8D Cost Trades With Entanglement

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Abstract—Many products have sizable components that form a large portion of their costs. In such cases, it is crucial for both the suppliers and the offerors of the ultimate products to be working together to common goals. It is possible to display such interactions in as few as two dimensions, and many firms might seek such answers as they are easily constructed and understood. However, as this paper demonstrates, seeking easy answers by artificially reducing the scope of the problem can lead producers astray. It is possible to get all the costs right and still sink a project. This paper proposes a construct with more dynamic elements, as it uses eight dimensions to understand how jets and their engines can work in tandem to enhance sales. This specific example generalizes to other markets.

Contents

1. Introduction

Jets and jet engines. One can't move without the other. What happens when these interactions are not fully understood? This issue is not a hypothetical question, nor one without an answer. Recently, an example occurred in the business jet market, with more than $1B lost on a single project.

Texas billionaire Robert Bass founded Aerion in 2003 and began developing what became their Aerion AS2 in 2004. In December 2020, I wrote on LinkedIn that the plane was worth every penny of its $120M price tag, but there were not enough pennies in the world to hit its demand target. Aerion wrote a firm retort days later, claiming new orders. I repeated my position, citing my evidence.

The company halted development in May 2021 and into liquidation that September.

They're not writing me anymore.

How can we validate both its cost and price but confidently invalidate the project in advance? It turns out that it is possible to describe the business jet market in four dimensions, and that for its engines with the same number, less the one common price dimension both share, for a total of seven. Time adds the eighth dimension.

This paper studies how these eight dimensions interact as they entangle.

2. Historic Context

Paul Samuelson, considered by many the father of modern economics and the 1970 winner of the Nobel Memorial Prize in Economic Sciences, had definite thoughts about price determination. He wrote that the law of supply and demand meant that "the equilibrium price, i.e., the only price that can last…must be at this intersection point of supply and demand curves."[1]

Every introductory text in economics has this paradigm in it, in one form or another, though those examples are uniformly hypothetical. Where do we find these relationships in the real world?

We can see a modern example in Figure 1, where costs rise from mine to mine in the market for iron ore.[2] After adding a profit margin above their costs, the mines collectively form an upward-sloping demand curve, a hallmark of modern economic analysis.

[1] Samuelson, Paul A., *Economics, 9th Edition,* McGraw Hill, 1971, p. 63

[2] https://www.rba.gov.au/publications/smp/2015/feb/graphs/graph-a2.html

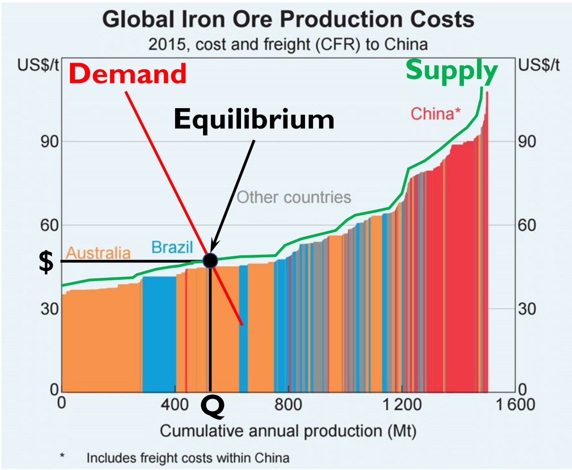

Figure 1 – Iron Ore Market Equilibrium

But the market for a commodity such as iron is quite different from other markets that use it. Aircraft have iron in them. The Wright Brothers made the first aircraft sale in February 1909, when they contracted with the US Army to provide one Model A Flyer for $25,000.[3] As in Figure 2, classical economics would say the market is in single-point equilibrium with this lone exchange in the market. But months later, when Glenn Curtis sold a second airplane[4] in June 1909 for $5,000, that put two distinct points in that market, as Figure 3 reveals. This observation is sufficient to negate the law of supply and demand. So, the question becomes: What replaces it?

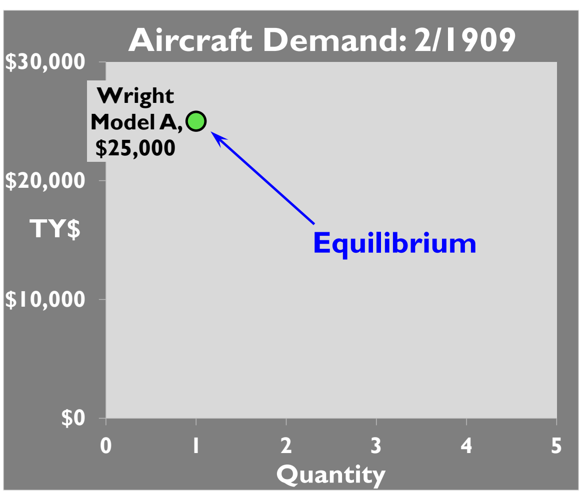

Figure 2 – Aircraft Market Equilibrium

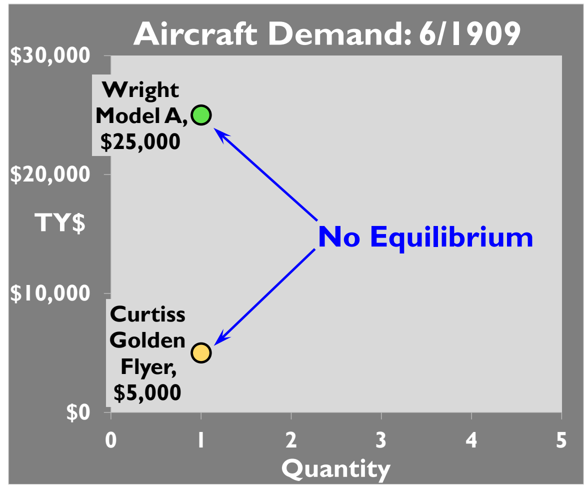

Figure 3 – Aircraft Market Disequilibrium

Observing the market is in disequilibrium does not suggest that it is in disarray. It merely notes that we have not accounted for other forces at work. We'll investigate those in a bit. Right now, we'll study demand in more detail.

3. The Known Twin

With any kind of luck, identical twins know their sister or brother their entire life. Typically, each of them would have a solid bond with the other. It would be hard for one to comprehend the other being unknown to them. In this modern world, despite their close bonds, though, it might be possible for one of them to have a much higher media presence than the other, making one of them effectively invisible in that realm, unknown to the public.

Let's give the twins' names and jobs to make this example more tangible.

At right, Cristina is an American expatriate living in Argentina. She models how costs and prices fall as quantities increase across a variety of markets. She is especially interested in the markets for business jets and the turbofan engines that power them.

Figure 4 – Cristina Models Learning & Demand

[3] https://www.britannica.com/technology/aerospace-industry/History

[4] https://en.wikipedia.org/wiki/Glenn_Curtiss

We get a unique type of estimator in Cristina, from Figure 4, as she's studying aerospace learning and demand curves. Based in Argentina, Cristina got a bad taste in her mouth when she heard about the law of supply and demand. No amount of locally sourced Argentinian Malbec could rinse it out. She couldn't shake it out of her limbs with a brisk ride across the Pampas, dancing the tango, or jumping in a stadium watching Lionel Messi playing football.

It stuck in her head as an anomaly. It was certainly something she wanted to explore in detail.

She never believed in the single-point equilibrium theory. She reasoned that there are dozens of business aircraft, from the smallest that can squeeze a few people into them up to converted airliners that seat hundreds. Also, there were ads for private turboprops for a few million dollars at the low end of this market and converted jumbo jets for nearly a quarter billion dollars up at its top, with speeds ranging from a couple of hundred miles per hour up to high subsonic models.

There couldn't be a sole point that described them all in a mathematically valuable way.

She plots the market's quantities sold by model and the prices they command. To make the analysis more complete, she adds turboprops to all the business jets she collects in her database in Figure 5, assembling data on 95 models over ten years.

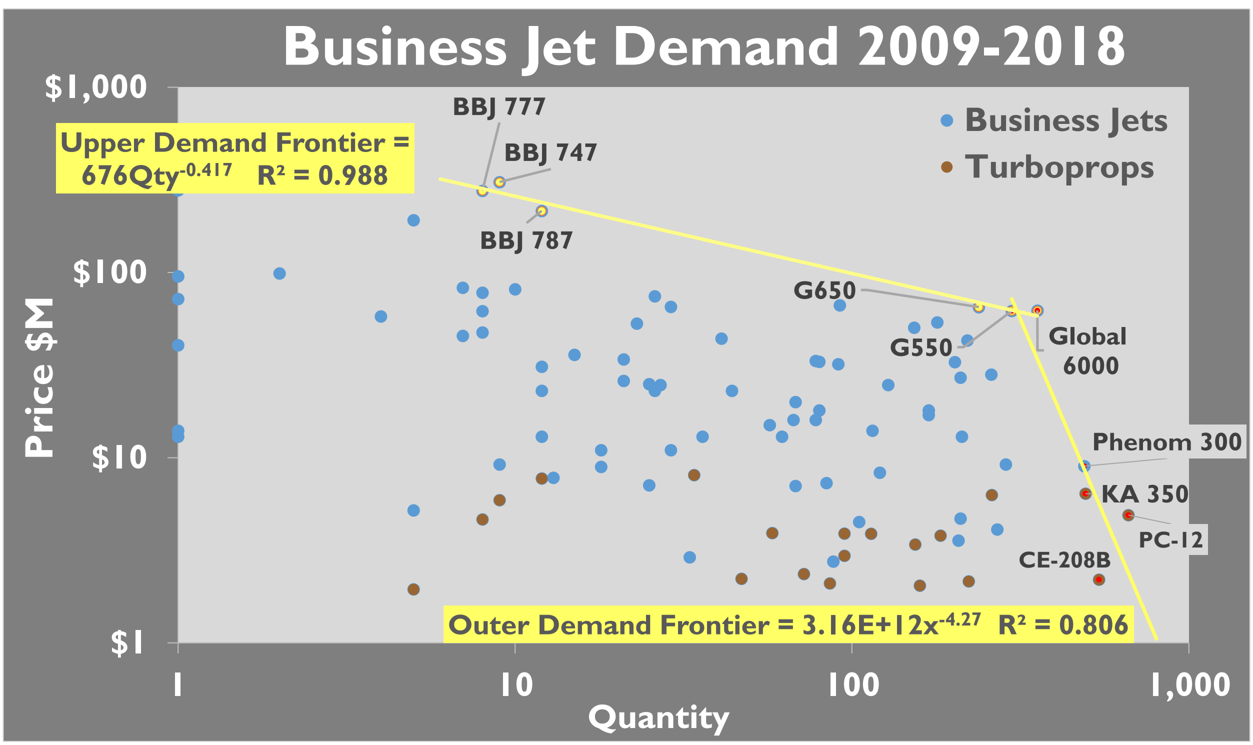

Figure 5 – The Business Aircraft Market has well-defined Upper and Outer Demand Frontiers

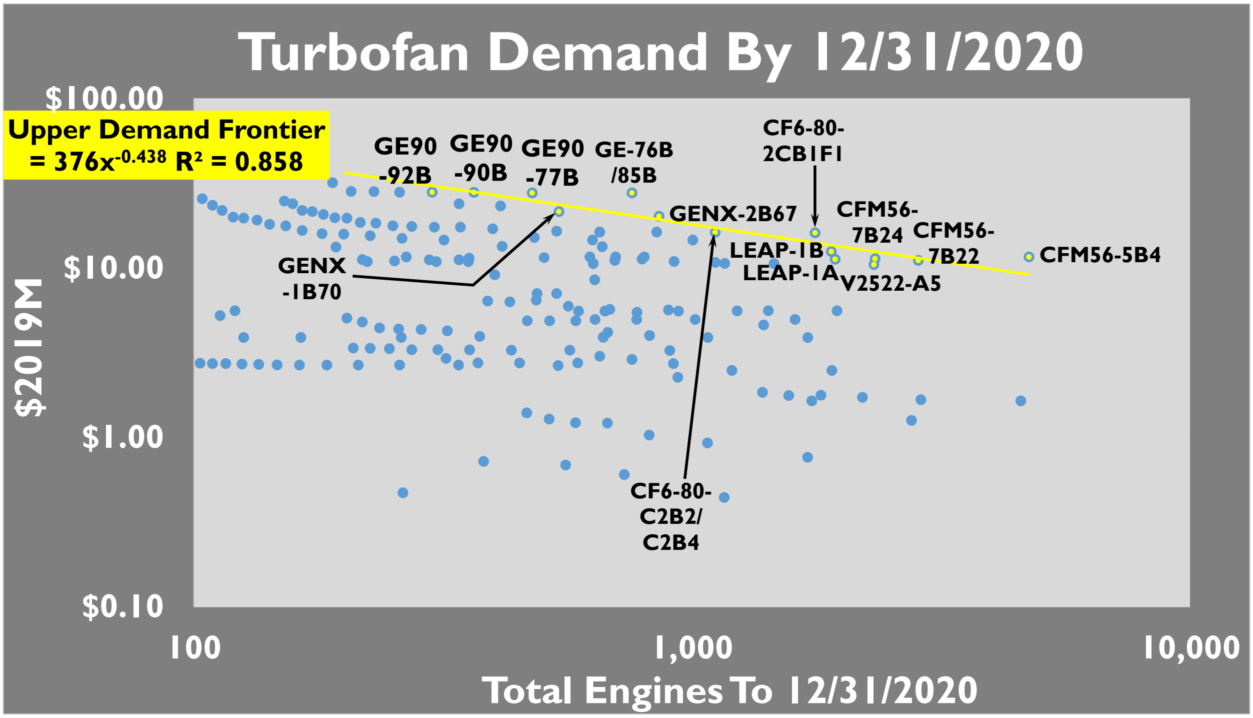

Figure 6 – The Upper Demand Frontier for Turbofan Engines is statistically significant

When Cristina plots her data, not only can she refute the hypothesis of a single-point equilibrium, but she also discovers some other interesting phenomena. Figure 5 reveals the market has at least a pair of self-organizing features, both of which are statistically significant. Along the higher reaches of the business aircraft market, there is an Upper Demand Frontier, as Equation 1.

$$2019\$M = \$676.4 * Quantity^{-0.417} * \varepsilon \qquad (1)$$

Where:

2019\$M = Predicted Upper Demand Frontier \$ for business aircraft
Quantity = Aircraft sales 1/1/2009 to 12/31/2018
ε = Error term for the equation

Adjusted for bias using the Ping Factor (as all equations are in this piece, thus, that factor won't be noted again), the Equation 1 curve represents a limit to how much money the market has within it to buy the highest-priced aircraft. With a P-Value of 5.39E-05, an adjusted R2 of 98.5%, a standard error of \$20.5M, and an especially low Mean Absolute Percentage Error (MAPE) of 6.9%, players in the market should not ignore it. Manufacturers operating in that region should also take note of its relatively flat slope of -0.417. That indicates that price reductions offer the chance for more than proportional increases in sales – cases in point include the Global 6000 at the right end of this curve generated more than eight times the revenue than the B777 at its left.

Cristina finds this market bounds itself concerning quantities sold with an Outer Demand Frontier, too, as represented by Equation 2.

$$2019\$M = \$3.16E{+}12 * Quantity^{-4.27} * \varepsilon \qquad (2)$$

Where:

2019\$M = Predicted Outer Demand Frontier \$ for business aircraft
Quantity = Aircraft sales 1/1/2009 to 12/31/2018
ε = Error term for the equation

Equation 2, with a P-Value of 1.52E-02, is statistically significant, though not nearly as well correlated as Equation 1. It has an adjusted R^2 of 75.7%, a standard deviation of \$14.3M, and a MAPE of 63.1%. This line means that at its limit, the market has absorbed as much product as it can for a given period, which is ten years for the case at hand.

Cristina finds the slope of the Upper Demand Frontier especially intriguing. If the manufacturer's cost structure can support the potential increase in revenue due to price decreases could improve profits. Of course, airframers will look to their suppliers to help to offer such prices. As engines make up a significant cost component of business aircraft, she decides to study the market for turbofan engines.

The engine manufacturers' prices are costs to the airframers.

The turbofan engine market has many more models, and Cristina finds 186 distinct models that were active at the time of the assembly of her database. Those points form the blue dots in Figure 6. She observes another self-organizing Upper Demand Frontier for the turbofan engine market, which we can characterize as Equation 3.

$$2019\$M = \$376.3 * Quantity^{-0.437} * \varepsilon \qquad (3)$$

Where:

2019$M = Predicted Upper Demand Frontier $ for turbofan engines
Quantity = Aircraft sales 1/1/2009 to 12/31/2018
ε = Error term for the equation

Equation 3 mimics the like one for business aircraft, as it too has a flat angle across log-log space. At the same time, its adjusted R^2 of 84.9% is not as well-correlated as the same curve for business aircraft. We need to recognize its deeper meaning with its P-Value of 1.76E-06, MAPE of 8.3%, and standard deviation of $2.72M. That is, in this market, as we found in the one for business aircraft, price reductions may be met with proportionally more significant revenue increases, as long, that is, as those engines can find willing airframers to use the models in question.

After all her work, Cristina found that no single point equilibrium exists for the business aircraft or turbofan engine markets. Such a curve would mean costs increase with the number of units (see Figure 1). However, if that were to apply to business jets and their turbofan engines, that would mean that the builders of such devices take more time with successive units; they lose learning as they go along, essentially becoming dumber.

Surely, Cristina reasoned, that cannot be the case. She knows people get smarter over time; that's what learning curves confirm.

But, if upward-sloping supply curves intersecting downward-sloping demand curves do not determine prices, what does, she wonders? She decides to contact her twin expatriate sister.

4. The Unknown Twin

Just as the southern hemisphere appealed to her sister, Sheila found herself drawn to the other side of the globe. In her case, she landed in the land down under. No wonder, then, when Cristina asked for help, Shiela piped back with a quick "no worries."

Sheila, Cristina's twin, is a US national living in Australia. She models how costs and prices rise with additional features in several different markets. She'll answer Cristina's questions about how better specifications drive both costs and prices in the markets for business jets and turbofans.

Figure 7 – Sheila studies features and their Value

Sheila, you see, is someone who works what we could reasonably call the unknown realm of economics. She had a hunch that the product features have something to do with sustainable prices. That means Sheila doesn't believe in upward-sloping supply curves for products that are not commodities. She studied a 1987 RAND Corporation aircraft cost model and found costs increased with both weight and speed, as Equations 4 and 5.[5]

$$Labr_{100} = 0.141EW^{0.820} * SP^{0.484} \qquad (4)$$
$$Matl_{100} = 0.241EW^{0.921} * SP^{0.621} \qquad (5)$$

Where:

$Labr_{100}$ = Cumulative Manufacturing Labor Hours for 100 Aircraft (in thousands)
EW = Aircraft Empty Weight (in pounds)
SP = Maximum Speed (in knots)
$Matl_{100}$ = Cumulative Manufacturing Material Dollars for 100 Aircraft (in thousands of 1977$)

RAND built Equations 4 and 5 on 13 observations. The labor equation, number 4, had an R^2 of 88%, with P-Values for empty weight and a maximum speed of less than 0.001 and 0.013, respectively. Aircraft material in equation 5 had a better correlation at 91%, and its P-Values for weight and knots were less than 0.001 and 0.003, in that order.

[5] Hess, R.W. and Romanoff, H.P, *Aircraft Airframe Cost Estimating Relationships: All Mission Types*, a RAND NOTE, 1987, p. 4

As someone who studies the business aircraft market, it makes sense to Sheila that both cost and Value should go up with speed. But, while cost models use weight to measure size or capacity, she reasons that the Value of space of business aircraft would be better estimated using some other metric. If adding weight were the best way to increase Value, all one would need to do would add lead to a plane to increase the sales price. Clearly, that's not the best option. She notes she could use maximum passenger limits but realizes that larger planes offer more space per traveler. Instead, she decides to see how the value changes with the cubic feet offered in each craft's cabin. After all, who doesn't want room to spread out?

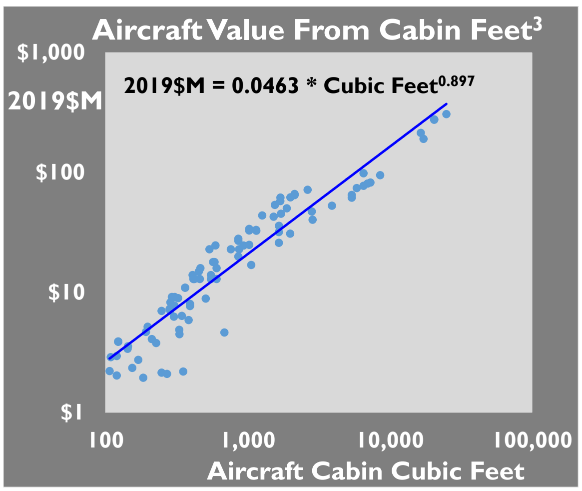

Figure 8 – Aircraft price goes up with cabin size

In Figure 8, we see her results. She finds that as cabin sizes increase, the sustainable prices do as well, according to Equation 6:

$$2019\$M = \$0.0463 * Cab\ Vol^{.897} * \varepsilon \quad (6)$$

Where:

2019$M = Predicted price for business aircraft
Cab Vol = Aircraft cabin volume (in cubic feet)
ε = Error term for the equation

Sheila finds Equation 6 is an excellent price estimator, with an Adjusted R^2 of 89.8%, but is concerned with the MAPE of 38.5%. She observes many of the smaller cabins belonging to turboprops fall below the line of best fit and decides to see how the market reacts to speed, as the prop-driven planes are slower.

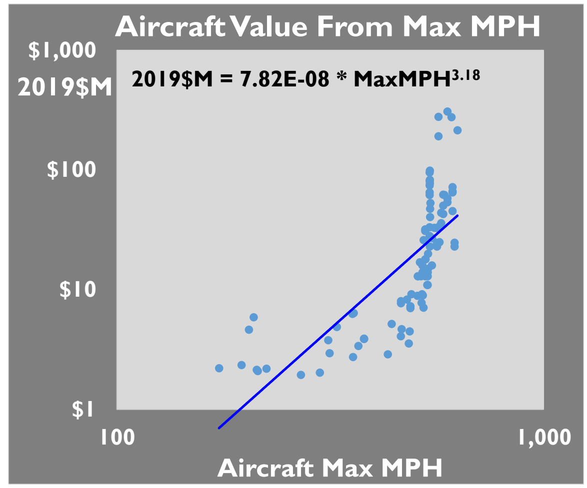

Figure 9 – Speed adds Value to business aircraft

Maximum Miles Per Hour, Shiela discovers, provides a viable estimator for the price of business aircraft, as shown in Equation 7.

$$2019\$M = \$7.82E\text{-}08 * MaxMPH^{0.897} * \varepsilon \quad (7)$$

Where:

2019$M = Predicted price for business aircraft
MaxMPH = Max aircraft speed (in miles per hour)
ε = Error term for the equation

Equation 7 is statistically significant (with a P-value of 1.85e-17). Its adjusted R^2 of 53.7% and MAPE of 78.8% aren't as good as those for Equation 6. She also notes the speed exponent is extraordinarily high. She combines the analyses, using cabin volume and maximum MPH simultaneously.

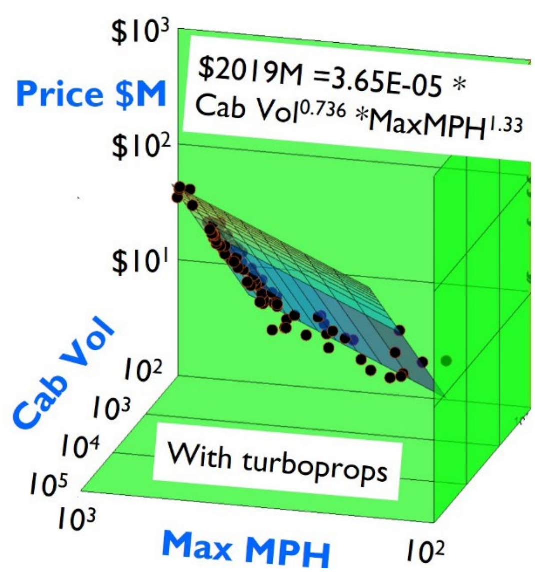

Figure 10 – Aircraft value (with turboprops) from Cabin Volume and Maximum Miles Per Hour

When she does, she gets Figure 10, expressed by Equation 8.

*\$2019M =3.65E-05 *Cab Vol*$^{0.736}$ **MxMPH*$^{1.33}$ * ε (8)

Where:

2019\$M = Predicted price for business aircraft
Cab Vol = Aircraft cabin volume (in cubic feet)
MaxMPH = Max aircraft speed (in miles per hour)
ε = Error term for the equation

With an adjusted R^2 of 96.4% and a MAPE of 19.3%, Equation 8 is a better predictor than either Equations 6 or 7. Sheila notes the speed exponent is still high, at 1.33. She also remembers Cristina wants to study jets, not turboprops, so she removes the latter group from the dataset and reruns her analysis in Figure 11, which uses Equation 9.

*\$2019M =2.46E-10 *Cab Vol*$^{0.671}$ **MxMPH*$^{3.29}$* ε (9)

Where:

2019\$M = Predicted price for business aircraft
Cab Vol = Aircraft cabin volume (in cubic feet)
MaxMPH = Max aircraft speed (in miles per hour)
ε = Error term for the equation

There are even better statistics for Equation 9, as its adjusted R^2 is 97.5%, while the MAPE falls to 13.7%, as it uses 75 observations, compared to the 95 used for Equations 6, 7, and 8. Note the dramatic difference in the slope for the speed component in Figure 11 compared to Figure 10. In the business jet market, buyers pay dearly for added speed.

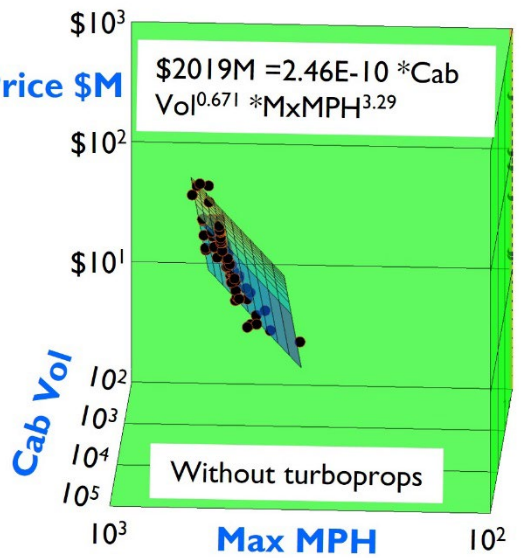

Figure 11 – Aircraft value (w/o turboprops) from Cabin Volume and Maximum Miles Per Hour

Since she did so well with aircraft, Sheila decides to see how the engines that power them behave. When she does, she discovers Figure 12.

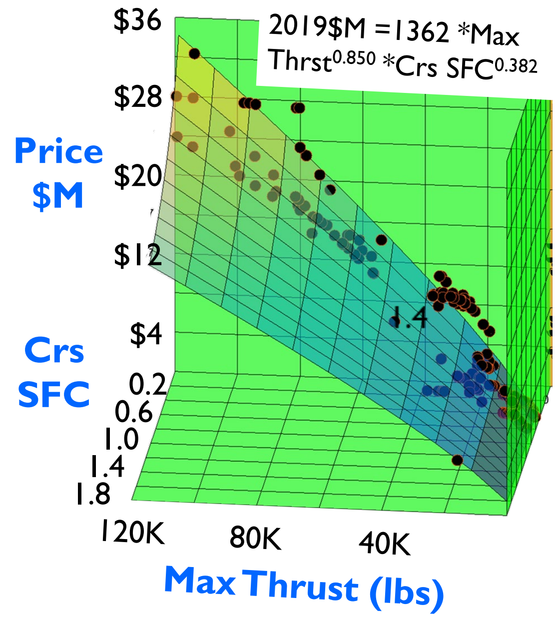

Figure 12 – Turbofan value from maximum thrust (in lbs) and Specific Fuel Consumption (lb/lbf/h)

She describes Figure 12 with Equation 10.

*2019\$M =1362 *Max Thrst*$^{0.850}$ **Crs SFC*$^{-0.382}$* ε (10)

Where:

2019\$M = Predicted price for turbofans
Max Thrst = Max turbofan thrust in pounds
Crs SFC = Specific Fuel Consumption in lb/lbf/h at cruise speed

Equation 10, derived from 186 observations, is well-correlated with an adjusted R^2 of 94.6% and a MAPE of 18.2%, with P-values of 9.51E-116 and 2.52E-06 for Max Thrust and Specific Fuel Consumption, respectively.

5. Pole Position

Now that they've completed some deep analysis on their problems, Cristina and Sheila wonder how they might be able to extend it. They remember how they used to share adjoining rooms as young girls living side by side and think about how they might recreate a similar environment for their work. At first blush, if they were to remain in their adopted countries, with

Cristina and Sheila living on widely distant continents, that appears problematic.

Then they ask themselves this: what if Australia and Argentina touched? After some reflection, they changed the question to this one: Where do any parts of Australia and Argentina meet? Clearly, with Australia as the world's only continental country surrounded by ocean, that looks to be a trick question.

And it is.

But it's one with some hidden mathematical meaning buried in its geography, something that may need only the slightest tweaks to offer a new, beneficial structure, one that is not widely known.

With just a little bit of recall and some research, they rediscovered those countries, specifically their territorial claims on Antarctica, touch at the South Pole, as shown in Figures 12, 13, and 14.[6,7,8]

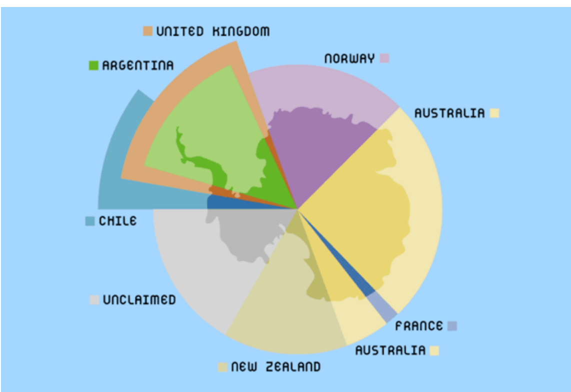

Figure 12 – The Argentine and Australian claims in Antarctica meet at the South Pole, with their airspaces separated by the Earth's axis

Figure 12 reveals a kind of border rarely seen globally – it's a vertical line in space meeting the ground at a point. The four corner states, Arizona, Colorado, New Mexico, and Utah, have these boundaries in the United States. But the South Pole in Antarctica is unique. Here, several countries share the Earth's axis as their common boundary.

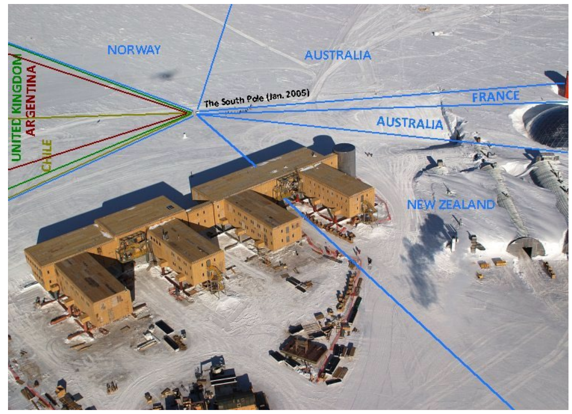

Figure 13 – All claims in Antarctica meet on a shifting point of ice above the South Pole

Figure 13 shows us that point is near the Amundsen-Scott South Pole Station, and we can get a close-up view of it in Figure 14.

Australia has a couple of Antarctic slices in its claim, while Argentina lies entirely within the UK's.

Figure 14 – The South Pole marker with a coffee cup; Amundsen–Scott South Pole Station behind

If Sheila were to go to the South Pole and walk into the Australian claim, one could not say that she was in negative Argentinian space. The same could be said for Cristina, as any movement of her part into the Argentine claim says nothing about the one Australia has. The twins note that, by convention, we call the point where all these claims meet 90° South latitude the South Pole. They wonder: What would happen if we called it something else?

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6 http://www.discoveringantarctica.org.uk/alevel_4_0.html

7 http://geosite.jankrogh.com/borders/ant/index.htm

8 https://commons.wikimedia.org/wiki/File:South_pole_geographic_el_station.jpg

6. 3 + 2 = 4

Given their analyses about business aircraft and observations about the South Pole, the twins decide to place the axes of their graphs near each, as shown in Figure 15. Cristina's Demand Plane, at right, needs only a vertical plane of two axes and is easily accommodated by Argentina's claim – observe again it has a horizontal quantity axis and a vertical price axis.

At the same time, in the exact Figure, Sheila's three Value axes, to the left, consisting of a pair of valued features, cabin volume, and maximum miles per hour, plotted on horizontal axes and a vertical price axis.

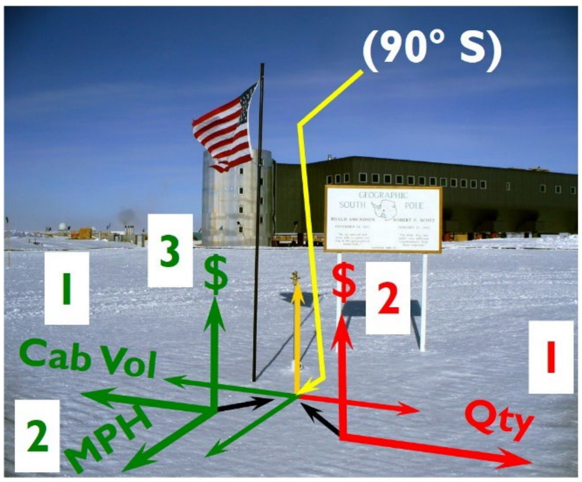

Figure 15 – What do the 3D Value Space and the 2D Demand Plane for Biz A/C have in common?

The twins have a thought. What, they wonder, would happen if we placed our horizontal axes in line with the Earth's axis at the South Pole?

In Figure 16, they do just that and note that since both the Value Space and the Demand Plane share the common vertical price axis, they need not replicate it in their axis count. Thus, the 3D Value Space and the 2D Demand Plane combine to form a 4D system, or, alternatively, 3 + 2 = 4. With this display system initiating at the South Pole, the twins think to rename its origin to (0, 0, 0, 0), with four axes representing (Valued Feature 1, Value Feature 2, Price, Quantity).

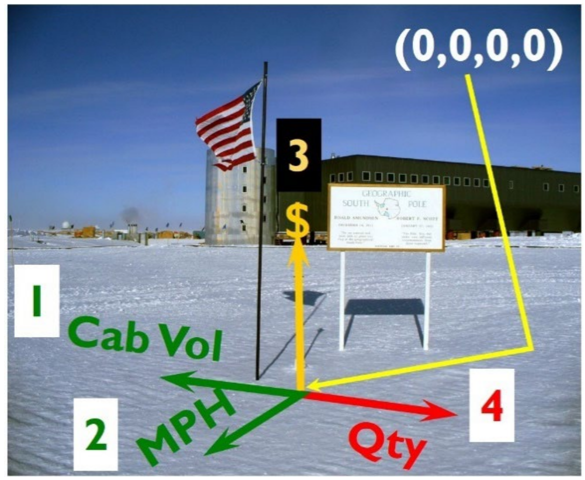

Figure 16 – 3D + 2D = 4D, since Value Space and Demand Plane share the Price Dimension

In Figure 17, they additionally note that while the South Pole drew their systems together, there is no need to depict them starting there, so they drop the geographic reference. They recognize all 4D market models form their own unique systems.

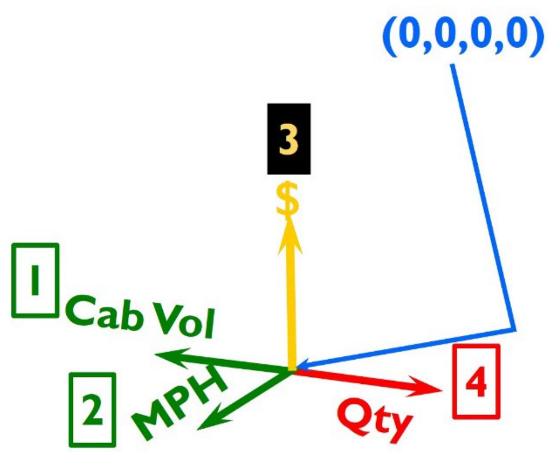

Figure 17 – 4D markets reside in their own space

Simply getting the axes in the right place is only the beginning of the analysis – they realize they must populate these systems in Figures 18 and 19.

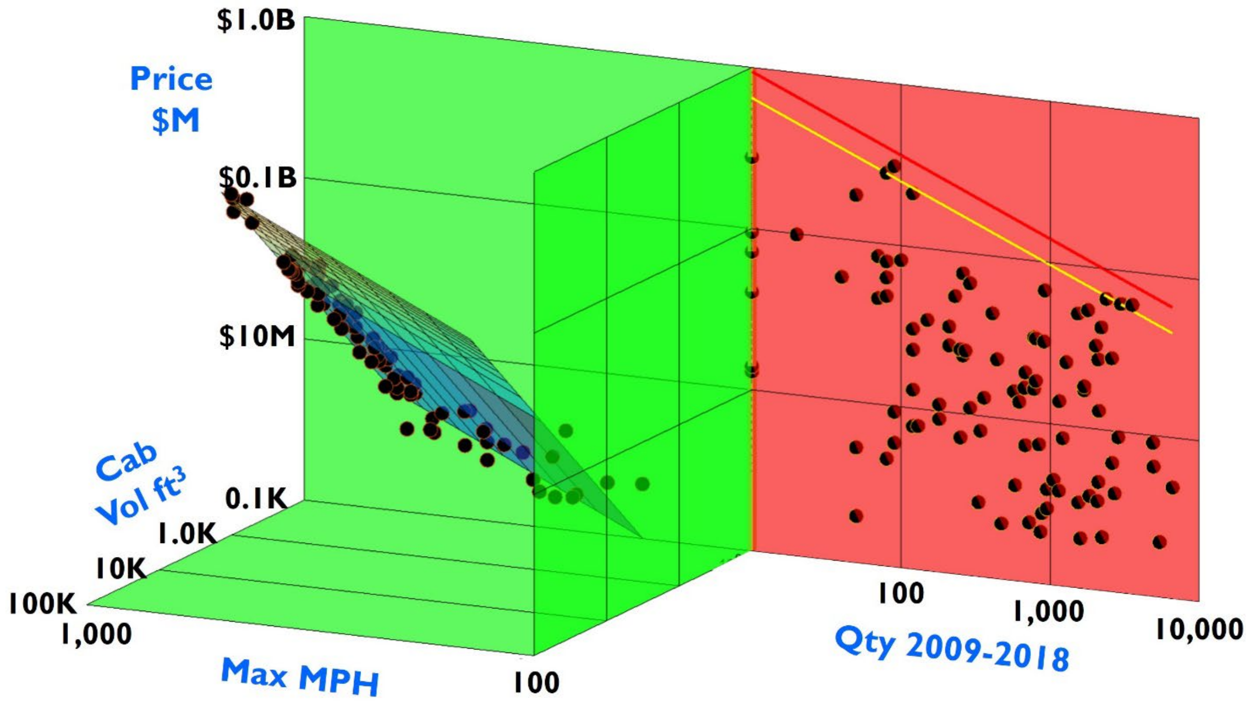

Figure 18 – The 4D market for business aircraft, with Value Space to the left, the Demand Plane at right

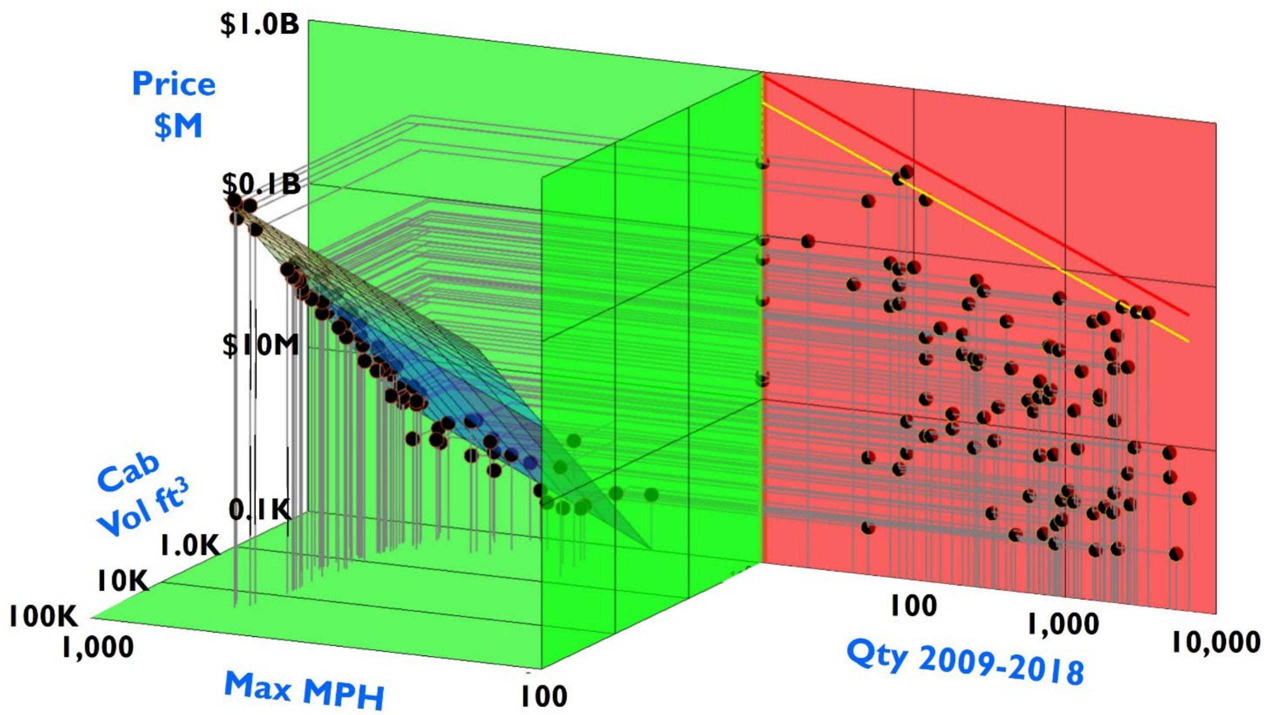

Figure 19 – All points in Business Aircraft Value Space have matches on the Demand Plane; Value and Demand entangle with each other in every market

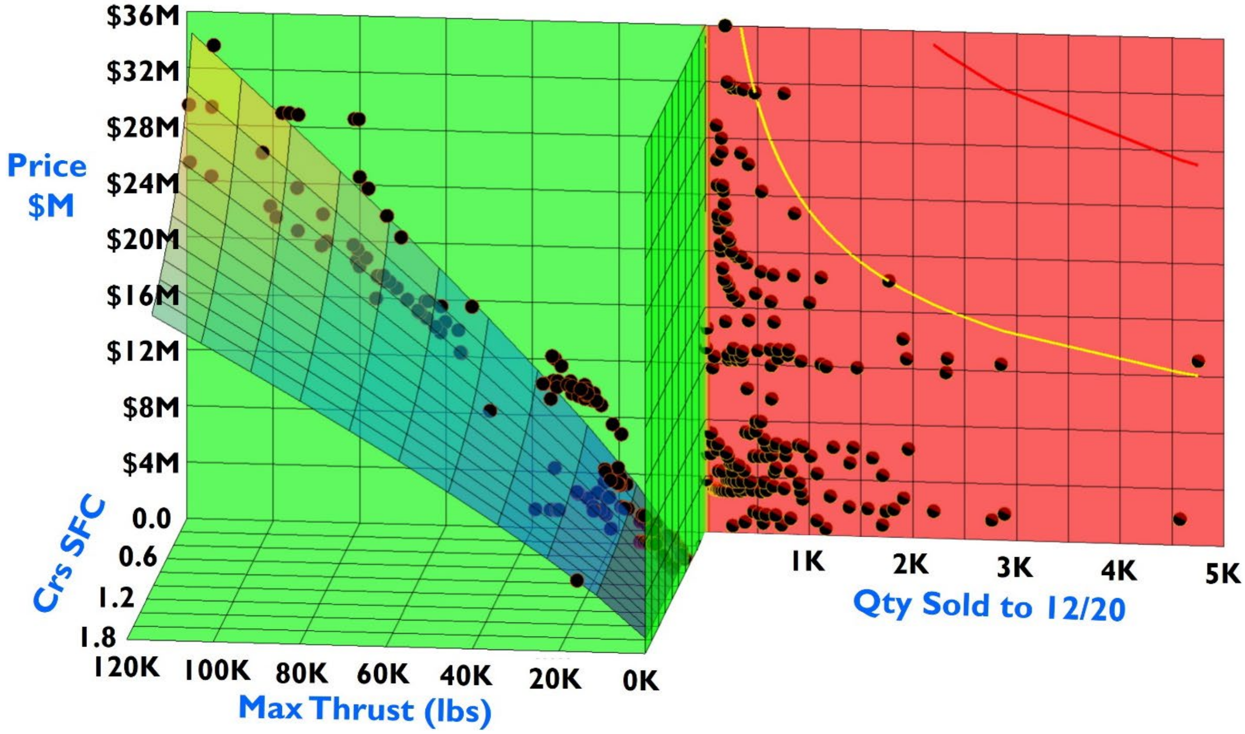

Figure 20 – The 4D market for turbofan engines, with Value Space to the left, its Demand Plane at right

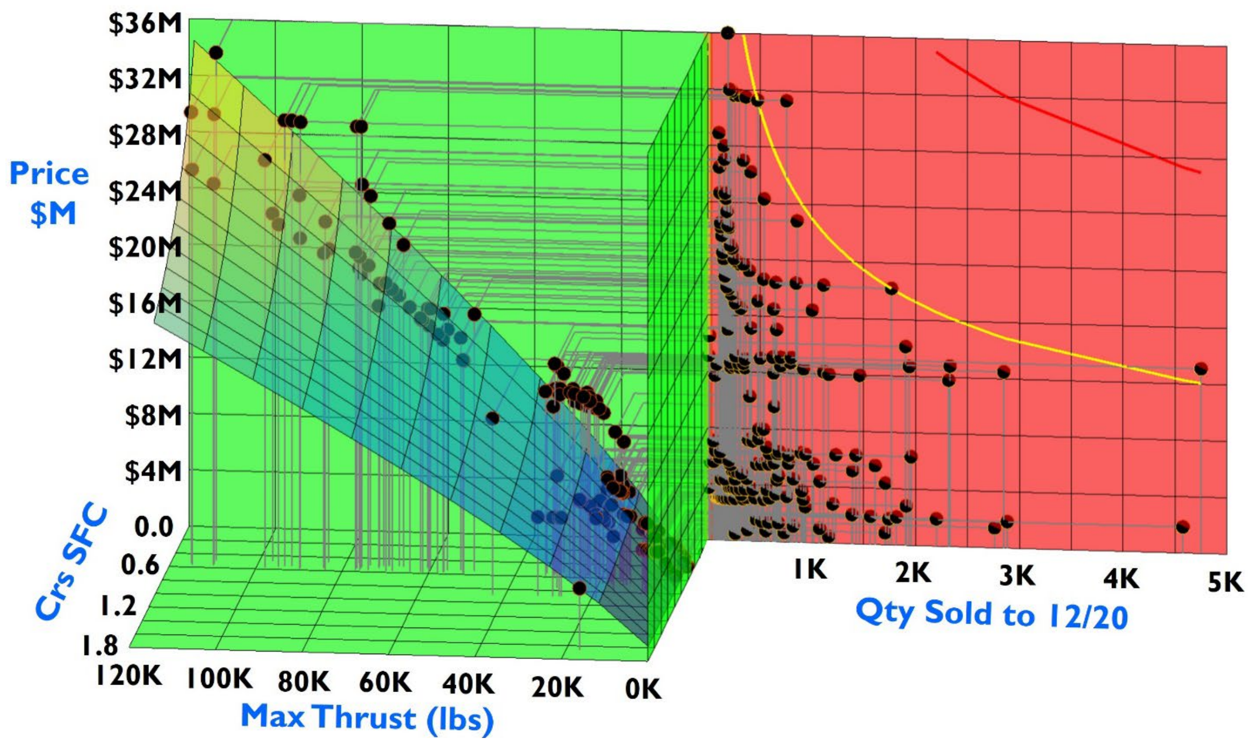

Figure 21 – Every point in the turbofan engine Value Space has a match on its Demand Plane as they entangle with one another

In Figure 18, with both sides of the system fully populated with business aircraft, they make an added insight. The Value points on the left and those on the right for Demand are not separate. Instead, they *entangle* with one another through the common price axis.

To drive this thought forward, they decide to draw what they call *point lines* that connect each Value Space position with its mate on the Demand Plane in Figure 19.

Figures 20 and 21 copy their business aircraft methodologies for turbofan engines and draw similar conclusions. They begin to wonder if they may be able to reveal even more entanglement.

7. 4 + 4 = 7

Once they break the convention of traditional land-based geometries, the twins realize there is little to prevent them from expanding their analyses. To that end, they observe that since the turbofan market, as depicted, takes 180° of arc, as does the one for business aircraft, there is nothing to prevent them from pairing them together, as shown in Figure 22.

There, we find the four dimensions of the turbofan market combined with the same number in the business aircraft market. Since each market shares the same price axis, we portray both markets simultaneously with only seven axes.

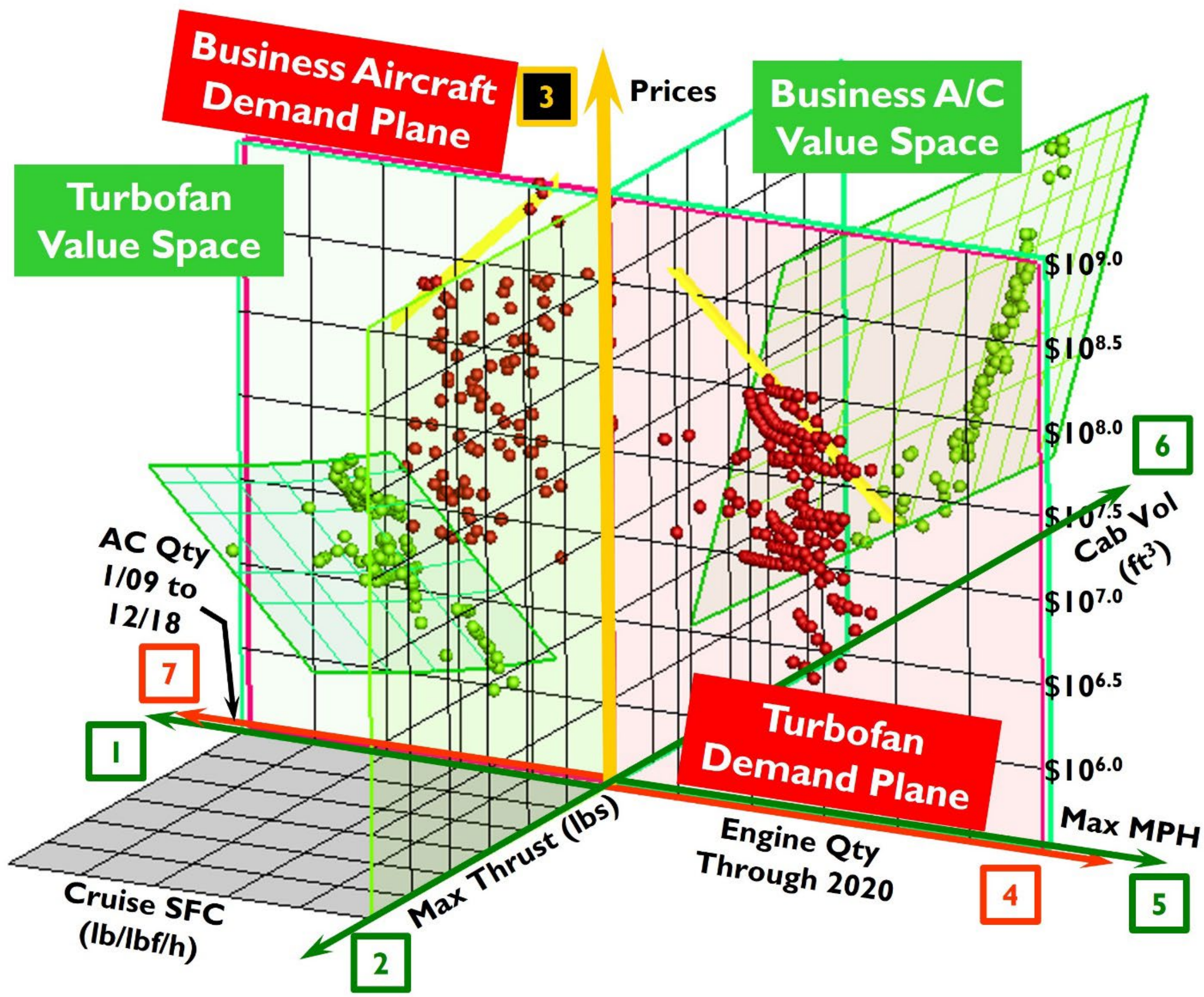

Figure 22 – Since the 4D turbofan and 4D business aircraft markets share a common price axis, they combine to form a 7D system (so, here, 4D + 4D = 7D)

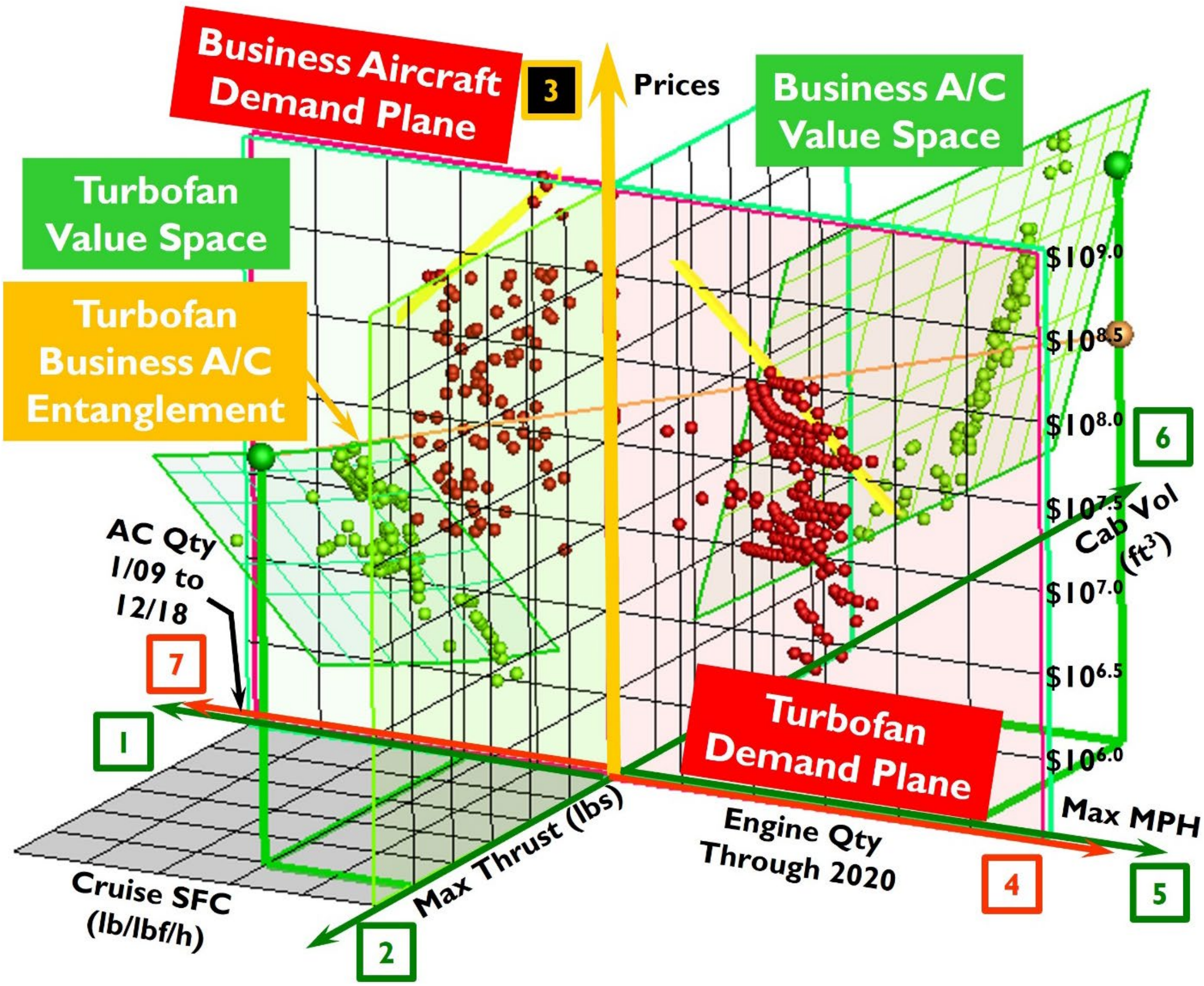

Figure 23 – Turbofans only sell to jet aircraft. Thus, every point (i.e., engine model) in the turbofan market entangles with one or more business aircraft models that use it; here, a new engine (the largest sphere at left) finds a need in the business aircraft market (the sphere partway up the rightmost vertical line, forming part of a new aircraft cost)

But there is more to Figure 22 than meets the casual eye. As we noted at the beginning of this piece, turbofan engines mate up with the jet aircraft that use them. The markets entangle with one another. Rather than show how every engine matches with one or more aircraft with which it may be paired, we can see this mutual relationship in Figure 23.

In Figure 23, at the behest of a manufacturer of business aircraft, a maker of turbofan engines has built a new engine, shown as the largest sphere on the left-hand side of the figure, placed on the turbofan Value Surface. Note that it has a connecting line that runs to a matching sphere partway up the rightmost vertical line in the diagram, representing the cost of a single engine. That rightmost vertical line represents all the value components of a new business aircraft. The total Value of the new aircraft model is the like-sized sphere above the turbofan engine component, lying on the business aircraft Value Surface. (Observe there is some distortion in the apparent contribution of the engine component of the aircraft due to the log-scaling – here, the engine portion of cost appears to exceed 50%; in practice, it typically runs from about 17% to 40%, depending on the paired models).

Thus, the engine manufacturers are entirely dependent upon their aircraft manufacturers to buy their products, and airframers face a significant cost component in their engines. It makes the twins wonder how to perform cost trades between these markets.

8. A 7D Entangled Trade

Far from a hypothetical construct, Cristina finds real-world issues suppliers from both markets could alleviate with benefits accruing to both manufacturers.

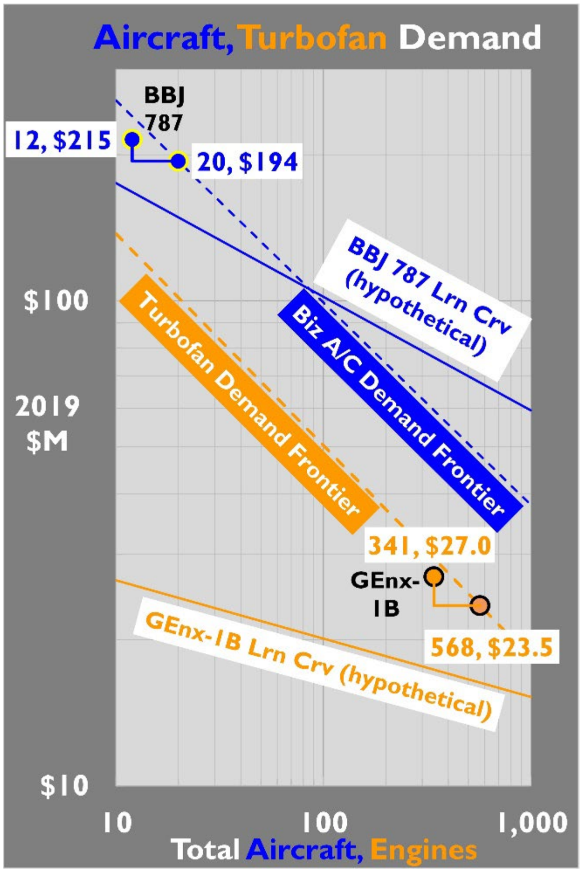

Figure 24 – The Boeing BBJ 787 might sell as many as 20 units in a decade, but only if its price falls in with its Demand Frontier; GE, with its GEnx-1B engine, might be able to offer relief

In Figure 5, she found twelve (12) sales of the Boeing BBJ 787 over a decade, while in Figure 6, her work showed that one of its engines, the GEnx-1B had sales of 341 over the same length of time (the BBJ 787 also uses the Rolls-Royce Trent 1000). What if Boeing wanted to make a push to sell 20 of these units in a decade? What would have to happen?

According to her calculations, if Boeing wants 20 of these units to sell in a decade, the price must fall nearly 10% to $194M as the Demand Frontier limits sales. Importantly, from Boeing's perspective, their GEnx-1B engines (they need two per plane) represent about 25% of the sales price at the current figures, and to get to this potential target, they'll want some supplier help. In all cases, Boeing will have to ensure the BBJ 787 price does not drop below its recurring cost, shown with its hypothetical BBJ 787 learning curve, for, in that case, they would be losing money.

General Electric, for its part, finds itself in a similar position. Its GEnx-1B is close to hitting its Demand Frontier. To get more sales, it will have to drop prices. To verify they can do that, they will have to compare their prices to their learning curve.

Crucially, if either firm were to refuse to lower its prices, despite their demonstrated abilities to do that comfortably, sales for both will not attain their ideal, maximized level. Intractability on any side could lead to decreased profits for both parties. Current profits may mask this condition – one firm or the other may believe they are doing well enough while not realizing they are not doing as well as possible.

9. Two Out Of Three Ain't Bad – Or Is It?

The recently deceased rock and roll singer, Meat Loaf told us in a song that "Two out of three ain't bad."[9]

A primary hypothesis held by the twins is that for any project to succeed, producers have 1) cost, 2) value (as sustainable prices), and 3) demand working in concert with one another.

Clearly, if cost were to exceed price, a program would stop. No one can stand to build at a loss consistently.

Values, again, sustainable prices for products, must align with the markets' view, as determined in methods examined above. Overpricing leads to decreased sales; underpricing results in monies left on the table.

A vital result that follows is that producers need to abide by the Demand Frontiers they face. These limits have error terms about them, and roughly speaking, about half of the products that form them will lie outside their bounds, the others within them. But, as producers begin their quest to launch a new product into their market, they must take the measure of their

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9 https://en.wikipedia.org/wiki/Two_Out_of_Three_Ain%27t_Bad

markets' limits. Not doing so can lead to economic disaster.

With that in mind, in 2020, Cristina and Sheila decided to analyze the Aerion AS2, shown in Figure 25.[10]

Figure 25 – The Aerion AS2

The Aerion AS2 is a supersonic business jet, and the twins first want to know if its costs are in line with history. While it was, in 2020, not yet in production, the firm did offer its development cost at $4B.

After some research, they construct Figure 26.

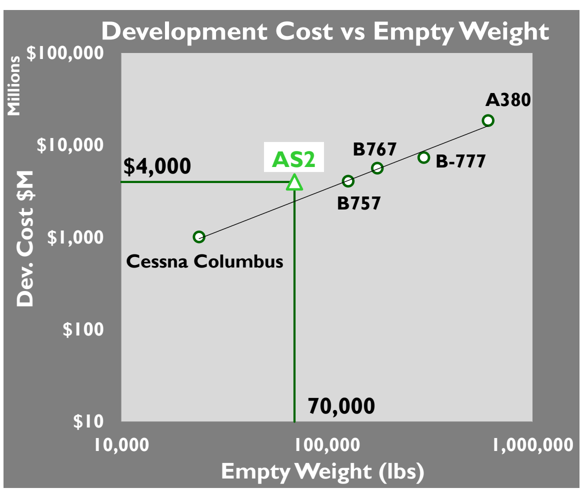

Figure 26 – Jet empty weight versus development costs, AS2 cost is 62% higher than that for subsonic models

Despite the low number of observations in Figure 26, they note the P-Value for the line of best fit is well below their criterion of 0.05, at 4.65E-04. At 70,000 pounds empty weight, the AS2, with its projected $4B development cost, is 62% higher than the projection for all subsonic aircraft in this database. They strive to see if that should be sufficient additional monies for development.

Returning to the RAND model,[11] they find a complete set of development equations and decide to compare the ratio of the projected cost components of the AS2 by discipline to each of those of the next fastest model in the dataset, the Boeing 777. When they do, they derive Figure 27.

Discipline	RAND Exponent	Base Factor	AS2 Factor	Ratio
Engineerng	1.030	616.1	983.9	160%
Tooling	0.609	44.6	58.8	132%
Mfg. Labor	0.429	14.5	17.6	122%
Material	0.811	157.2	227.3	145%
Design Supt.	1.280	2929.4	5241.1	179%
Flight Test	1.270	2752.3	4901.9	178%
Program	0.745	104.2	146.2	140%

Figure 27 – Ratio of cost differences due to speed by discipline for AS2 relative to Boeing 777

Given Boeing's historical cost breakdowns,[12] the added cost of going from 511 knots (the Boeing 777's top speed) to 805 nautical miles per hour (the AS2's maximum), the added 62% cost above subsonic development programs seems to be reasonable.

Turning her attention to the Value of AS2, Sheila decides to put in its projected cabin volume (1146 cubic feet) and top speed (925 miles per hour) into Equation 8; she finds the projected AS2 Value at $57 million. Realizing there is a premium for speed among business jet owners, she runs the same variables into Equation 9 (the one that removes turboprops); Sheila finds the market might support a price of $160M.

Since the company priced its AS2 at $120M and received some firm orders, the market had proved that it was worth that much.

So, Aerion had passed the initial 1) cost and 2) value tests, or two out of three key measures. Meatloaf would say that's already not bad. But what, the twins wondered, could they say about demand?

10 https://en.wikipedia.org/wiki/Aerion_AS2

11 RAND, *op.cit.,* Table 14, p. 88

12 Willcox, Karen, 16.885 Aircraft Systems Engineering Cost Analysis, MIT, slides 16-17, September 19, 2004

10. 7 + 1 = 8

The twins hypothesize that Demand is something quite different from Value or Cost. Cost, they discovered, falls upon manufacturers, and they incur additional charges if they build larger or more complicated products or if they have a newer, inexperienced labor force. Costs fall with learning or added experience over time.

Despite manufacturers setting initial prices, thereby putting their stamp on Value, they've found that the buyers will set ultimate prices based on how they assign Value to all the features offered in goods and services. Value often falls over time; some new buyers only enter the market through lower prices. Learning, which drops costs over time, enables these price reductions. When she comes across the data forming Figure 28, Sheila discovers that a television price of $300 in 2000 dropped to below $10 in 2019 (adjusting for resolution, refresh rate, warranty, etc.).

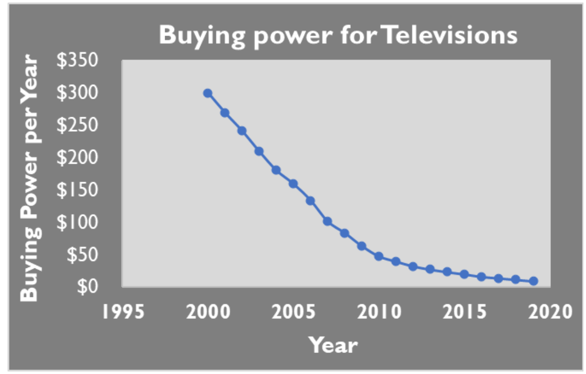

Figure 28 – Television buying power, 2000-2019

The twins take note of a physical trajectory in Figure 29. They wonder if they might track market movements in like fashion. In Figure 30, they depict the direction of the electric call market for a decade, from January 1, 2009, through December 31, 2018.

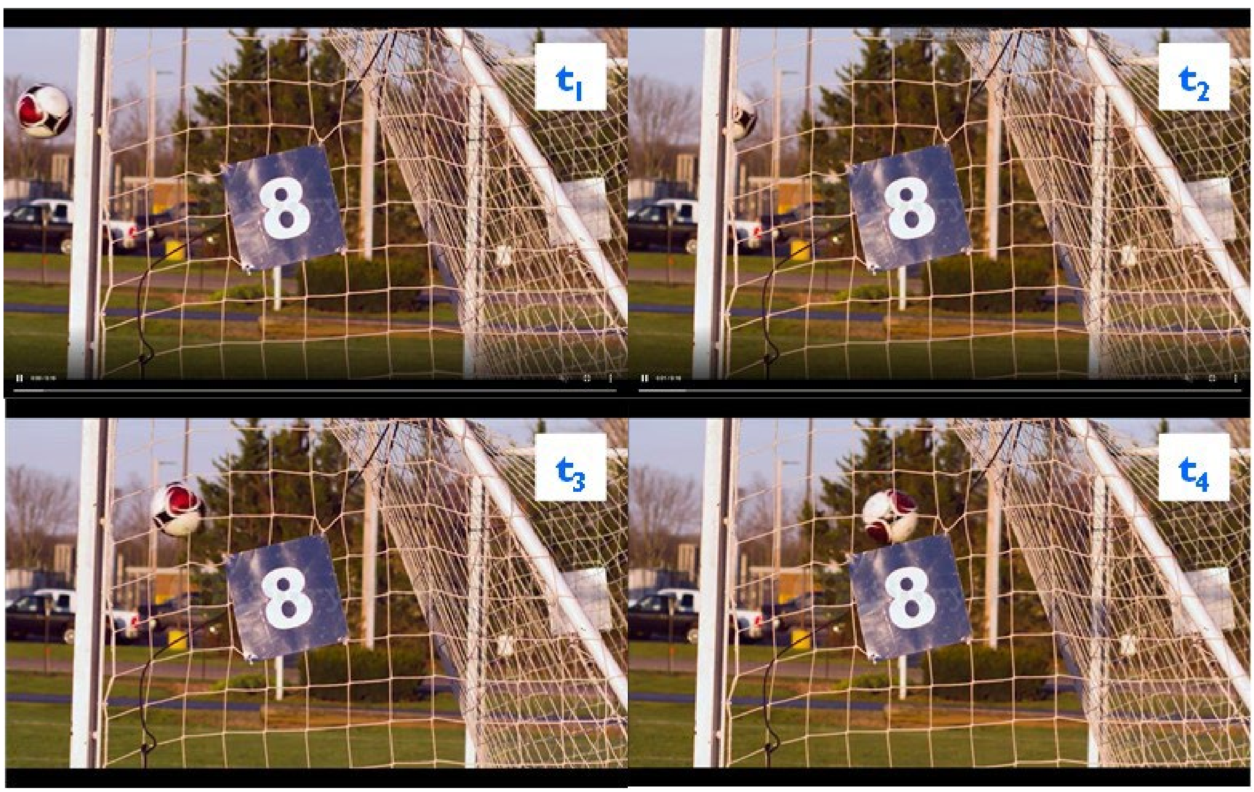

Figure 29 – An object placed in motion (here, a soccer ball) stays in motion unless acted up by other forces. Market forces mimic physical forces.

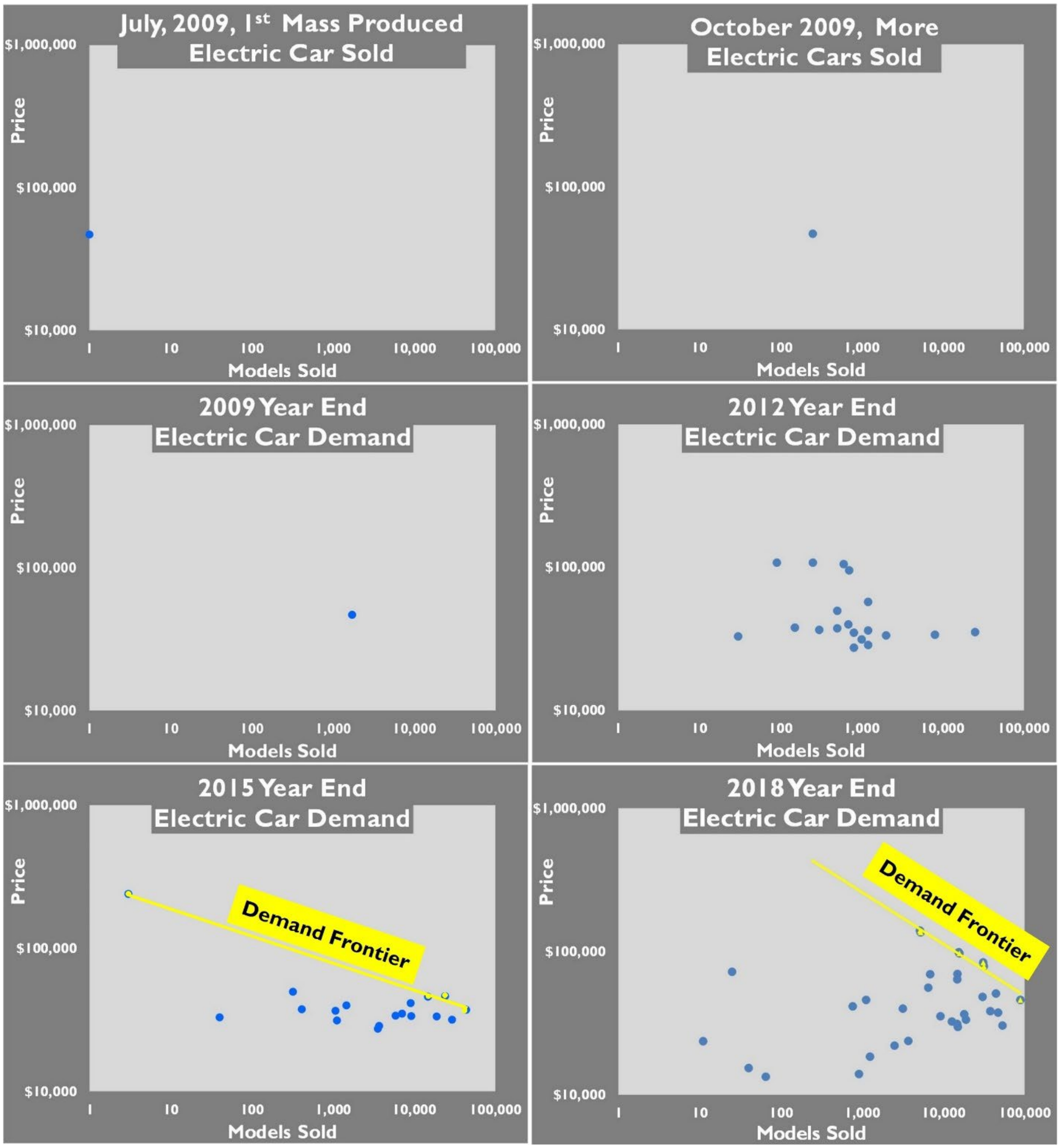

Figure 30 – The electric car market demand changed rapidly over time, beginning with a single model in 2009, growing to dozens of market entrants by the end of 2018, when a clear Demand Frontier formed. Here, each point represents the quantity of a model sold (the horizontal component) and its price (the vertical part)

With a single sale, the market for modern mass-produced electric cars began the same way aircraft did. As shown in the upper left chart in Figure 30, a single sale of the Mitsubishi i-MiEV in July 2009 launched the market, as it was the only entrant in the field in that year. As 2009 ended, Mitsubishi sold more of its ground-breaking machine. But by 2012 (center right), many more models made their way into the market – observe too, with the constant scaling between all six graphs, that Demand had soared. By 2015, this market had organized at its limit into a Demand Frontier, which moved dramatically in the next three years to the chart in the lower right-hand corner.

But do all markets expand so quickly?

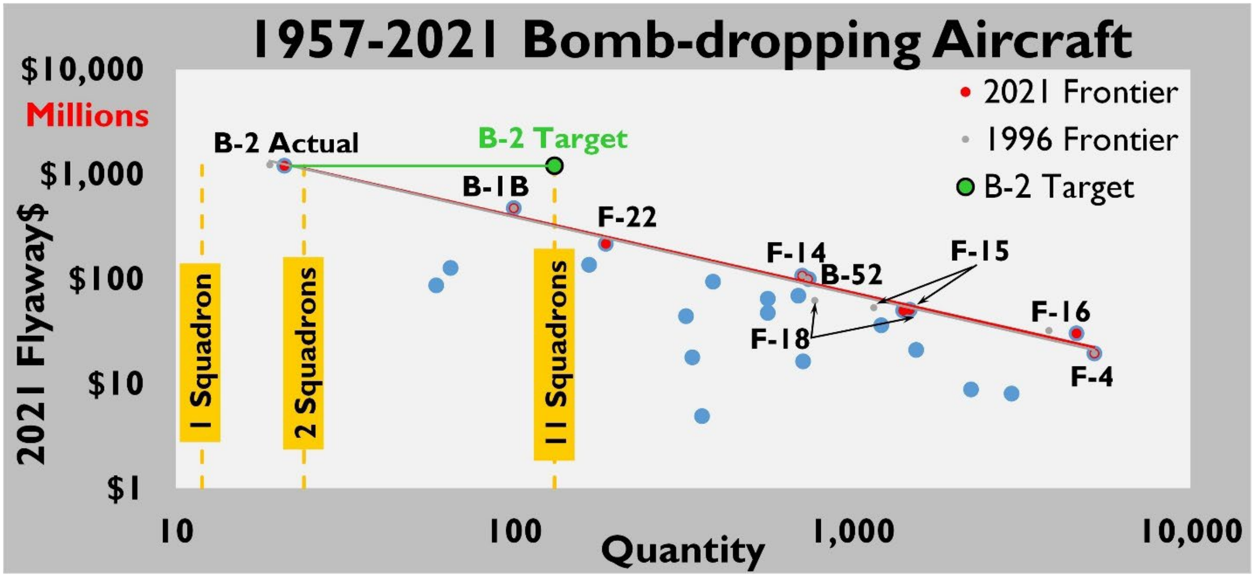

Figure 31 – The United States Government market for bomb-dropping aircraft is very stable. Its Demand Frontier in 1996 (the gray line) changed about 2% in 25 years as it reached its 2021 Frontier (in red). Not seeing this limit confounded the B-2 program, as the United States Air Force only received 21 of the 132 units it wanted when the US Congress stopped their buy. At its Demand Frontier, this market is effectively at rest.

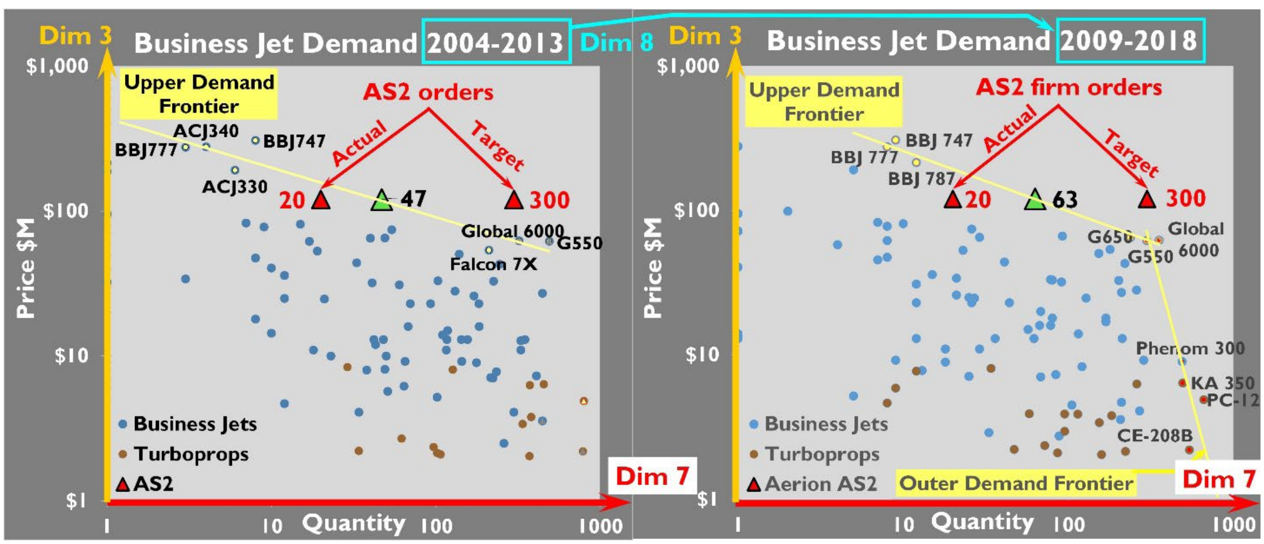

Figure 32 – At right, Dimensions 3 and 7 make up the Business Aircraft Demand Plane from 2009-2018 from Figure 23, above other dimensions are removed for clarity). As we add another dimension, Time, to the mix, we go back five years into the market's history to the figure at left while looking at the identical dimensions. Five years before, the market supported 47 models over ten years at $120 million each at the Demand Frontier. Five years later, the market could carry 63 models, indicating slow growth in this market.

In Figure 31, they find the United States Government has a self-imposed limit on the number of fighter, bomber, and attack aircraft they can buy. The amount they purchased in 1996 (the gray points and line) moved only about 2% in the 25 years up to the 2021 Frontier. Failure to observe this limit led planners to assume more buys of the B-2 bomber than the market would support (the same type of actions applied to the F-22, which started with 750 units but settled for 187).

Figure 32 adds time to the seven dimensions we used in Figure 23, for a total of 8. Since half of the right-hand side of the chart uses half of the left-hand side, these analyses are necessarily entangled. It is as if the soccer ball in Figure 28 had only advanced half of its diameter. For the period at left, at the target price of $120M, the market could support 47 units; five years later, at right, it might absorb 63. The market was going in the right direction for the Aerion AS2.

Crucially, though, the standard deviation about the 47-unit projection in the 2004-2013 projection was more extensive than that for 2009-2018. That meant the chance of Aerion making their targeted quantity of 300 units over ten years fell from one in ten in the left-hand chart to about one in 40 in the latter.

Another way to look at market capacity comes with Figure 33. This chart presents a different view of the ability of the business aircraft market to absorb expensive models.

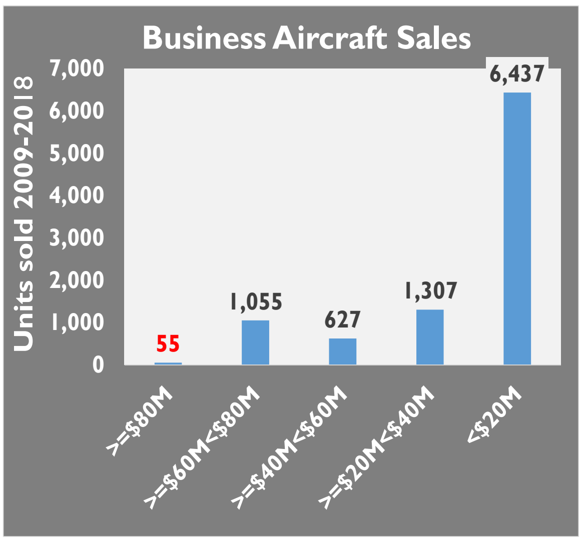

Figure 33: Aerion wanted to sell 300 AS2s at $120M in a decade, but the market only supported 55 units at $80M or more over ten years

Some may argue that the AS2 is so unique it forms its market with its own rules. But, when we compare electric cars to those with internal combustion engines, as we do in Figure 34, we find both classes of equipment abide by the same Demand Frontier.

Figure 34 plots the 2018 quantities sold and prices for all 36 electric car models then in production, compared to 43 gas-powered designs (a fraction of those on offer in 2018). Purposely included in the gasoline group were that year's most popular (the Toyota Corolla) and expensive (the Bugatti Chiron) cars to help discover market limits. Interestingly, several electric and gas vehicles combined to form a relatively flat and highly correlated curve: The Demand Frontier $ = 14.2M* Qty-0.484, Adjusted R2 = 99.8%, MAPE = 6.0%). While this does not comprise the entire market, this study, by design, attempts to model its Demand. What is clear here is that both gas and electric models abide by the same Demand Frontier.

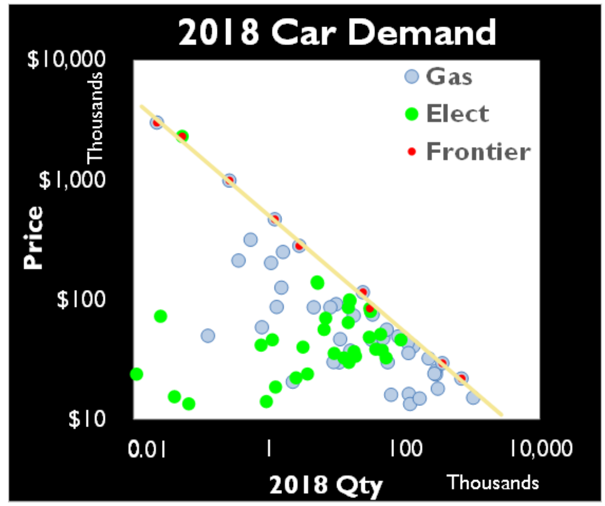

Figure 34: In 2018, gas and electric cars abided by the same Demand Frontier

Thus, for these reasons, it seems clear that the Aerion AS2 would not make its targeted quantity of 300 units in a decade and 500 overall. That's what the twins concluded. I did too.

So, in December 2020, I said the vehicle was worth every penny (which came from the Value Analysis), but there weren't enough pennies in the world to make its target (the Twins' Demand Analysis conclusion). They went into receivership in September 2021. Its final tally seems much like that of the Concorde, as Figure 35 reveals.

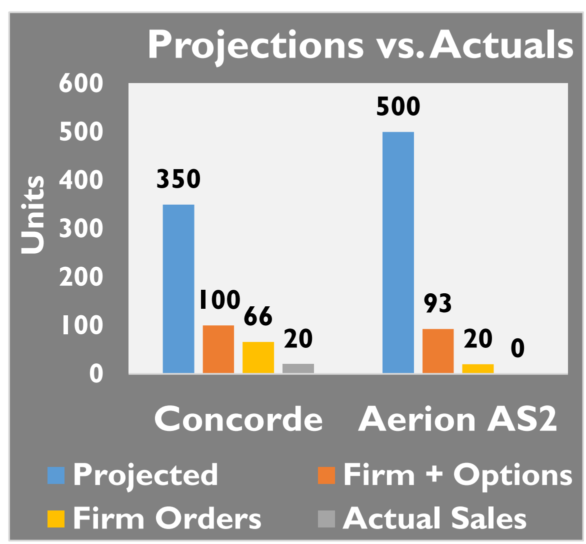

Figure 35: The Aerion AS2 had a sales history that mimicked the Concorde. Both supersonic programs were overly optimistic.

11. Conclusions

Life is intricate. Market interactions reflect and define the complexities of our economic life. Significantly, however, we as makers and buyers of goods and services demonstrate self-organization in all facets of the economy.

RAND and others have derived statistically significant models which reflect these self-organizing features as they apply to costs. Producers across different companies and industries work in much the same way. While there are variations, in the aggregate, the result of their efforts is a series of cost relationships that become predictable over time. Estimators can assemble broad paradigms that forecast how manufacturers will behave based on their work in the past.

Buyers ultimately set Value, the sustainable prices of products based on their specifications and Demand, the relationship between quantities sold of products and their prices. At first, a new market such as that for the first airplane or mass-produced electric car will not reveal any organization. But, when those markets gain new models, they begin to form collective Demand limits, known as Demand Frontiers, and reactions to the features offered to them, which are Value Response Surfaces. These reactions are often more highly correlated than their corresponding cost equations, as the market effectively dismisses goods and services that are too expensive or bids up the prices of too cheap products.

The Aerion AS2 had a reasonable development cost estimate and likely had a defensible recurring cost number based on that. It was worth the $120 million they charged for it, as evidenced by the firm order for 20 units they received for it at that price. It had two of three key parameters nailed down.

But Aerion completely misjudged Demand. Often Demand projections for new products take one of two routes.

In the first method, producers take a poll of potential buyers and ask them to lay down the required amount of cash as a down payment if they would like to make a purchase. If they agree in principle, that will form part of their basis for Demand estimate, against which the firm in question would apply some form of discount in the total, perhaps taking away as many as half of those who paid from their projection, based on their historical records.

A second way would be to form an operating cost model to flush out the new system's efficiencies over the old ones, thus providing a method by which they could forecast how many of the latest models the market would want.

As shown here, existing markets *always* reveal what they want, and when it comes to Demand, how many of a new product they may be able to absorb. Getting the data that enables to make such predictions is time-consuming but typically costs only a tiny fraction of the monies a firm stands to lose by not doing it.

Estimators need to do the same exercises concerning Value – DeLorean thought he could sell his car based on looks, but at the beginning neglected to put in the requisite horsepower, and it cost him his company.

Entangled markets, such as those for jets and jet engines, move in concert with one another. Only by recognizing collective benefits might one firm convince its partner of the usefulness to drop prices to increase both revenue and profits to both parties.

Cost trades across eight or more dimensions occur every day. This paper provides some methods to uncover the intricacies of those details.

It is incumbent upon estimators to address the programs upon which they work and analyze their market and that of their key suppliers to enable maximum possible profitability. Analysts should study cost, schedule, and risk, as they've done for decades, but need to add Value, Demand, and Time effects to gain a broader grasp of their markets.