The Missing Link: An Evolution of Portfolio Natural Selection

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ABSTRACT

"It is not the strongest of the species that survives, nor the most intelligent that survives. It is the one that is the most adaptable to change." –Charles Darwin

This research presents a conceptual framework and methodology for solving the *missing link* between learning curve estimates, prediction intervals, S-curves and generating analytically-based affordable cost constraints to *naturally select* trade space in a portfolio. This holistic approach expands the *evolution* of the S-curve.

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Introduction

This research presents a conceptual framework and methodology for solving the *missing link* between cost improvement curves, prediction intervals, S-curves, and generating analytically-based affordable cost constraints to *naturally select* trade space in a portfolio. This holistic approach expands the evolution of the S-curve.

The left side of *Figure 1* depicts the inherent tensions and challenges in getting needed capability downrange on-budget and on-schedule, with these constraints:

- Warfighters demand systems that are operationally effective, suitable, developed, and fielded on time.
- Resource sponsors and acquisition leadership face limited funds and multiple, competing demands in a portfolio, which consequently lead to issues of program affordability. The totality of warfighter demands for superiority on the battlefield can never be fully met. Subsequently, tradeoffs need to be made and risks need to be managed.

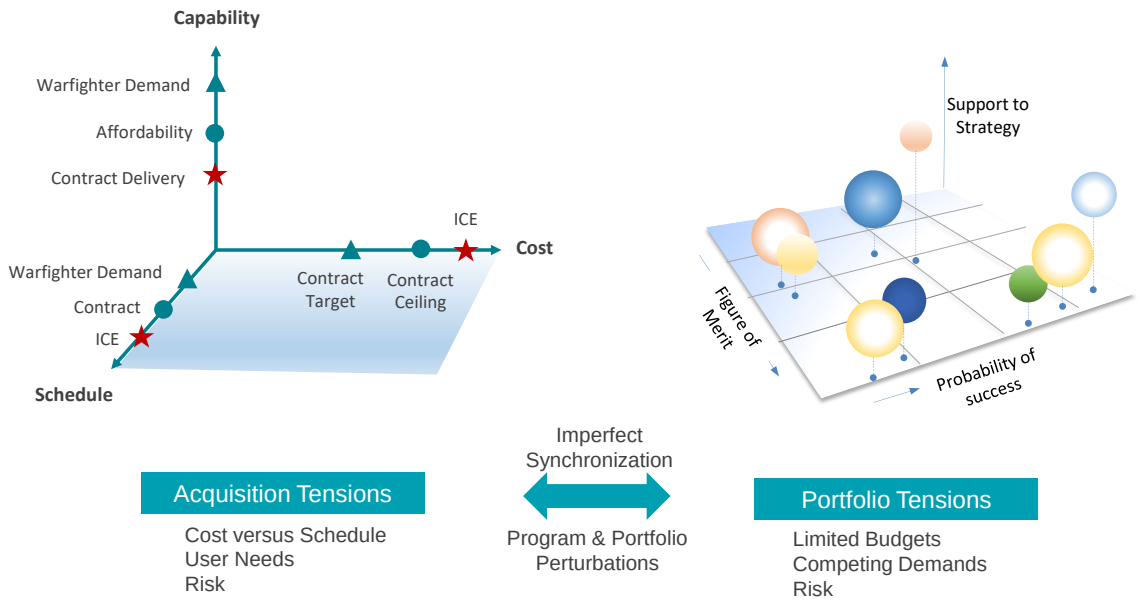

Figure 1: Depiction of the acquisition strains under consideration against the portfolio strains which leadership is often faced with balancing; Independent Cost Estimate (ICE).

Too often acquisition programs operate with a very limited view and understanding of the defense enterprise. They focus myopically on funding their programs over the Future Years Defense Program (FYDP). For example, program offices use cost improvement curves to generate point estimates, usually at or near a 50/50 level of risk for use in the budget, however they do so without:

- Offering senior leadership a menu of pricing and risk combinations;
- Adequately addressing economic-order quantities and facility capacity;
- An enterprise view of affordability constraints and opportunity costs in the portfolio.

Former Undersecretary of Defense (Acquisition, Technology and Logistics), Mr. Frank Kendall, addressed these shortcomings in his series of Better Buying Power (BBP) initiatives.

Unfortunately, the acquisition and cost analysis communities have lacked an integrated, data-driven framework for integrating all of the necessary steps to achieve genuine acquisition reform, and the promises of BBP. The result has been unremitting cost growth since the 1970's, as depicted in *Figure 2* with Navy examples. The same picture holds for all defense components.

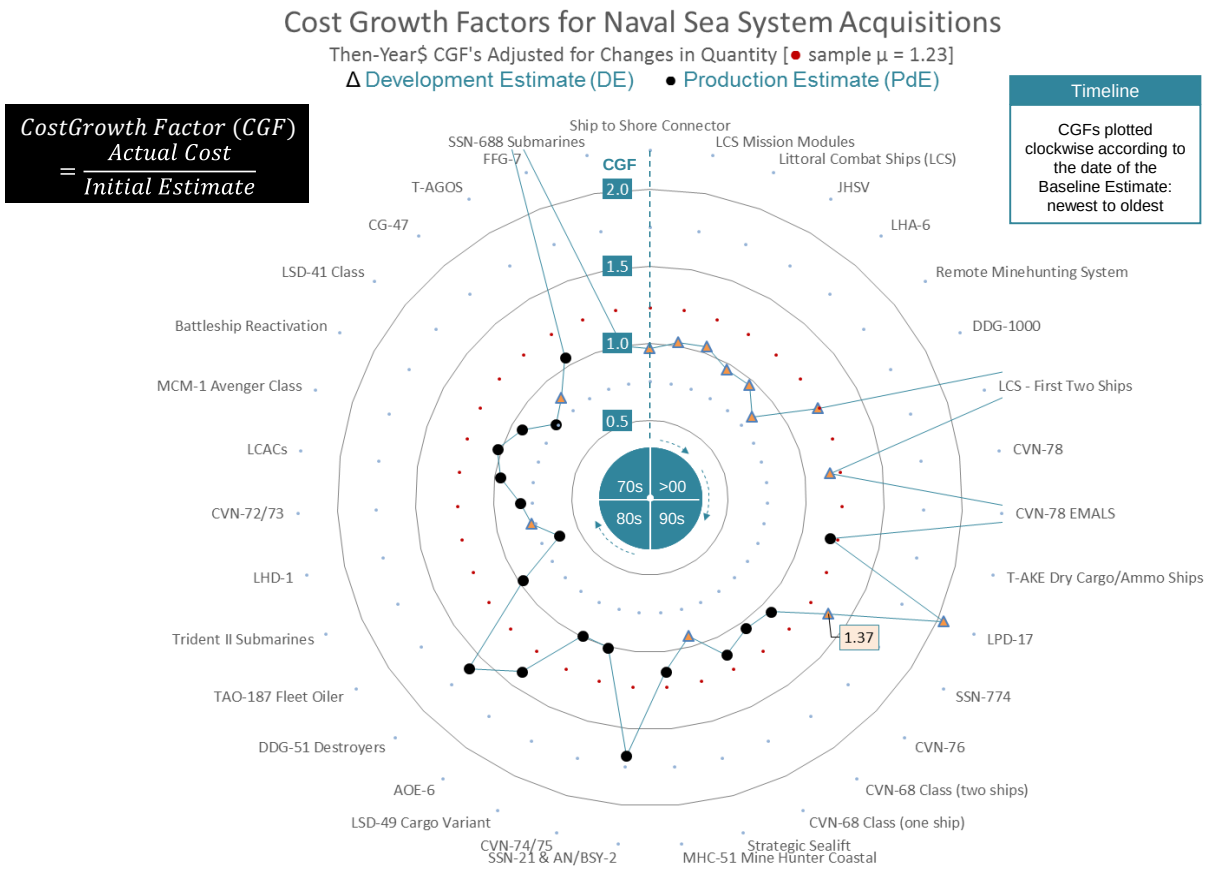

Figure 2: Depiction of cost growth in Navy Systems since the 1970's

The graph plots data culled from Selected Acquisition Reports (SARs). The SARs are a convenient, comprehensive, official source of data on the costs, schedule, and technical performance of major defense acquisition programs. Importantly, they are tied to acquisition milestones; e.g., Development or Milestone B, and Production, or Milestone C. In the graph, these milestones are denoted by DE, development estimate, and PdE, production estimate, respectively, using standard SAR lexicon. For a given program, the SAR provides two estimates of cost.

- Baseline estimate (BE) are usually made when the system nears a major milestone.
- Current estimate (CE), which is based on best-available information and includes all known and anticipated revisions and changes to the program.

The ratio of CE to BE is a Cost Growth Factor (CGF). The graph plots CGFs, adjusted for changes in acquisition quantity. The concentric circles represent the locus of points of constant cost growth:

- 1.0 = no cost growth
- 1.5 = 50% cost growth
- 2.0 = 100% cost growth, or a doubling of costs from the baseline estimate.

Points are plotted counter-clockwise (beginning at true North) according to the currency of the program: most recent in the North East quadrant to most historical in the North West quadrant. Finally, the plot uses the latest type of estimate available – hence a prevalence of PdE's for ship programs in the 1970's and 1980's. DE's in these cases are available from the authors.

The take-away is that cost growth continues today despite decades of acquisition reform. Even in recent times, more than 100% cost growth has occurred on several programs such as Landing Platform Dock (LPD-17) and Electromagnetic Aircraft Launch System (EMALS), while other programs such as Remote Minehunting System (RMS) have been cancelled because of performance issues and cost problems.

This paper attempts to mitigate the tensions of *Figure 1* and the cost growth of *Figure 2* through improved, data-driven linkages between some of the cost-estimating steps of program acquisition, opportunity costs, and tradeoffs in the portfolio. *Figure 3* serves as a banner for this initiative.

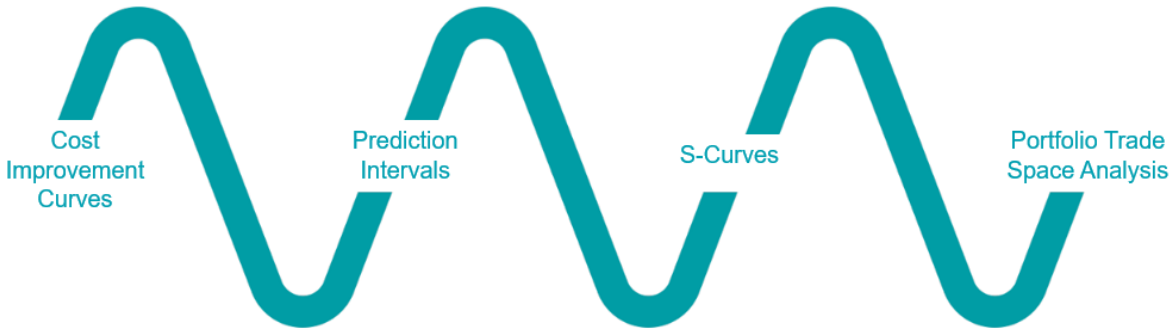

Figure 3: Missing links to be addressed

In addition to making the hard choices early, even slight improvement of Department of Defense (DOD) resource allocation processes can have enormous impacts on saving taxpayers' money and ensuring fiscal responsibility. Forging the *missing links* between cost improvement curves to prediction intervals, prediction intervals to S-curves, and finally, those S-curves to affordable cost constraints, could contribute significantly to improving these allocation processes. That said, if establishing these links were simple, the solution this paper suggests would be standard DOD practice today. It must be acknowledged that challenges within the cost community and national security domain have reasonably hindered the implementation of this evolution to portfolio selection. However, if decision makers were provided with a more thorough understanding of the value of the *missing links*, these challenges could perhaps be overcome.

Background and Challenges

This paper's purpose is to provide the much needed, more thorough understanding of the value of the *missing links* and how to overcome the challenges associated with them. The challenges presented in this paper fall into two general categories:

- Cost community challenges attributed to non-existent guidance in the national security domain, which have resulted in a lack of specifics on how affordability analysis is to be conducted, resulting in a "Law of the Land" mentality;
- Technical challenges associated with variation in the level of statistical and technical expertise of the average professional in the community resulting in a general lack of awareness of statistical biases caused by current techniques.

Figure 4 below illustrates the authors' perception of the current state of cost estimating and resource allocation. As one can see, the elements are not all connected and seldom used sequentially, which is necessary for making the most informed decision. For example, while cost improvement curves are widely utilized, they often do not include the generation of prediction intervals, and even less often are those prediction intervals used to generate S-curves. Experience has shown these authors that there are *missing links* between cost improvement curves, their corresponding prediction intervals and S-curves, and the generation of analytically based affordable cost constraints to *naturally select* trade space in a portfolio.

Overall, these individual analysis techniques, or partial combinations thereof, do not provide a comprehensive approach to informing leadership of the trade space within a portfolio. This paper will provide an evolutionary analytical solution that not only bridges the gaps between the *missing links*, but also improves upon the weaknesses of a limited set of cost estimating techniques. The desired end state, a systematic, iterative process that facilitates understanding is illustrated below in *Figure 5* addressing the technical challenges.

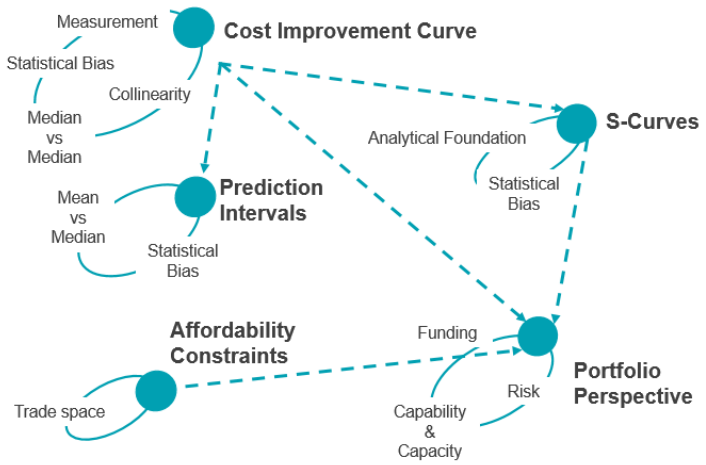

Figure 4: Current State of cost estimating and resource allocation, which includes disjointed efforts seldom applied in succession (as indicated by the dashed lines) due to loose guidance and lack of analytical framework

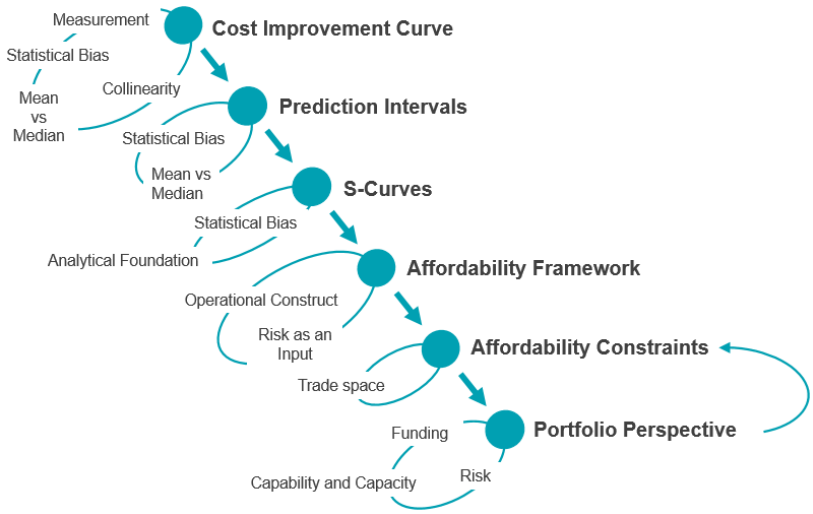

Figure 5: Future State cost estimating and resource allocation. The loop back between Affordability Constraints and Portfolio Perspective is indicative of an iterative process to be worked together.

As mentioned above and illustrated in *Figure 4* and *Figure 5*, this evolution faces technical and other challenges demanding advances. The following section will briefly describe these challenges before stepping into more detail later.

Cost Improvement Curves

Cost improvement curves (CIC) establish relationships to create estimates and take the form of a power function. CIC are commonly established through transforming the space surrounding the dependent (i.e. cost) and independent (e.g. cumulative quantity or factory throughput (rate)) to log-space and performing linear regression. Cost estimating relationships (CER) through regression can be a powerful tool for generating cost point estimates, but can be laden with bias which is routinely ignored or unknown in practice.

An important form of bias arises from running an Ordinary Least Squares (OLS) regression on data transformed from unit space to log space. Since the CIC is a power function, it is not linear in its parameters. Therefore, the parameters cannot be estimated by linear regression. The usual solution is to transform the equation into log space, which then makes OLS regression possible. However, taking the antilog of both sides of the log-log equation to generate the unit space equation and corresponding estimates for the theoretical first unit production cost (T1) or any other Y unit cost leads to bias. The least squares estimates obtained from the regression performed in log-log space are unbiased and have minimum variance of any unbiased estimators, but unfortunately these properties only apply to the parameters themselves and not their exponential functions. Due to the convexity of exponential functions, upward bias occurs, which can be quite significant.

Consistent with the aforementioned challenges stemming from lack of awareness surrounding these issues, cost analysts who find a point estimate for Y using the antilog of the CIC are not only neglecting bias, but also the true nature of this value. Performing this calculation actually results in the median, rather than the mean value, that many practitioners believe they are calculating, inadvertently shifting

the focus from the mean to the median. Therefore, in addition to being a biased estimate, Y often has a mistaken identity as well. Unfortunately, these challenges do not represent the most significant power function challenge the community faces – simply knowing that the problems exists is the biggest challenge. Therefore, to overcome these challenges, more widespread awareness of their existence is needed first.

Other forms of bias present in the parameters derived from OLS regressions are multicollinearity or omitted variable bias. Multicollinearity occurs when two independent regressors included in the equation are correlated with each other and both predict the dependent variable, such as a learning and rate variable. Omitted variable bias occurs when a regressor is correlated with a variable not included in the regression, such as a rate variable. The most frequently observed occurrence of these biases comes from the inclusion or exclusion of a rate variable in the CIC. Good practice in CER and CIC development requires an assessment of its severity, since the consequences of the condition can undermine the very foundations of the analysis.

The rate curve, already suffering from the challenges mentioned above, comes with its own unique set of additional challenges. Rate is generally accepted as the ratio of two measureable quantities, such as miles per hour or cost per pound. However, when applied in most cost estimates, the intent is for the rate parameter to capture the change in unit cost corresponding to an amount of work performed, within a defined period of time, within a fixed environment. Certainly, this is a little more than the ratio of two measureable quantities. For example, consider the change in cost of building a single engine when the number of engines being produced within one week increases from 20 to 30, without additional assembly lines being activated or other changes in use of the facility. Given the number of potential variables that can change in a factory from day to day or week to week, it can be very difficult to account for variability in the environment and accurately measure rate. On top of that, if one is measuring the rate of an assembled part, in theory the rate effect of purchased parts used in the assembled part would also need to be measured. Cost practitioners typically assume that analysis of the prime contractor's rate curve is representative of supplier behavior and operations, but this may not be the case.

Prediction Intervals

Many of the challenges addressed in this paper are compounding. For example, the challenges of bias and the misunderstanding of the shift from estimating the mean to the median when deriving CIC parameters compound upon the difficult calculation of prediction intervals.

Those issues aside, practitioners often times do not calculate them at all in favor of confidence intervals or coefficients of variation, despite prediction intervals being the best tool to use when it comes to predicting future values. This is because prediction intervals account for both the error associated with the mean of the historical dataset as well as the individual observations, whereas a confidence interval only accounts for the error associated with the mean of the historical data set. This suggests an additional challenge to be addressed beyond bias – community awareness.

S-curves

S-curves are sometimes neglected due to the convenience of presenting a point estimate, making the link between S-curves and setting affordable cost constraints essentially nonexistent – a major challenge for implementation.

Regardless, S-curves are an effective method of conveying the relationship between a predicted dollar value and the likelihood of completing a project for less than or equal to a stated dollar value, the basis for determining affordability. Unfortunately, when used, the most common current practice is generating S-curves from the confidence intervals, not the prediction intervals, corresponding to CIC. But in order to create an S-curve capable of representing a comprehensive portfolio, prediction intervals must be employed. Generating an S-curve from a prediction interval is only modestly more challenging than generating an S-curve from a confidence interval. However the observed lack of implementation of this best practice reinforces that some challenges addressed in this paper simply stem from a lack of knowledge or situational awareness, as opposed to technical complexity.

Additionally the compounding challenge of bias continues to persist. Accurate S-curves must be derived from unbiased estimates of $\hat{Y}$ and Var($\hat{Y}$) (the variability associated with $\hat{Y}$). However, as mentioned earlier, depending on the cost estimating methodology, often times these parameters are biased.

Unfortunately, the challenges associated with S-curves do not end with their derivation; they extend into communication with decision makers. The S-curve is limited in conveying a single price and risk at one time. When evaluating a portfolio of options and enabling decision makers to trade between combinations of products with different price-to-risk relationships, the S-curve is limited in its ability to adequately convey everything a decision maker needs to know relative to cost-risk.

Affordability Framework

This is not a new endeavor. Under direction from the Obama administration,

> "The Director for Acquisition Resources and Analysis in the Office of the Under Secretary of Defense for Acquisition, Technology and Logistics (OUSD (AT&L)) asked the Institute for Defense Analyses (IDA) to, conduct a study of ways to establish an analytical framework that will inform decisions by DOD acquisition executives regarding affordability." (Porter et al 2013).

Many of the methodologies, challenges, and issues highlighted in their findings are reiterated and expanded on here. The first challenge both the Porter et al. (2013) study and this paper acknowledge is a lack of definition of basic terms such as "affordability analysis", a lack of consistent approach to its execution across defense components (i.e., an operational construct), and lack of metrics and visibility of metrics (tripwires) to determine if a program is on track to meet its affordability objective.

For example, Chief of Naval Operations (CNO) issued direction in 2013 that programs include Key Performance Parameters (KPPs) for cost and schedule. One objective of the cost KPP was to ensure that the discussion of available trade space was realistically bounded by the amount of money the project was projected to have available through the FYDP. The guidance goes on to say that the cost KPPs should include objective and threshold values based on appropriate measures of unit cost (e.gs. Average Procurement Unit Cost (APUC) or Program Acquisition Unit Cost (PAUC)), but provides no further guidance as to what level of confidence should be placed on these values.

Affordability Constraints

Agencies and Departments predominantly focus on the degree to which a program is funded in a department or component's future year's program (as CNO direction dictated). However, the lack of affordability framework results in challenges in setting affordability constraints. Decision makers are unaware of the analytical underpinnings and the risk distributions associated with the point estimates corresponding to their portfolio of programs. Consequently, programs are too often priced at a mean or median point estimate, without visibility of their point on a cumulative distribution of cost, or an S-curve. Objective and threshold values for cost are set by resource sponsors without knowledge of the analytical underpinnings of the cost distribution and without an evaluation of cost and capability tradeoffs in the portfolio.

This *missing link* between the portfolio and the analysis that supports it can result in cost constraints that are not realistically bounded. Programs can be funded at levels below the theoretical 0% confidence level by direction of management challenges, leaving Program Managers scrambling to de-scope the capabilities of their programs, extend schedule, cut capacity (planned quantity) or add funds. In light of these factors, it is a real possibility that programs do not deliver their intended capability.

Portfolio Perspective

Once presented with a portfolio of costs and risk, the challenges faced by the cost estimator do not disappear. Because no framework exists, there is nothing that dictates the point at which funding is sufficient and risk sufficiently reduced. Decision makers must determine the level of risk they are comfortable or uncomfortable taking, given affordability constraints. There is also no framework for making trades between dissimilar products and metrics. For example, how does one determine whether to purchase more current generation tanks or invest in developing a more capable tank?

With decisions ideally evaluated in the context of other priorities, the next challenge from a lack of affordability framework perspective is a linkage between the cost constraints and the portfolio, including other meaningful factors to be considered, like measures of military value and total lifecycle cost beyond the FYDP for operations, sustainment, and demilitarization and disposal.

Lastly, the final, unavoidable challenge is always time; decision makers want their answers now! The challenges illuminated and addressed in this paper do not lend themselves to speed. They require additional work beyond the status quo. Fortunately, much of the work is upfront and automated processes can be implemented.

Evolutionary Steps

BBP initiatives in the U.S. DOD mandate driving productivity improvements in acquisition programs, making production rates economical, and addressing affordability and tradeoffs in the portfolio. Too many acquisition programs today focus on only part of the decision space while lacking an enterprise perspective.

Opening the aperture, former USD (AT&L) Frank Kendall posits that

"Affordability means conducting a program at a cost constrained by the maximum resources the Department can allocate for that capability."

From his perch as DOD's acquisition leader, he noted that:

"Many of our programs flunk this basic test from their inception."

The evolutionary steps of this section present an analytical framework to address not only Secretary Kendall's acquisition issues, but other methodological issues, with the end goal of getting capability downrange to the warfighter at an affordable price with acceptable risk.

Cost Improvement Curves

Equation (1) specifies the model under consideration, a CIC with a rate variable

$(1)\ Y_i = \alpha\ Q_i^b\ R_i^c\ e^{\varepsilon_i},\ \varepsilon_i\ \backsim N(0,\sigma_\varepsilon^2), i = 1\ to\ n$, where:

$Y_i =$ an observation on the dependant variable (e.g., unit cost or hours),

$Q_i =$ an observation of cumulative quantity,

$R_i =$ an observation of rate of production, and where α, b, and c are population parameters to be estimated.

Quantity captures the phenomenon of organizational learning by doing, as emphasized by Arrow (1962), Wright (1936), and McCarthy (2020). An Original Equipment Manufacturer (OEM) or one of its vendors gains experience or knowledge, at a company or enterprise level, in the production of component, subassembly, or end item, and thus achieves a reduction in unit cost.

Importantly, the quantity variable, in a broadly defined sense, allows for a range of sources of gains in productivity in addition to traditional passive learning by doing (Thompson, 2009). It captures the results of deliberate actions made by an organization to reduce unit costs through modifications to product design, capital investments in facilities, and use of specialized tooling. Importantly, all of these factors are implicitly captured in historical values of Y_i.

Inclusion of the rate variable in the CIC is intended to capture the phenomenon of changes in unit cost in a given time period, such as a fiscal year or over the course of a production lot, due solely to the number of units produced during that same period (Goldberg, 2003). Proponents of a rate variable offer this rationale:

- OEM's and vendors offer economic-order quantity (EOQ) discounts in the form of step-ladder pricing;
- Economies of scale on the factory floor (optimal staffing and optimal employment of tooling);
- Broader allocation of fixed costs.

As the International Cost Estimating and Analysis Association (ICEAA) Cost Estimating Body of Knowledge (CEBoK) suggests, program offices advocate the use of a rate variable to support major reviews by the Office of the Secretary of Defense, the Office of Management and Budget, and the U.S. Congress. These entities frequently make changes to a baseline number of quantities of weapon

systems to be acquired. This perturbation, in turn, requires changes to the dollars included in a component's budget – with rapid cost-analysis turnaround demanded. Inclusion of a rate variable facilitates the analysis.

This paper focuses on the use of equation (1) in building an S-curve that serves as an analytical foundation to support affordability analyses, with linkage to a capability portfolio. Nevertheless, it's worth highlighting some of the profound issues that arise in applied applications; i.e., actually trying to use the CIC to generate cost estimates. Each of these issues should be examined in detail to ensure the accuracy and credibility of the estimate. They're briefly summarized here, with remedial action suggested.

Measurement of Rate

Issue: Hands-on experience supporting defense components, program offices, and independent cost-analysis organizations reveals that the rate variable is usually measured as quantity authorized per year or per production lot. In terms of equation (1), it's expressed as $Q_i - Q_{i-1}$, or the delta in cumulative quantity from one time period (lot, or observation) to the previous one.

As Goldberg posits, this is likely incorrect for several reasons. First, outlay rates for major acquisition defense programs (MDAPs) extend over many years, not just the year of authorization. For any one year, the contractor is likely working on platforms and components authorized many years previously. Furthermore, the contractor is likely working on multiple projects at the same time, with many components in common. Rate should capture this phenomenon, especially as it relates to economies of scale.

Resolution: Consider Goldberg's proposition to

> "… vertically sum the number of units across all lots in progress during each fiscal year" (Goldberg, 2003).

That is, measure total work in a contractor's production facility. Other important considerations include:

- If plant capacity is exceeded, then unit costs may increase with rate due to overtime, cost of hiring and training new workers, and increasing tooling and test equipment,
- If rate should apply only to work at the OEM level or also to vendors.

Collinearity

Issue: Multicollinearity may be weak or strong, but it's always present. Good practice in CER and CIC development requires an assessment of its severity since the consequences of the condition can undermine the very foundations of the analysis. The critical question from the practitioner's perspective is:

> At what point does Multicollinearity become harmful, and what can be done about it?

If quantity is always increasing, then there's often a high linear relationship between Rate and Cumulative Quantity, as shown in *Figure 6*, using the OLS procedure of taking the logs of both sides of equation (1) to generate estimates.

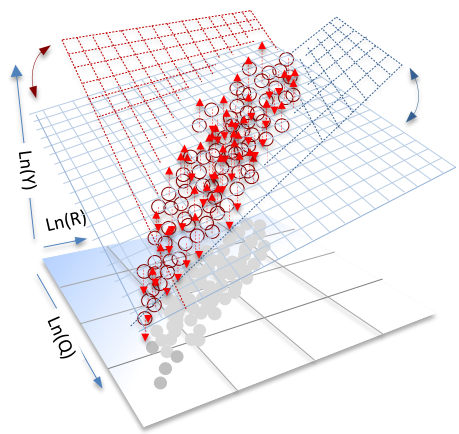

Figure 6: High Correlation between Quantity and Rate

The observations are in a tube. The estimated regression plane wobbles in successive samples. This renders estimates of the unknown population parameters (the exponents of the CIC) imprecise (Flynn, 2017).

Resolution: The engagement sequence for addressing the issue of Multicollinearity requires the execution of these steps, with details provided by Flynn (2017):

- Detection of symptoms, heuristics, and formal tests,
- Classification to determine levels of severity,
- Localization to discern the affected variables,
- Treatment to create a menu of remedies.

Statistical Bias of T1 Estimates

Issue: Estimates of the parameters are customarily obtained by performing OLS on the logarithmic transformation of equation (1), or

$$(2)\ \ln(Y_i) = \ln(\alpha) + b\ln(Q_i) + c\ln(R_i) + \varepsilon_i.$$

Least squares estimates of *b* and *c* are Best Linear Unbiased Estimate (BLUE). Problems arise, however, in estimating the constant term, α (Goldberger, 1968). The traditional method is to un-log the logarithmic term; that is

$$(3)\ \hat{\alpha} = e^{\widehat{\ln(\alpha)}},$$

presuming falsely, that this yields α. Note that this estimate of α is biased by a multiplicative variance factor since

$$(4)\ E(\hat{\alpha}) = E\left(e^{\widehat{\ln(\alpha)}}\right) \neq e^{E(\widehat{\ln(\alpha)})}, \text{ but instead}$$

$$= e^{E(\widehat{\ln(\alpha)})+\frac{1}{2}\sigma^2_{\widehat{\ln(\alpha)}}} = e^{\ln(\alpha)+\frac{1}{2}\sigma^2_{\widehat{\ln(\alpha)}}} = \alpha e^{\frac{1}{2}\sigma^2_{\widehat{\ln(\alpha)}}},$$

The expression $\sigma^2_{\widehat{\ln(\alpha)}}$ is the variance of the estimate of the constant term, in log space. Since this value is greater than zero in finite-sized samples, $e^{\frac{1}{2}\sigma^2_{\widehat{\ln(\alpha)}}}$ is greater than one, and $\hat{\alpha}$ therefore biased upward, with the bias proportional to the variance. The greater the variance, the greater the bias, all other things being equal.

Resolution: Following Goldberger (1968), a minimum-variance, unbiased estimator of α in equation (1), or the level or conditional median of the function Y_i, is

(5) $\tilde{\alpha} = e^{\hat{\gamma}} F$, where

$\hat{\gamma} = \widehat{\ln(\alpha)}$ or the OLS estimate of $\ln(\alpha)$, and $F = \sum_{j=0}^{\infty} \frac{f_j (cw)^j}{j!}$, where

$c = -\frac{1}{2} m^{00}$, with m^{00} denoting the estimated, unscaled variance of $\widehat{\ln(\alpha)}$, i.e., the upper left corner element of the inverse of $\boldsymbol{X}'\boldsymbol{X}$, with $\boldsymbol{X}$ the data matrix.

$w = s^2$, or the residual variance from OLS estimation

$$f_j = (0.5v)^j \frac{\Gamma(0.5v)}{\Gamma(0.5v + j)}$$

$v = n - k - 1$ degrees of freedom, where

n = number of observation and k = number of explanatory variables.

Statistical Bias of Medians and Means

Issue: As Goldberger posits in his seminal work on the estimation of power-function regression equations, and as mentioned in the challenges section, analysts are generally unaware that they are not estimating mean values of Y but rather median values:

> "For empirical implementation of the Cobb-Douglas [power] function, it is customary to append a multiplicative log normal disturbance and fit a linear regression in the logarithmic variables. When this is done, attention is shifted (apparently unwittingly) to the conditional median from the conditional mean which is ordinarily the prime target of study."

The Joint Agency Cost Schedule Risk and Uncertainty Handbook (JA CSRUH) also emphasizes this point. However, it does not indicate the estimates of the conditional median, using OLS estimation of equation (1), are biased. To complicate matters, estimates of both the conditional median and conditional mean, using OLS estimation of equation (1) are biased.

Resolution: Determine which measure of central tendency is the focus of attention: the conditional median or conditional mean. Note that Y in equation (1) is log normally distributed. The median value will fall to the left of the mean in a probability density function.

Note that the mean value of Y can be interpreted as the average value of Y observed from repeated observations on the same set of values of Q and R. It is usually denoted by the letter E. The median of Y is the "middle value" in repeated sampling. In a sorted sequence of estimates, half the values are less than the median, denoted by the letter M, and half are higher.

1) $Y = \alpha\ Q_i^b\ R_i^c\ e^{\varepsilon_i}$ where $\varepsilon_i \backsim N(0, \sigma_\varepsilon^2)$

2) $E(Y) = \alpha\ Q_i^b\ R_i^c\ e^{0.5\sigma_\varepsilon^2}$

3) $M(Y) = \alpha\ Q_i^b\ R_i^c$

Though the OLS estimates, $\widehat{ln\alpha}, \hat{b}$, and $\hat{c}$, obtained by transforming equation (1) into log space are unbiased, the exponential functions of these parameters are not due to the convexity of the exponential function

4) $E(\hat{Y}) = E(e^{\hat{w}}) = e^{w+0.5Var(\widehat{w})}$

5) $= aQ^b R^c e^{m^*\sigma^2}$

where $m^*\sigma^2$ is the variance of a predicted value of any given value of Q and R. This is the variance of the familiar variance-covariance result

6) $m^*\sigma^2 = Var(\widehat{ln\alpha}) + (lnQ)^2 Var(\hat{b}) + (lnR)^2 Var(\hat{c}) + 2(lnQ)Cov(\widehat{ln\alpha}, \hat{b}) +$

$$2(lnR)Cov(\widehat{ln\alpha}, \hat{c}) + 2(lnQ)(lnR)Cov(ln\hat{b}, ln\hat{c})$$

Therefore, the challenge here is to develop unbiased estimators of the median and mean. Following Goldberger, the equations that produce unbiased predictions for median and mean, respectively, are:

7) $E(F_M) = e^{-0.5m^*\sigma^2}$

8) $E(F_E) = e^{0.5\sigma^2} e^{-0.5m^*\sigma^2}$

That is, the products of the estimator $e^{\hat{w}}$ and the correction factors result in unbiased estimators of the median and mean as follows:

9) $E(e^{\hat{w}} F_M) = [e^w e^{0.5m^*\sigma^2}] * [e^{-0.5m^*\sigma^2}]$

$= e^w = \alpha X^b R^c$

10) $E(E(e^{\hat{w}} F_E) = [e^w e^{0.5m^*\sigma^2}] * [e^{-0.5\sigma^2} e^{-0.5m^*\sigma^2}]$

$= e^w e^{0.5\sigma^2} = \alpha X^b R^c e^{0.5\sigma^2}$

Note that (7) and (8) present <u>expected</u> values. The actual correction factor, as with the T1 bias correction from Goldberger, is of the form:

$$F(w; v, c) = \sum_{j=0}^{\infty} \frac{f_j (cw)^j}{j!}$$

Where:

$$f_j = (0.5v)^j \frac{\Gamma(0.5v)}{\Gamma(0.5v + j)}$$

w = variance of the estimator

c = a constant
v = degrees of freedom

The correction factor for the median of Y is:

$F_M(w; v, c)$, where $w = s^2$ and $c = -\frac{1}{2}m^*$

The correction factor for the mean of Y is:

$F_E(w; v, c)$, where $w = s^2$ and $c = \frac{1}{2}(1 - m^*)$

Note:

$$m^* = [1 \quad 1 \quad 1]\,(X'X)^{-1}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

After correcting for the biases in T1 and conditional mean and median estimates, the final step towards significantly improved prediction interval accuracy involves addressing the variance component of the prediction interval. Currently, the common method for calculating this component is based on outputs from OLS regression that are used in the following equation:

$$Var(forecast\ error) \equiv \sigma_F^2 \equiv s^2 + \frac{s^2}{n} + (X_1^* - \overline{X_1})^2\, Est.Var(b_1) + (X_2^* - \overline{X_2})^2\, Est.Var(b_2) + 2(X_1^* - \overline{X_1})(X_2^* - \overline{X_2})\, Est.Cov(b_1 b_2)$$

However, to remain consistent with the corrective action of eliminating bias, this calculation needs an expression for variance of unbiased estimators. Consider the conditional mean to be the unbiased estimator of choice, in which case the variance of unbiased mean predictions can be expressed knowing that

$$\lim_{n\to\infty} E(F_E^2) = 1,$$

Where F_E is the Goldberger adjustment factor for the conditional mean.

Therefore, it follows that the asymptotic variance of the unbiased prediction of the conditional mean of Y_i is:

$$Asy\ Var(\tilde{Y}) =\propto X^b R^c exp(0.5\sigma_\varepsilon^2 x_o (X'X)^{-1} x_o exp[0.5\sigma_\varepsilon^2 x_o (X'X)^{-1} x_o - 1]$$

Hence, the unbiased estimator is consistent. That is, its distribution collapses to the true value as the sample size increases to infinity. Estimates of small-sample variance of the unbiased mean and median can be computed through repeated random sampling, or Monte Carlo simulation. This procedure uses the formulas for unbiased predictions to generate about 5,000 samples and determine the variance of the sequence for mean and median estimates. The authors have a repeated sampling procedure for the variance component in development.

Prediction Intervals

Statistical Bias of Prediction Intervals

Issue: The components of calculating a 95% prediction interval are seen in the expression:

$$\widehat{Y} - tstatistic * \sigma_F \;\leq\; Y \;\leq\; \widehat{Y} + tstatistic * \sigma_F, \text{ where } \sigma_F \text{ is the square root of } \sigma_F^2$$

The issue here is that the elements of this prediction interval expression are often based on biased calculations and possible misidentification of the Y estimate, as discussed previously. The estimate for Y is usually unknowingly based on the median, and biases in this estimate lead to inaccurate prediction intervals regardless of whether it is representing the mean or median. In turn, the component of variance is not based on an unbiased estimator. The effects of these flawed calculations result in an inaccurate prediction interval.

Resolution: Use Goldberger's correction factors for the conditional mean or median, depending on choice of unbiased estimator, to correct the estimate for Y used in finding the prediction interval. Also, perform the prediction interval calculation with an accurate expression of variance, which can be found through Monte Carlo simulation for small sample variance. Ensure that this expression of variance is consistent with the unbiased estimator of choice, mean or median, in agreement with the Y estimate component of the prediction interval.

S-curve

Issue: Traditional generation of S-curves occurs through OLS derivations (which result in biased parameters), incorporates the use of confidence intervals (as opposed to prediction intervals), and only show one cost against one risk at a time.

Resolution: Generating the S-curve from the CIC (with the inclusion of learning and rate variables), changing the focus from confidence intervals to prediction intervals, and showing more than one cost vs risk parameter at a time.

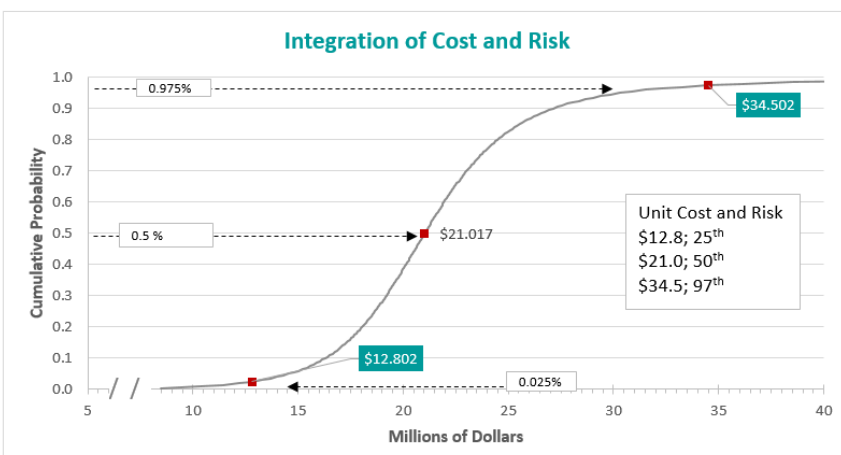

Figure 7: S-curve generated with prediction intervals and highlighting multiple cost vs risk outputs

Affordability Framework

Affordability analysis is a data driven process for evaluating the relative merits (life-cycle costs, military value, and risk) of a material solution or program in a capability portfolio for various levels of resource availability, given strategic priorities of senior leadership.

Through the illumination of trade space, affordability analysis seeks to address these two critical questions in building and maintaining a defense force:

- Can the program afford to buy the material solution?
- If bought can the program afford to own it?

Affordability cost constraints for procurement link the S-curve of *Figure 7* with a portfolio's budget. Ideally, the constraints are determined in a top-down manner based on the resources that a program can allocate to an acquisition element given the priority of the capability, inventory objectives, procurement and deployment schedules, and competing needs, or opportunity costs. *Figure 8* depicts the general process of evaluation, from defining a set of prioritized capabilities and programs to providing decision support to individual projects.

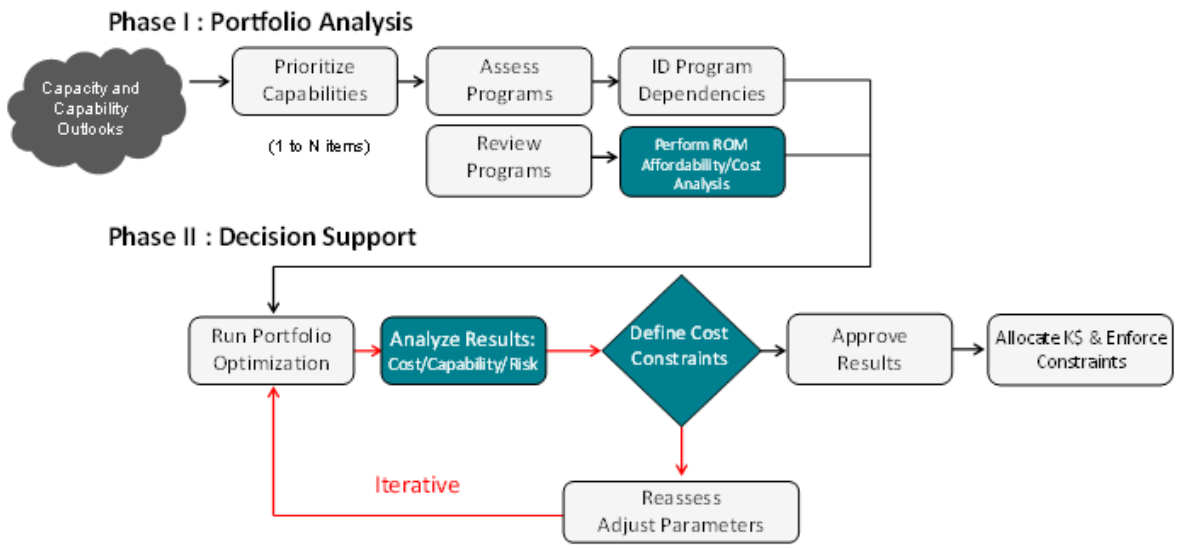

Figure 8: Depiction of portfolio analysis process with decision support utilizing the iterative process of cost constraints to analyze the portfolio perspective to an optimized state.

As an operational construct, a system or program or capability is deemed affordable if it:

- Supports the achievement of strategic goals,
- Syncs with enterprise-level tradeoffs between current operational requirements and long-term needs,
- Reduces risk in the portfolio and accords with leadership's tolerance for risk,
- Yields a risk-based return on investment (figure of merit/life-cycle cost) greater than alternatives or, more precisely, yields a marginal value greater than the opportunity cost's marginal value,

- Fits within a budget and programming "topline," not only in the FYDP but over the lifecycle.

Affordability Constraints

Two types of constraints are common: goals, which include threshold and objective values, and caps. The constraints, in turn, are informed by life-cycle costs estimates for the acquisition. A goal is a target cost, typically expressed in terms of a metric such as average acquisition cost or yearly operating and support (O&S) cost. Early in the acquisition phase of a program, the goal might be set in terms of ranges of values rather than a fixed number. Goals, in turn, are set for both investment costs and sustainment costs.

A threshold value is the maximum allowed cost, while an objective value is the desired cost, given maximum contractor efficiency, and usually set through should-cost studies. Investment costs, as stated earlier, come in the form of APUC, total acquisition cost divided by delivered quantities, and PAUC, total procurement costs divided by delivered quantity. Sustainment costs on the other hand, are usually the average yearly operating and sustainment costs, such as cost per flight hour.

A cap is an affordability cost typically set at a point beyond which the decision calculus regarding the material or capability solution needs to be revisited. Tradeoffs must be considered in applying a cap to an individual program. Setting the cap entails analysis of this key issue:

At what point does the cost of a new or enhanced capability become so high that portfolio leadership would rather cancel the program, significantly reduce quantity or operational capability, or cut back other programs in the portfolio rather than continue?

Building upon previous steps (CIC, to prediction intervals, to S-curves), *Figure 9* links cost, risk, and affordability. The notional threshold and objective unit prices, to the left and right of the mean, define the core of trade space. The cap is set at the 80th percentile. Under this schema, decision makers are presented with a menu of options (cost, risk, and funding combinations) for consideration.

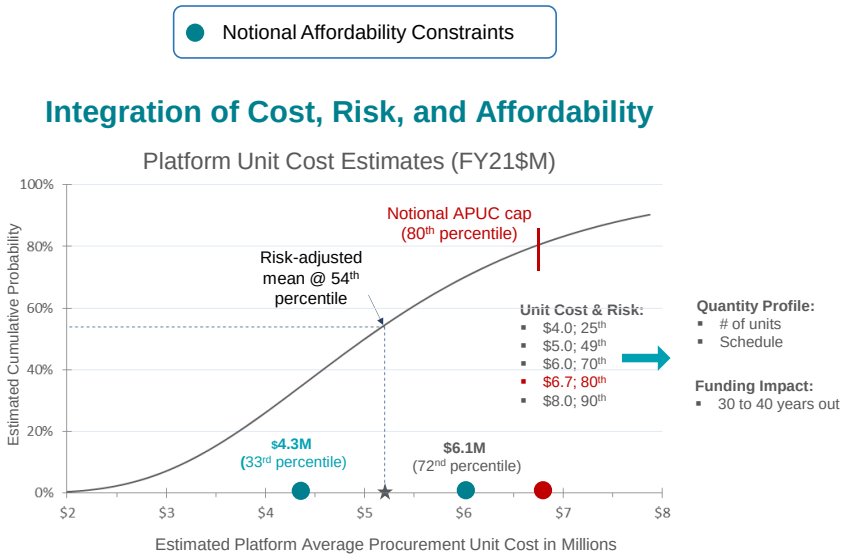

Figure 9: Linkages between cost, risk, and affordability with a menu of options for leadership to choose from.

Importantly, affordability constraints should be based on the anticipated level of future budgets, or Total Obligational Authority (TOA), available to procure and support a product within a relevant portfolio of products, with the sand chart of *Figure 10* providing an example. In general, affordability constraints are the product of budget, inventory, and product life-cycle analysis within a portfolio context.

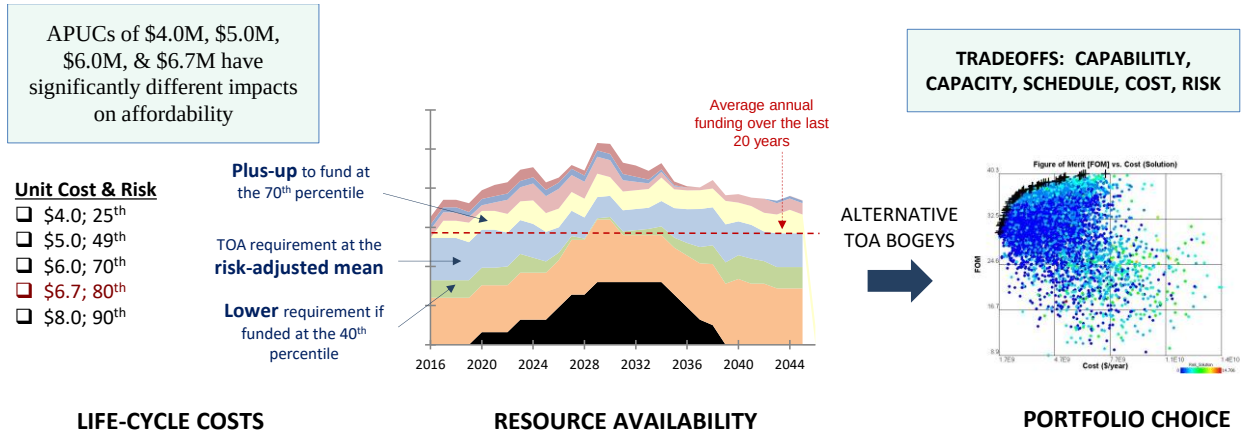

Figure 10: Linkage of cost estimates, cost constraints, and affordability trade space in the portfolio

Affordability constraints force prioritization of requirements, drive performance and cost trades, and ensure that unaffordable programs do not enter the acquisition process. If affordability caps are breached, costs must be reduced or else program cancelation can be expected. Constraints stem from long-term affordability planning and analysis made with a portfolio perspective, as *Figure 10* shows.

Portfolio Tradeoffs

Tradeoffs within the portfolio leverage the insights from the previous steps. For various level of funding and unit prices, options are considered based on figure of merit (military value or capability), capacity (or procurement quantity), schedule, and risk (from the S-curve). *Figure 11* provides a schematic of the decision space.

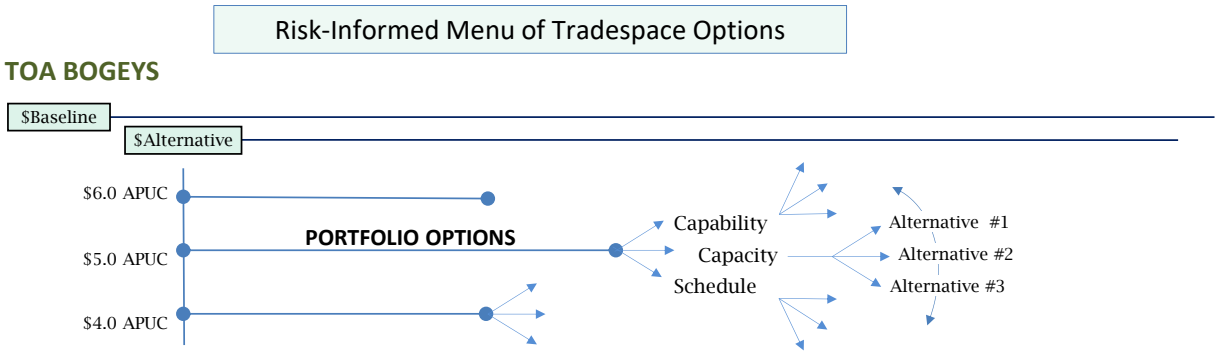

Figure 11: Cost, Capability, and Schedule Trade Space

Many organizations employ sophisticated optimization techniques to assess value in the portfolio. Many use the tools commonly found in the discipline of Multi-Objective Decision Analysis (MODA). MODA uses an attribute hierarchy to assign a value score to each alternative or Course of Action (CoA).

Alternatives are then compared based on an attribute score and life-cycle costs to determine which are most efficient (i.e., provide greatest performance for the cost, or lowest cost for given performance). Overall objectives of the analysis are to:

- Assess what capability is lost or gained from one alternative to another and at what cost,
- Provide a method to down-select alternatives based on affordability and minimum acceptable capability,
- Identify cost and operational effectiveness drivers,
- Identify relative value in terms of warfighting capability (i.e. mission tasks, measure of effectiveness, etc.),
- Integrate cost and military utility to illuminate the trade space.

Importantly, any such models should emphasize the linkages between weapon systems and strategy to ensure that any and all potential investments accord with the overall requirements of the defense component (Downes-Martin, 2011).

While beyond the scope of this study to conduct a MODA, a sample (albeit simplistic) output of this analysis is presented *Figure 12*, with bubbles sized to represent life-cycle costs in constant dollars. That is, budgeting an acquisition program at the 40th percentile (the S-Curve of *Figure 9*) increases cost risk for that program, but enables procurement of an additional program in the portfolio. Budgeting at the 80th percentile buys down cost risk but only at the expense of defunding a program in the portfolio.

The central tenet of this research is to present decision makes with a menu of data-driven, analytically-informed options from which to choose.

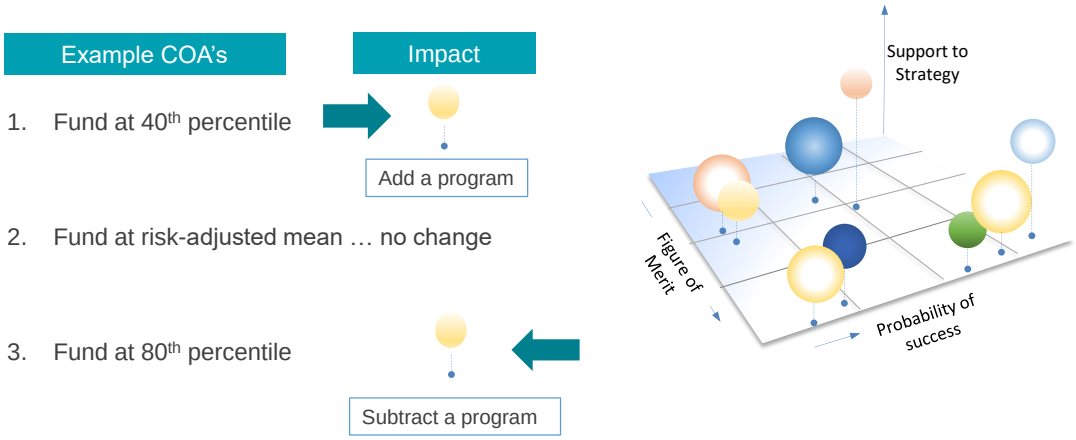

Figure 12: Portfolio Tradeoffs

Value Added

When estimated requirements are greater than the budget, decision makers must decide what does not get funded. The authors have frequently witnessed that decisions are made without an understanding of the affordability constraints associated with the requirements. That is, often time decision makers do

not know the confidence levels of the requirements they are allocating budget towards, and even less often do they know the distributions of possible costs and corresponding confidence levels for those requirements. This leads to a potential outcome of trading away too much cost through underfunding a requirement below the one percent confidence level, and all but ensuring the requirement will be in-executable.

Alternatively, equipped with the output of the analytical framework presented here, decision makers would understand the distribution of costs and confidence levels associated with each requirement in the portfolio. It presents a framework for smoother synchronization of acquisition, resourcing, and planning. Instead of focusing on programs in stovepipe fashion, with broken linkages all along the way from learning curves to the portfolio, it engenders more informed resource-allocation decisions through:

- Better Cost Estimates
    - Applies to CIC and, more generally, to any CER in power-function form,
    - Flags inadvertent focus on the median rather than the mean, with Y, the dependent variable, log-normally distributed,
    - Replaces biased OLS estimates of the constant term and Y with unbiased estimates,
    - Present formulas for the unbiased conditional mean and the unbiased conditional median,
    - Provides statistical insights on other key issues related to CIC such as collinearity and use of a rate variable – relative to plant capacity.
- S-Curves from Predictions
    - Uses unbiased predictions of Y,
    - Generates unbiased prediction intervals using an asymptotic forecast variance,
    - Enables treatment of risk as both an output variable from regression analysis and an input variable in tradeoffs with funding availability and unit cost.
- Affordability Constraints
    - Links with the S-curve thereby providing an analytical, data-driven foundation to minimize the guess work in setting constraints,
    - Presents decision makers with menu of options for the establishment of threshold and objective unit costs and cost caps,
    - Re-establishes the principles of good governance: cost analysts illuminate trade space while leadership decides where to place their bets.
- Portfolio Tradeoffs
    - Links affordability constraints with cost, capability, and funding options in the portfolio,

- Addresses head-on the opportunity cost of choices in a menu of pricing options for acquisition programs,
- Determines holistically the overall fiscal feasibility of a program in a portfolio,
- Supports the establishment of realistic program baselines,
- Helps control life-cycle costs,
- Instills more cost-conscious management in the defense component.

In closing, this paper offers cost analysts an analytical process that represents an opportunity for the profession to deliver greater value to decision makers in the form of more informed portfolio decisions. The steps in the process are quite simple: create CIC, apply these to the prediction intervals, use the prediction intervals to calculate the S-curves, and from there generate the affordability constraints to generate trade space within the portfolio. In describing the process, the paper illuminated an important problem ignored or simply overlooked by cost practitioners -- the inherent biases created by the traditional OLS methods. Consistent correction of bias errors will contribute to more informed portfolio decisions.

Appendix

Derivation of the Expected Value of an Exponential Expression

Let X denote a random variable; i.e.

(1) $X \sim N(0,\sigma^2)$, where

$$X \equiv \ln(Y)$$

This implies that $Y = e^X$. Therefore, since X is normally distributed, it has a probability density function

(2) $$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{(x-u)^2}{2\sigma^2}\right) dx.$$

Since Y is a function of the random variable X, the mean of Y is specified by the relationship

(3) $$E(Y) = \int_{-\infty}^{+\infty} Y f_X(x) dx$$

$$= \int_{-\infty}^{+\infty} e^X \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{(x-u)^2}{2\sigma^2}\right) dx$$

$$= \int_{-\infty}^{+\infty} e^X \frac{1}{\sigma\sqrt{2\pi}} exp\left(\frac{-x^2+2ux-u^2}{2\sigma^2}\right) dx$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} exp\left(\frac{2\sigma^2 x - x^2 + 2xu - u^2}{2\sigma^2}\right) dx$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{-2\sigma^2 x + x^2 - 2xu + u^2}{2\sigma^2}\right) dx.$$

Completing the square in the numerator of the exponential ratio yields

(4) $$E(Y) = \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{-2\sigma^2 x + x^2 - 2xu + u^2 + 2\sigma^2 u + \sigma^4 - 2\sigma^2 u - \sigma^4}{2\sigma^2}\right) dx$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{(x-u-\sigma^2 x)^2 - 2\sigma^2 u - \sigma^4}{2\sigma^2}\right) dx$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}}\, exp\left(\frac{2\sigma^2 u + \sigma^4}{2\sigma^2}\right) exp\left(-\frac{(x - u - \sigma^2 x)^2}{2\sigma^2}\right) dx$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}}\, exp\left(\frac{\sigma^2}{2} + \mu\right) exp\left(-\frac{(x - u - \sigma^2 x)^2}{2\sigma^2}\right) dx$$

$$= e^{0.5\sigma^2 + \mu} \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}}\, exp\left(-\frac{(x - u - \sigma^2 x)^2}{2\sigma^2}\right) dx.$$

The integrand of the last expression is a normal distribution with mean = $\mu + \sigma^2$ and variance = σ^2 and hence is equal to 1. Thus,

(5) $E(Y) = e^{0.5\sigma^2 + \mu}$.

Derivation of Unbiased Estimators of the Conditional Mean and Median of Y

Background on Chi-Square (χ^2) Distribution

Let X represent a normally distributed random variable; i.e.

(1) $X \sim N(0,\sigma^2)$, where

μ = population mean

σ^2 = population variance

Let s^2 denote the variance of a sample of data drawn from the population. That is,

(2) $s^2 = \sum_{i=1}^{n} \frac{(x_i-\bar{x})^2}{n-1}$, where

n = sample size

x_i= the ith observation

$\bar{x}$ = sample mean

(3) $(n-1)s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2$

Dividing each side by the population variance yields a sample statistic that follows a χ^2 distribution.

(4) $\frac{(n-1)s^2}{\sigma^2} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})}{\sigma^2} \sim \chi^2{}_{n-1}$, with $n-1$ degrees of freedom. Adjustment factors that yield unbiased estimators of the conditional mean and median of Y, denoted F_E and F_M respectively, both use the exponential term, $e^{c\sigma^2}$. Hence, the task becomes one of finding an unbiased estimator of this term.

Lemma: For a given constant c, an unbiased estimator of $e^{c\sigma^2}$ is

$\sum_{j=0}^{\infty} \frac{f_j (cw)^j}{j!}$, where

w = a random variable; i.e., an estimator of σ^2, say s^2

$f_j = \frac{\left(\frac{1}{2}v\right)^j \Gamma(\frac{1}{2}v)}{\Gamma(\frac{1}{2}v+j)}$

v = degrees of freedom

Hence,

(5) $\frac{vw}{\sigma^2} \sim \chi^2{}_v$, since w is a sample variance which is "standardized" by the population variance.

Proof of Lemma:

The moments of the χ^2 distribution based on Abramowitz & Stegun (1972) are

$$(6)E(\chi^2{}_v)^j = \frac{2^j\Gamma(\frac{1}{2}v+j)}{\Gamma\left(\frac{1}{2}v\right)} = \frac{v^j}{f_j}$$, for j=0, 1, 2-….

Recall that "c" is a constant, either $\frac{1}{2}(1-m^*)$ or $-\frac{1}{2}m^{*x}$, depending upon whether the mean or median is sought. In either event, this generalization holds, multiplying by "one" and rearranging terms.

$$(7)cw = cw\left(\frac{v\sigma^2}{v\sigma^2}\right) = \underbrace{\left(\frac{c\sigma^2}{v}\right)}_{\text{Constant}} \cdot \underbrace{\left(\frac{vw}{\sigma^2}\right)}_{\text{Follows the Chi-Square distribution}}$$

Hence, following equation (6), the moments of cw are

$$(8)E(cw)^j = \left(\frac{c\sigma^2}{v}\right)^j E\left(\frac{vw}{\sigma^2}\right)^j = \left(\frac{c\sigma^2}{v}\right)^j \cdot \frac{v^j}{f_j} = \frac{(c\sigma^2)^j}{f_j}$$ for j=0, 1, 2… Now,

$$(9)E\left[\sum_{j=0}^{\infty}\frac{f_j(cw)^j}{j!}\right] = \sum_{j=0}^{\infty}\frac{f_jE[(cw)^j]}{j!} = \sum_{j=0}^{\infty}\frac{f_j[\frac{(c\sigma^2)^j}{f_j}]}{j!} = \sum_{j=0}^{\infty}\frac{(c\sigma^2)^j}{j} = e^{c\sigma^2}$$, by series expansion of the exponential function using Maclaurin's Formula

$$e^x = \sum_{j=0}^{\infty}\frac{x^j}{j!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^j}{j!},$$ which is identical to equation (9), where $x = c\sigma^2$

List of Acronyms

APUC - Average Procurement Unit Cost

BBP - Better Buying Power

BE - Baseline Estimate

CE - Current Estimate

CEBoK - Cost Estimating Body of Knowledge

CER - Cost Estimating Relationships

CGF - Cost Growth Factor

CIC - Cost improvement curves

CNO - Chief of Naval Operations

CoA - Course of Action

DE - Development Estimate

DOD - Department of Defense

EMALS - Electro Magnetic Aircraft Launch System

EOQ - Economic-Order Quantity

FYDP - Future Years Defense Program

ICE - Independent Cost Estimate

ICEAA - International Cost Estimating and Analysis Association

IDA - Institute for Defense Analyses

JA CSRUH - Joint Agency Cost Schedule Risk and Uncertainty Handbook

KPP - Key Performance Parameters

LPD-17 - Landing Platform Dock

MDAPs - Major Acquisition Defense Programs

MODA - Multi-Objective Decision Analysis

O&S - Operating and Support

OEM - Original Equipment Manufacturer

OLS - Ordinary Least Squares

OUSD (AT&L) - Office of the Under Secretary of Defense for Acquisition, Technology and Logistics

PAUC - Program Acquisition Unit Cost

PdE -Production Estimate

RMS - Remote Minehunting System

SARs - Selected Acquisition Reports

T1 – theoretical first unit production cost

TOA - Total Obligational Authority

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