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# Multiproduct Cost-Volume-Profit Model: A Resource Reallocation Approach for Decision Making

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*This work addresses the problem of reallocating productive resources to maximize profit. Most contributions to the topic focus on developing or improving the Cost-Volume-Profit model to obtain solutions that provide an ideal mix of products before the data is given. In particular, some algorithms are available for the problem, such as the ones proposed by Kakumanu and Shao and Feng. However, these proposals do not consider the minimum number of units to be produced, and the reallocation of productive resources for each product is a problem found in these studies. Bearing this in mind, a new algorithm based on individual financial revenue is proposed. Computational results indicate that the proposed method can be utilized as a decision support system.*

## Introduction

The Industrial Revolution brought with it technological advancement in various fields of knowledge. This process provided a continuous quest to achieve more efficient operations, in other words, using productive resources to produce in the best possible way. With this in mind, various techniques and management systems became commonly used which, in turn, led to an increase in the number of organizations in various sectors (Hillier & Lieberman, 2001, p. 1).

Consequently, to deal with this situation, organizations began to provide or produce various products, the commonly known product mix. This process can be clearly seen in manufacturing organizations as it attempts to provide a set of similar products, but with specific features, observing the characteristics and needs of their customers (Kakumanu, 1998, p. 87).

Concerning this context mainly for manufacturing organizations, the definition of production is of great importance because there are not always opportunities for adjustments in production levels without financial loss. Consequently, considering market information, manufacturing organizations define their master production plan based on the Cost-Volume-Profit (CVP) model, which, in turn, defines the optimum volume for a

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product, considering the cost and profit information. Although the purpose of the model is simple, its implementation is difficult because manufacturing organizations produce various products; i.e., there are many variables to be considered and, especially in practice, there is not enough time to find a good solution or a product mix with productive resources to maximize profit.

Some algorithms have been proposed to deal with this problem, such as those by Kakumanu (1998), Shao and Feng (2007) and, more recently, Milanovic, Milanovic, Misita, Klarin, and Zunjic (2010) Although these discussions have advanced in addressing the problem of having more than one product, some gaps still remain. Considering this, focusing on complementing or corroborating these studies, the purpose of this article is to present a new algorithm that helps to define the optimal production volume, considering the CVP model in situations with various products and minimum quantities to be produced. More specifically, an algorithm attempts to determine the required volume for each product in order to achieve the highest possible profit, considering that:

- the minimum production or minimum capacity of all products, which is usually determined by the market demand or machinery capacity, is predefined by the decision makers; and
- productive resources can be reallocated from one product to another.

To fulfil this purpose, the rest of this work is organized as follows. The next section shows a brief background of the CVP. The section following that introduces the new algorithm and its proposal. After that the computational experiment and the main results are presented. Finally, the last section provides the conclusions.

## Cost-Volume-Profit Model

The Cost-Volume-Profit model is understood to be a model that determines the volume required to achieve the balance between costs and revenues, as defined well by Chan (1990, p. 253). It is worth mentioning that the CVP model is usually used for short-term planning (Phillips, 1994, p. 31). Kakumanu (1998, p. 88) argues that most studies about this definition classify the CVP model by considering the number of products involved, the behavior of variables entailed, and the number of considered periods in the following classes:

- Single and multiproduct problems: For this category, problems with one product are defined as single. On the other hand, problems with more than two products are called multiproduct problems.
- Deterministic and stochastic problems: When all variables of the problem are known and do not have random elements, these problems are described as deterministic. On the other hand, problems are considered stochastic, as in a real situation values should change during the period of production.
- Single and multiperiod problems: In this category, problems for a single period are termed single and problems for the other situations are considered multiperiod problems.

By observing the classification proposed by Kakumanu (1998, p. 88), the main articles about CVP are summarized in Table 1. Consequently, it can be clearly observed that there are few works utilizing the application of the CVP model for multiproduct problems. Overall, considering the concept of CVP, Equations (1) to (5) present the idea of the original CVP model for a single product, single period, and deterministic variables.

**TABLE 1** Selected studies on CVP models

| Research | Research summary | Category | | |
|---|---|---|---|---|
| | | Products | Variables | Periods |
| Kakumanu (1998) | He develops and tests a CVP model with product limitations for multiproduct, which optimizes the rate of return on sales revenue. | Multiproduct | Deterministic | Single |
| Shao and Feng (2007) | They present a stochastic CVP model based on the Economic Value Added model (EVA) for uncertain situations, in which companies currently experience. | Single | Stochastic | Single |
| Milanovic et al. (2010) | He indicates a universal equation that shows the influential variables and their impact on the profit based on the cost-volume-profit equation. | Single | Deterministic/Stochastic | Single |
| Chan (1990) | He discusses the sensitive analysis for the cost-volume-profit mode by using incremental analysis using case studies and graphs. | Single | Deterministic | Single |
| Yuan (2009) | He presents an application of fuzzy logic in the CVP analysis to handle the imprecisions in the original model. | Single | Stochastic | Single |
| González (2001) | He develops an alternative model for the CVP multiproduct, by using data provided by ABC systems to keep track of some variables, to reach the required profit. | Multiproduct | Deterministic | Single |
| Jaedicke and Robichek (1964) | They include some concepts of probability in the CVP model creating uncertainty and making some variables no longer fixed and work with some approximations of values. | Single | Stochastic | Single |
| Phillips (1994) | This article examines the basic CVP model and describes how to include uncertainty during the decision-making process. | Single | Stochastic | Single |
| Yunker and Schofield (2005) | They analyze and apply a stochastic CVP model specifically geared towards the determination of enrollment fees for training and development. | Single | Stochastic | Single |
| Yunker and Yunker (2003) | They analyze and apply a CVP model under uncertainty specifically geared towards classroom instruction. | Single | Stochastic | Single |

$$P = TR - TC \tag{1}$$

$$TC = TVC + FC \tag{2}$$

$$TVC = Q \cdot VC \tag{3}$$

$$TR = Q \cdot SP \tag{4}$$

$$P = (SP - VC) \cdot Q - FC \tag{5}$$

wherein:
$P$ represents profit;
$TR$ represents total revenue;
$TC$ represents total costs;
$TVC$ represents total variable costs;
$FC$ represents fixed costs;
$Q$ represents units sold;
$VC$ represents variable costs; and
$SP$ represents selling price.

According to Phillips (1994, p. 31), it is important to emphasize that the original CVP model considers the following as its premise: (a) fixed cost will remain unchanged and variable cost will change proportionately with sales volume; (b) revenue is only affected by units sold; and (c) efficiency levels remain unchanged.

## Models and Algorithm

### *Notation*

In this work, the following notation and decision variables are used.

*Indices*.
$i = 1,2,3,\ldots, n$ represent product $i$.

*Variables*.
$n$ represents the number of products;
$M_i$ represents the production volume of product $i$;
$QR$ represents the remaining quantity of resources;
$MPC_i$ represents the minimum capacity to produce product $i$;
$IM_i$ represents the initial product volume of product $i$;
$PM_i$ represents the profit margin of product $i$;
$P_i$ represents the price of product $i$;
$C_i$ represents the cost of product $i$;
$S_i$ represents the production scheduling of product $i$;
$MD_i$ represents the market demand of product $i$;
$L_i$ represents the maximum capacity of production of product $i$; and
$FM_i$ represents the final volume of product $i$.

### *Kakumanu's CVP Model*

Kakumanu's CVP model was chosen because it was one of the principal articles to present a mathematician model to deal with the issue of multiproduct problems in the CVP model.

The CPV model with limited product for multiple products, as proposed by Kakumanu (1998), identifies the production volumes for each product, i.e., the optimal product mix. It considers the respective limits with the aim of optimizing the defined rate of return on sales. In summary, the pseudocode of the CVP model proposed by Kakumanu (1998) is presented below:

1. **Step 1.** Calculating the volume. Using the equations shown in his work, he computes the required volume for all of the products.
2. **Step 2.** Checking the first possibility of the solution. By checking whether all of the required volumes computed on the previous step are the same or smaller than the limit, he determinates whether the procedure stops or goes on. In case they are all the same or smaller, the best solution is found. Otherwise, he proceeds to the next step.
3. **Step 3.** Checking the second possibility of solution. By checking whether all of the required volumes computed on the previous step are the same or bigger than the limit, he determinates whether the procedure stops or goes on. In case they are all the same or bigger, the best solution is found. Otherwise, he proceeds to the next step.
4. **Step 4.** Checking the third possibility of solution. If at least one product has the required volume smaller than the limit and another one has it bigger than the limit, then a new required volume needs to be calculated.
5. **Step 5.** Calculating the new mixes. Utilizing the reminisces of the products that are over the limit, he creates new mixes of products from the ones that are under.
6. **Step 6.** Calculating the required new volume. Using the equations and the new values found earlier, he calculates an optimum mix of products.

***Proposed Model***

The purpose of the proposed method is to develop a mix of production based on the individual contribution of each product, considering the maximum and minimum capacity of production and, especially, the market demand. This new method differs from Kakumanu's because it relocates the excess material on the more profitable products. To meet the objective of the proposed model, the GBV model, we establish an initial solution from the minimum production. Bearing this in mind, it identifies the difference between the market demand and the initial solution, aiming at reallocating excess capacity of products that did not meet their demand. Subsequently, based on the calculation of unit profit, a new solution is defined and the total profit is calculated, comparing it with the initial income of the problem. Taking this into account, the GBV model is shown in the next section.

*Step 1. Adjusting Production to the Minimum Capacity.* At first, the minimum necessary should be produced so that from the remainder, the new volumes are adjusted.

$$M_i = MPC_i \quad \forall i \tag{6}$$

*Step 2. Calculate the Remainder.* Knowing the values of the original production and the minimum capacity of all products, the remainder is calculated from the initial production of the problem minus the minimum capacity of each product.

$$QR = \sum_{i=1}^{n} IM_i - M_i \tag{7}$$

*Step 3. Calculate the Profit Margin of Each Product.* In order for an optimal production sequence to be created, the profit margins of all products need to be calculated first so that the more productive ones can be defined. The profit margin can be calculated as shown in Equation (8).

$$PM_i = P_i - C_i \tag{8}$$

*Step 4. Identify the Optimal Production Sequence.* Based on the profit margins calculated in Step 5, the ideal sequence of production can be calculated. The following must be determined, $S_1 = \{PM_1, PM_2, \ldots, PM_i\}$, considering $i \leq n$ and that $S_1$ is the production scheduling of products with $PM_i \geq 0$ and $PM_1 \geq PM_2 \geq, \ldots, \geq PM_n$ and $S_2 = i$, for all products.

*Step 5. Determine the Final Volume for Each Product.* The final volume for each product can be calculated using Algorithm 1. It considers the production sequence defined in Step 4. Overall, the algorithm reallocates the remainder in the products until it reaches a maximum production or market demand.

**Algorithm 1:** Pseudocode to calculate the final mix

**for** $i = 1$ **to** $n$ **do**
    $FM_{S_2(i)} = \min\left(MD_{S_2(i)}; L_{S_2(i)}\right)$
    **if** $QR < \min\left(MD_{S_2(i)}; L_{S_2(i)}\right) - MPC_{S_2(i)}$ **then**
        $FM_{S_2(i)} = MPC_{S_2(i)} + QR$
        $QR = 0$
    **else**
        $QR = QR - \left(\min\left(MD_{S_2(i)}; L_{S_2(i)}\right) - MPC_{S_2(i)}\right)$
    **end**
**end**

## Numerical Example

The proposed method will be exemplified in a problem with three products ($n = 3$), considering the information in Table 2.

Based on these data, the first two steps are taken, which define the amount of resources that will be reallocated in order to adjust the volume manufactured to the ideal volume production. This amount is obtained by the difference between the initial product volume that is determined using the characteristics of the products to be fabricated, the set-up of production, and the minimum capacity of production. In this case, it is shown in Table 3.

In Steps 3 and 4, the order of optimal production is defined with the objective of prioritizing and ranking products by using the individual profit of each product as a parameter. The result of this step is shown in Table 4.

Finally, considering the algorithm shown in Algorithm 1, the production volume required for each product is determined by optimizing resources. In other words, aiming at maximizing profit, the resources were reallocated according to the optimal production sequence, aiming at maximizing profit. The final result of the GBV model for this example is shown in Table 5.

**TABLE 2** Data of the numerical example

| Products ($i$) | Price ($P_i$) | Cost ($C_i$) | Initial product volume ($IM_i$) | Market demand ($MD_i$) | Maximum capacity of production ($L_i$) | Minimum capacity of production ($MPC_i$) |
|---|---|---|---|---|---|---|
| 1 | 67.58 | 24.51 | 1,500 | 2,500 | 1,750 | 950 |
| 2 | 79.66 | 61.04 | 4,800 | 4,500 | 5,000 | 1,750 |
| 3 | 47.38 | 12.85 | 2,500 | 3,000 | 2,750 | 1,450 |

**TABLE 3** Steps One and Two

| Products ($i$) | Initial product volume ($IM_i$) | Minimum capacity of production ($MPC_i$) | Difference between ($IM_i$) and ($MPC_i$) | Remain quantity of resources ($QR$) |
|---|---|---|---|---|
| 1 | 1,500 | 950 | 550 | |
| 2 | 4,800 | 1,750 | 3,050 | 4,650 |
| 3 | 2,500 | 1,450 | 1,050 | |

**TABLE 4** Steps Three and Four

| Products ($i$) | Price ($P_i$) | Cost ($C_i$) | Profit margin of product ($PM_i$) | Production scheduling ($S_2$) |
|---|---|---|---|---|
| 1 | 67.58 | 24.51 | 43.07 | 1 |
| 2 | 79.66 | 61.04 | 18.62 | 3 |
| 3 | 47.38 | 12.85 | 34.52 | 2 |

**TABLE 5** Final solution

| Products ($i$) | Initial product volume ($IM_i$) | Final volume of product ($FMs_2(i)$) |
|---|---|---|
| 1 | 1,500 | 1,750 |
| 2 | 4,800 | 4,300 |
| 3 | 2,500 | 2,750 |

### *Computational Experiment*

It is important to highlight that the computational experiment was utilized to verify whether the GBV model presents good performance in response to the variability of the number of products. To perform the computational experiments of the proposed model, 50 different problems were generated in a random way. Overall, the data simulated different situations with different numbers of products and different possible scenarios regarding the maximum and minimum capacity of production. This diversification of problems was created for it to be tested in different situations. Taking this into account, Table 6 presents the data of problems. Consequently, the main results are shown in Table 7 and Figure 1.

Considering the computational experiment realized in this section, the results show that the application of the GBV algorithm can improve the financial performance in most of the cases. For example, we can see in problem number 2 that the initial production and sale of products are equal but when the algorithm is applied the profit gain has a significant increase.

In practical terms, the GBV algorithm can be used on small and medium organizations that have not integrated administration and control systems as a decision support system, especially regarding cost system.

### *Conclusions*

In this work, we address the problem of reallocating productive resources to maximize profit, considering multiproduct CVP. In order to fulfil this purpose, we proposed the GBV

**TABLE 6** Data of problems

| Problem | Product | Price | Cost | Mix (initial production) (Unit) | Market demand (Unit) | Maximum capacity (Unit) | Minimum capacity (Unit) | N. products |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 29.00 | 15.00 | 6150 | 4500 | 145000 | 0 | |
| | 2 | 22.00 | 15.50 | 14350 | 14000 | 145000 | 0 | |
| | 3 | 8.50 | 4.00 | 4100 | 5500 | 145000 | 0 | |
| | 4 | 11.00 | 8.50 | 4100 | 7000 | 145000 | 0 | |
| | 5 | 9.50 | 6.25 | 12300 | 10000 | 145000 | 0 | 5 |
| | 1 | 37.44 | 25.42 | 8000 | 12500 | 13000 | 6000 | |
| 2 | 2 | 50.13 | 46.09 | 2000 | 3000 | 2500 | 1000 | 2 |
| | 1 | 62.56 | 40.84 | 1750 | 1650 | 1800 | 950 | |
| | 2 | 2.83 | 0.61 | 600 | 1000 | 750 | 350 | |
| | 3 | 48.19 | 22.70 | 1100 | 950 | 1200 | 500 | |
| 3 | 4 | 81.14 | 18.75 | 700 | 1250 | 1000 | 500 | 4 |
| | 1 | 48.66 | 13.31 | 1150 | 1000 | 1200 | 600 | |
| | 2 | 9.55 | 5.32 | 500 | 750 | 600 | 200 | |
| | 3 | 93.62 | 62.69 | 1400 | 1600 | 1800 | 850 | |
| | 4 | 81.31 | 38.69 | 550 | 850 | 800 | 200 | |
| 4 | 5 | 92.89 | 69.89 | 900 | 850 | 1000 | 500 | 5 |
| | 1 | 14.74 | 3.23 | 500 | 650 | 550 | 300 | |
| | 2 | 76.37 | 3.87 | 1000 | 1400 | 1750 | 600 | |
| | 3 | 88.55 | 0.51 | 850 | 1200 | 1000 | 700 | |
| 5 | 4 | 24.93 | 18.38 | 1100 | 1550 | 2000 | 500 | 4 |
| | 1 | 67.58 | 24.51 | 1500 | 2500 | 1750 | 950 | |
| | 2 | 79.66 | 61.04 | 4800 | 4500 | 5000 | 1750 | |
| 6 | 3 | 47.38 | 12.85 | 2500 | 3000 | 2750 | 1450 | 3 |
| | 1 | 94.88 | 16.28 | 400 | 500 | 600 | 300 | |
| | 2 | 16.90 | 3.77 | 650 | 1300 | 1000 | 500 | |
| | 3 | 30.21 | 5.82 | 500 | 650 | 700 | 400 | |
| | 4 | 57.72 | 41.48 | 650 | 950 | 800 | 550 | |
| | 5 | 94.58 | 77.70 | 1750 | 1500 | 2000 | 950 | |
| | 6 | 71.60 | 25.40 | 1200 | 1400 | 1050 | 850 | |
| | 7 | 89.01 | 72.03 | 500 | 950 | 1100 | 400 | |
| 7 | 8 | 29.70 | 22.78 | 1500 | 1150 | 2400 | 800 | 8 |
| | 1 | 16.68 | 2.35 | 800 | 1000 | 1200 | 500 | |
| | 2 | 49.85 | 43.11 | 600 | 800 | 700 | 450 | |
| | 3 | 40.75 | 1.37 | 1500 | 1350 | 1900 | 650 | |
| 8 | 4 | 12.71 | 7.02 | 2250 | 2450 | 2300 | 1600 | 4 |
| | 1 | 16.42 | 13.70 | 8000 | 10000 | 8500 | 6000 | |
| 9 | 2 | 76.02 | 57.41 | 6400 | 6000 | 8000 | 3000 | 2 |
| | 1 | 32.61 | 6.99 | 1200 | 1100 | 1500 | 650 | |
| | 2 | 4.45 | 0.19 | 1850 | 1400 | 1200 | 600 | |
| | 3 | 98.57 | 71.56 | 1650 | 1800 | 2000 | 1000 | |
| | 4 | 6.82 | 0.07 | 650 | 950 | 700 | 500 | |
| | 5 | 9.71 | 7.13 | 1000 | 850 | 1300 | 450 | |
| 10 | 6 | 20.41 | 9.69 | 900 | 1150 | 1200 | 500 | 6 |

(*Continued*)

**TABLE 6** (*Continued*)

| Problem | Product | Price | Cost | Mix (initial production) (Unit) | Market demand (Unit) | Maximum capacity (Unit) | Minimum capacity (Unit) | N. products |
|---|---|---|---|---|---|---|---|---|
| | 1 | 21.87 | 14.55 | 2000 | 1100 | 1000 | 750 | |
| 11 | 2 | 32.00 | 20.00 | 6000 | 7000 | 8200 | 5500 | 2 |
| | 1 | 88.81 | 14.95 | 1200 | 1150 | 2000 | 600 | |
| | 2 | 83.88 | 8.35 | 600 | 850 | 1000 | 300 | |
| | 3 | 47.46 | 19.16 | 600 | 400 | 350 | 150 | |
| 12 | 4 | 19.52 | 13.89 | 250 | 350 | 300 | 150 | 4 |
| | 1 | 92.42 | 86.45 | 120 | 150 | 200 | 100 | |
| | 2 | 97.99 | 1.80 | 850 | 750 | 800 | 600 | |
| | 3 | 30.34 | 18.98 | 1000 | 1100 | 1200 | 800 | |
| | 4 | 90.40 | 3.73 | 200 | 150 | 220 | 80 | |
| | 5 | 22.81 | 12.88 | 950 | 1350 | 1000 | 600 | |
| | 6 | 21.25 | 2.99 | 400 | 650 | 420 | 200 | |
| 13 | 7 | 4.00 | 2.32 | 1100 | 950 | 1200 | 350 | 7 |
| | 1 | 71.57 | 48.42 | 2000 | 1150 | 1300 | 500 | |
| | 2 | 41.96 | 28.85 | 900 | 1250 | 1000 | 500 | |
| 14 | 3 | 54.94 | 31.00 | 750 | 950 | 1000 | 400 | 3 |
| | 1 | 69.28 | 16.58 | 1000 | 700 | 1000 | 400 | |
| | 2 | 46.89 | 41.97 | 800 | 1050 | 900 | 600 | |
| | 3 | 39.84 | 3.25 | 900 | 750 | 900 | 450 | |
| | 4 | 49.73 | 11.60 | 500 | 350 | 300 | 100 | |
| 15 | 5 | 95.76 | 64.78 | 900 | 1400 | 1500 | 650 | 5 |
| | 1 | 33.01 | 19.46 | 550 | 750 | 600 | 300 | |
| | 2 | 83.47 | 11.92 | 500 | 650 | 800 | 400 | |
| | 3 | 94.18 | 37.23 | 450 | 300 | 150 | 100 | |
| | 4 | 68.09 | 22.01 | 400 | 650 | 700 | 350 | |
| | 5 | 82.65 | 52.27 | 835 | 800 | 650 | 400 | |
| 16 | 6 | 31.37 | 27.62 | 800 | 1150 | 1200 | 500 | 6 |
| | 1 | 9.81 | 6.84 | 800 | 1150 | 900 | 500 | |
| | 2 | 32.38 | 26.55 | 850 | 600 | 750 | 250 | |
| | 3 | 83.63 | 27.05 | 700 | 1050 | 1000 | 500 | |
| | 4 | 20.31 | 17.11 | 600 | 750 | 800 | 300 | |
| | 5 | 12.33 | 8.63 | 850 | 600 | 400 | 250 | |
| | 6 | 90.26 | 3.02 | 600 | 1000 | 1200 | 400 | |
| | 7 | 62.39 | 53.55 | 1250 | 800 | 600 | 300 | |
| | 8 | 57.52 | 53.37 | 500 | 850 | 900 | 450 | |
| 17 | 9 | 5.21 | 1.88 | 600 | 900 | 650 | 500 | 9 |
| | 1 | 35.26 | 4.13 | 650 | 300 | 500 | 150 | |
| | 2 | 71.50 | 44.06 | 1150 | 700 | 650 | 400 | |
| | 3 | 52.90 | 7.28 | 650 | 900 | 1000 | 500 | |
| 18 | 4 | 62.49 | 1.72 | 700 | 1000 | 900 | 500 | 4 |
| | 1 | 67.29 | 47.19 | 800 | 900 | 1100 | 600 | |
| | 2 | 46.19 | 41.01 | 1550 | 850 | 800 | 600 | |
| | 3 | 69.13 | 12.84 | 900 | 1050 | 1200 | 400 | |

(*Continued*)

**TABLE 6** (*Continued*)

| Problem | Product | Price | Cost | Mix (initial production) (Unit) | Market demand (Unit) | Maximum capacity (Unit) | Minimum capacity (Unit) | N. products |
|---|---|---|---|---|---|---|---|---|
| | 4 | 84.60 | 10.31 | 800 | 1200 | 950 | 550 | |
| | 5 | 52.40 | 36.22 | 950 | 750 | 800 | 450 | |
| | 6 | 46.46 | 1.78 | 550 | 450 | 400 | 200 | |
| 19 | 7 | 30.14 | 21.79 | 650 | 850 | 900 | 450 | 7 |
| | 1 | 18.40 | 5.48 | 1550 | 1250 | 1100 | 750 | |
| | 2 | 76.90 | 39.60 | 900 | 1100 | 1200 | 650 | |
| 20 | 3 | 58.89 | 33.35 | 1200 | 1500 | 1400 | 800 | 3 |
| | 1 | 67.46 | 42.68 | 1100 | 1350 | 1400 | 600 | |
| | 2 | 59.13 | 44.01 | 650 | 550 | 420 | 300 | |
| | 3 | 45.57 | 40.38 | 1000 | 1400 | 1100 | 700 | |
| | 4 | 33.28 | 12.41 | 700 | 650 | 800 | 250 | |
| 21 | 5 | 42.98 | 16.42 | 150 | 200 | 250 | 100 | 5 |
| | 1 | 67.03 | 43.06 | 700 | 1050 | 1200 | 500 | |
| | 2 | 94.91 | 24.01 | 600 | 1000 | 750 | 450 | |
| | 3 | 53.15 | 49.81 | 900 | 1400 | 1000 | 750 | |
| | 4 | 14.37 | 9.62 | 550 | 350 | 500 | 100 | |
| | 5 | 21.74 | 7.93 | 400 | 600 | 500 | 300 | |
| | 6 | 26.00 | 13.69 | 700 | 1050 | 800 | 400 | |
| | 7 | 32.07 | 3.71 | 400 | 150 | 300 | 50 | |
| 22 | 8 | 13.51 | 2.35 | 850 | 700 | 650 | 450 | 8 |
| | 1 | 42.25 | 28.65 | 600 | 1050 | 750 | 400 | |
| 23 | 2 | 42.45 | 13.50 | 5750 | 5500 | 6000 | 3000 | 2 |
| | 1 | 42.28 | 24.49 | 1500 | 1200 | 2000 | 800 | |
| | 2 | 36.79 | 18.43 | 350 | 500 | 400 | 200 | |
| | 3 | 45.33 | 20.57 | 700 | 1050 | 1200 | 500 | |
| | 4 | 65.27 | 8.04 | 500 | 700 | 600 | 400 | |
| | 5 | 34.16 | 25.06 | 1000 | 900 | 1200 | 450 | |
| | 6 | 93.67 | 1.56 | 750 | 1200 | 1700 | 550 | |
| | 7 | 100.42 | 76.96 | 1100 | 1350 | 1300 | 700 | |
| | 8 | 91.52 | 10.18 | 500 | 700 | 850 | 400 | |
| 24 | 9 | 96.22 | 62.55 | 600 | 550 | 700 | 300 | 9 |
| | 1 | 17.68 | 0.28 | 800 | 1500 | 1800 | 650 | |
| | 2 | 52.68 | 22.55 | 1500 | 2000 | 1900 | 1000 | |
| | 3 | 30.19 | 15.54 | 1300 | 2000 | 1800 | 750 | |
| 25 | 4 | 80.93 | 73.02 | 1700 | 2000 | 2200 | 900 | 4 |
| | 1 | 89.84 | 50.08 | 1500 | 1400 | 1650 | 500 | |
| 26 | 2 | 70.80 | 35.78 | 1200 | 1500 | 1350 | 800 | 2 |
| | 1 | 11.74 | 6.24 | 800 | 1050 | 900 | 650 | |
| | 2 | 26.34 | 1.93 | 750 | 950 | 1100 | 550 | |
| | 3 | 22.99 | 21.65 | 3000 | 2650 | 3800 | 1200 | |
| 27 | 4 | 23.38 | 2.75 | 700 | 1500 | 1750 | 500 | 6 |
| | 5 | 57.24 | 5.46 | 1300 | 1250 | 1400 | 600 | |
| | 6 | 69.58 | 45.01 | 800 | 950 | 1200 | 550 | |

(*Continued*)

**TABLE 6** (*Continued*)

| Problem | Product | Price | Cost | Mix (initial production) (Unit) | Market demand (Unit) | Maximum capacity (Unit) | Minimum capacity (Unit) | N. products |
|---|---|---|---|---|---|---|---|---|
| | 1 | 99.38 | 91.60 | 800 | 1050 | 900 | 400 | |
| | 2 | 28.37 | 7.65 | 800 | 750 | 1000 | 450 | |
| | 3 | 40.36 | 25.48 | 450 | 650 | 600 | 300 | |
| | 4 | 43.05 | 40.77 | 900 | 1150 | 1300 | 600 | |
| | 5 | 74.98 | 31.13 | 500 | 900 | 800 | 350 | |
| | 6 | 28.34 | 25.95 | 1200 | 1000 | 1300 | 800 | |
| | 7 | 53.58 | 3.05 | 400 | 600 | 800 | 300 | |
| | 8 | 7.43 | 3.36 | 900 | 1150 | 1100 | 650 | |
| | 9 | 67.07 | 7.63 | 450 | 550 | 700 | 200 | |
| 28 | 10 | 100.44 | 65.68 | 700 | 1000 | 900 | 450 | 10 |
| | 1 | 21.93 | 19.10 | 7600 | 7450 | 8000 | 4000 | |
| 29 | 2 | 87.99 | 82.17 | 6000 | 6600 | 7000 | 3500 | 2 |
| | 1 | 11.22 | 2.32 | 650 | 900 | 800 | 500 | |
| | 2 | 33.71 | 21.66 | 500 | 600 | 800 | 300 | |
| | 3 | 94.49 | 35.65 | 750 | 1000 | 1200 | 550 | |
| | 4 | 95.72 | 44.06 | 1000 | 800 | 1200 | 650 | |
| 30 | 5 | 80.49 | 64.20 | 1250 | 1750 | 1600 | 850 | 5 |
| | 1 | 79.54 | 72.50 | 800 | 1050 | 1000 | 600 | |
| | 2 | 12.69 | 7.71 | 800 | 900 | 1100 | 650 | |
| | 3 | 60.60 | 40.12 | 950 | 1200 | 1300 | 750 | |
| | 4 | 42.25 | 30.33 | 900 | 1000 | 1200 | 650 | |
| | 5 | 100.67 | 1.98 | 1100 | 1400 | 1200 | 900 | |
| 31 | 6 | 66.21 | 35.10 | 750 | 950 | 850 | 450 | 6 |
| | 1 | 56.73 | 21.77 | 5800 | 6500 | 6000 | 4500 | |
| | 2 | 27.14 | 6.41 | 800 | 950 | 900 | 650 | |
| 32 | 3 | 26.70 | 10.28 | 1200 | 1000 | 1500 | 850 | 3 |
| | 1 | 79.44 | 20.25 | 3000 | 2500 | 3200 | 2000 | |
| | 2 | 15.08 | 2.65 | 700 | 850 | 800 | 400 | |
| | 3 | 27.13 | 9.18 | 900 | 1100 | 1000 | 600 | |
| 33 | 4 | 24.05 | 2.18 | 1200 | 900 | 1500 | 500 | 4 |
| | 1 | 67.30 | 5.26 | 800 | 1000 | 900 | 400 | |
| 34 | 2 | 17.92 | 16.96 | 950 | 1200 | 1400 | 500 | 7 |
| | 3 | 3.85 | 1.92 | 1300 | 1150 | 1500 | 600 | |
| | 4 | 8.84 | 0.89 | 1500 | 1350 | 1800 | 1000 | |
| | 5 | 77.11 | 59.26 | 5000 | 6300 | 5800 | 4000 | |
| | 6 | 3.93 | 3.04 | 800 | 1350 | 1200 | 500 | |
| 34 | 7 | 4.91 | 2.70 | 3200 | 4000 | 3600 | 2500 | 7 |
| | 1 | 13.25 | 6.58 | 6500 | 8000 | 8600 | 5000 | |
| | 2 | 7.80 | 7.20 | 5200 | 7000 | 6800 | 3500 | |
| 35 | 3 | 10.20 | 9.70 | 6700 | 5500 | 7000 | 4000 | 3 |
| | 1 | 29.60 | 23.87 | 650 | 800 | 1000 | 400 | |
| | 2 | 44.91 | 19.88 | 500 | 750 | 650 | 300 | |
| | 3 | 92.00 | 33.94 | 1100 | 900 | 1400 | 750 | |

(*Continued*)

**TABLE 6** (*Continued*)

| Problem | Product | Price | Cost | Mix (initial production) (Unit) | Market demand (Unit) | Maximum capacity (Unit) | Minimum capacity (Unit) | N. products |
|---|---|---|---|---|---|---|---|---|
| | 4 | 79.61 | 17.85 | 500 | 800 | 750 | 350 | |
| | 5 | 79.78 | 70.04 | 1200 | 1050 | 1600 | 750 | |
| | 6 | 50.48 | 22.73 | 500 | 600 | 800 | 300 | |
| | 7 | 96.87 | 17.42 | 550 | 650 | 700 | 350 | |
| 36 | 8 | 38.28 | 15.18 | 600 | 700 | 900 | 400 | 8 |
| | 1 | 37.52 | 5.38 | 3800 | 3500 | 4000 | 2500 | |
| 37 | 2 | 87.21 | 61.43 | 1800 | 1950 | 2000 | 1200 | 2 |
| | 1 | 53.01 | 38.53 | 1200 | 1500 | 1400 | 750 | |
| | 2 | 76.09 | 21.02 | 1000 | 1300 | 1100 | 650 | |
| | 3 | 58.56 | 16.40 | 1100 | 1250 | 1500 | 800 | |
| | 4 | 81.92 | 55.46 | 600 | 950 | 800 | 350 | |
| 38 | 5 | 15.56 | 4.90 | 800 | 1250 | 1000 | 500 | 5 |
| | 1 | 22.56 | 9.66 | 1500 | 1850 | 1600 | 800 | |
| | 2 | 14.17 | 3.84 | 900 | 800 | 1200 | 450 | |
| | 3 | 74.47 | 46.23 | 1550 | 1400 | 1800 | 950 | |
| 39 | 4 | 98.21 | 49.82 | 500 | 700 | 600 | 300 | 4 |
| | 1 | 30.41 | 25.51 | 1800 | 1500 | 2000 | 1000 | |
| | 2 | 16.68 | 14.75 | 2000 | 2300 | 2200 | 1600 | |
| | 3 | 32.71 | 21.12 | 4000 | 5200 | 4800 | 3550 | |
| 40 | 4 | 43.19 | 35.80 | 4500 | 4800 | 5000 | 3850 | 4 |
| | 1 | 61.72 | 16.77 | 1300 | 1500 | 1700 | 950 | |
| | 2 | 33.60 | 29.19 | 1100 | 950 | 1350 | 700 | |
| 41 | 3 | 91.78 | 60.86 | 1450 | 1350 | 1600 | 900 | 6 |
| | 4 | 61.54 | 47.60 | 900 | 1200 | 1000 | 650 | |
| | 5 | 62.94 | 46.76 | 900 | 1100 | 950 | 600 | |
| 41 | 6 | 3.87 | 1.57 | 1500 | 1400 | 1750 | 1000 | 6 |
| | 1 | 75.52 | 17.01 | 900 | 700 | 1000 | 450 | |
| | 2 | 88.45 | 8.06 | 700 | 950 | 800 | 450 | |
| | 3 | 1.44 | 0.92 | 850 | 950 | 900 | 500 | |
| | 4 | 70.07 | 59.39 | 900 | 800 | 1100 | 550 | |
| | 5 | 56.78 | 26.91 | 400 | 600 | 500 | 250 | |
| | 6 | 17.28 | 10.67 | 1000 | 1150 | 1100 | 650 | |
| | 7 | 72.86 | 0.54 | 650 | 700 | 800 | 500 | |
| | 8 | 77.22 | 55.20 | 400 | 600 | 500 | 250 | |
| 42 | 9 | 57.27 | 27.93 | 300 | 350 | 500 | 150 | 9 |
| | 1 | 58.65 | 49.04 | 3000 | 4000 | 3600 | 2350 | |
| | 2 | 13.42 | 5.00 | 2400 | 2100 | 2800 | 1500 | |
| 43 | 3 | 31.12 | 22.84 | 5200 | 5500 | 5800 | 4350 | 3 |
| | 1 | 21.20 | 5.25 | 1000 | 800 | 1200 | 550 | |
| | 2 | 77.09 | 9.32 | 800 | 950 | 900 | 600 | |
| | 3 | 39.60 | 28.90 | 1400 | 1200 | 1500 | 750 | |
| | 4 | 23.60 | 20.14 | 800 | 600 | 1000 | 450 | |
| | 5 | 5.15 | 1.33 | 1000 | 1350 | 1200 | 650 | |

(*Continued*)

**TABLE 6** (*Continued*)

| Problem | Product | Price | Cost | Mix (initial production) (Unit) | Market demand (Unit) | Maximum capacity (Unit) | Minimum capacity (Unit) | N. products |
|---|---|---|---|---|---|---|---|---|
| | 6 | 37.14 | 21.45 | 800 | 1000 | 900 | 650 | |
| 44 | 7 | 35.68 | 25.97 | 600 | 750 | 650 | 450 | 7 |
| | 1 | 11.16 | 6.69 | 450 | 550 | 700 | 200 | |
| | 2 | 61.18 | 9.48 | 650 | 900 | 800 | 550 | |
| | 3 | 64.55 | 45.93 | 600 | 650 | 800 | 400 | |
| | 4 | 75.63 | 22.08 | 500 | 600 | 700 | 300 | |
| | 5 | 4.05 | 1.17 | 1400 | 1500 | 1800 | 950 | |
| | 6 | 44.27 | 41.16 | 800 | 1100 | 950 | 650 | |
| | 7 | 71.08 | 0.01 | 550 | 800 | 700 | 350 | |
| | 8 | 19.54 | 6.65 | 800 | 850 | 1000 | 550 | |
| | 9 | 10.37 | 8.90 | 1000 | 1250 | 1200 | 650 | |
| 45 | 10 | 28.34 | 13.17 | 600 | 750 | 700 | 450 | 10 |
| | 1 | 66.41 | 18.56 | 1300 | 1450 | 1600 | 850 | |
| 46 | 2 | 55.13 | 30.40 | 800 | 900 | 1100 | 650 | 5 |
| | 3 | 1.74 | 0.14 | 1100 | 950 | 1300 | 650 | |
| | 4 | 59.01 | 26.14 | 1800 | 1650 | 2000 | 1150 | |
| 46 | 5 | 9.65 | 5.26 | 1200 | 1450 | 1300 | 850 | 5 |
| | 1 | 65.79 | 56.54 | 1000 | 1100 | 1300 | 650 | |
| | 2 | 11.68 | 9.58 | 2800 | 3000 | 3200 | 2250 | |
| | 3 | 28.14 | 8.79 | 2000 | 2500 | 2250 | 1450 | |
| | 4 | 68.23 | 57.65 | 2500 | 2300 | 2600 | 1850 | |
| | 5 | 64.86 | 55.25 | 1200 | 950 | 1400 | 700 | |
| | 6 | 52.18 | 41.30 | 2500 | 3000 | 2700 | 1625 | |
| | 7 | 49.31 | 39.92 | 900 | 1050 | 1000 | 650 | |
| 47 | 8 | 24.41 | 1.81 | 2500 | 3000 | 2650 | 1350 | 8 |
| | 1 | 93.84 | 48.52 | 1800 | 2000 | 2300 | 1250 | |
| | 2 | 52.97 | 30.35 | 1000 | 750 | 1200 | 500 | |
| | 3 | 72.74 | 27.19 | 600 | 500 | 750 | 350 | |
| | 4 | 16.21 | 4.72 | 700 | 850 | 750 | 300 | |
| | 5 | 87.74 | 49.55 | 2000 | 2500 | 2250 | 1150 | |
| 48 | 6 | 44.72 | 13.35 | 1000 | 800 | 1150 | 550 | 6 |
| | 1 | 5.20 | 4.64 | 800 | 1100 | 950 | 550 | |
| | 2 | 79.20 | 5.86 | 800 | 550 | 900 | 300 | |
| | 3 | 76.72 | 55.88 | 3200 | 2500 | 3500 | 1650 | |
| | 4 | 56.72 | 1.49 | 2200 | 3000 | 2400 | 1750 | |
| | 5 | 85.08 | 78.86 | 600 | 800 | 700 | 350 | |
| | 6 | 45.08 | 35.50 | 800 | 850 | 900 | 500 | |
| 49 | 7 | 99.66 | 56.23 | 800 | 650 | 1200 | 400 | 7 |
| | 1 | 84.49 | 14.96 | 2700 | 3000 | 3900 | 2050 | |
| 50 | 2 | 18.95 | 3.17 | 2550 | 1950 | 2200 | 1100 | 2 |

**TABLE 7** Compared results

| Problem | Initial situation (production) | Initial situation (selling) | Future situation |
|---|---|---|---|
| 1 | $248050.00 | $215200.00 | $228750.00 |
| 2 | $104201.08 | $104201.08 | $112183.97 |
| 3 | $111056.05 | $105059.82 | $123665.64 |
| 4 | $130206.33 | $123754.82 | $139536.42 |
| 5 | $160299.34 | $160299.34 | $199150.91 |
| 6 | $240290.42 | $234704.26 | $250378.61 |
| 7 | $166589.69 | $159946.24 | $175473.06 |
| 8 | $87387.08 | $81480.66 | $84164.77 |
| 9 | $140881.51 | $133438.02 | $134527.30 |
| 10 | $99796.87 | $94933.05 | $101150.78 |
| 11 | $86646.82 | $80055.75 | $91323.41 |
| 12 | $152334.06 | $142981.45 | $160730.75 |
| 13 | $129767.24 | $115562.86 | $117740.45 |
| 14 | $76045.37 | $56370.41 | $62468.91 |
| 15 | $136519.33 | $109499.23 | $123572.56 |
| 16 | $115647.44 | $106041.01 | $116001.51 |
| 17 | $119450.59 | $113094.18 | $164851.17 |
| 18 | $123978.03 | $100734.68 | $122920.22 |
| 19 | $179580.48 | $168247.90 | $189021.07 |
| 20 | $84259.57 | $80382.07 | $91012.87 |
| 21 | $60867.67 | $58311.71 | $63764.14 |
| 22 | $99908.67 | $90196.48 | $111109.84 |
| 23 | $174643.88 | $167405.53 | $169446.03 |
| 24 | $243938.05 | $236005.32 | $301592.78 |
| 25 | $91598.16 | $91598.16 | $105111.74 |
| 26 | $101676.09 | $97699.28 | $101201.28 |
| 27 | $128130.14 | $125073.21 | $149555.92 |
| 28 | $131289.26 | $129776.99 | $166373.86 |
| 29 | $56436.57 | $56012.46 | $58234.97 |
| 30 | $127960.95 | $117630.55 | $133486.36 |
| 31 | $171704.19 | $171704.19 | $186459.45 |
| 32 | $239012.10 | $235728.35 | $243150.09 |
| 33 | $228643.88 | $192492.59 | $195530.72 |
| 34 | $162006.53 | $160523.75 | $180706.58 |
| 35 | $49814.26 | $49214.26 | $59186.78 |
| 36 | $194112.94 | $181039.64 | $210857.42 |
| 37 | $168537.03 | $158894.90 | $162761.84 |
| 38 | $143202.65 | $143202.65 | $154956.07 |
| 39 | $96623.88 | $91354.55 | $97484.62 |
| 40 | $92289.03 | $90820.02 | $99578.35 |
| 41 | $138683.55 | $134699.12 | $145891.89 |
| 42 | $202157.75 | $189388.50 | $208255.01 |
| 43 | $92052.42 | $89527.07 | $92808.79 |
| 44 | $110091.82 | $104069.97 | $113662.46 |

(*Continued*)

**TABLE 7** (*Continued*)

| Problem | Initial situation (production) | Initial situation (selling) | Future situation |
|---|---|---|---|
| 45 | $140053.96 | $140053.96 | $165863.80 |
| 46 | $148193.07 | $143021.88 | $153031.32 |
| 47 | $183947.11 | $179428.45 | $190962.55 |
| 48 | $247334.82 | $230851.69 | $250040.24 |
| 49 | $293443.95 | $254007.27 | $266237.59 |
| 50 | $227953.80 | $218490.63 | $239350.11 |

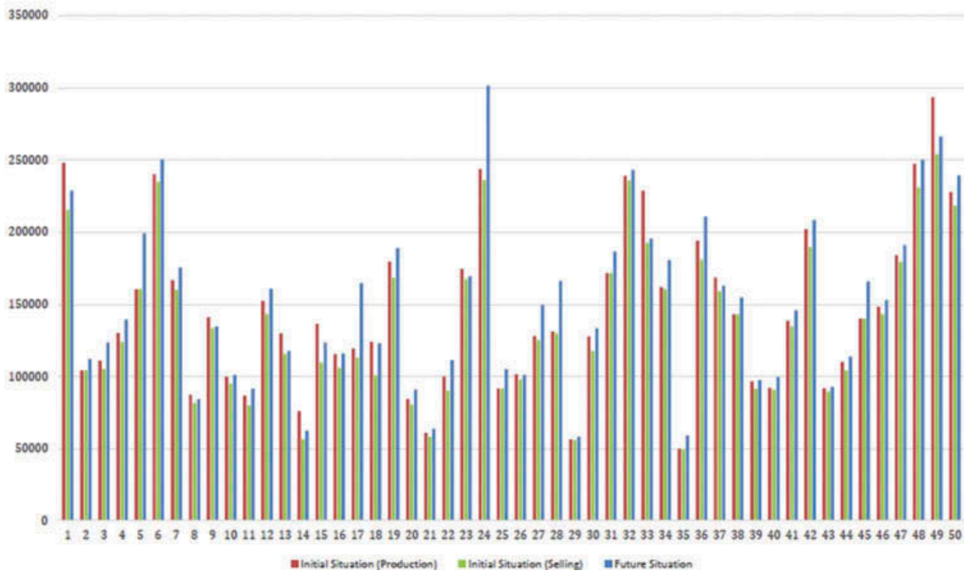


**FIGURE 1** Results for each problem.

model and we carried out a computational experiment. Based on the results shown in the previous section, it can be concluded that the proposed model can be used as an important tool in decision making when it comes to balancing or adjusting production capacities, using the ideas or the assumption of the CVP model for a product and perspectives presented by Kakumanu (1998). Considering this, we believe our work contributes to the discussion in this research area because it contemplates and presents a solution to the situation in which there are multiple products. Furthermore, it is important to highlight that this situation can often be found in small manufacturers. Although our contribution to this discussion has been significant, we highlight some issues that still remain to be addressed in future studies. Among these issues, we emphasize that future studies should focus on the following problems:

- Multiple products should be considered in environments with stochastic variability, because the productive system does not always present a deterministic behaviour.

- The GBV model should be integrated with the costing systems, such as activity based costing or time-driven activity-based costing.
- The GBV model should be adjusted to consider different markets, such as perfect competition, monopolistic competition, oligopoly, and monopoly.

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