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Minimize risk. Maximize potential.

Modeling Unit Learning Curves from Lot Data

Dr. Steve Van Drew

ICEAA Technology Showcase Webinar Series

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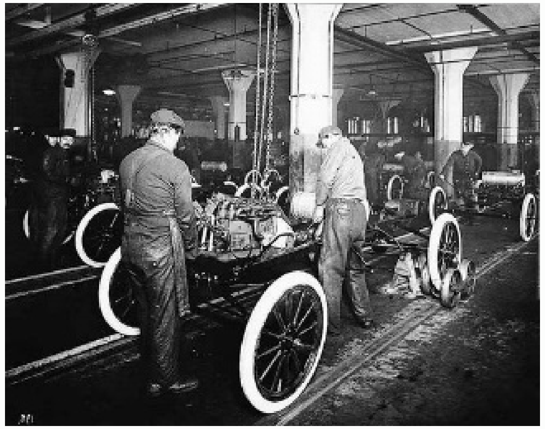
Webinar Topics

- Learning Curve Fundamentals
- Power Model Review
- Working with Lot Data
  - Lot midpoint approximation
- Demonstration
  - Data transformation
  - Excel LINEST function and Goal Seek
  - @RISK implementation



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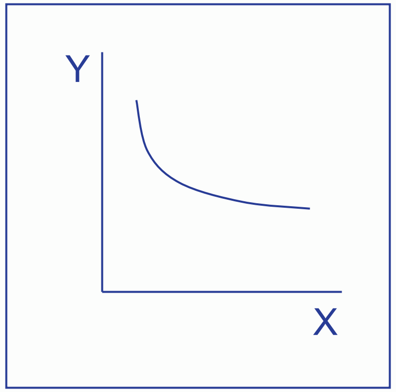
Unit (Crawford / Boeing) Learning Curve Theory

As the number of units produced doubles, the **unit** cost/time to produce the doubled unit decreases by a constant percentage.

Unit (X)	Unit Cost (Y) or Time
1	100
2	90
4	81
8	72.9



$$Y = aX^b$$

where:

Y = cost/time of *Xth* unit

a = cost/time of first unit (T_1)

X = number of unit produced

b = learning curve exponent

$$b = \frac{\ln(slope)}{\ln(2)} = \frac{\ln(.90)}{\ln(2)}$$
$$= -.1520$$

$$Y = 100X^{-.1520}$$

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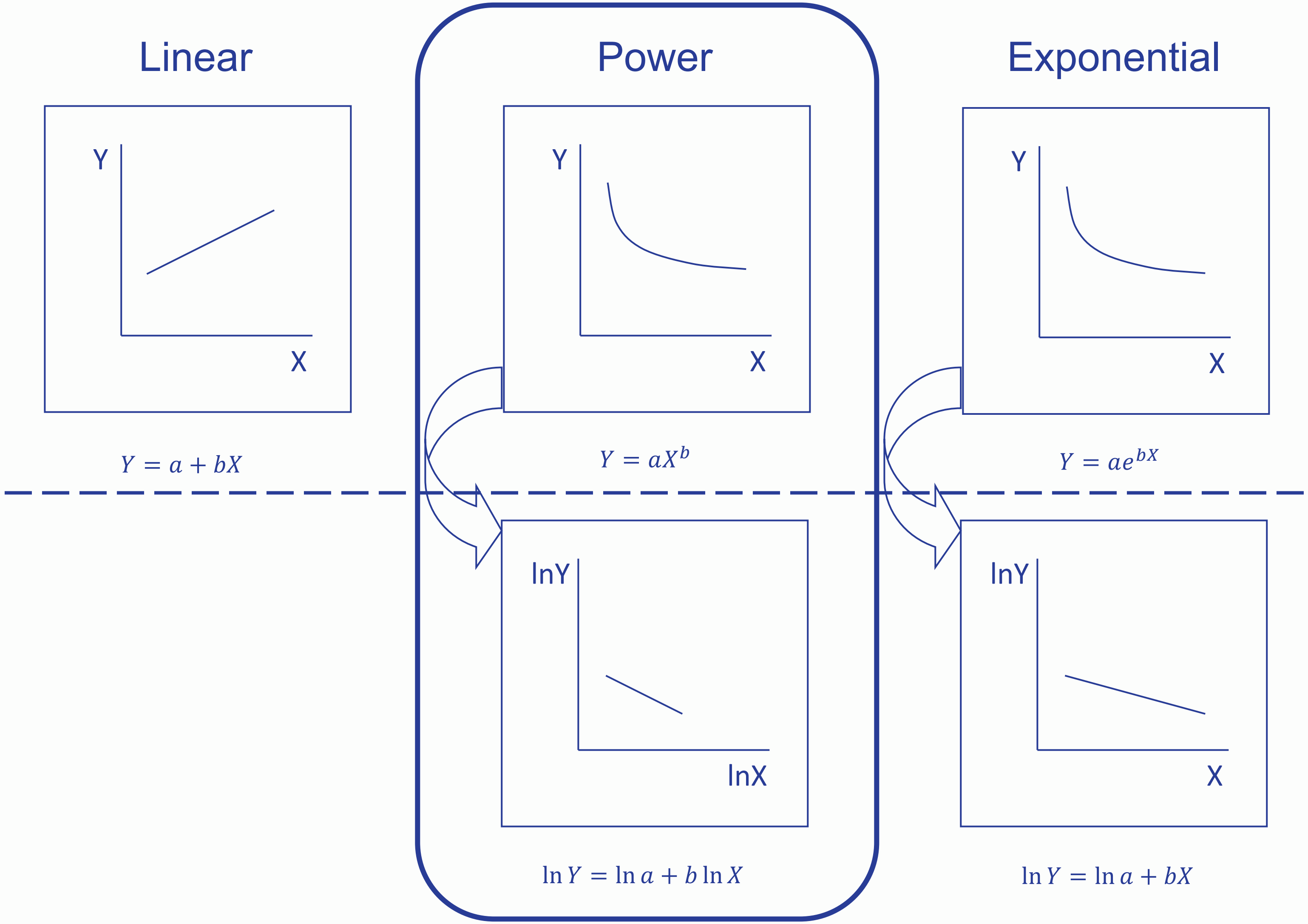
Intrinsically Linear Transformations



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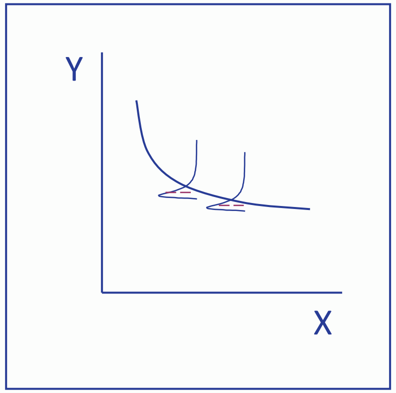
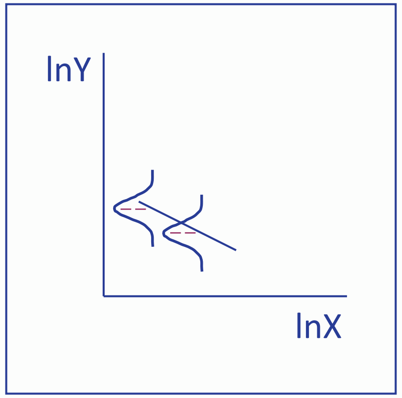
Power Model Basics

Power Model



$$Y = aX^b\varepsilon'$$

Transformed Model



$$\ln Y = \ln a + b \ln X + \varepsilon$$

ε’ in power model is a multiplicative error term that is lognormally distributed, but in the transformed model ε is additive and normally distributed with same characteristics assumed as an ordinary least squares linear regression error term

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Power Model Lognormal Error Term

if $Y \sim N(\mu,\sigma)$

then

$X = e^Y \sim \mathit{ln}(\mu,\sigma)$

and

$E(X) = e\text{^}(\mu+\sigma^2/2)$

$V(X) = [e\text{^}(2\mu+\sigma^2)](e^{\sigma^2}-1)$

to model in @RISK use:
RiskLognorm $(E(X), \{V(X)\}^{.5})$
or
RiskLognorm2 (μ,σ)

For fuller coverage of modeling regression error terms go to
https://go.palisade.com/Modeling-Regression-Errors-with-RISK.html

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Determining T_1 and Slope from Lot Data: Problem Statement

A critical subassembly has experienced the following production hours for the first five lots. Assuming learning follows the unit theory, find the T1 and slope for the appropriate learning curve and estimate the production hours needed for the 45 units in Lot 6.

Lot	Size	Units	Cost (man–hrs)
1	15	1 - 15	35000
2	10	16 – 25	20500
3	60	26 – 85	85000
4	30	86 – 115	39500
5	50	116 - 165	58000

Determining T_1 and Slope from Lot Data: Lot Midpoint (LMP)

- The Lot Midpoint (LMP), also called the Algebraic Lot Midpoint or Lot Plot Point, is defined as the theoretical unit whose cost (or time) is equal to the Average Unit Cost (AUC) or Lot Average Cost for that lot on the learning curve
- The LMP value represents the X value and the AUC represents the Y value when working with lot data

"The difficulty in finding the LMP is that you must know the slope of the learning curve in order to determine the LMP, but you need all the LMPs to find the slope of the learning curve."

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Determining T_1 and Slope from Lot Data: LMP Approximation

$$LMP = \left[\frac{(L+.5)^{b+1} - (F-.5)^{b+1}}{(b+1)(L-F+1)}\right]^{1/b}$$

where:
b = slope coefficient
F = first unit # in lot
L = last unit # in lot

See backup slides for a six-term alternate LMP approximation formula that ACE-IT’s CO$TAT 7.5 uses

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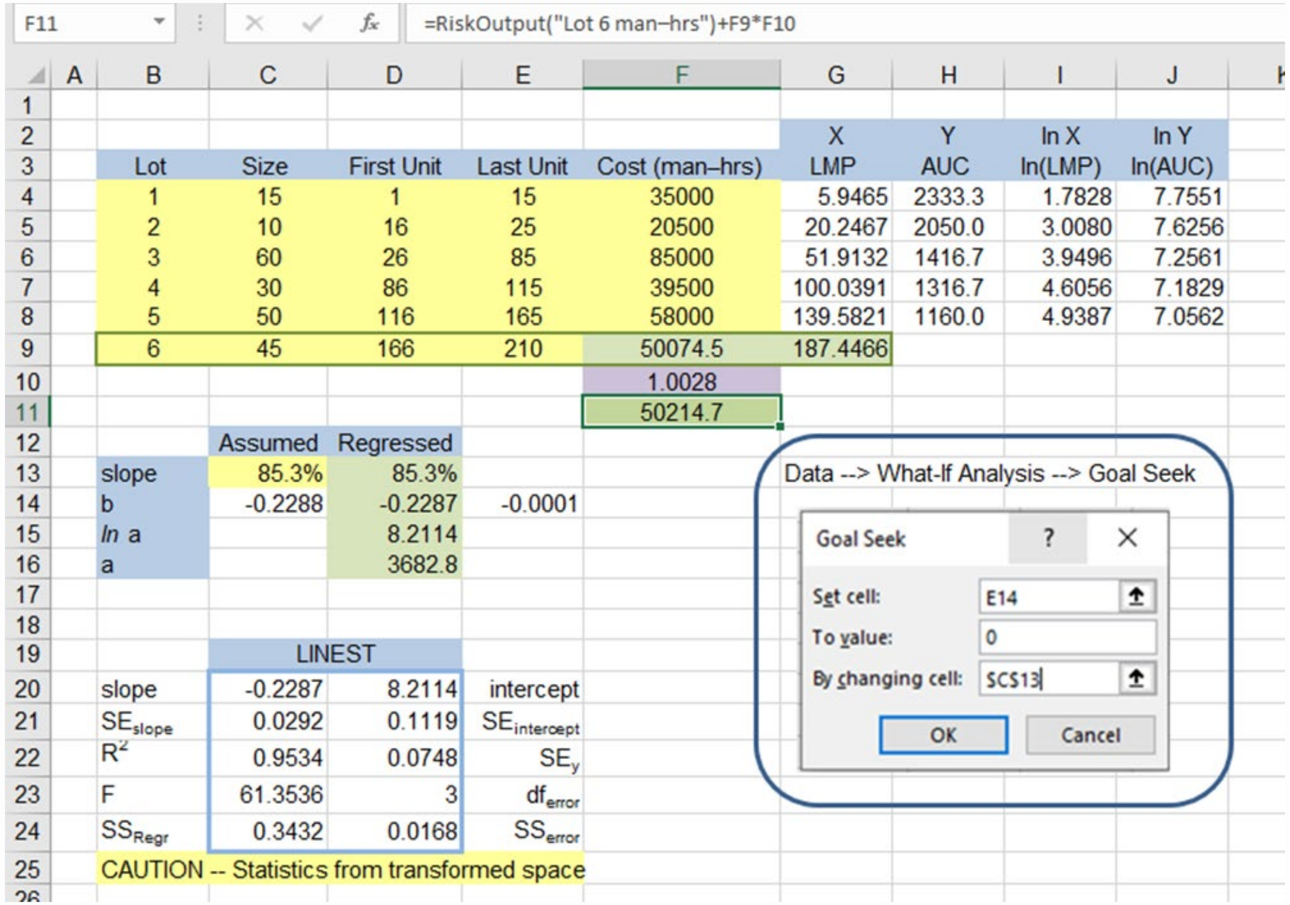
Demonstration

General Steps:

1. Enter lot data in B4:F8
2. Enter starting slope assumption in C13
3. Run Goal Seek as shown in dialog box image
4. Enter new lot data in B9:E9
5. Run @RISK simulation with F11 as output



F11 =RiskOutput("Lot 6 man–hrs")+F9*F10

	A	B	C	D	E	F	G	H	I	J
1										
2							X	Y	ln X	ln Y
3		Lot	Size	First Unit	Last Unit	Cost (man–hrs)	LMP	AUC	ln(LMP)	ln(AUC)
4		1	15	1	15	35000	5.9465	2333.3	1.7828	7.7551
5		2	10	16	25	20500	20.2467	2050.0	3.0080	7.6256
6		3	60	26	85	85000	51.9132	1416.7	3.9496	7.2561
7		4	30	86	115	39500	100.0391	1316.7	4.6056	7.1829
8		5	50	116	165	58000	139.5821	1160.0	4.9387	7.0562
9		6	45	166	210	50074.5	187.4466			
10						1.0028				
11						50214.7				
12			Assumed	Regressed						
13		slope	85.3%	85.3%						
14		b	-0.2288	-0.2287	-0.0001					
15		*ln* a		8.2114						
16		a		3682.8						
17										
18										
19			LINEST							
20		slope	-0.2287	8.2114	intercept					
21		SE_{slope}	0.0292	0.1119	$SE_{intercept}$					
22		R^2	0.9534	0.0748	SE_y					
23		F	61.3536	3	df_{error}					
24		SS_{Regr}	0.3432	0.0168	SS_{error}					
25		CAUTION -- Statistics from transformed space								

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Thank You!

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Back Up Slides

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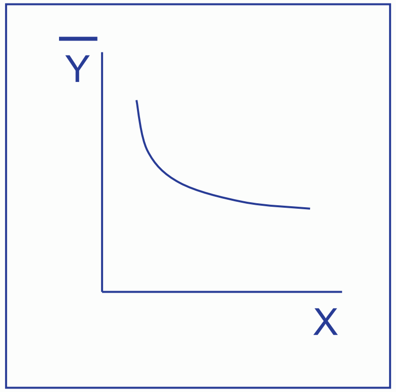
Cum Average (Wright) Learning Curve Theory

As the number of units produced doubles, the **cumulative average** cost/time to produce the doubled number of units decreases by a constant percentage.

Unit (X)	Cum Avg Cost ($\bar{Y}$) or Time
1	100
2	90
4	81
8	72.9



$$\bar{Y} = aX^{b}$$

where:

$\bar{Y}$ = cum avg cost/time for X units

a = cost/time of first unit (T_1)

X = cum number of units produced

b = learning curve exponent

$$b = \frac{\ln(slope)}{\ln(2)} = \frac{\ln(.90)}{\ln(2)}$$
$$= -.1520$$

$$\bar{Y} = 100X^{-.1520}$$

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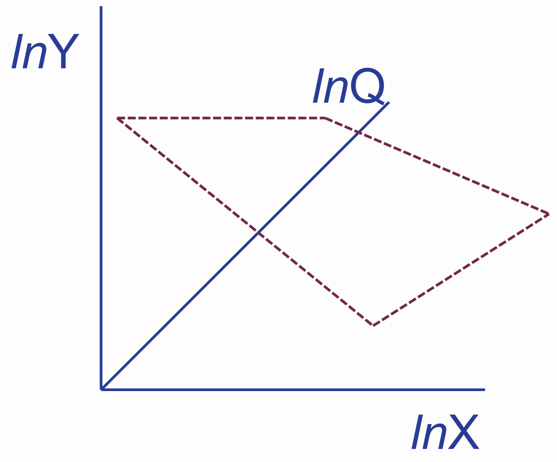
Unit Learning Curve “Rate” Model

The rationale for the Rate Model is that unit cost may not only decrease as more units are produced, as modeled by Unit learning, but may also decrease as the rate of production increases and economies of scale are realized.



$$Y = aX^bQ^c$$

where:

Y = cost/time of *Xth* unit

a = cost/time of first unit (T_1)

X = number of unit produced

b = learning curve exponent

Q = rate of production (qty per time period or lot)

c = rate exponent (rate slope = 2^c)

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CO$TAT 7.5 Six-Term LMP Approximation Formula

$$\text{If } PQ > 0 \Rightarrow LTQ_i = \sum_{Q=PQ+1}^{PQ+LQ} Q^b \cong \frac{(PQ+LQ)^{b+1} - (PQ)^{b+1}}{b+1} + \frac{(PQ+LQ)^b - (PQ)^b}{2} + \frac{b}{12}\left((PQ+LQ)^{b-1} - (PQ)^{b-1}\right)$$

$$\text{If } PQ = 0 \Rightarrow LTQ_i = \sum_{Q=1}^{LQ} Q^b \cong \frac{(LQ)^{b+1}}{b+1} + \frac{(LQ)^b}{2} + \frac{b}{12}(LQ)^{b-1} - \frac{1}{b+1} + \frac{1}{2} - \frac{b}{12}$$

The true lot midpoint is then given by:

$$LMP_i = \left(\frac{LTQ_i}{LQ}\right)^{1/b}$$

From Shu-Ping Hu & Alfred Smith (2013), Accuracy Matters: Selecting a Lot-Based Cost Improvement Curve, Journal of Cost Analysis and Parametrics, 6:1, 23-42

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