the other hand, focus on expectations inferred from the prices and yields of financial instruments actively traded on Wall Street, such as Treasuries and, more recently, inflation derivatives.

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[33] While the focus here is the U.S., similar financial instruments exist in other countries. For example, the U.K.'s version of U.S. "Treasury Breakevens" is the delta between nominal gilts (bonds) and index-linked gilts, with the latter using their Retail Price Index (RPI) rather than their CPI. This market was created in 1981.

Further, both the U.K. and the European Union have robust options markets for inflation protection. Both pre-date the U.S.'s, having reached critical mass in 2003 versus 2009 in the U.S. Interestingly, the European Central Bank (ECB) sponsors a "Survey of Professional Forecasters" for EU countries, as does the U.S. Philly FED, with both surveys identically named.

As Economists note, focus sharpens and credibility rises when prices are set by market agents that bear financial risk, such as pension, insurance, and hedge-fund managers.

- Survey-Based Measures
    - The Survey of Professional Forecasters. Administered by the Federal Reserve Bank of Philadelphia, this survey provides explicit observations on long-term *mean* inflation expectations, and, importantly, enables construction of near-term *probability distributions* of inflation as well.
- Market-Based Measures
    - Treasury "Breakevens." The delta between nominal and real yields, *after* adjusting for risk and liquidity premiums, provides observations on the market's expectation of inflation up to 30 years out.
    - Inflation Derivatives. Zero-coupon swaps in this nascent, *laissez faire* options market provide data on the expectations of market players hedging against the risk of inflation. "Cap and floor" data enable the construction of inflation probability density functions based on the daily interaction of agents who buy and sell protection against rising and falling price levels in the macro economy.[34]

These distinct but related areas of investigation yield different but complementary measures of *mean* inflation expectations, and, in two cases, legitimate measures of *variance*. Together, they enable the development of a range of scientifically-sound, historically-based, and *market-consistent* values to employ in risk and uncertainty analyses, such as depicted in Figure 5.

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[34] Estimating option-implied PDFs for inflation futures or equities in general relies upon seminal work by Breeden and Litzenberger. The discounted, present value of the price of a call option, c(*K*), with a strike price *K* and an expiry data *t*, is related to the probability density function $f(K) = e^{rt} \frac{\partial^2 c(K)}{\partial K^2}$, where *r* is the discount rate. That is, the probability density function is proportional to the second derivative of the call-price function with respect to the strike price. For most assets, including inflation caps and floors, option prices are observed at a finite number of strike prices and expiry dates. Once a call-price function has been interpolated, implied PDF's can be obtained by numerically calculating the second derivative. Mathematical derivations are available from the authors in addition to a real-world example of the calculations using 25-year inflation puts and calls, with data obtained courtesy of Bloomberg and the Federal Reserve Bank of Cleveland.

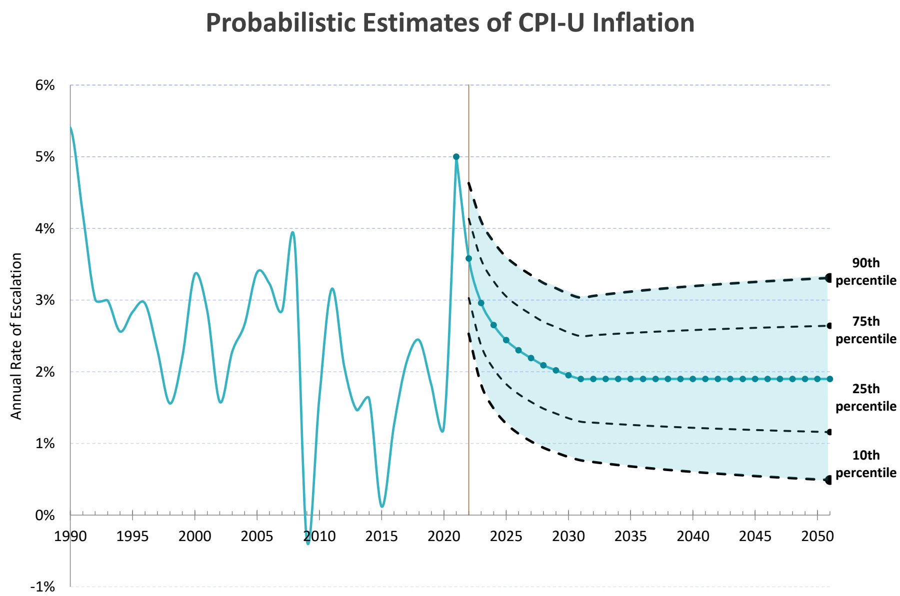


*Figure 5: Probabilistic Estimate of Macro Inflation*

These values, in turn, support estimation of realistic S-curves, in *then-year or out-turned currency*, for life-cycle cost estimates.

Importantly, the market-based measures yield Compound Annual Growth Rates, say five or ten years out, with the value of 2.8% shown in Figure 4. But with inflation rates running higher near term, this implies that values must eventually *fall below* the 2.8% mark for the average to hold over the five-year period.

Mathematically, as equation (12) shows, the level of prices is increasing monotonically throughout the forecast period. But, as noted in equation (13), the rate of increase *decreases* over the next decade. The second derivative is roughly zero thereafter, upon which time the markets reflect a long-term inflation target of slightly less than 2% per annum, in accordance with central bank objectives in the U.S. and abroad.

11) $\frac{dCPI}{dt} > 0 \; for \; all \; t, where \; t = time.$

12) $\frac{d^2CPI}{dt^2} < 0 \; until \; \approx t = 2030.$

13) $\frac{d^2CPI}{dt^2} = 0 \; for \; t > 2030.$

Variance, on the other hand, steadily increases as demonstrated in Figure 5 by wider and wider numerical bands of inflation rates (on the y-axis) covered by a confidence interval, say, for example, between the 90th and 10th percentiles. That is, the longer the forecast horizon, the more uncertain are the estimates of inflation, or of any economic aggregate, for that matter.[35]

## Linkage to Unique Indices

In many cases, the CPI-U point and probabilistic forecasts of Figure 5 can be used without adjustment. In other cases, however, deltas to the point estimates must be applied. Figure 6 shows a nominal 100 basis point increase over the 2% CPI-U baseline, to yield a steady-state value of 3% per annum.

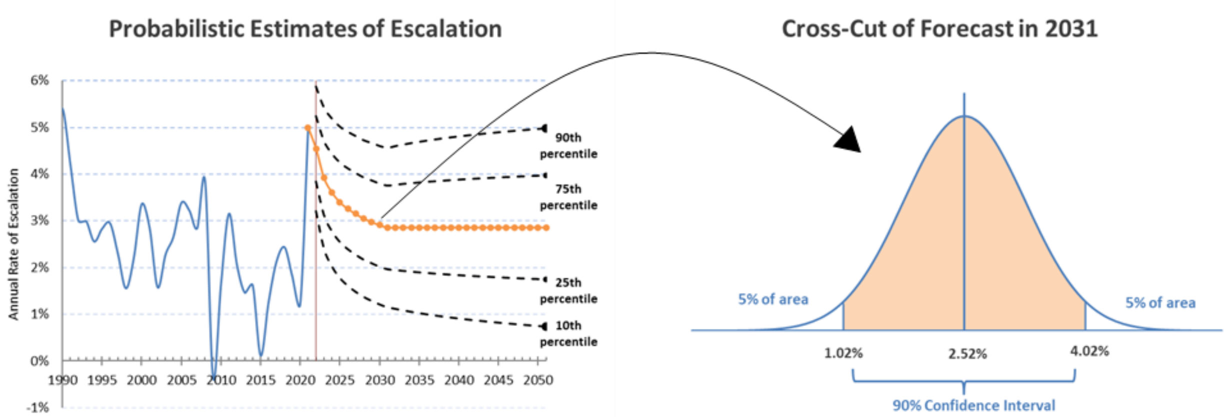


*Figure 6: Linkage to Unique Indices*

## Applications

Our research has important and varied applications that enable better decision-making at different levels of management and points in time. These applications include:

- Planning, Planning, Programming, and Budgeting
    - Enables risk-based funding decisions at the portfolio level. That is, a menu of escalation rates and their probabilities of occurrence are

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[35] This salient fact was made clear to Dr. Flynn several years ago in a meeting with Drs. D'Angelo and Lee at the Federal Reserve in Washington, D.C. They provide analytical support to the Board of Governors.

presented. Decision makers then choose which point estimate to use, and with what level of risk, as illustrated in Figure 7.

  - Supports the establishment of contingency funding for escalation, defined as the funding delta associated with a P50 (e.g., a baseline) and a higher cumulative percentage, say at the P80. The choice of the upper bound is highly dependent upon the volatility of inflation in the economy
  - Enables benchmarking between sites, projects, and business segments, and organizations

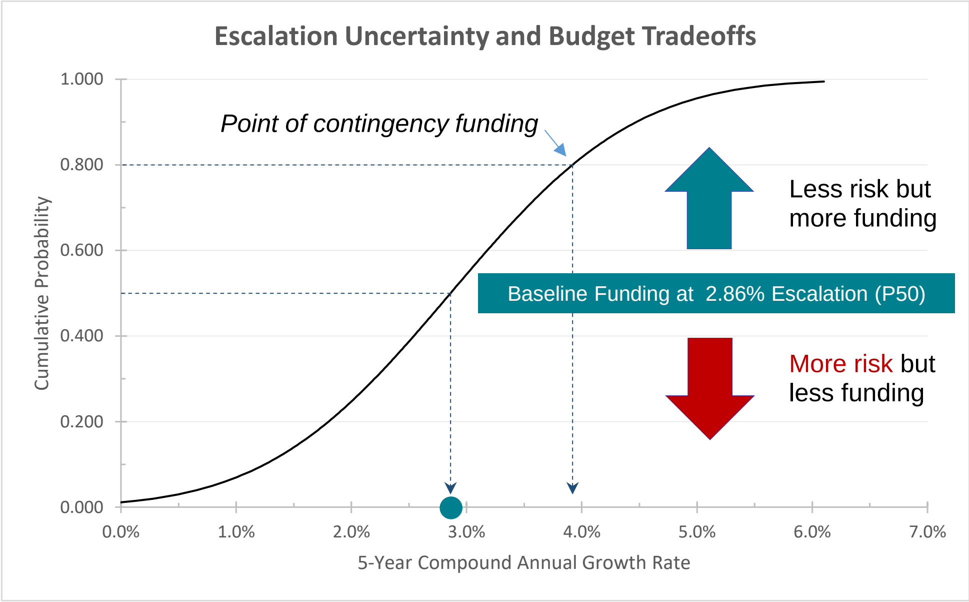



*Figure 7: Risk and Funding Tradeoffs*

- Analysis of Alternatives (AoA's)
  - Supports point estimates and risk analysis
  - Better differentiates between alternatives, especially when their compositions and time spans vary

- Life-Cycle Cost Estimates
  - Supports point estimates and risk analysis

- Cost Controls
  - Supports the creation of cost controls for programs and projects, such as threshold and objective values for labor and material escalation.

The escalation methodology, techniques, and best practices presented in this paper will help organizations develop estimates that exhibit the four characteristics of a reliable estimate -- accurate, credible, comprehensive, and well-documented -- per the Government Accountability Office (GAO) *Cost Estimating and Assessment Guide.*[36]

- Accuracy. Escalation models are ideally built using actual historical data for labor and material. They are site or business-segment specific, taking into consideration different labor markets and high-cost versus low-cost areas.
- Credible. The models use data-driven methodology. They eschew the use of 3rd party "black box" indices for escalation.
- Comprehensive. Methodology addresses all components of escalation across different types of labor and material and projects.
- Well-documented. Methodology emphasizes visibility, transparency, and repeatability, fully documented in point papers, workbooks, and slide decks.

## Summary and Conclusions

### Summary

*"A currency, to be perfect, should be absolutely invariable in value,"* said David Ricardo in 1817. Stable money means money with stable value. In theory, the most important feature of the modern world's fiat money (e.g., the dollar, euro, Russian ruble, and Chinese yuan) is the supposed stability of its value, unlike the more volatile commodity-based money (e.g., gold, copper, silver, and more recently crypto-currency).

But, as our research has emphasized, money is anything but stable, the phenomenon of Money Illusion notwithstanding. A dollar is not a dollar, and a euro is not a euro in terms of what they can buy. There's only one clear and unequivocal indication by which

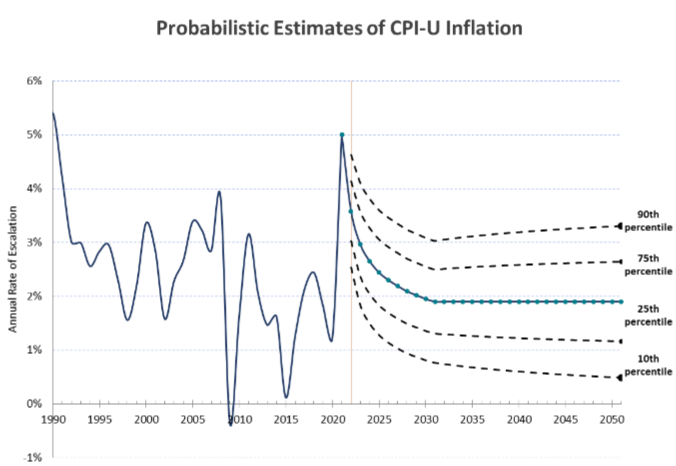
[36] "Cost Estimating and Assessment Guide," U.S. Government Accountability Office.

to judge whether a currency is depreciated, and to what extent.  That indication is an index number.

That said, the over-all problem of the construction of reliable and unbiased index numbers, or “*Minding Your P’s and Q’s*,” is not an easy one.  The main difficulties lie in the selection of the basis of the comparison, in the choice of the commodities or other quantities which are to be included, and in the selection of the most appropriate weights.

Some of these issues are of a mathematical nature, such as the formulaic construct employed and its statistical properties.  Our research recommends use of Fisher’s index for most practical purposes for creating and comparing economic data – labor rates, material prices, and construction projects.

Most of the issues, however, relate to stiff challenges in data collection.  Our research has related the authors’ experience developing unique escalation indices for domestic and international clients, and forecasting their future values.  Figure 8 summarizes our approach.




**Historical Data:**

- Collected historic national CPI-U indexes and calculated Compound Annual Growth Rate (CAGR)
- Calculated a delta between historical escalation to general inflation (Consumer Price Index – Urban Workers [CPI-U]) for 2011-2021

| | CPI-U CAGR | Average Delta |
|---|---|---|
| 10-Yr (2011 – 2021) | 1.82% | **+.50%** |
| 5-Yr (2016 – 2021) | 2.34% | **N/A** |

**Projections:**

- Applied market-based models to calculate point estimates for CPI-U inflation out 30 years
- Also produced probability distributions of projected inflation rates
- Used the historical deltas to forecast composite escalation (labor, materials and overhead)

*Figure 8: Escalation Measurement and Projections*

## Conclusions

Our paper identifies a clear, groundbreaking path to advance the treatment of inflation on future DoD and other federal government budget formulations and investment planning initiatives. While data to support rigorous program-specific evaluation of inflation is often not readily available, a concerted data collection plan to capture price pressures specific to federal programs should yield further advances in the methodologies.

At the margin, it's far more important to expend resources in the collection of an additional year's worth of *actual* data than using an imprecise "canned" index as a substitute.

The continued impact of economic policies that directly impact the global reserve currencies should be monitored, and future analysis may focus on the impacts of further fiscal stimulus or the dampening effects of tighter monetary policies in the near term. The changing influence of emerging reserve currencies, such as the Chinese Yuan, may further impact inflation considerations in the future. Supply chain pressures are another key consideration for future research. While there is optimism that infrastructure investments will correct the inefficiencies that have been exposed due to the ongoing pandemic and a rise in global economic nationalism,[37] continued supply-chain bottlenecks may have long-term impacts on purchasing power and may lead to a re-allocation of investment resources within the U.S. (potentially putting addition upward pressure on prices).

As the topic of inflation becomes more commonplace in the public sphere, the proper treatment of changing price levels will become ever more important when planning for future public sector investments. As has been demonstrated, out-year planning for large-scale programs is critically dependent on a thorough understanding of prices and their corresponding quantities.

[37] "Why the impressive page of investment growth looks likely to endure." The Economist, Feb 5, 2022.