A 3-Market, 10-Dimension Trade

by Doug Howarth

ABSTRACT:

Any person, company, or government working across three or more related markets decides how to divide the costs between them. Often decision-makers give little thought as to how those resource splits need to work when working in conjunction to a common goal. Using the example of the Prompt Global Strike (PGS) initiative, this paper studies ways to optimize costs in three connected markets (air-to-surface missiles, bombers, tanker aircraft) across ten dimensions.

1.0 A Change In Perspective

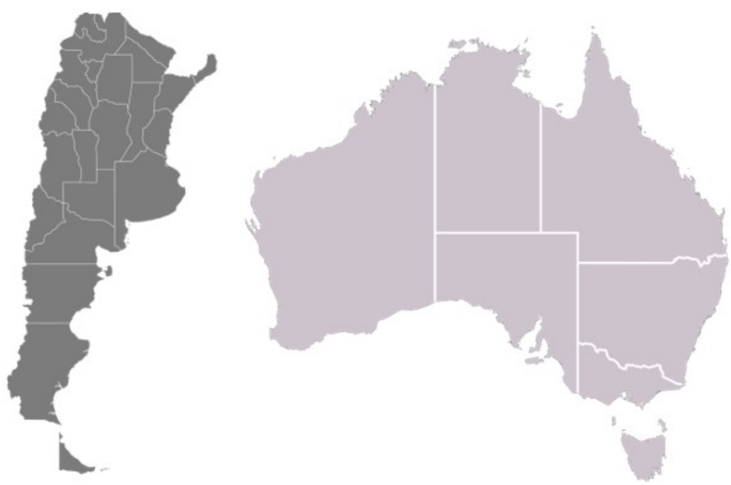


Fig. 1 Which two countries are these? Where do they touch?

Often a change in the way we view a problem can enhance how we approach it. We all know the standard definition of "dimension" as it applies to physical world, as a measure of space, as height, width, and length. But, in the world of mathematics, dimension can mean "[t]he least number of independent coordinates required to specify the points uniquely in a space." [1] What if we wanted to display ten dimensions? What to do? We get a clue with Figures 1 and 2.[2-4]

While the right-hand picture in Figure 1 might be instantly recognizable as Australia (and it is), the one on the left looks like the state of Delaware turned on its head. It is, instead, Argentina. So, where do they meet?

Yes, this is a trick question. No country touches Australia. But the claims Australia makes as its territories, that's another matter. Australia claims two large swaths of Antarctica, as we can see in Figure 2. They abut Argentina's Antarctic claim along the Earth's axis, meeting at the South Pole. What of it? How does this help us?

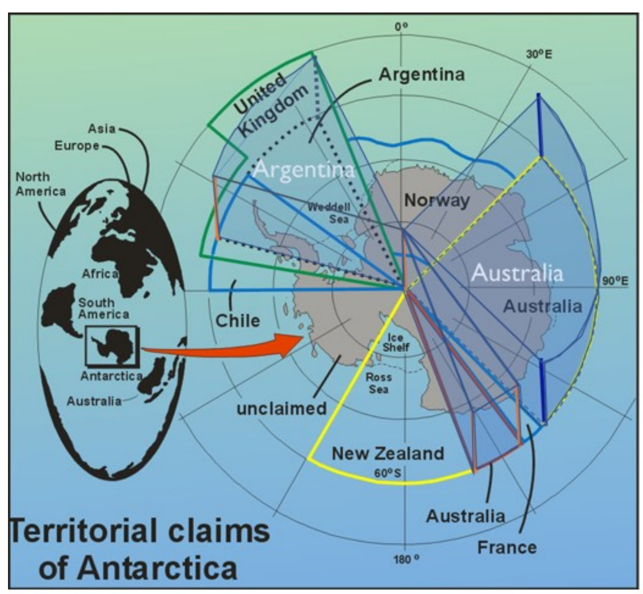


Fig. 2 Argentinian and Australian claims meet at the South Pole, their airspaces abutting the Earth's axis

We, of course, denote the South Pole as 90° south latitude. But what if we didn't? If we called it "0," then every point moving away from it would be positive. That's because geography is never negative. If started at the South Pole and walked one step into the Argentinian claim, we could say we moved into positive Argentinian space, but we wouldn't say we were in negative Australian space. If we turned around, retraced our step onto the South Pole, and then took another step into the Australian claim, we would be in positive Australian space and no other.

This seemingly odd construct will prove useful. We'll create an n-dimensional system with it.

2.0 A Common Problem

When complicated cost requirements face decision-makers, often, the solutions require the resources of three or more related markets. The United States Prompt Global Strike (PGS) program may have such conditions. "Prompt Global Strike (PGS) is a United States military effort to develop a system that can deliver a precision-guided conventional weapon airstrike anywhere in the world within one hour." [5] While some PGS concepts called for land or submarine-based missiles to be used in such a capacity, or to perhaps use kinetic weapons dropped from space, both of those models are fraught with difficulties. Land and sub-based long-range missiles look like Intercontinental Ballistic Missile (ICBMs), which tend to excite major adversaries unfavorably and could lead to increased tensions. Kinetic weapons released from space may violate treaties prohibiting such actions. If PGS is to work within the current political constraints it faces, this paper assumes it will require an air-to-surface weapon launched by an aircraft, which may, in turn, need mid-air refueling. Thus, we need to examine the costs of three interconnected markets, those for 1) air-launched missiles, 2) bombers and 3) tanker aircraft.

3.0 United States Air-Launched Missiles And Bombs

3.1 Missile And Bomb Database

Glide Bomb/ Missile	1997 - 2016 Qty	2016 $K	Max V kph	Pay-load Kg	Max Rng Km	Launch Kg
BLU-109	18,556	$51	1,605	240	28	924
BLU-110	6,565	$36	1,605	202	28	447
BLU-111	33,330	$32	1,605	87	28	227
BLU-117	24,506	$40	1,605	429	28	948
SBD I	16,577	$59	1,200	93	111	129
SBD II	2,417	$78	1,200	93	72	129
AGM-158	635	$1,352	1,200	450	370	1,021
AGM-158-I	275	$1,912	1,200	450	1,000	1,021
AGM-88E	643	$892	2,280	66	150	355
AGM-84	4,152	$528	855	221	270	675
AGM-130	102	$804	1,200	907	75	1,323
AGM-154A	1,742	$557	1,200	42	110	450
AGM-154B	3,893	$501	1,200	177	110	450
AGM-154C	6,599	$442	1,200	250	110	450
AGM-142	46	$1,773	1,482	350	80	1,361
AGM-114P	14,886	$113	1,591	9	8	47
AGM-114N	8,741	$123	1,591	9	8	47
AGM-114R	25,238	$161	1,591	9	8	47

Fig. 3 US market for air to ground glide bombs and missiles, in 2016$K

If the United States develops the PGS as a weapon launched from a fighter, bomber, or attack aircraft, the PGS becomes an air-to-surface weapon. Its market for publicly acknowledged air to surface weapons is a matter of public record. We observe the US purchases of such devices over the 20 years beginning January 1, 1997 and ending on December 31, 2016, in Figure.[6-8] Glide bombs form the first six entries (BLU-109, -110, -111, -117 along with the Small Diameter Bombs (SBD) 1 and II) while missiles make up the rest. Note the database accounts for the models, their quantities sold from the beginning of 1997 to the end of 2016, their prices, maximum velocity, payload, range, and launch mass.

3.1.1 US Missile and Bomb Demand

The second and third columns from Figure 3 contain the quantities purchased and the average prices, respectively, for the US air-to-ground missile and bomb purchases from 1997 to 2016. We plot these ordered pairs as Figure 4.

The blue points in Figure 4 are for missiles; the brown ones are for glide bombs. Note that not only do the missiles uniformly sell for more than the bombs, there also appears to be an outer limit for sales in this market. These six outermost models (for the AGM-84, AGM-114R, AGM-84C, AGM84D, AGM-158, and AGMA-158-1), marked with yellow markers over the blue points, form a statistically significant boundary called the Demand Frontier, defined by Equation (1).

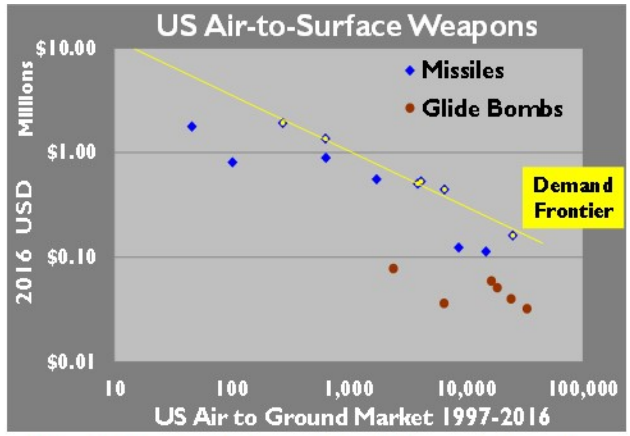


Fig. 4 US missile demand and Demand Frontier

$$Price=\$4.17E10^{7} * Quantity^{-0.533} * \epsilon \quad (1)$$

Where:

Price = estimated projectile price, in 2016$
Quantity = missiles/bombs sold, 1957-2016
ϵ = the error for this equation

Equation 1, an unbiased estimator using the Ping Factor[5], has an adjusted R^2 of 98.2%, a P-value of 0.01, and a standard error of $91,800 (we remove the recurring multiplicative error term, ϵ, in subsequent equations for convenience). The Demand Frontier reveals limiting quantities given prices. For an air-launched Prompt Global Strike system, the Frontier limits quantities given target prices of the weapon system. We discover a pair of these limits in Figure 3.

Figure 5 reveals that if the United States government were willing to spend up to $7.49 million per PGS system in 2016$, might be able to afford up to 25 of these devices. If instead, they limited themselves to $5.18 million (again, in 2016$), they could buy up to 50 such systems.

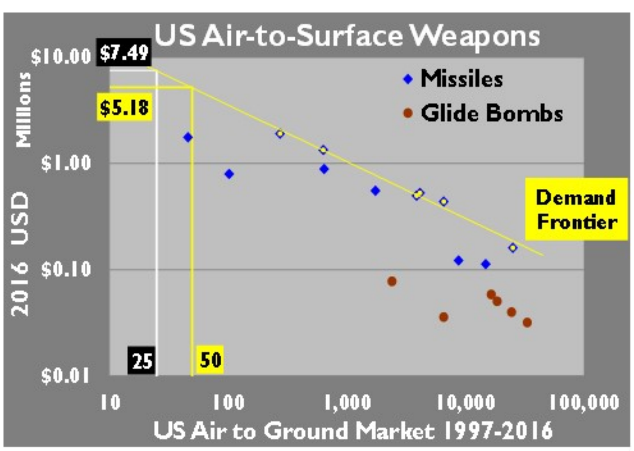


Fig. 5 US missile prices set quantity limits

Given that this market forces constraints upon its buyers, we might ask ourselves what it is we would get for such prices, this notion addresses the idea of Value, which we address in the next section.

3.1.2 US Missile and Bomb Value

We hypothesize that the features of the missiles and bombs we have in Figure 1 might have something to do with their sustainable Value. We find we can estimate the Value of missiles and bombs using their features, as in calculated in Equation (2) and displayed in Figure 6.

$Price = 977 * R\,Km^{0.452} * Grv1,Pow^{3.00} * MV^{0.167}$ (2)

Where:

Price = estimated projectile price, in 2016$

RKm = range in kilometers

Grv1, Pow2 = gravity bombs have a value of 1; missiles have a value of 2

MV = launch mass in kilograms times maximum speed in kilometers per hour (momentum)

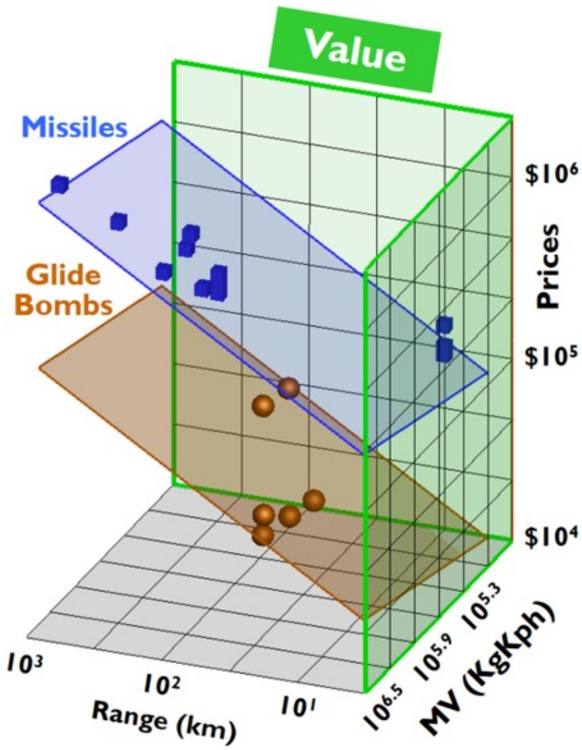


Fig. 6 US missile 3D Value Space

Equation 2 (which excluded AGM-142 as an outlier) is unbiased, adjusted by the Ping Factor (as are all equations that follow), has an adjusted R^2 of 96.9%, P-values of 1.84E-06, 5.69E-10 and 3.06% for range, grv1pwr2, and mv, respectively, and a standard error of $147,000. The term grv1, pow2, is a step function term that discovers powered (pow) projectiles (missiles) have eight times (2^3) more value than do gravity (grv) bombs. The mv term is launch mass times maximum velocity (KgKph), which is an expression of momentum.

We can discover the graphical meaning of Equation (2) in Figure 6. Log-linear in all three directions, Figure 6 shows us how the market rewards (indicated by their demonstrated willingness to pay) added momentum (the MV term, KgKph) and range (in kilometers). Note that the curvilinear responses (range raised to the 0.452 power, momentum raised to the 0.167 power) appear planar in a three-dimensional log-linear environment. Observe that the market readily pays for the added flexibility missiles offer, at a rate eight times that for glide bombs.

Name	Max R km	MV KgKph	2016 Price	1997-2016 Qty
BLU-111	28	364,389	$32,000	33,330
AGM-158-1	1,000	1,225,200	$1,912,000	275
AGM-84	270	577,125	$528,000	4,152

Fig. 7 Ordered quads for 3 observations

Let's consider a subset of the missile and bomb database in Figure 7.

It is at this juncture we appeal to the hypothetical construct we imagined at the South Pole. What if we had a system that began at 0 and then went positive in all directions away from it?

Figures 4 and 5 depicted a two-variable system, with quantity on the horizontal axis and price on the vertical axis. In Figure 8, we see quantity as a horizontal axis and price as a vertical axis on the red, right-hand side of the diagram, with the three observations of Figure 7. Observe from the red, right-hand Demand Plane perspective, all points right or above the origin are positive.

Figure 6 used range (a horizontal dimension), mv (another horizontal axis, momentum), and price (the vertical axis). We can place them at the left-hand side of Figure 8 in the green Value Space. As with the right-hand side of Figure 8, all points in Value Space, moving away from the origin, are likewise positive.

Thus, we can plot the four variables from Figure 7 as ordered quads, which we do in Figure 8. In a market setting, the general format for ordered quads is (valued feature 1, valued feature 2, price, and quantity). The origin of such systems, using the Cartesian systems as a baseline, is (0,0,0,0). The left-hand side of such plots is green and depicts Value Space. The red-hand side of the graph is red, revealing the Demand Plane. Value Spaces and Demand Planes connect in linked, dual states across four dimensions; *all markets work and have always worked in this way.*

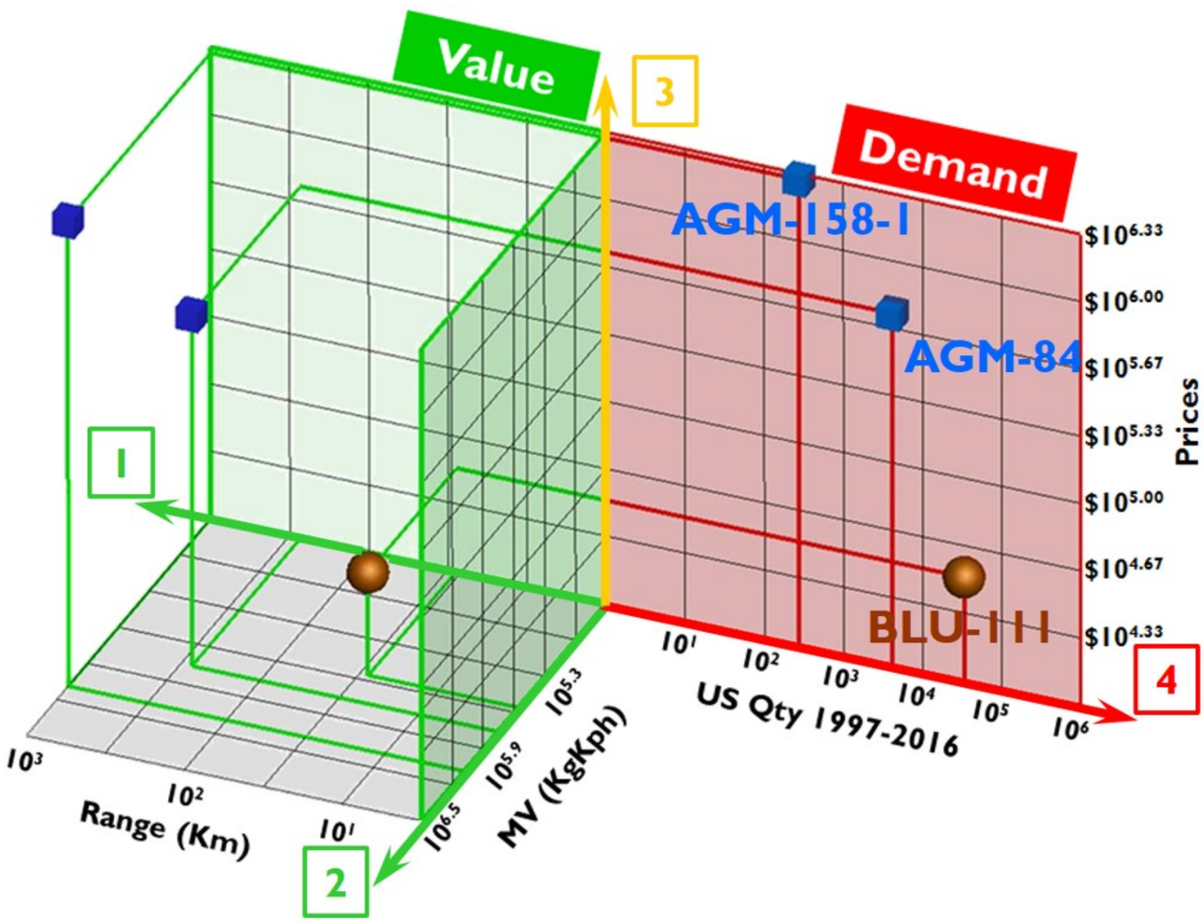


Fig. 8 Models in the US missile/bomb market plot as ordered quads in linked, dual, states

4.1 US Fighter, Bomber and Attack Aircraft

The market for unclassified fighter, bomber, and attack aircraft in the United States is well known and fully documented. There are only a couple of dozen of such planes were or have been in wide use over sixty years, as shown in Figure 9.

Aircraft	1957–2016 Quantity	2016 Flyaway $	Max Kph	Payload Kgs	Range (Km)	Aircraft	1957–2016 Quantity	2016 Flyaway $	Max Kph	Payload Kgs	Range (Km)
B-52	740	$80,200,000	1046	36287	16327	A-10	716	$19,600,000	707	7257	4152
B-1B	100	$424,600,000	1336	56699	11999	F-14	712	$57,100,000	2485	6577	2961
AV-8B	323	$42,400,000	1064	6003	1101	F-111	563	$106,000,000	2655	14288	6759
F/A-18A-D	1480	$50,900,000	1915	6337	2012	F-4	5195	$18,700,000	2369	8459	2599
F/A-18E/F	563	$64,900,000	1915	8051	2443	A-7	1569	$13,300,000	1123	6804	4603
F-15A-E	1415	$51,600,000	3018	11113	2543	F-8	1219	$11,900,000	1971	1814	2792
F-117A	64	$85,300,000	993	1814	1721	A-4	2960	$7,200,000	1083	4491	3219
F-16C/D	4540	$28,200,000	2128	7711	4216	F-5	2246	$9,700,000	1706	3175	3718
F-22	195	$171,900,000	2414	9539	2961	F-35A	177	$177,300,000	1713	8165	2221
B-2	21	$1,143,400,000	1014	22680	11104	F-35B	36	$185,000,000	1712	6804	1666
A-6	693	$64,600,000	1043	8165	5222	F-35C	63	$221,900,000	1712	8165	2221

Fig. 9 The US market for fighters, bombers and attack aircraft from 1/1/1957 to 12/31/2016

The market for unclassified fighter, bomber, and attack aircraft in the United States is well known and fully documented. There are only a couple of dozen of such planes were or have been in wide use over sixty years, as shown in Figure 9.

The US used many of these planes for roles other than bombing. However, we need to have them all incorporated into this study not only to provide us with a sufficient number of data points but also to remind us that there are many ways to deliver ordnance from the air – to be thorough, we include them all.[6]

4.1.1 Military Aircraft Demand

If we plot the quantity column from Figure 9 as the horizontal amounts, and the prices as the vertical amounts, we get Figure 10, which is the United States market for bomb-dropping aircraft over 60 years. The outermost points, highlighted in yellow, form this market's 2016 Demand Frontier, described by Equation 3, depicted in Figure 8.[7-12]

$$Price=\$1.11E10 * Qty^{-0.733} \quad (3)$$

Where:

Price = aircraft price, in 2016$

Quantity = number of aircraft sold, 1957-2016

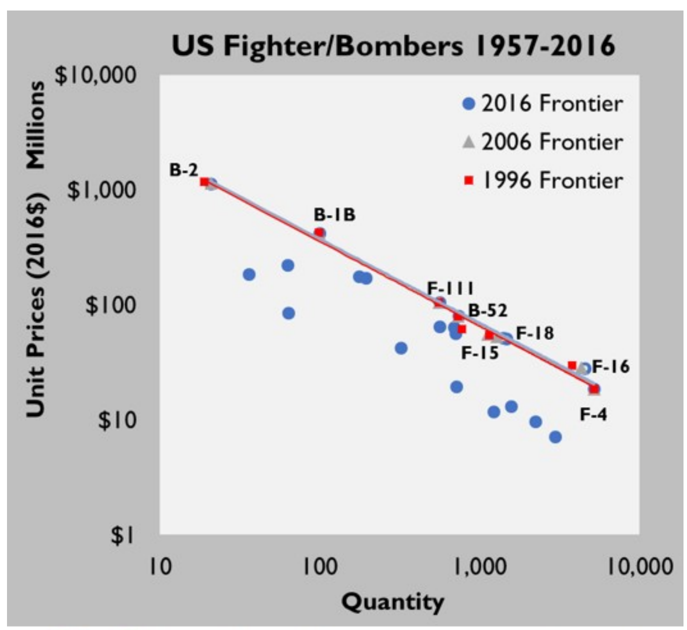


Fig. 10 The US bomb-dropping aircraft Demand Frontier is stable over the last 20+ years

Equation 3, an unbiased estimator using the Ping Factor, has an adjusted R^2 of 99.3%, a P-value of 6.23E-08, and a standard error of $25.5 million. The P-factor indicates the chance of this equation coming about due to chance is very low; we can feel confident in using it.

Equation 3's slope is within 1% of the slopes of the equations describing the 1996 and 2006 Demand Frontier's, while its constant was within 2.5% for the same two equations. Thus, this market, at its limit, demonstrates stability.

Recently, the US began work on their B-21 Raider bomber, with the expectation that they will buy a minimum of 100 of them[13] a unit cost of $550 million in FY 2010 dollars.[14] Given we know Equation 3, we might want to confirm that quantity-price combination of 100 units at $550M in 2010 dollars is feasible for the B-21 bomber. We examine this in Figure 11.

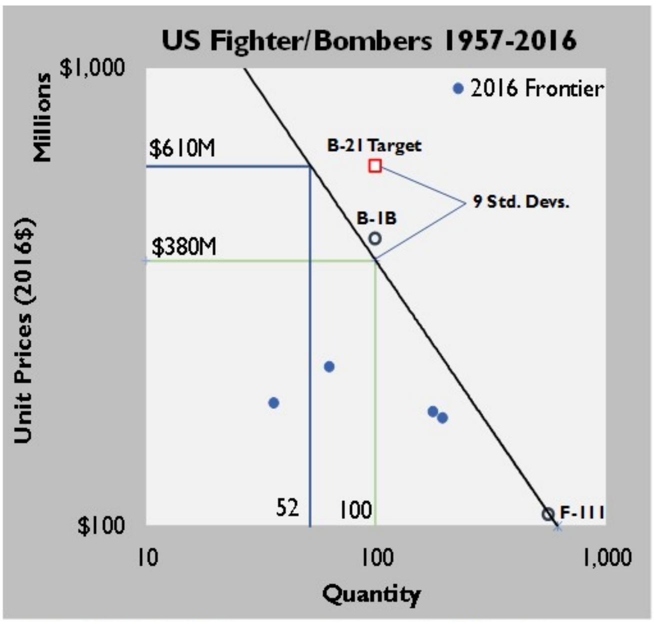


Fig. 11 The B-21 target is 9 standard deviations past the highly stable 2016 Bomber Demand Frontier

Figure 11 reveals that at $610 million in 2016$ (the inflated value of the $550 million in 2010), the Demand Frontier only supports 52 units (the ordered pair (52 units, $610 million)). Conversely, if the US government sets its requirements for 100 units, it will have to get the price of the B-21 down to $380 million each. Given the standard error of $25.5 million, the target price is 9.0 standard deviations away from the predicted limit price at 100 units ((target price of $610 million – limit price of $380 million)/standard error of $25.5 million). Using another statistical metric, we can note the 2016 Demand Frontier has a Mean Absolute Percentage Error (MAPE) of 8.8%, and that the most any program exceeded the Demand Frontier was by 17.8%. The B-21 program proposes to go past the Demand Frontier by 60.5%

Unless there is a large change in the procurement approach to military planes capable of carrying bombs, the chance of getting 100 vehicles at the posted price is unfathomably low.

Given this, we may want to see what is possible regarding the features we would like to get compared to those for which the US government has sufficient monies.

We do this in the next section.

4.1.2 Military Aircraft Value

We hypothesize that bomber features support their Value. Equation 4 shows we can predict their sustainable prices from their features.

$$Price = 45703\ Qty^{-0.664} * PL\ Kgs^{0.659} * Max\ Kph^{0.737} \quad (4)$$

Where:

Price = aircraft price, in 2016$

Quantity = number of aircraft sold, 1957-2016

PL Kgs = max payload, in kilograms

Kph = max speed, in kilometers per hour

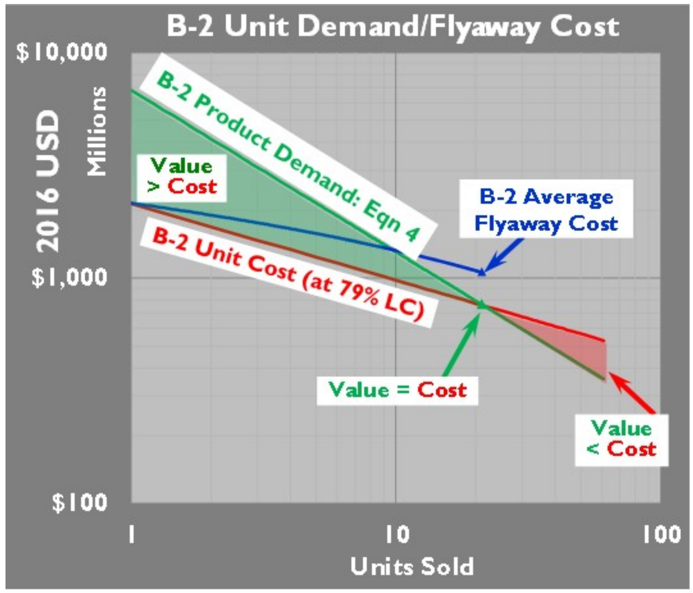


Fig. 12 The Quantity term for bomber Value (Eqn. 4) lets us draw a product demand curve for each model

Equation 4 is an unbiased estimator adjusted by the Ping Factor. Its adjusted R^2 is 92.3%, with P-values of 2.57E-10, 3.26E-06, and 0.18% for quantity, max payload, and max speed, respectively, and it has a standard error of $111.2 million. Figure 10 shows a useful result from Equation (4). After the constant, the equation's first term, $Qty^{-0.664}$, reveals the action of quantity on the Value of any model in the market. Each unit the United States Government gets is worth progressively less to them. Thus, the quantity term in Equation 4 describes a product demand curve, shown as the green line. As its slope (equivalent to a learning curve of 63.1%) is steeper than the learning curves demonstrated in this industry, it follows that since costs do not fall as fast, at a certain point, aircraft cost will eventually equal aircraft value, an equilibrium point in Multidimensional Economics (the other equilibrium condition, often seen in commercial environments, is the breakeven point, where initial cost, originally higher than the price, finally meets and then falls below the sustainable price, which we call Value). Before the point where the B-2 cost equaled B-2 Value, in the region where the aircraft value exceeds its cost, the program was in *sustainable disequilibrium*. After that, when cost exceed Value, the program reached *unsustainable disequilibrium*, which is unsupportable in the long run.

Combining the 4D structure we discovered in Figure 8, the bomber Demand Frontier we found in Figures 10 and 11 and the Value response surfaces we calculated for Equation 4, we derive bomber market trade possibilities in Figure 13. The brown plane, the bomber value at 100 units, crosses the yellow plane (at $382 million in 2016$), forming a straight line in log space, but a curvilinear production possibility curve in linear space in the left-hand picture of Figure 12. At the same time, bomber value at 52 units, the light blue plane intersects the purple plane ($610 million in 2016$), offering the higher production possibility curve in the left side of Figure 12.

Figure 11 shows us that the B-2 Unit Value, using Equation 4, started at nearly $7 billion for the first unit and instantly began to fall. In this market, the quantity exponent for incremental Value, at -0.768, is very steep; this means that recurring aircraft costs, which likely have much flatter slopes (that is, learning curves with slopes greater than -0.768) can eventually catch up to aircraft value. In Figure 11, the recurring Value of the B-2 exceeded its cost until it reached the 21st unit, at which point the two curves equated to one another. Beyond this point, costs would be greater than Value; that is why the B-2 program stopped. In any industry, product lines cease to run when cost exceeds Value.

Combining the 4D structure we discovered in Figure 6, the bomber Demand Frontier we found in Figure 9 and the Value response surfaces we calculated in Equation 4, we derive bomber market trade possibilities in Figure 12. The brown plane, the bomber value at 100 units, crosses the yellow plane (at $382 million in 2016$), forming a straight line in log space, but a curvilinear production possibility curve in linear space in the left-hand picture of Figure 13. At the same time, bomber value at 52 units, the light blue plane intersects the purple plane ($610 million in 2016$), offering the higher production possibility curve in the left side of Figure 13.

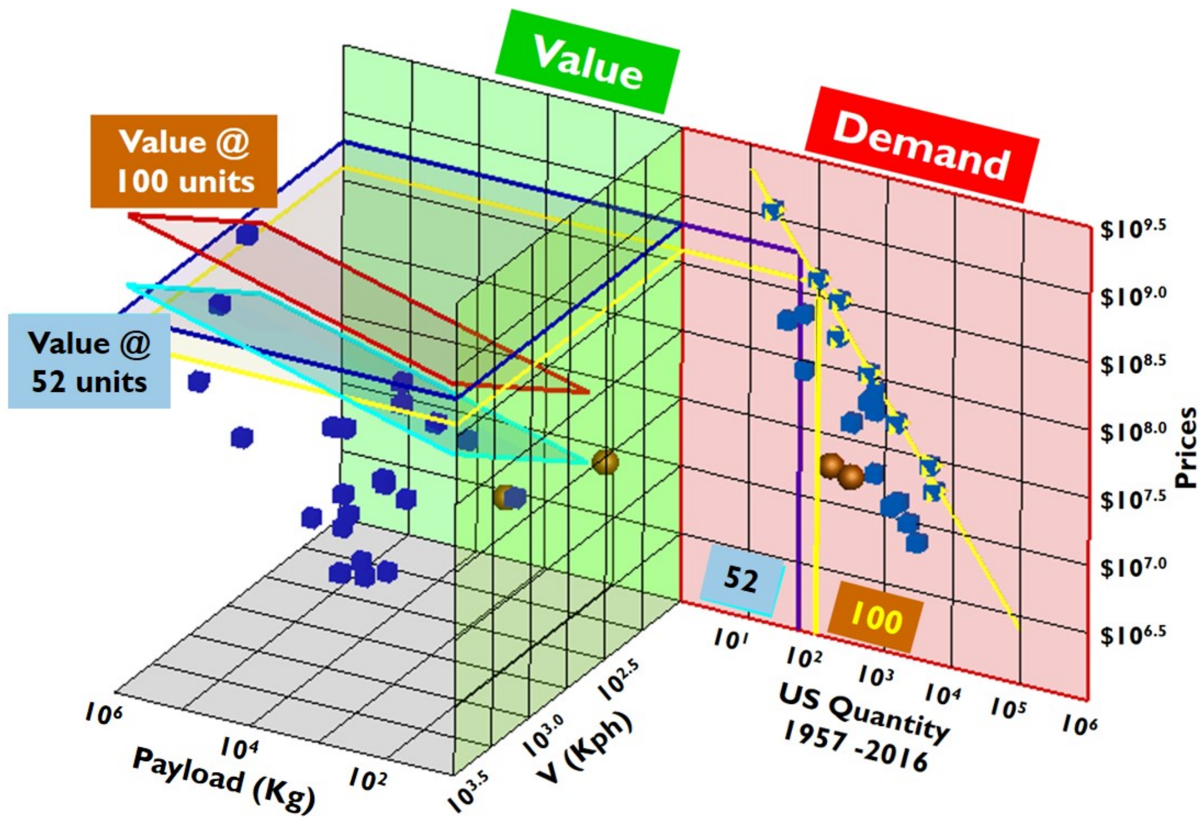


Fig. 13 The US market for fighters, bombers and attack aircraft has varying possibilities for differing quantities

5.1 US And NATO Tanker Aircraft

The market for United States and NATO tanker aircraft is small but fully documented. There are twelve such models in Western Bloc, as shown in Figure 14.

Maker and Model	Total Q	NATO Active Q	2019$M	1st Yr	Op yrs	MxPLlbs	R nm	MxMPH	Engs
Lockheed HC-130	75	45	$83.28	1959	61	36500	1050	380	4
McDonnell Douglas KC-10	62	59	$139.57	1981	39	356000	3826	619	3
Lockheed Martin KC-130B	6	6	$76.78	1958	62	45000	1000	362	4
Lockheed Martin KC-130H	33	33	$76.78	1965	55	45000	1000	362	4
Lockheed Martin KC-130T	28	28	$76.78	1983	37	45000	1000	362	4
Lockheed Martin KC-130J	53	53	$76.74	2004	16	57000	2835	417	4
Boeing KC-135	803	398	$62.52	1957	63	150000	1304	580	4
Boeing F/A-18E/F	600	120	$74.07	2001	19	13040	390	1190	2
Bell Boeing CMV-22B	39	1	$79.18	2007	13	10000	390	351	2
Airbus A310 MRTT	6	6	$134.40	2009	11	62000	972	608	2
Airbus A330 MRTT	39	41	$253.43	2011	9	143000	972	547	2
Airbus A400M Atlas	87	87	$184.93	2013	7	111300	1800	485	4

Fig. 14 Active US and NATO Air Refueling Aircraft

If we take the Figure 14 data and analyze tanker value and Demand, we get Figure 15 below.

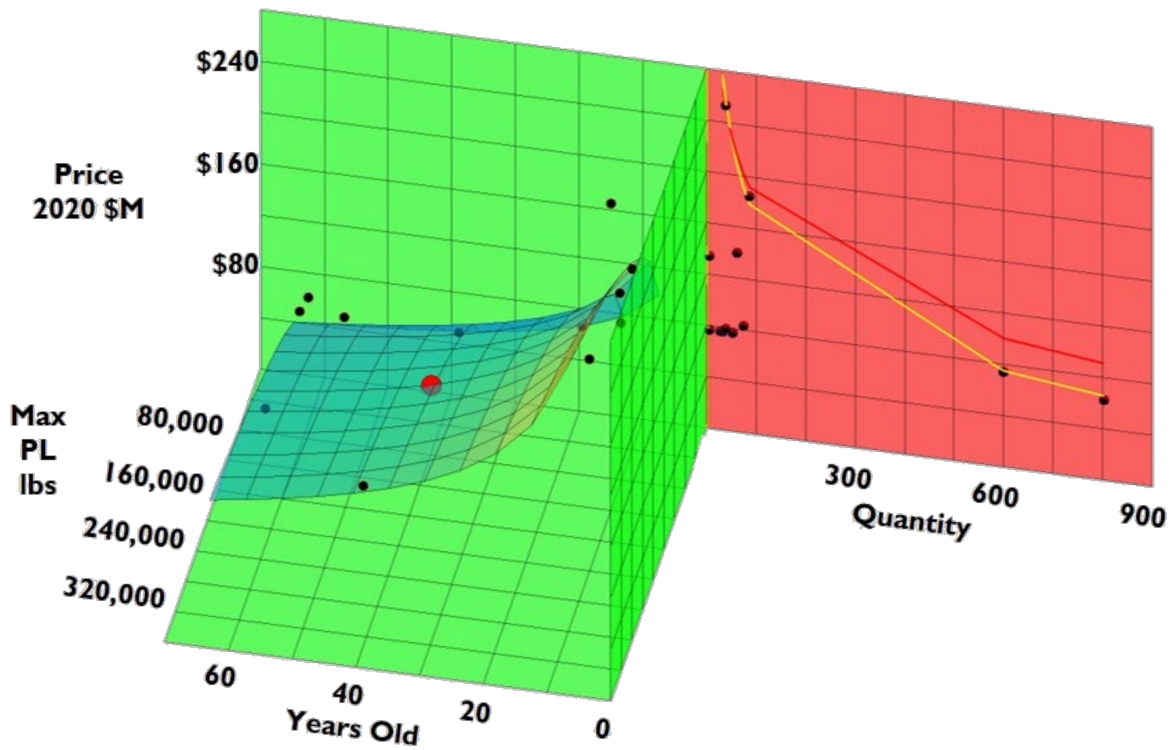


Fig. 15 Value (left) and Demand (right) are well-correlated for Refueling Aircraft

With only 12 vehicles to model, we get significant insight for Tanker Value as Equation 5.

$Price = 22.2 * Op\ yrs^{-0.3670} * MaxPLlbs^{0.246}$ (5)

Where:

Price = aircraft price, in 2020
Op yrs = years since a model first became operational
MaxPLlbs = maximum fuel payload in payload

Equation 5 is unbiased and has an adjusted R^2 of 69.8%, Pearson' s^2 of 80.6%, and MAPE of 15.6% and a P-Value of 0.19%. We can use it to estimate the Value of tankers. We cannot derive a statistically significant equation for tanker demand.

Importantly, given the current one-hour time constraint for the PGS mission, we soon realize we will not use tankers to accomplish it; this may change if the time required for PGS changes.

6.1 Creating A Multidimensional View

Since all markets share the price axis, if we abut one 4D system against another, we get a 7D view. We see one from our work in bombers and missiles in Figure 16, below.

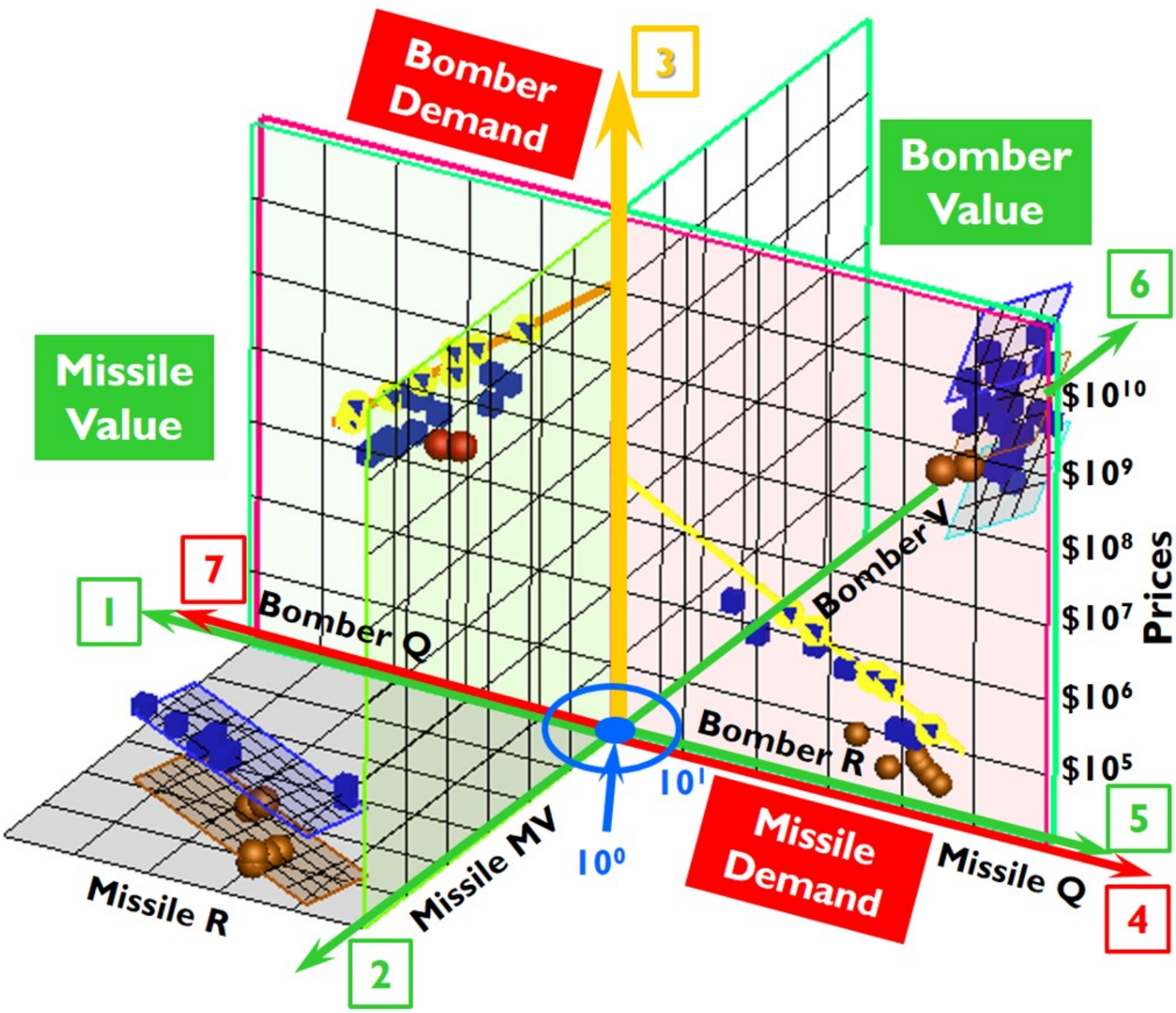


Fig. 16 Abutting one 4D system with another creates a 7D system, as each shares the price axis

The front half of Figure 16 addresses missiles, while its back half concerns itself with bombers. Observe that both markets share the price axis. Thus, while missiles using Dimension 1-4, but since bombers also use the price axis, bombers only add dimensions 5, 6, and 7. If we want to consider more dimensions, we will need to use some meaningful way to compress the data. We see the first part of this process in Figure 17.

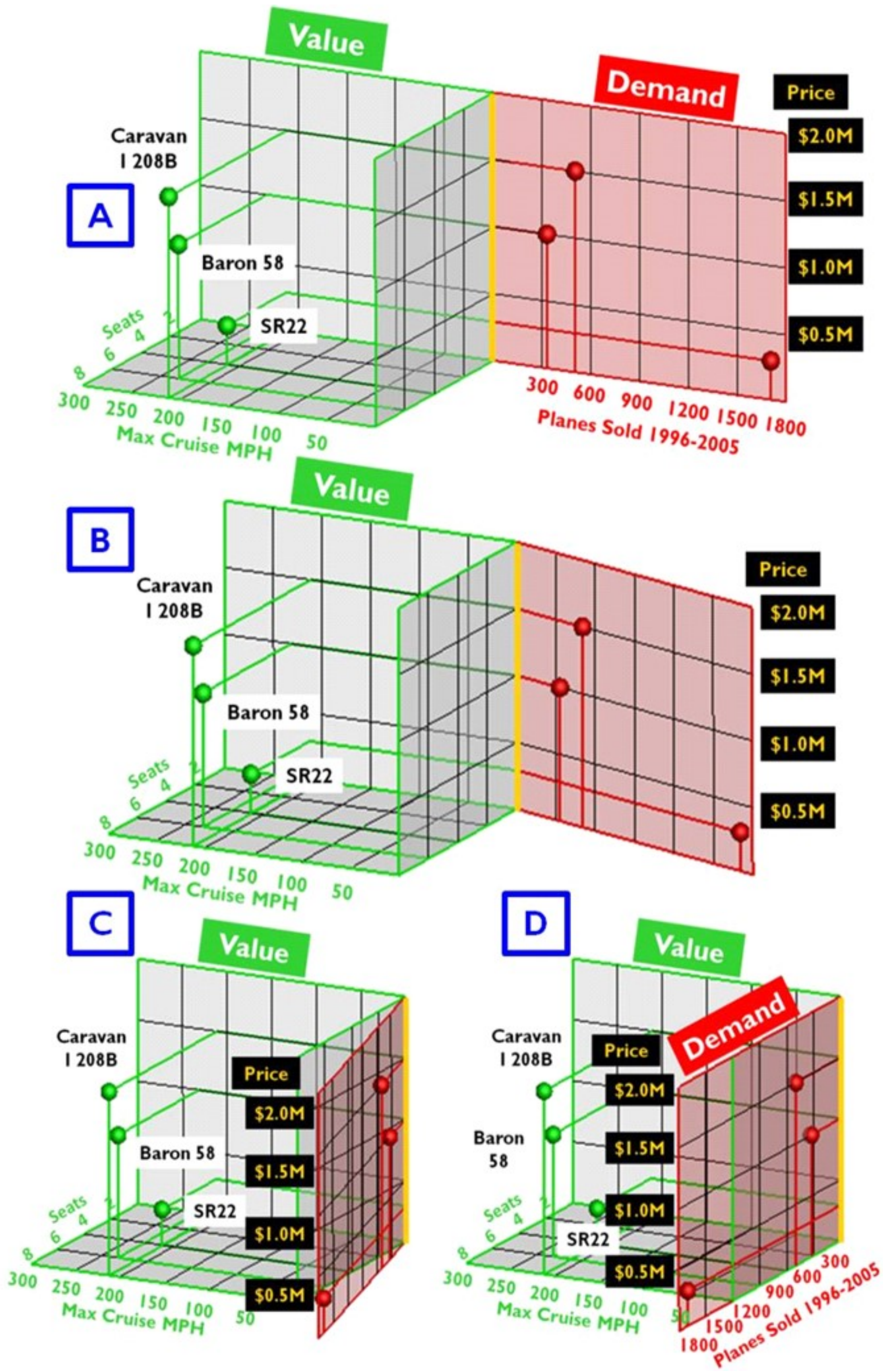


Fig. 17 Demand Plane data relates to Value Space data only through the price axis. Thus, we can rotate the Demand Plane from its default position A through B and C to D, where it lies flat against its Demand Plane.

Figure 17 (using some general aviation data much like missiles in Figure 8) begins with a standard portrayal of three models in a 4D system, with orthogonality ruling the model, as all dimensions are at right angles to all others. But consider the Demand Plane. Note that all the information about Demand is on the plane itself. It is not *required* to be orthogonal to its companion Value Space. Since this is the case, we can move it from its default position A though positions B and C until it comes to rest against its associated Value Space. We've lost no meaning about Demand – all of its information relates to its height from quantity axis and its distance from the price axis.

Markets	Dimensions Required For			
	Value	Quantity	Currency	Total
1	2	1	1	4
2	4	2	1	7
3	6	3	1	10
4	8	4	1	13
n	2n	n	1	3n+1

Fig. 18 There is a pattern for the number of markets considered and the number of dimensions required to display them.

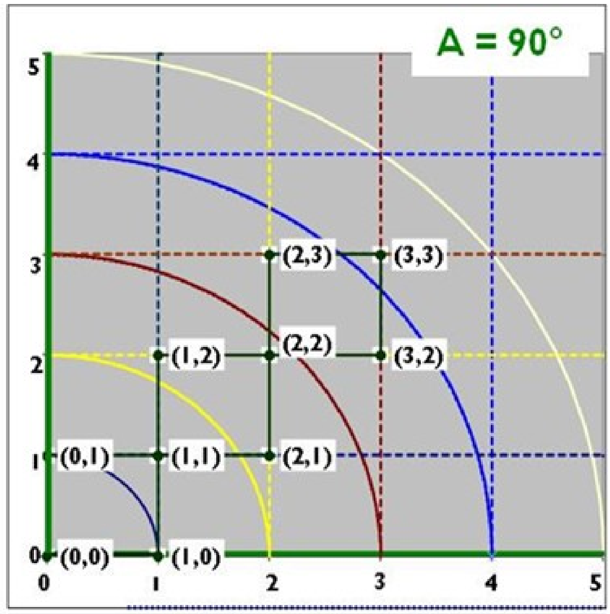
As we saw that a single market takes four dimensions, and two of them require seven, we might hazard a guess as to how dimensions grow in concert with the number of markets studied. We get a glimpse of this in Figure 18. There, as the first column counts markets, note the second column observes we need two primary value axes for each market. We needed two for one market, and four for two, and so on, 2n value dimensions for n markets, as the last row reveals. We had one quantity dimension for our first market and added another for the next one. Thus, we need as many quantity dimensions, n, as we have markets. At the same time, we keep the same single currency (price or cost) dimension for markets. Thus, in summing up the required dimensions, we find if we need n markets, we need 3n+1 dimensions. To portray more markets, we will need to gain more data compression.

In Figure 19, we plot a standard quadrant of the Cartesian coordinate system in the upper right-hand corner, as 19A. Here, the axes are at right angles to one another. But what if they weren't? With 19B, 19C, 19D, and19E, we find out we can easily track the position of points arrayed in a space by accounting for their distances from the origin and their angularities.

Encouraged, with Figure 20, we show how to portray a system of five markets. The angle created by the Value Dimensions of Market 1 show its contribution to a hypothetical GDP, as the inner green circle stand for GDP with a horizontal extent or radius of 10^1 or 1 (note, because of the use of logarithmic scaling, the origin is 10^{-1}, where it stands in for 0). With the Market 1 Demand axis lying flat against its respectively Value Space, we are free to abut Market 1 with Market 2. Market 2's portion of GDP covers the angle between its Value Axes, and its Demand Plane lies flat against its Value Space. We repeat the same procedure for Market 3 and know that we could use it again for Markets 4 and 5, or any number of markets at the same time.

We now have enough knowledge and the framework to display all ten dimensions of our trade study in one view, we see in Figure 21. Going out from the center horizontally, each circle, beginning at 10^{-4} (our proxy for zero), increases the Value by a factor of ten. The green cylinder marked by 10^0 represents world GDP in 2019,[19] with a height in base ten log space of 13.44.

Figure 19 The whole number ordered pairs in Figure A are oriented in a Cartesian system, with a 90° angle between the axes for the Value Features. As we move to Figures B, C, D and E, with angles between those axes of 70°, 50°, 30° and 10°, respectively, we can keep track of these whole numbers in the same way that we keep an accounting of positions in the pantographs that support extendable mirrors or scissor lifts. The data is not lost, but merely compressed. We can track all nonnegative numbers in this way, including the irrational ones.



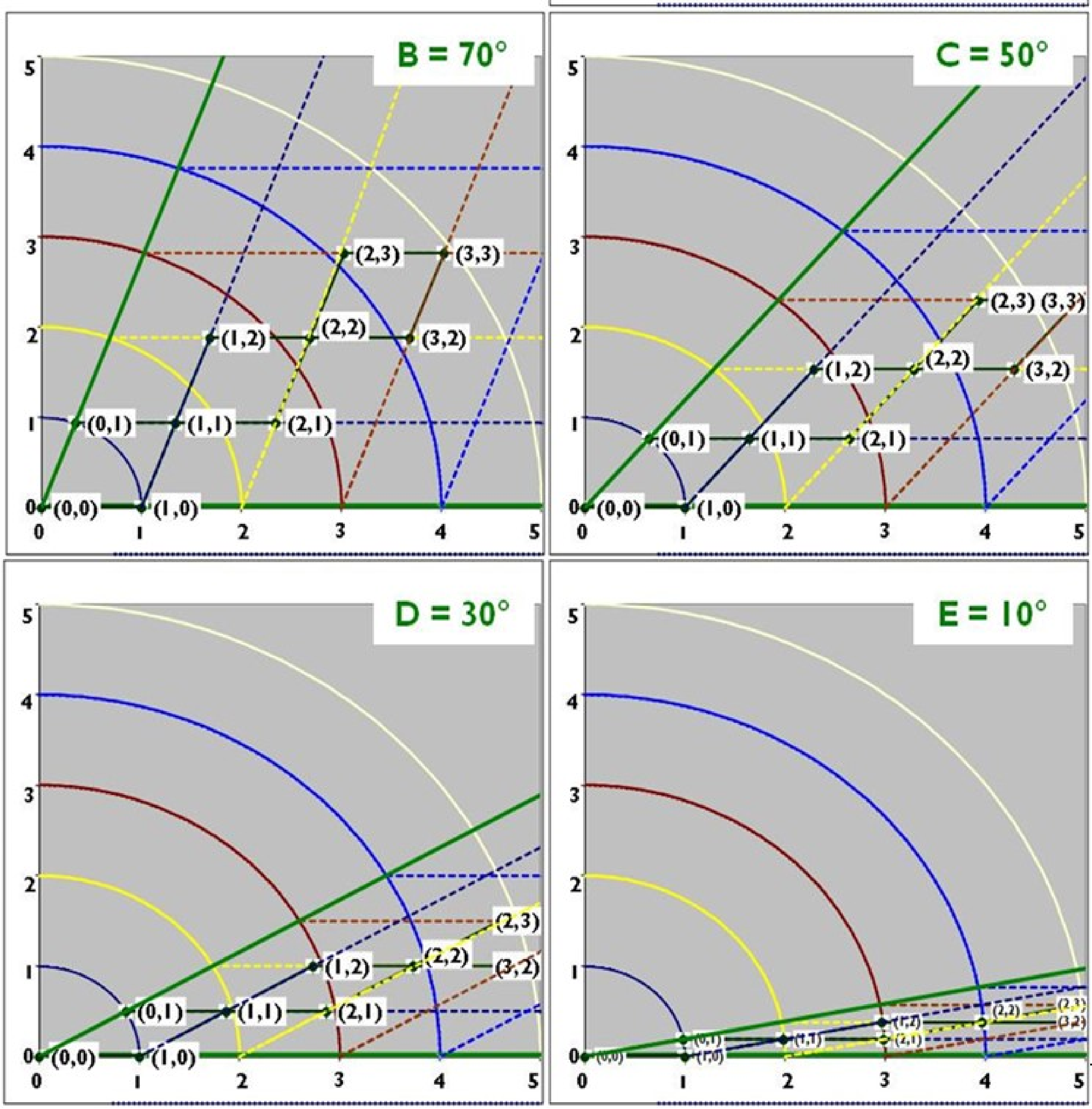
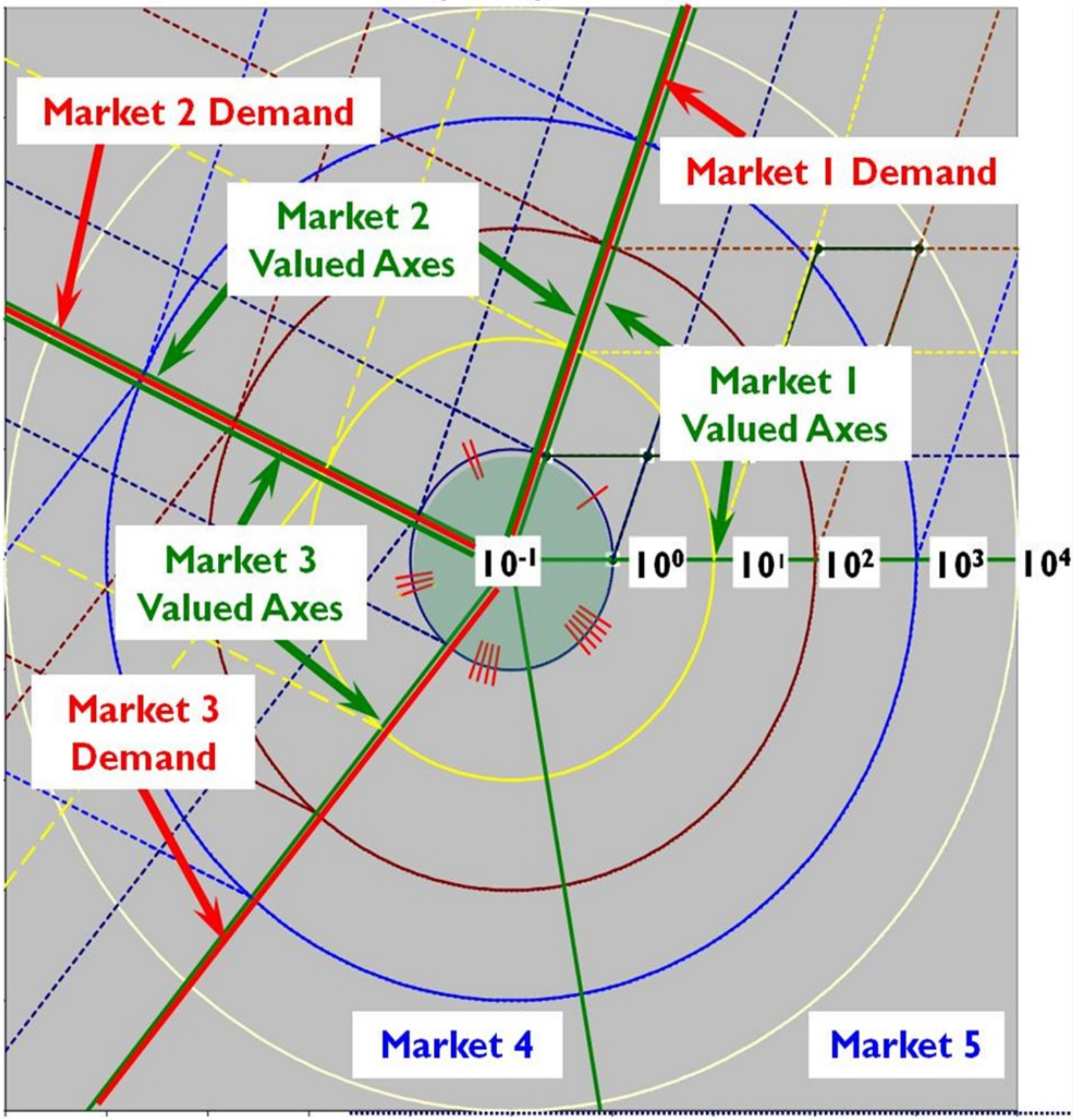


Figure 20 We cut our GDP pie column into 5 pieces, one for each of five markets making up a fictitious GDP total, from the top, it would look like this. Each market would have two Valued Feature Axes defining it, along with an adjacent Demand Plane.



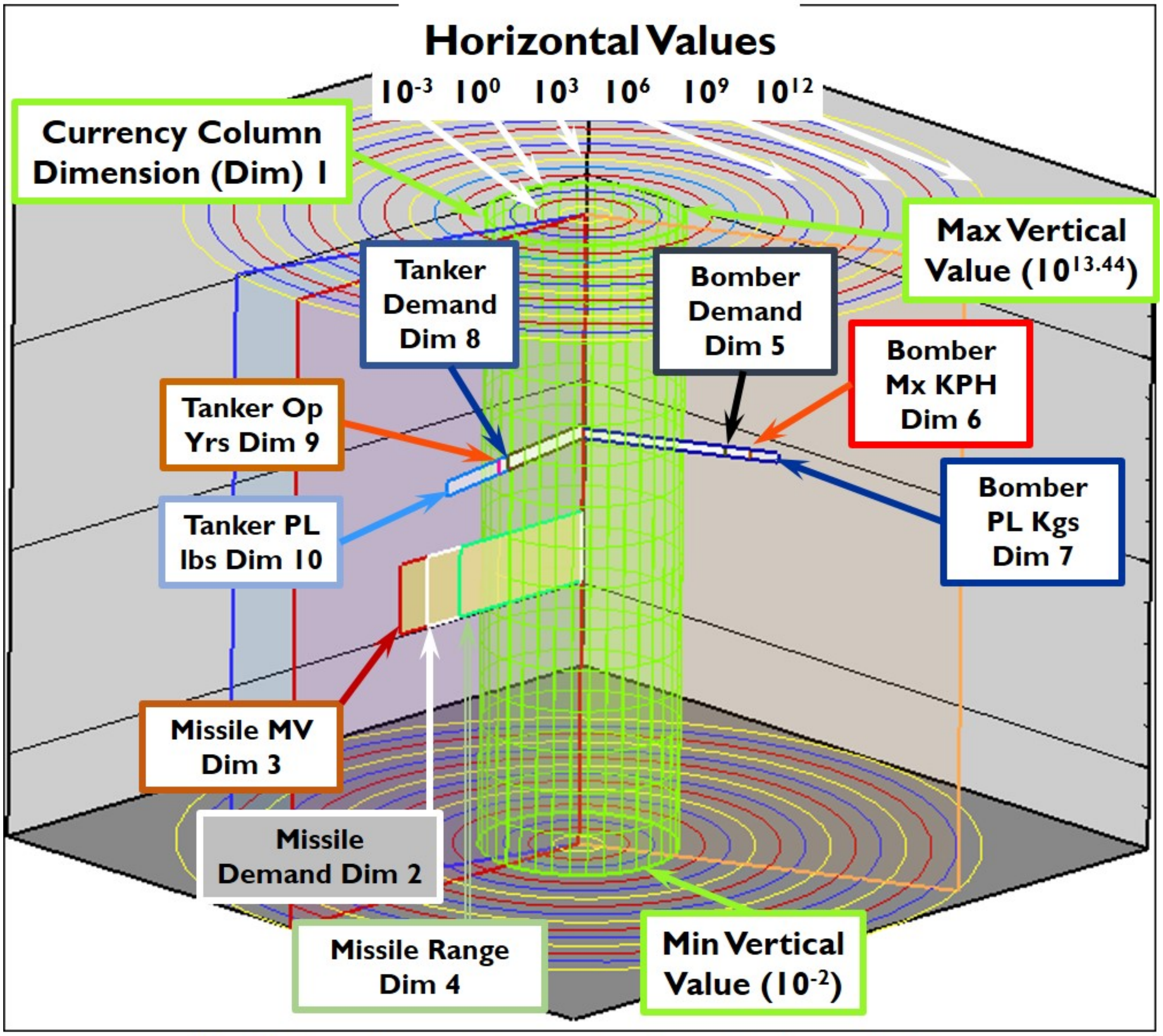
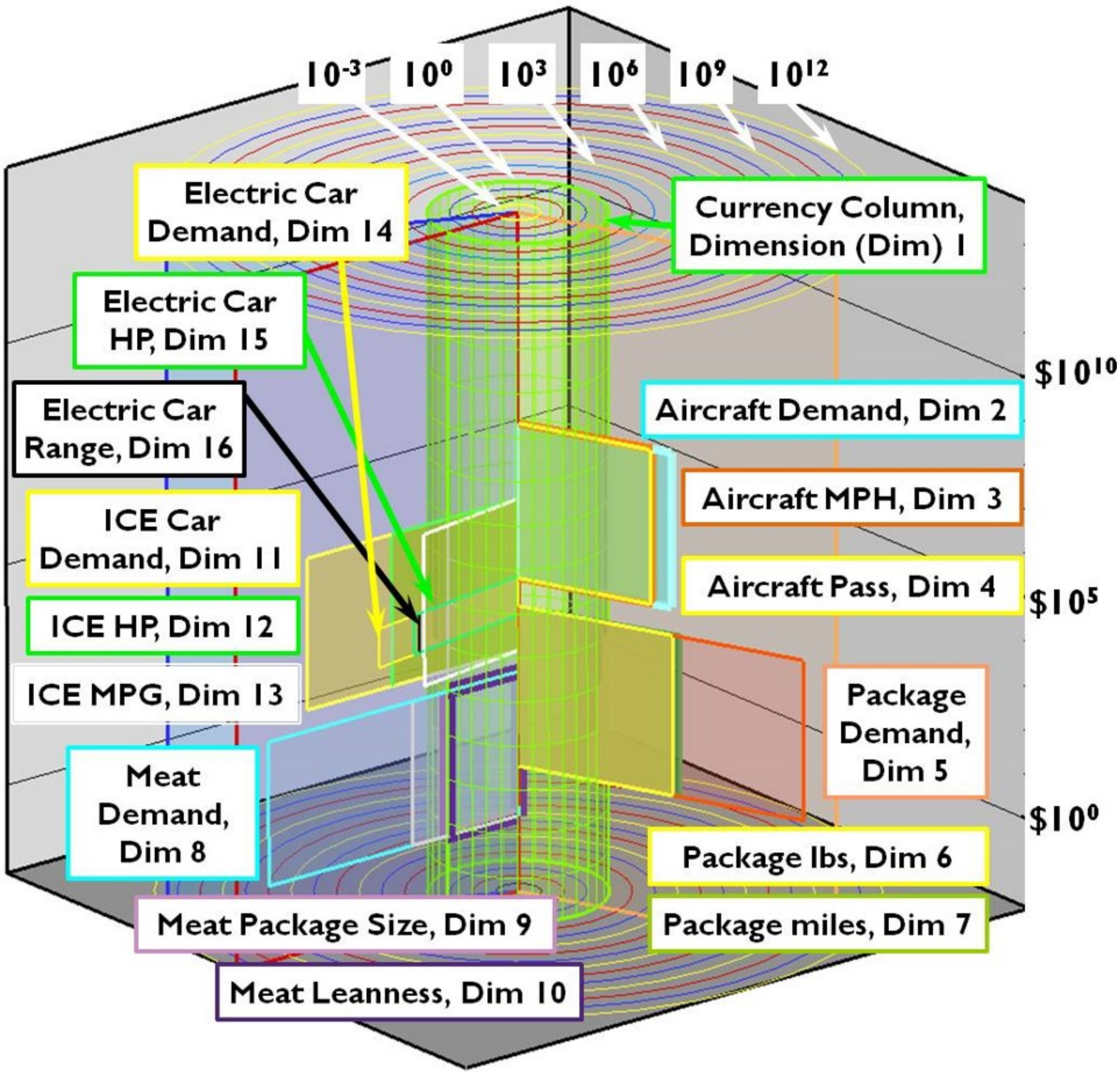


Fig. 21 This is 10D view of three markets: Missiles, Bombers, and Tankers

Vertically, each circular line on the green cylinder goes up by a factor of 10, beginning with 10^2 and reaching 10^{13} and finally $10^{13.44}$. The volume of the cylinder represents World GDP. Each market has three planes, two for Value and one for Demand. For all three market planes, their lower reach represents the least expensive product in the market, while their upper edge signifies the most expensive product in it. The outer edge for the Value Planes represents the greatest amount of the units of measure in that category in that market, while the outer edge for the Demand Planes represents its projected sales quantities in 2020. The angle between each markets' Value Planes defines its share of GDP.

For clarity's sake, we see another view of larger, unrelated markets with broader price ranges in Figure 22. Note the relatively wide angle between Internal Combustion Engine (ICE) car value planes for horsepower and miles per gallon. That market comprised 2.35% of world GDP in 2016 or almost 8.5° of arc.

Figure 22 | Using some of the recent techniques that we just learned, we swing the Demand Planes onto compressed Value Spaces for five markets, and employ 16 mathematical dimensions. This consists of two Valued Feature axes for each market, one Quantity axis for each market, along with a single currency axis common to all markets. In theory, there is no limit to the number of markets that we characterize in this fashion. Note that the market for cars with Internal Combustion Engines (ICE) forms the largest market in this study, with 2.35% of world GDP, over 347 times that for electric cars.



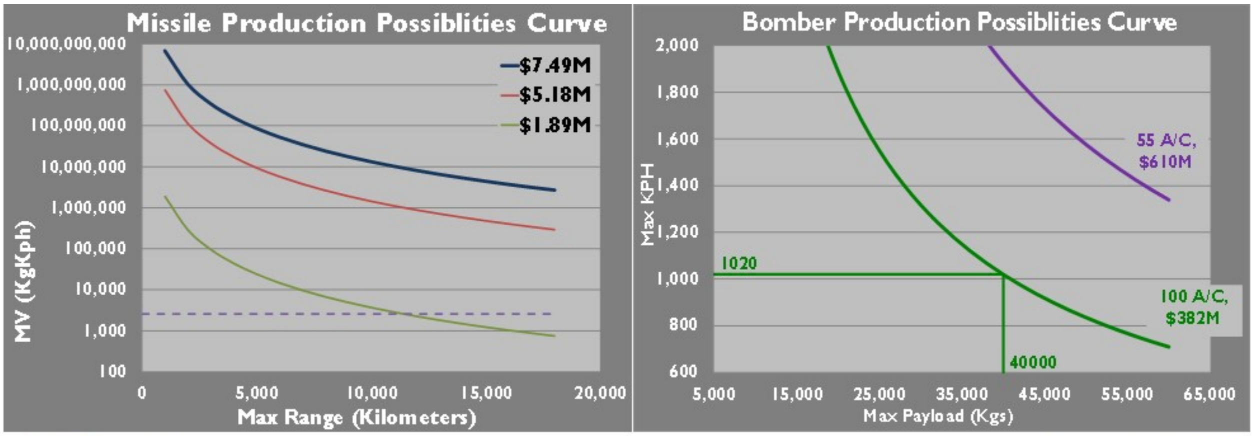


Fig 23 At left, production possibility curves for hypersonic missiles, an extrapolation of the data. The US could afford a few dozen very expensive air-to-surface missiles, but to accomplish the Prompt Global Strike mission, they would have to be hypersonic, and the data used here does not draw out the speed component separately. At right, with no consideration of stealth, the data indicates we could buy 100 bombers with a maximum speed just below Mach 1, and a payload of 40,000 kilograms.

With tanker aircraft out the picture for a one-hour mission, we focus our attention on bombers and missiles. In Figure 23, at right, we see the production possibility curves that fall out of our analysis in Figure 13. The upper line in the right side of Figure 23 shows the possibilities we have with a 55 aircraft buy. Without considering stealth (which, of course, is major feature of the B-21, B-2, and other aircraft in the database), if we dropped our average flyaway cost to $382M in 2016 dollars, we could buy 100. Importantly, we could afford a 40,000-kilogram payload with a maximum speed of about Mach 0.9.

In similar fashion to the bombers, we work our production possibility curves for 25 and 50 hypersonic missiles (for $7.49M and $5.18M average cost, respectively). Note here that with a database limited to 1000 kilometers in range, having more range for an air-to-surface missile amounts to a large extrapolation. Importantly, we have no hypersonic data in this analysis, something that may be available in the future, but which is not at this time. With most of the models in this database having similar speeds, the data does not support a separate analysis of the Value of speed. That would likely change if we developed hypersonic missiles, which we could analyze. It might also be the case that merging air-to-surface missiles with surface to surface missiles, some of which are hypersonic, would allow that analysis. In any event, the data indicates we will only be able to field a few dozen of these new, still hypothetical hypersonic devices unless there is a massive shift in the way we procure these devices.

Observe the lower line for devices costing $1.89M, the same cost as the most expensive model in the database currently. At a certain point, marked by the horizontal dashed line, a new hypothetical device costing that amount or less would offer less momentum than that offered by the best punch by a super lightweight boxer.[20]

7.1 Summary

To bet our major adversaries would instantly distinguish a Prompt Global Strike targeting a region near them from an Intercontinental Ballistic Missile launched at them is untenable. If PGS systems are to work, they need to air-launched. The problem calls for more forward-based aircraft and a new fleet of hypersonic missiles. The market does not support the stated desired 100 B-21 aircraft at the desired cost. The cost needs to come down or the quantity will. A significant cost reduction could come about by example, dramatically reducing its range. Placing more tankers at forward bases along routes a smaller B-21 might take would let it perform long-range missions at an affordable cost. Shorted bomber range worked and continues to work for the B-52.

To study cost implications in multiple related markets, we need a system that offers the flexibility to study as many markets as needed simultaneously to understand. This paper offers ways to address any number of markets at the same time. It is useful in working integrated solutions between markets.

References:

1. https://www.thefreedictionary.com/dimension, Retrieved February 14, 2020
2. Author Andrés Baldrich, September 5, 2005. Map of Argentina showing all the territories recognized by that country's government as their own. This file has a Creative Commons Attribution-Share Alike 2.5 Generic license: https://commons.wikimedia.org/wiki/File:Argentine_map_of_Argentina.png, retrieved February 14, 2020
3. Australia map from Papayoung, a derivative work from Svgalbertian. This file has a Creative Commons Attribution-Share Alike 3.0 Unported, https://commons.wikimedia.org/wiki/File:Australia_Locator_Map_(alt).svg, retrieved February 14, 2020
4. The territorial claims on Antarctica: http://www.atsummit50.org/session/the_antarctic_treaty-1.html, retrieved February 14, 2020
5. https://en.wikipedia.org/wiki/Prompt_Global_Strike Retrieved January 30, 2020
6. Howarth, Douglas K. "A 7D Trade," presented to the 31st Congress of the International Council of the Aeronautical Sciences (ICAS), Belo Horizonte, Brazil, September 11, 2018, ISBN: 978-3-932182-88-4
7. Missile specifications from http://www.af.mil/About-Us/Fact-Sheets/, retrieved October 15, 2016
8. Missile and bomb market data retrieved from this site http://www.saffm.hq.af.mil/shared/media/document/AFD-150309-005.pdf, on October 15, 2016

9. Hu, Shu-Ping. “The Impact of Using Log-Error CERS Outside the Data Range and Ping Factor,” ISPA/SCEA Joint International Conference 2005, ISBN 0-9720204-4-6, p. 1279, Denver Colorado, June 14-17, 2005. The Ping Factor removes downward bias in power-form regression equations. The lower the correlation, the higher the Ping Factor.
10. Manned and unmanned aircraft specifications from http://www.af.mil/About-Us/Fact-Sheets/, retrieved October 15, 2016
11. F-35A data from https://fas.org/sgp/crs/weapons/RL30563.pdf, retrieved October 15, 2016
12. F-16 data from https://news.lockheedmartin.com/2006-04-21-Lockheed-Martin-Delivers-4-300th-F-16 and https://www.codeonemagazine.com/article.html?item_id=141, retrieved February 10, 2020
13. F-18 data from *Jane's All The World's Aircraft Development and Production 2013-2014,* page 720, and https://web.archive.org/web/20120106065148/http://www.boeing.com/defense-space/military/fa18/fa18_milestones.htm, retrieved February 10, 2020
14. F-15 data from http://tealgroup.com/images/TGCTOC/sample-wmcab2.pdf, retrieved February 10, 2020
15. B-2 data from https://www.globalsecurity.org/wmd/systems/b-2-production.htm, retrieved February 10, 2020
16. B-1 data from https://en.wikipedia.org/wiki/Rockwell_B-1_Lancer, retrieved February 10, 2020
17. Drew, James (April 20, 2016). “USAF basing revised bomber count on ‘minimum’ of 100 B-21s”. Flight Global. Retrieved February 8, 2018, from https://www.flightglobal.com/news/articles/usaf-basing-revised-bomber-count-on-minimum-of-100-424433/.
18. Gertler, Jeremiah, “Air Force B-21 Raider Long-Range Strike Bomber,” Congressional Research Service, June 7, 2017, Retrieved February 8, 2018, from https://fas.org/sgp/crs/weapons/R44463.pdf, retrieved October 15, 2016
19. 2019 International Monetary Fund (IMF) Gross Domestic Product (GDP) estimate from https://en.wikipedia.org/wiki/List_of_countries_by_GDP_(nominal) Retrieved February 24, 2020\
20. https://www.quora.com/If-the-power-of-a-punch-comes-mostly-from-speed-why-would-heavyweight-boxers-fighters-most-likely-win-against-lightweights Retrieved February 25, 2020. “According to Scientific American, Ricky Hatton's punch speed clocked in at about 25 mph, or 11.2 m/s, and the mass of 63.5 kg. So, in hypothetically, he'd have a momentum of 711.2 kg m/s.”