Minimize risk. Maximize potential.
Modeling Unit Learning Curves from Lot Data

Dr. Steve Van Drew
Webinar Topics

• Learning Curve Fundamentals
• Power Model Review
• Working with Lot Data
  – Lot midpoint approximation
• Demonstration
  – Data transformation
  – Excel LINEST function and Goal Seek
  – @RISK implementation
Unit (Crawford / Boeing) Learning Curve Theory

As the number of units produced doubles, the unit cost/time to produce the doubled unit decreases by a constant percentage.

\[ Y = aX^b \]

where:

- \( Y \) = cost/time of \( Xth \) unit
- \( a \) = cost/time of first unit (\( T_1 \))
- \( X \) = number of unit produced
- \( b \) = learning curve exponent

\[
b = \frac{\ln(slope)}{\ln(2)} = \frac{\ln(0.90)}{\ln(2)} = -0.1520
\]

\[ Y = 100X^{-0.1520} \]
Intrinsically Linear Transformations

- **Linear**
  \[ Y = a + bX \]

- **Power**
  \[ Y = aX^b \]
  \[ \ln Y = \ln a + b \ln X \]

- **Exponential**
  \[ Y = ae^{bX} \]
  \[ \ln Y = \ln a + bX \]
ε’ in power model is a multiplicative error term that is lognormally distributed, but in the transformed model ε is additive and normally distributed with same characteristics assumed as an ordinary least squares linear regression error term.
if $Y \sim N(\mu, \sigma)$

then

$X = e^Y \sim \ln(\mu, \sigma)$

and

$E(X) = e^{\mu+\sigma^2/2}$

$V(X) = [e^{(2\mu+\sigma^2)}](e^{\sigma^2}-1)$

to model in @RISK use:
RiskLognorm (E(X), \{V(X)\}^{.5})

or
RiskLognorm2 (\mu, \sigma)

For fuller coverage of modeling regression error terms go to
https://go.palisade.com/Modeling-Regression-Errors-with-RISK.html
Determining $T_1$ and Slope from Lot Data: Problem Statement

A critical subassembly has experienced the following production hours for the first five lots. Assuming learning follows the unit theory, find the $T_1$ and slope for the appropriate learning curve and estimate the production hours needed for the 45 units in Lot 6.

<table>
<thead>
<tr>
<th>Lot</th>
<th>Size</th>
<th>Units</th>
<th>Cost (man–hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>1 - 15</td>
<td>35000</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>16 – 25</td>
<td>20500</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>26 – 85</td>
<td>85000</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>86 – 115</td>
<td>39500</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>116 - 165</td>
<td>58000</td>
</tr>
</tbody>
</table>
Determining $T_1$ and Slope from Lot Data: Lot Midpoint (LMP)

• The Lot Midpoint (LMP), also called the Algebraic Lot Midpoint or Lot Plot Point, is defined as the theoretical unit whose cost (or time) is equal to the Average Unit Cost (AUC) or Lot Average Cost for that lot on the learning curve.

• The LMP value represents the X value and the AUC represents the Y value when working with lot data.

“The difficulty in finding the LMP is that you must know the slope of the learning curve in order to determine the LMP, but you need all the LMPs to find the slope of the learning curve.”
Determining $T_1$ and Slope from Lot Data: LMP Approximation

\[ LMP = \left[ \frac{(L + .5)^{b+1} - (F - .5)^{b+1}}{(b + 1)(L - F + 1)} \right]^{1/b} \]

where:
- $b$ = slope coefficient
- $F$ = first unit # in lot
- $L$ = last unit # in lot

See backup slides for a six-term alternate LMP approximation formula that ACE-IT’s CO$T$AT 7.5 uses.
Demonstration

General Steps:
1. Enter lot data in B4:F8
2. Enter starting slope assumption in C13
3. Run Goal Seek as shown in dialog box image
4. Enter new lot data in B9:E9
5. Run @RISK simulation with F11 as output
Thank You!

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Back Up Slides
Cum Average (Wright) Learning Curve Theory

As the number of units produced doubles, the cumulative average cost/time to produce the doubled number of units decreases by a constant percentage.

\[ Y = aX^b \]

where:

- \( Y \) = cum avg cost/time for \( X \) units
- \( a \) = cost/time of first unit (\( T_1 \))
- \( X \) = cum number of units produced
- \( b \) = learning curve exponent

\[ b = \frac{\ln(slope)}{\ln(2)} = \frac{\ln(.90)}{\ln(2)} \]

= -.1520

\[ \bar{Y} = 100X^{-1.1520} \]
The rationale for the Rate Model is that unit cost may not only decrease as more units are produced, as modeled by Unit learning, but may also decrease as the rate of production increases and economies of scale are realized.

\[ Y = a X^b Q^c \]

where:
- \( Y \) = cost/time of \( Xth \) unit
- \( a \) = cost/time of first unit (\( T_1 \))
- \( X \) = number of unit produced
- \( b \) = learning curve exponent
- \( Q \) = rate of production (qty per time period or lot)
- \( c \) = rate exponent (rate slope = 2^c)
CO$T$AT 7.5 Six-Term LMP Approximation Formula

If $PQ > 0 \Rightarrow LTQ_i = \sum_{Q=PQ+1}^{PQ+LQ} Q^b \cong \frac{(PQ + LQ)^{b+1} - (PQ)^{b+1}}{b + 1} + \frac{(PQ + LQ)^b - (PQ)^b}{2} + \frac{b}{12} ((PQ + LQ)^{b-1} - (PQ)^{b-1})$

If $PQ = 0 \Rightarrow LTQ_i = \sum_{Q=1}^{LQ} Q^b \cong \frac{(LQ)^{b+1}}{b + 1} + \frac{(LQ)^b}{2} + \frac{b}{12} (LQ)^{b-1} - \frac{1}{b + 1} + \frac{1}{2} - \frac{b}{12}$

The true lot midpoint is then given by:

$LMP_i = \left( \frac{LTQ_i}{LQ} \right)^{1/b}$