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Applications of a Parsimonious Model of Development Programs' Costs and Schedules

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A model of the cost and schedule of a development program, characterized by three non-dimensional parameters, gives means for estimating the cost and schedule impacts of constraining funding below planned levels, as well as for assessing the realism of the costs and schedules of planned programs. In contrast to models of the Norden-Rayleigh-Weibull class, the model explicitly considers specific components of cost, and captures the distinction between a development program's value (i.e., the things delivered) and its cost (i.e., the money paid to acquire the value). Treating staff levels, staff productivity and cost, overhead, purchased material costs, and the burdens imposed by staff coordination and by allocating a program's effort to individual workers or teams and collating the results, the model reflects effects of management actions to make programs optimal by such criteria as minimal cost, minimal time, or minimal cost subject to a maximum-time constraint.

Introduction

People responsible for managing or assessing programs to develop complex equipment in times of budget uncertainty regularly need quantitative estimates for the effects of reduced funding over a period of years. If the program's outlays are reduced from planned amounts for certain years, what will be the impacts on total cost and schedule? The same people also need quantitative ways to judge the realism of planned costs and schedules. Is a plan that calls for developing a certain equipment in a period of Y years, with an outlay pattern of \$$M_i$ for year i, $1 \leq i \leq Y$, realistic in view of other programs' experience?

Porter and Gallagher (2004) applied the Norden-Rayleigh-Weibull model of development programs' outlays to address the first need: the cost and schedule impacts of reducing funding for one year. They develop a revised budget corresponding to funding curtailed in a single year, assuming that total constant-dollar outlays do not change, and given the length of the revised program. The Norden-Rayleigh model (Norden, 1963; Putnam, 1978) and its generalization to the Norden-Rayleigh-Weibull model (Brown, White, & Gallagher, 2002) address the second need: cost-schedule plans whose constant-dollar outlays do not follow a Rayleigh distribution (or another Weibull distribution with a maximum at positive time and a fat right tail) do not follow the experience of many development programs and so may be considered unrealistic. Indeed, other distributions can be used, such as Parr (1980), but those models typically exhibit similar features to those of the Rayleigh distribution.

Due to the sensitive, development program-specific nature of the data that the example results are based on, we regret we cannot share the underlying data with others.

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The Norden-Rayleigh-Weibull model is an ad hoc model, justified by the fact that outlay profiles of many development programs have been seen to follow it (Gallagher and Lee, 1996; Unger, Gallagher, & White, 2004). A model that treated specific components of development programs' costs and schedules, as well as management actions to balance pressures for low cost and quick schedules, may give a helpful alternative way to address the two needs mentioned above. We have found a certain parsimonious model of this kind helpful. The model treats staff levels, staff productivity, staff cost rates, costs of allocating an effort to tasks for individuals or teams, and collating the outputs, overhead, purchased materials, and the burden of coordinating individual efforts. The model's chief distinguishing feature is its explicit, quantitative accounting for the distinction between the value of a development program—the things that the program delivers—and the program's cost, which is the money paid to acquire the value. This article explains the model and shows its applications to the two needs.

The Model

Let the object of a development effort, or a part of one, be to produce a certain value V. By value we mean the set of things delivered by the effort: work packages, released drawings, tested components, etc. Value is distinct from cost, which is the money paid to acquire value. The cost to get a specific value varies with the management of a program's resources. Acquiring a required value in a way that meets an appropriate criterion (such as least cost, least time, or least cost subject to a maximum-time constraint) is a key objective of program management. Value may be measured by a variety of units. Generally, it may be measured in work packages. For an effort in which a specific product, such as released Computer-Aided Design-Computer-Aided Manufacturing (CAD-CAM) drawings, is to be produced, value may be measured in units of that product. The value of a development program to the purchaser is, of course, complete specifications for the manufacture of a cost-effective system that satisfies certain requirements. The things we describe as units measure the elements of such specifications. Costs of those things are represented in earned-value management (EVM) systems, and EVM data, specifically actual cost of work performed (ACWP) data, are available for calibrating and testing the model. Porter and Gallagher (2004) and others using Norden-Rayleigh-Weibull analyses also work with these costs.

To model the cost and schedule of a development effort, let workers' productivity be q value units per worker per year. A "worker" may be either an individual, or a team of individual workers who deal with a specific task, and whose work is coordinated collectively with the work of other teams. We take q to be invariant during each phase of a given development effort, though, as detailed below, we do treat variations in productivity and the other model parameters with phase. We do not expect significant improvements in productivity over a phase, believing that development workers generally apply familiar skills and available technology over a phase. This is distinct from the case of productivity improvement during production, where investments in product design and in production technology reduce unit costs over sufficiently lengthy production runs.

When N workers are employed, the project will incur a burden for coordinating their efforts. With N workers, $N(N-1)/2 \approx N^2/2$ interactions are possible. Let the coordination burden absorb workers' output equivalent to rN^2 units per year. When an effort takes T years to generate value V with N workers, then the workers produce value qNT and coordination absorbs value rN^2T, so that,

$$V = (qN - rN^2)T. \quad (1)$$

Let workers' compensation have the rate p dollars per worker per year, and let the project's overhead rate be D dollars per year. Let the cost of allocating the projects' requirements to N workers, and collating the resulting outputs, be aN dollars. If the project spends m dollars on purchased material, the effort's cost C will be:

$$C = pNT + DT + aN + m. \tag{2}$$

Equations (1) and (2) are the model. Its parsimony is evident; nevertheless, we find it capable of giving useful insights. These may be seen more clearly if the model is put into a non-dimensional form. To do this, we need reference values for the number of workers, for time, and for cost.

The effort will take the least time when the number of workers N makes $(qN - rN^2)$ a maximum; that number is $q/2r$, and the minimum time is $4rV/q^2$. Accordingly, we introduce a non-dimensional number of workers ρ as:

$$\rho \equiv \frac{2r}{q}N, \tag{3}$$

and a non-dimensional time τ as:

$$\tau \equiv \frac{q^2}{4rV}T. \tag{4}$$

An absolute minimum cost for V would be seen if D, a, m, and r were all zero. That cost would be pV/q. With this in view, we introduce a non-dimensional cost γ as:

$$\gamma \equiv \frac{q}{pV}C. \tag{5}$$

In the non-dimensional variables ρ, τ, and γ, the model is:

$$\tau = \frac{1}{2\rho - \rho^2} \tag{6}$$

and

$$\gamma = 2(\rho + \hat{D})\tau + \hat{a}\rho + \hat{m}. \tag{7}$$

Equations (6) and (7) show that three non-dimensional parameters, $\hat{D}, \hat{a},$ *and* $\hat{m}$, defined by

$$\hat{D} \equiv 2\frac{rD}{pq}, \quad \hat{a} \equiv \frac{1}{2}\frac{aq^2}{rpV}, \quad \hat{m} \equiv \frac{qm}{pV}, \tag{8 -- 10}$$

characterize the model.

The non-dimensional parameters have simple interpretations: $\hat{D}$ is equal to the ratio of overhead cost to workers' compensation, when the effort is staffed to complete in minimum time; $\hat{a}$ is equal to half the ratio of allocation-collation cost to workers' compensation for the minimum-time effort, and $\hat{m}$ is equal to the ratio of expenditures on purchased material to the effort's absolute minimum cost. Replacing τ in (7) by its value from (6) gives:

$$\gamma = \frac{2(\rho + \hat{D})}{2\rho - \rho^2} + \hat{a}\rho + \hat{m}, \tag{11}$$

Equations (6) and (11) are parametric equations for a curve in the (γ, τ) plane, with non-dimensional staff level ρ as parameter. The curve shows how management can control a project's cost and schedule by varying staffing. FIGURE 1 gives an example.

Considering values of γ and τ as ρ tends to the two limiting values 0 and 2 shows that the curve lies between the two asymptotes $\tau/\gamma = 1/(2\hat{D})$ and $\tau/\gamma = 1/(4 + 2\hat{D})$. Staffing increases along the curve in the direction shown by the arrow. At points above and to the right of point A, staffing is too low for efficient generation of value. Because of the overhead burden, both cost and time are larger than they need to be. The curve has a vertical tangent at point A, the point of minimum cost, and a horizontal tangent at point B, the point of minimum time. Arc AB is the region of the curve in which management may, by varying staffing, trade cost for time. In that region, increasing staff decreases time while increasing cost.

Staff levels greater than that of point B increase cost; and because of the coordination and allocation-coordination burdens, those levels also increase time rather than decreasing it. This is the model's reflection of the mythical man-month (Brooks, 1975). By virtue of the definitions of non-dimensional staffing and time, point B corresponds to non-dimensional staff value 1, and to non-dimensional time 1. The value of ρ for point A follows from setting the derivative of the right side of (11) equal to zero, which leads to the quartic equation:

$$\rho^4 - 4\rho^3 + \left(\frac{2}{\hat{a}} + 4\right)\rho^2 + \frac{4\hat{D}}{\hat{a}}\rho - \frac{4\hat{D}}{\hat{a}} = 0. \tag{12}$$

Straightforward analyses, given in the appendix, show that there is just one solution of (12) in the relevant region $0 < \rho < 1$.

Phases of a Development Program

Because the ratios determining parameters $\hat{D}$, $\hat{a}$, and $\hat{m}$ will typically not be the same constants at all times in the evolution of a development program, we believe the model is

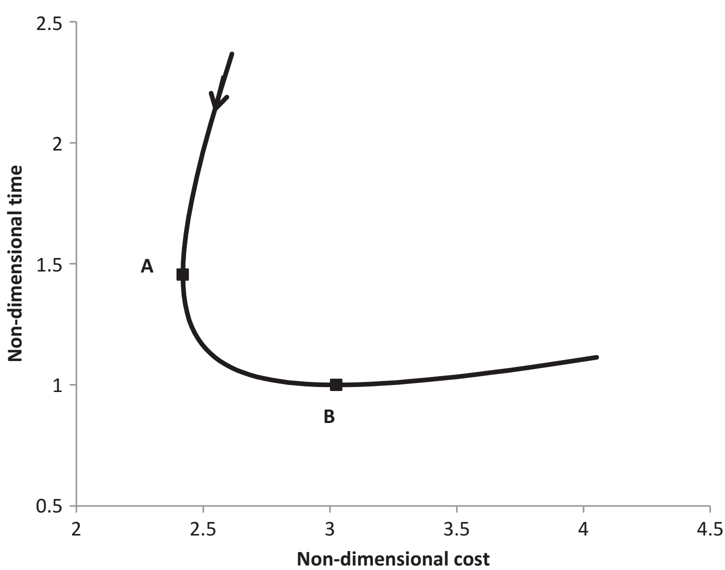

FIGURE 1 Staffing on non-dimensional cost-time curve.

best applied separately to certain phases of a program. Norden (1970) treated five phases: planning, design, construction and development test, initial operational testing, and release.

In the planning phase, a typical program will involve a fairly small number of senior people, who determine the product's overall characteristics. After this work is done, in a subsequent phase a considerably larger number of workers do detailed design. A third phase carries out prototype construction and development testing, and may involve re-engineering and re-manufacturing to correct detected flaws. Many of the workers active in the second and third phases remain active during initial operational testing, to cope with any deficiencies revealed. The release phase, which may continue after production begins, provides the transition to manufacturing.

Division into phases necessarily involves an analyst's judgment, and the appropriate number of phases depends on the level of program detail available to support that decision. In the following, we will consider the first four of Norden's phases when estimating the impact of constrained funding, a division in which the details available for a program that has operated for some time may support. For assessing the realism of planned outlay profiles, we will consider three phases, a division which may be appropriate when less program information is available.

Estimating Parameter Values

Those closely involved with a development program may be able to assign values to all of *p, q, a, D, r*, and *m*, and so calculate the model's parameters at each of the program's phases. Otherwise, useful estimates follow from the parameters' interpretations as ratios of quantities with which managers and cost analysts are generally familiar. In addition to the values of $\hat{D}$ *and* $\hat{a}$ as ratios to personnel costs of a minimum-time effort, and the value of $\hat{m}$ as a ratio to the effort's absolute minimum cost given above, model parameters also have simple expressions in terms of the fractions of the phase's cost contributed by overhead, personnel costs, allocation-collation costs, and purchased material, with the relative burden of coordination.

Let overhead be a fraction d of the phase's cost, let workers' compensation be a fraction w of that cost, let allocation-collation contribute a fraction s of the total cost, and let purchased material be equal to a fraction y of that cost. Of course, d, w, s, and y must sum to one. Also, let the relative coordination burden $(rN^2T)/V$ have the value z. Then,

$$\hat{D} \equiv 2\frac{rD}{pq} = 2\frac{DT}{pNT}\frac{rN}{q} = 2\frac{d}{w}\frac{z}{z+1}, \tag{13}$$

$$\hat{a} \equiv \frac{1}{2}\frac{aq^2}{rpV} = \frac{1}{2}\frac{aN}{pNT}\frac{q}{rN}\frac{qNT}{V} = \frac{1}{2}\frac{aN}{pNT}\frac{z+1}{z}(1+z) = \frac{1}{2}\frac{s}{w}\frac{1+z}{z}(1+z), \tag{14}$$

$$\hat{m} = (1+\mathrm{z})\left(\frac{\mathrm{y}}{\mathrm{w}}\right). \tag{15}$$

In treating an actual program, managers and cost people would, we believe, have quite reliable estimates for values of the parameters d, w, s, y, and z. To illustrate the relation of those parameters to the model's non-dimensional parameters, we give some examples. We expect the planning phase to have a substantial coordination burden, low overhead and allocation-collation costs, and negligible purchased material cost. This suggests values of z around 0.2, s around 0.01, d about 0.2, w about 0.79, and a zero value for y, giving:

$$\hat{D} = 0.084,\ \hat{a} = 0.046,\ \hat{m} = 0. \tag{16}$$

In a detailed design phase, we expect more significant allocation-collation costs, significant coordination burden, overhead costs more significant than in the planning phase, and modest costs of purchased material. Taking $d = 0.4$, $s = 0.1$, $w = 0.4$, and $y = 0.1$, with $z = 0.2$, gives:

$$\hat{D} = 0.33,\ \hat{a} = 0.9,\ \hat{m} = 0.3. \tag{17}$$

Compared to a detailed design phase, we expect that a phase of prototype construction and development testing would have increased coordination burden, very significant overhead costs, significant allocation-collation costs, and larger costs for purchased material. Choosing $d = 0.5$, $s = 0.15$, $y = 0.2$, and $w = 0.15$, with $z = 0.25$ gives:

$$\hat{D} = 1.33,\ \hat{a} = 3.13,\ \hat{m} = 1.67. \tag{18}$$

In initial operational testing, we expect substantial overhead cost, allocation-collation costs reduced from the preceding phase, insignificant costs of purchased material, and coordination costs somewhat reduced from the preceding phase. Taking $d = 0.5$, $s = 0.05$, $w = 0.4$, and $y = 0.05$, with $z = 0.25$, gives:

$$\hat{D} = 0.5,\ \hat{a} = 0.39,\ \hat{m} = 0.16. \tag{19}$$

Because a release phase involves interactions among manufacturing and development personnel, we believe the model may require further development for use in a release phase.

Application to Estimating Impacts of Funding Cuts

Managers of a development effort may react to reduced funding by reducing staff from planned levels. Reducing other components of cost, such as overhead, may be much more difficult than staff reduction. Assuming that the original plan's staffing was optimal in some sense—minimum cost, minimum time, or minimum cost subject to a maximum-time constraint—the model may be used to predict how the effort's total cost and schedule will change.

Our intention here is to understand those cost and schedule impacts of funding changes that are endogenous to the operations of development programs. We do not address here such exogenous effects as lost production due to transfer of staff between projects, or to delays in finding new staff, or costs of training new staff. To analyze impacts of reduced funding, we introduce another non-dimensional variable, $\hat{R} \equiv \frac{4r}{pq}\frac{C}{T}$, for expenditure rate. An equation for $\hat{R}$ follows from the definitions given previously:

$$\hat{R} = \gamma(\rho) \cdot (2\rho - \rho^2). \tag{20}$$

Consider now the challenge faced by the manager of a program planned for minimum-cost operation, when the funding rate for a certain year has been reduced by a relative amount σ, from R_0 to R_1. Electing to achieve the rate reduction by staff reduction, management will in effect seek a value of non-dimensional staffing ρ_1, such that,

$$\frac{\hat{R}(\rho_1) - \hat{R}(\rho^*)}{\hat{R}(\rho^*)} = -\sigma, \tag{21}$$

where we have written ρ^* for the value of ρ that makes γ a minimum. Changing ρ from ρ^* to ρ_1 will imply a staffing change from planned value N_0 to value $N_{1,}$ cost from planned value C_0 to C_1, and time from planned value T_0 to $T_{1,}$ as follows:

$$\frac{N_1}{N_0} = \frac{\rho_1}{\rho_0}, \tag{22}$$

$$\frac{\mathrm{C}_1}{\mathrm{C}_0} = \frac{\gamma(\rho_1)}{\gamma(\rho^*)}, \tag{23}$$

and, by virtue of (6),

$$\frac{T_1}{T_0} = \frac{\tau(\mathrm{N}_1)}{\tau(\mathrm{N}^*)} = \frac{2\rho^* - \rho^{*^2}}{2\rho_1 - \rho_1^2}. \tag{24}$$

The model has an interesting implication for the variation of cost with change in funding rate. To see this, let us consider the local behavior of $\hat{R}(\rho_1) - \hat{R}(\rho^*)$. For $\rho < 2$, $\hat{R}$ is continuously differentiable, so for ρ sufficiently near ρ^*,

$$\frac{\hat{R}(\rho) - \hat{R}(\rho^*)}{\hat{R}(\rho^*)} \approx \frac{\hat{R}'(\rho^*)}{\hat{R}(\rho^*)}(\rho - \rho^*). \tag{25}$$

where prime denotes differentiation with respect to ρ. Solving (25) for $\rho - \rho^*$ gives:

$$\rho - \rho^* \approx -\sigma \frac{\hat{R}(\rho^*)}{\hat{R}'(\rho^*)}. \tag{26}$$

Recall that $\gamma'(\rho^*) = 0$. Consequently, for ρ sufficiently near ρ^*,

$$\frac{\gamma(\rho) - \gamma(\rho^*)}{\gamma(\rho^*)} \approx \frac{1}{2}\frac{\gamma''(\rho^*)}{\gamma(\rho^*)}(\rho - \rho^*)^2. \tag{27}$$

Using (26) in (27) gives the local behavior of the relative change in cost as a function of the relative change in rate, σ:

$$\frac{\gamma(\rho) - \gamma(\rho^*)}{\gamma(\rho^*)} \approx \frac{1}{2}\frac{\gamma''(\rho^*)}{\gamma(\rho^*)}\sigma^2\left[\frac{\hat{R}(\rho^*)}{\hat{R}'(\rho^*)}\right]^2.$$

Equation (28) shows that relative changes in cost will vary as the square of relative changes in rate, for sufficiently small relative changes in rate. Consequently, relative cost changes will tend to 0 faster than relative changes in rate, as the latter changes tend to 0. Thus, the current model supports Porter and Gallagher's assumption that funding restrictions do not change a product's total base-year cost, for sufficiently small relative funding restrictions. Cost changes for specific cases, in particular, when funding restrictions are not relatively small, depend on details of the cases.

Let us consider an example. The fifth and sixth columns of TABLE 1 show planned and modified outlay rates for tasks in a program originally planned for 10 years,[1] and values for model parameters in the phases of the individual years' work.

Outlay rates for the task of year 3 are to be reduced from \$134.4 million per year ("Planned outlay rate" column) to \$100.0 million per year ("Revised outlay rate" column), a

TABLE 1 Original and changed program plans

Program Phase	Program year	$\hat{D}$	$\hat{a}$	$\hat{m}$	Planned rate, By \$M/Y	Revised rate, By \$M/Y
Plan	1	0.084	0.046	0.000	36.0	36.0
Plan	2	0.084	0.046	0.000	97.1	97.1
Det. Des.	3	0.333	0.900	0.300	134.4	100.0
Det. Des.	4	0.333	0.900	0.300	152.6	120.0
Det. Des.	5	0.333	0.900	0.300	149.5	120.0
PC/DT	6	1.333	3.125	1.667	127.2	100.0
PC/DT	7	1.333	3.125	1.667	108.2	100.0
PC/DT	8	1.333	3.125	1.667	60.0	60.0
IOT	9	0.500	0.391	0.156	27.5	27.5
IOT	10	0.500	0.391	0.156	8.7	8.7

relative change of −26% ($\sigma = 0.26$). Accomplishing this by reducing staff implies reducing nondimensional staff from the value ρ^* for minimum cost to the value ρ_1, where, by virtue of (21), ρ^* and ρ_1 are related by:

$$\frac{\hat{R}(\rho_1) - \hat{R}(\rho^*)}{\hat{R}(\rho^*)} = -0.26. \tag{29}$$

The value of ρ^* follows from the solution of (21), for values of $\hat{a}$ and $\hat{D}$ for the work of program year 3. With ρ^* and ρ_1 in hand, relative changes in staffing, cost, and time to earn the value planned for year 3 follow from Equations (22), (23), and (24). Columns "Modified Time" and "Modified Cost" in TABLE 2 show the resulting modified times and costs for year 3; the same procedure gives modified times and costs for the remaining years' tasks.

As indicated in TABLE 2, the modified times for accomplishing the work planned for the years affected by funding restrictions are greater than one year. To provide yearly outlays, we generate values of cumulative modified cost, at the cumulative times implied

TABLE 2 Modified times and costs

Pgm Yr	$-\sigma$	ρ^*	$\hat{R}(\rho^*)$	$\gamma(\rho^*)$	$\hat{R}1$	$\rho1$	$\gamma(\rho_1)$	Modified Cost BY \$M	Modified Time Years
1	0.00	0.33	0.83	1.52	0.83	0.33	1.52	36.00	1.00
2	0.00	0.33	0.83	1.52	0.83	0.33	1.52	97.06	1.00
3	−0.26	0.43	1.98	2.95	1.47	0.27	3.12	142.19	1.42
4	−0.21	0.43	1.98	2.95	1.55	0.30	3.06	158.14	1.32
5	−0.20	0.43	1.98	2.95	1.59	0.31	3.04	153.97	1.28
6	−0.21	0.53	6.35	8.11	4.99	0.35	8.57	134.45	1.34
7	−0.08	0.53	6.35	8.11	5.86	0.47	8.15	108.84	1.09
8	0.00	0.53	6.35	8.11	6.35	0.53	8.11	60.04	1.00
9	0.00	0.56	2.42	3.00	2.42	0.56	3.00	27.55	1.00
10	0.00	0.56	2.42	3.00	2.42	0.56	3.00	8.70	1.00

by the modified times of TABLE 2, interpolate the result with a piecewise linear continuous function, and provide yearly outlays as differences of that function. TABLE 3 compares the original and modified outlays, and FIGURE 2 presents the comparison graphically.

As TABLE 3 and FIGURE 2 show, the modified program significantly reduces outlays in program years 3 through 5, while increasing outlays substantially in years 8 through 10, and adding two years (program years 11 and 12) to the effort. Total base year outlays increase modestly, by roughly 3%. As noted above, the present work assesses only those cost and schedule impacts of constrained funding that are endogenous to a project. We believe that a data-based assessment of the cost and schedule impacts of such exogenous effects as inefficiencies due to staff transfers, delays in finding new staff, and training new staff would be a worthwhile study.

Now let us turn to total obligation authority (TOA). The planned outlays of TABLE 2 and TABLE 3 were generated by the 7-year TOA stream [90, 180, 170, 160, 145, 110, 100], with a 4-year spendout pattern chosen for simplicity of [0.4, 0.3, 0.2, 0.1], and a base year of 1995, using Air Force RDT&E deflators. TABLE 4 shows the resulting then-year (TY) outlays, the deflators, and the base-year (BY) outlays.

We wish to find the TOA stream corresponding to the modified outlays of TABLE 3, for comparison with the planned TOA stream. In general, mapping outlays into TOA

TABLE 3 Original and modified outlays, BY $M

36	36
97.05882	97.05882
134.4318	99.99975
152.5684	111.5627
149.4656	120.0002
127.2186	120.0001
108.2446	100.4569
60.0416	99.99998
27.54854	78.25104
8.703846	42.35601
	17.29156
	3.966423

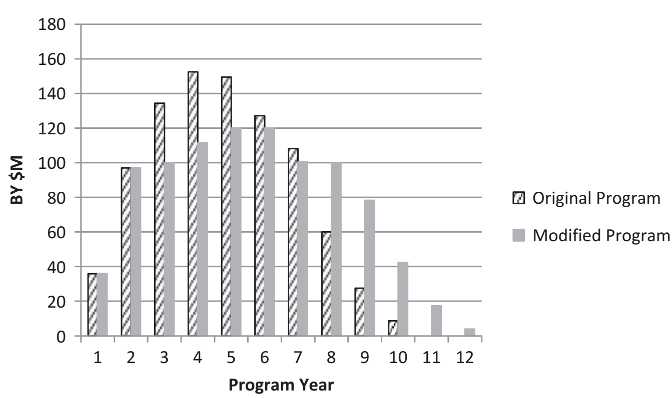

FIGURE 2 Original and modified outlays.

TABLE 4 Then-year outlays, deflators, and base-year outlays

Outlays ($M TY)	Deflators	Outlays ($M BY)
36.0	1.0000	36.00
99.0	1.0200	97.06
140.0	1.0414	134.43
160.0	1.0487	152.57
158.0	1.0571	149.47
136.5	1.0729	127.22
118.0	1.0901	108.24
66.5	1.1076	60.04
31.0	1.1253	27.59
10.0	1.1489	8.70

leads to an overdetermined system of linear algebraic equations. When the spendout pattern covers Y years, mapping from TOA to base-year outlays maps a set of K TOA values into $K + Y - 1$ outlays. Thus, if there are M years of outlays, a set of $M - Y + 1$ TOA values must satisfy a set of M linear algebraic equations, to match the given outlays. Only if the M outlay values happen to lie in the subset of R_M defined by the mapping from R_{M-Y+1} to R_M determined by the spendout pattern will the M equations have a solution. We deal with this difficulty by taking the $M - Y + 1$ values of TOA that minimize the sum of the squares of the errors in the M equations, and compute those TOA values with the Moore-Penrose generalized inverse. Others, for example Porter and Gallagher, also determine TOA from outlays as squared-error minimizing values.

At this point we have $M - Y + 1$ values of TOA, determined by adjustments to the original $K + Y - 1$ outlays, which stemmed from the original K values of TOA. Since reducing outlays will always generate relative values of the time required to earn the value planned for a given year that exceed 1, M will not be less than $K + Y - 1$, so there will be no fewer than K values of TOA for the modified program. Generally, $M - Y + 1$ will exceed K. TABLE 5 compares the original and modified TOA streams, and FIGURE 3 shows the comparison graphically.

The rate reductions of TABLE 1 result in significantly reduced TOA for program Years 3, 4, and 7, a significantly increased TOA for Year 6, and two additional years' TOA. The modified programs' total TOA is 4% larger than that of the original program. This increase

TABLE 5 Original and modified TOA streams

90	89.97
180	179.89
170	80.56
160	119.97
145	141.48
110	135.09
100	72.77
	120.66
	56.04

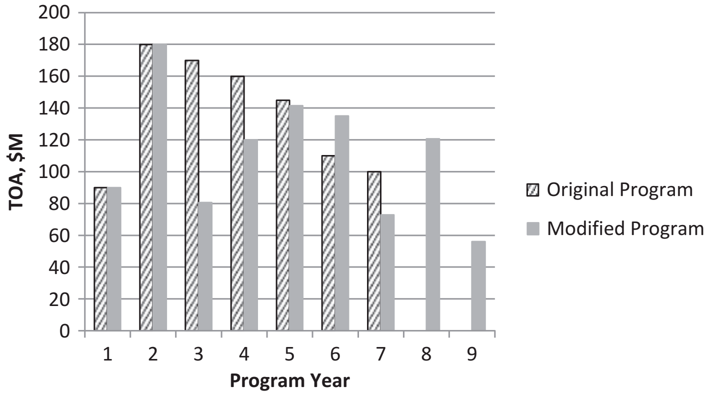

FIGURE 3 Graphical comparison of original and modified TOA streams.

is relatively greater than that of the corresponding BY streams, because of inflation in the added two years. Thus, with the Staffing Efficiency Model (SEM) and mappings both from and to TOA, we have been able to calculate the effects on an Major Defense Acquisition Program (MDAP) program's costs and schedule of a sequence of outlay reductions beginning in the third year of a program originally planned for 10 years. A key result of this work is the indication that the relative increase in both total BY outlays and total TOA can be modest, even when substantial year-over-year changes occur in both streams, when only endogenous effects are modeled. We believe that this result indicates that the substantial increases sometimes mentioned may be due to exogenous causes.

Comparing the Porter and Gallagher (P-G) method with the SEM is not straightforward, because a key input to the P-G method, the length of the revised program, is an output of the SEM. Also, the P-G method treats a single year of constrained funding, while the SEM can deal with adjusted funding in many years. To put the two methods on a nearly level field, we applied both to data for a certain case in which the original program was given by the outlays implied by the TOA of the 2002 Selected Acquisition Reports (SAR) budget. The single year's constraint was given by the 2003 outlays implied for Fiscal Year (FY) 2003 by the TOA budget of the 2006 SAR. We obtained the Norton-Rayleigh-Weibull (NRW) forecast by implementing the eight-step procedure given in Porter and Gallagher (2004, pp. 70–71).

SEM inputs for years following FY 2003 were the outlays of the original program, that is, no constraints for the work to be done year-by-year after FY 2003 were imposed. The program length input to the P-G program was to keep the revised program length equal to the original program length of 15 years. That turned out also to be essentially the length of the SEM revised program; the SEM revised program had a 16th year, but its amount, one ten-millionth of the total program outlays, was negligible and we neglected it. FIGURE 4 compares the P-G model and SEM outlays to those implied by the 2009 SAR budget. FIGURE 4 shows that SEM and P-G model outlays are qualitatively quite similar for this near level-field comparison. The SEM offers greater flexibility than the P-G model, and is not constrained by the NWR profile.

Application to Assessing Realism of Cost and Schedule Plans

Planned costs and schedules for phases of a development program may be converted to the model's non-dimensional costs and times, and the result compared with results for other

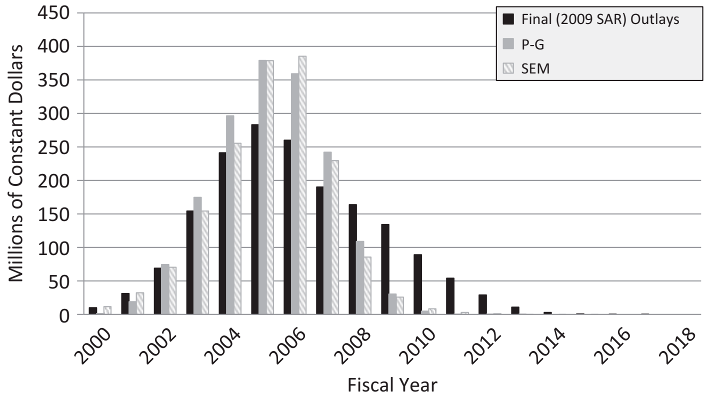

FIGURE 4 Comparison of SEM and NRW forecasts.

programs to determine if the plan implies operation on favorable parts of an appropriate cost-time curve. A program that proposes to operate for costs and schedules outside the arc between the minimum-cost and minimum-time points (arc AB of FIGURE 1) may be deemed unrealistic. In fact, operation near the minimum-time point may be unrealistic, because significant cost increases lead to very little decrease in time in that region.

With detailed information about a set of development programs—that is, given values of a, D, m, p, q, and r as well as values of C, V, and T—one could evaluate the non-dimensional parameters of Equations (8), (9), and (10), and see how the programs' non-dimensional costs and times fell in the (γ, τ) plane. If those data collapsed the observations near some common curve, planned programs' costs and schedules could be assessed with a diagram like that of FIGURE 5.

Data for that approach are potentially available from contractors' work plans and/or should-cost studies. Lacking that information, we made an initial evaluation of the model's potential for assessing reasonableness of planned programs' costs and schedules in this way: By virtue of Equations (4) and (5), the quotient γ/τ may be related to the quotient C/T with a single constant:

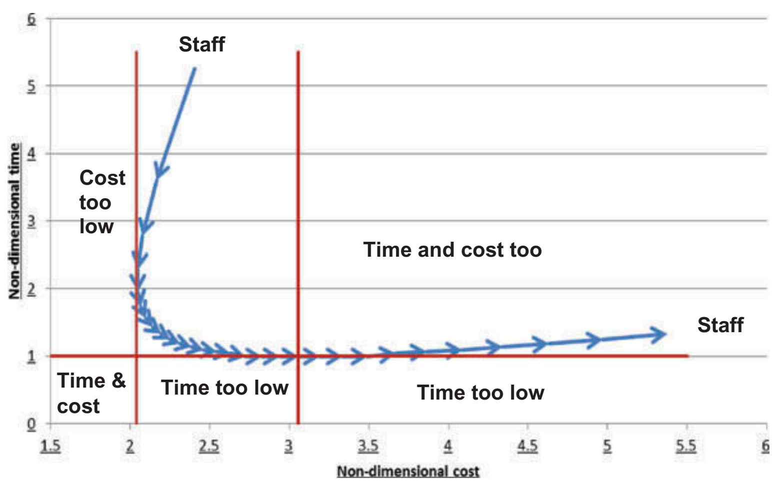

FIGURE 5 Example of SEM test.

$$\frac{\gamma}{\tau} = \frac{q/pV}{q^2/4rV}\frac{C}{T} = \frac{1}{C_1}\frac{C}{T}, \tag{30}$$

where

$$C_1 \equiv \frac{1}{4}\frac{pq}{r}. \tag{31}$$

To estimate the parameter C_1, we note that:

$$C_1 \equiv \frac{1}{4}\frac{pq}{r} = \frac{1}{4}\frac{pNT}{T}\frac{qN}{rN^2} = \frac{1}{4}\frac{wC}{T}\frac{1+z}{z} = \frac{1}{4}w\frac{1+z}{z}\frac{C}{T}. \tag{32}$$

We evaluated C_1 in terms of a reference value of C/T, as:

$$C_1 = \frac{1}{4}w\frac{1+z}{z}\left(\frac{C}{T}\right)_{ref}. \tag{33}$$

We believe it appropriate to use a value near the peak value of C/T for $(C/T)_{ref}$, so that C_1 reflects the capacity of the facility. We chose to express $\left(\frac{C}{T}\right)_{\text{ref}}$ in terms of the mean expenditure rate, since managers will generally know a project's planned total cost and schedule. To evaluate peak expenditure rate from values of mean expenditure rate, we observed that for any Rayleigh distribution, the maximum outlay rate is equal to a constant multiple of the mean outlay rate; to five significant figures, the multiplier's value is 1.6559. The mean outlay rate for a program under examination is known, either exactly in the case of a plan, or by estimation from Rayleigh analysis in the case of an observed, partially executed program.

FIGURE 6 shows that ACWP data for 17 programs, dating from the 1970s through 2011, and comprising missile, aircraft, and electronics systems, exhibit a proportional relation between peak rate and mean rate. We chose the programs without reference to their relations of peak and mean rate, requiring only that the data indicated monomodal variation of ACWP with time, covered a completed program, and did not indicate a level-of-effort

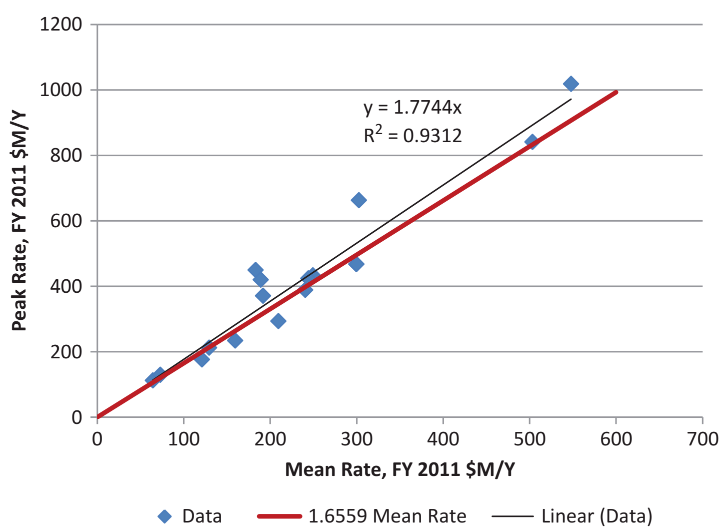

FIGURE 6 Relation between mean and peak outlay rates.

program. We used a proportional model, considering a program with non-zero peak rate and zero mean rate unrealistic.

The t statistic for the slope of the regression line in FIGURE 6 is 30.91, and the Cook distances (Cook, 1977) of the date in FIGURE 6 are all less than 0.35. We take these facts, with the R^2 of 0.93, as reasonable justification for using a proportional relation of peak rate to mean rate. In the following, we used the Rayleigh proportionality constant rather than the regression value of FIGURE 6, wishing to tie the constant to a general model that has been found to be widely applicable, rather than to a result from a specific data set.

In view of FIGURE 6, we evaluated $C1$ with Equation (32), choosing:

$$\left(\frac{C}{T}\right)_{ref} = 1.6559\frac{\text{Total outlay, \$}BY}{\text{Program length, years}}. \tag{34}$$

With these considerations, we were able to evaluate non-dimensional ratios γ/τ from observations of C/T. Then, using Equations (13), (14), and (15) as a guide to evaluating parameters $\hat{a}$, $\hat{D}$, and $\hat{m}$, we found values of ρ with:

$$\frac{\gamma}{\tau} = 2\rho + \hat{D} + (\hat{a\rho} + \hat{m})(2\rho - \rho^2), \tag{35}$$

which follows from dividing the right side of Equation (5) by the right side of Equation (4). As explained below, we adjusted the choice of Equation (34) for the version of the SEM test used in applications.

In view of our limited information on the programs analyzed, we considered only three phases: a "growing" phase, characterized by relatively large coordination costs and little cost of purchased material; a "peak" phase, exhibiting lower coordination costs than the growing phase and significant cost of purchased material; and a "declining" phase, in which coordination costs remained low and costs of purchased equipment were not significant. TABLE 6 shows values of the non-dimensional parameters we assumed for these three phases, with associated values of the ratios *w, d, s, y*, and *z*.

Programs that follow the typical development program's Norden-Rayleigh-Weibull outlay pattern may readily be divided into growing, peak, and declining phases. FIGURE 7 shows division of a program's actual cost of work performed (ACWP) data into the three phases.

FIGURE 7 also illustrates our smoothing of the C/T data. Inevitably, program data have perturbations which, followed exactly, obscure the overall trends that we wish to analyze. Our smoothing was done with polynomial splines. For each phase of each program, we adjusted the number of knots and order of the smoothing spline to capture what, in our opinion, were the data's essential features.

In the leftmost chart of FIGURE 8, the analyst believes that only the overall linear trend of the data is meaningful. In the center chart, she or he believes that the data exhibit a meaningful change in slope, between time intervals (0.5, 1.5) and (1.5, 2.5). The rightmost

TABLE 6 Values of non-dimensional parameters

	w	d	s	y	z	$\hat{a}$	$\hat{D}$	$\hat{m}$
Growing	0.750	0.2	0.050	0.00	0.25	0.24	0.089	0.00
Peak	0.425	0.5	0.025	0.05	0.11	0.36	0.21	0.13
Declining	0.475	0.5	0.025	0.00	0.11	0.32	0.19	0.00

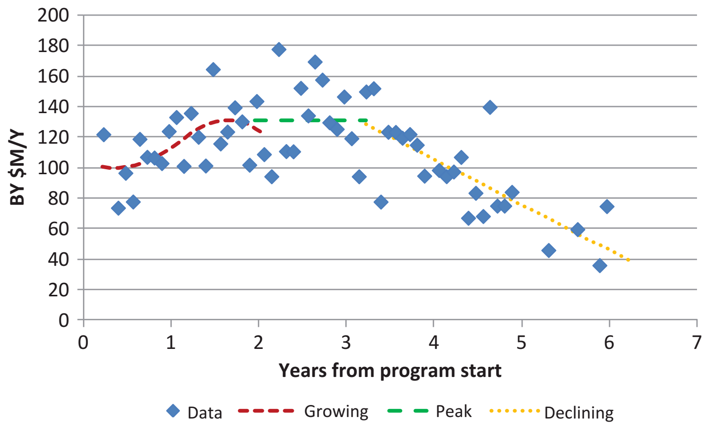

FIGURE 7 Division of ACWP data into phases.

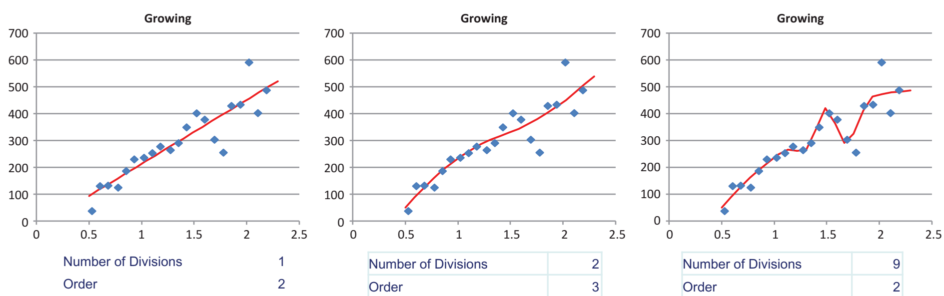

FIGURE 8 Examples of smoothing.

chart of FIGURE 8 shows appropriate smoothing when, in the analyst's judgment, the upward and downward excursions between times 1.5 and 1.8 are meaningful.

When data for a complete development program are available, applying the SEM test for realism of planned costs and schedules will generate three plots in the (γ, τ) plane, showing how a program's costs and schedules appear in the growing, peak, and declining phases. When these plots all exhibit points not far from the curves' "noses"—that is, not far from the minimum-cost point—the test as presently developed indicates a reasonable program.

Points on the growing phase plot from early in program development and points on the declining phase from late in the program development may generate large non-dimensional times and costs. Generally associated with very small increments in ACWP, those points represent events that are quite likely not significant for analysis of a whole program, and may represent staffing levels so small that the SEM does not apply. We remove such points from the plots in this article where it makes discussion of the results easier, but they are never removed from analyses.

FIGURE 9 illustrates an example of such output (generated for a 10-year program expending $1 billion FY2011) that exactly follows the Rayleigh distribution. As the program ramps up, its plot points move down into the minimum-cost area. In the peak phase, the plot points are on the segment (A, B), near the minimum cost point. The declining points start near the minimum-cost point.

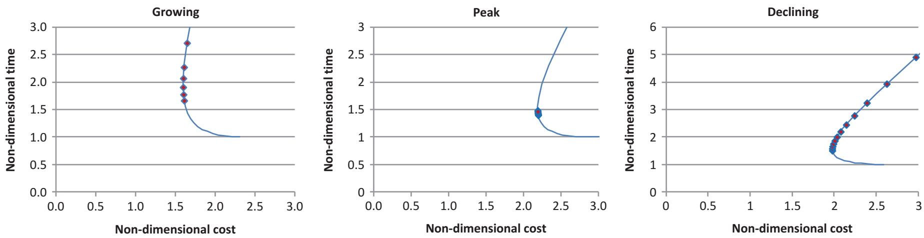

FIGURE 9 SEM test on well-behaved program.

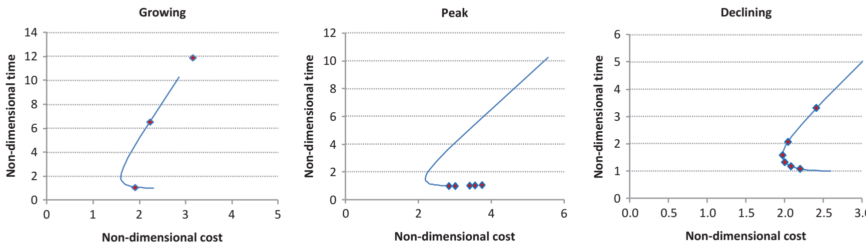

FIGURE 10 SEM test on front-loaded program.

For contrast, FIGURE 10 illustrates an example of a 10-year program that also expends \$1 billion FY2011, but with outlays narrowly concentrated in the first 4 years.

As this program ramps up, its plot points move well into the region of inefficient trades between cost and time. The plot point for the peak phase remains in that region, as do several of the points of the declining phase. These results and the following ones were obtained by multiplying the value of $(C/T)_{ref}$ from Equation (34) by 0.6 for the peak and declining phases. We made this ad hoc modification because we wished the new test to show that profiles following Rayleigh distributions were realistic. In addition to the two theoretical programs discussed above, we analyzed data for two real programs.

As one might expect for programs that actually execute, there are no marked departures from points of efficient operation. As Figure 11 shows, both programs operate near the minimum-cost point during the growing, peak, and declining phases, but Program B presses further into the region of inefficient cost-time trades than Program A does, during the peak and declining phases. The expanded scales of FIGURE 12 show the discrepancy between Program A and Program B in the peak phase more clearly.

In testing the realism of planned outlay profiles, the SEM has the advantage of modeling actual constraints on program execution, such as those imposed by allocating work and collating the results, and by coordination among workers or worker-teams (the "mythical man-month"). A plan can conform perfectly to a Rayleigh or Weibull profile, yet be unexecutable in the intended facility.

Consider, for example, a program planned for constant-dollar outlays of \$5B over a period of 11 years, with exactly a Rayleigh profile. A modified program, intended to execute in the same facility but either including more content (greater constant-dollar outlays) or executing in less time would pass the NWR test of realism, so long as its constant-dollar outlays conformed to the Rayleigh pattern. In contrast, the SEM would show that either of those changes, if great enough, forces operation into inefficient regions of the (γ, τ) plane. FIGURE 13 shows how the SEM operating points would vary when a program planned to

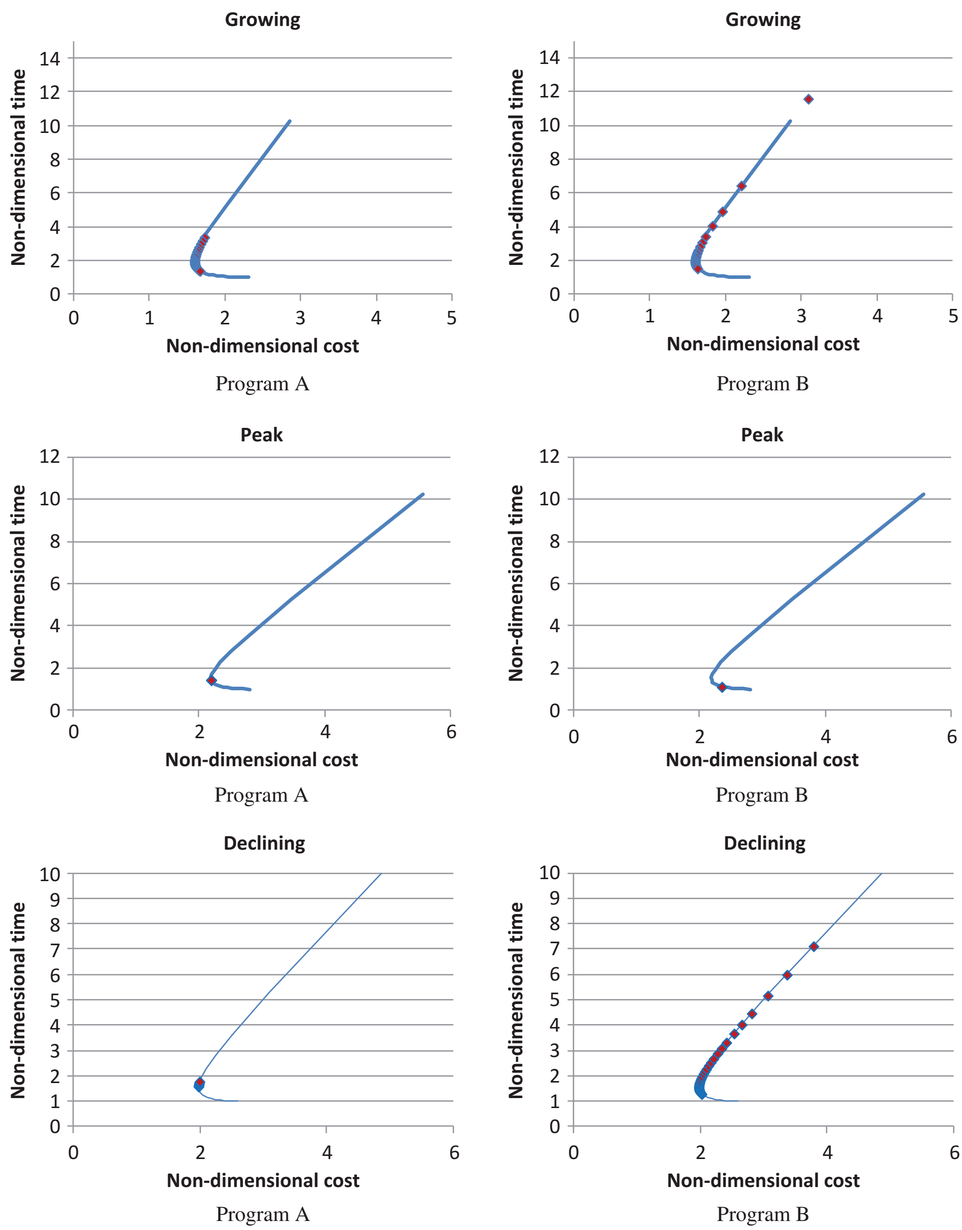

FIGURE 11 SEM results for real programs.

expend \$5B in constant dollars is modified to expend \$6B and to expend \$7.5B. The operating points shift steadily into the less-efficient parts of the (γ, τ) plane, raising concern about executability.

Moreover, considering the variation of non-dimensional time, and the fact that, by Equation (4), for operations with fixed labor productivity and rate of coordination burden calendar time is proportional to value, one sees that calendar times corresponding to the operating points raise even greater concerns about executability: for a value of \$6B, the

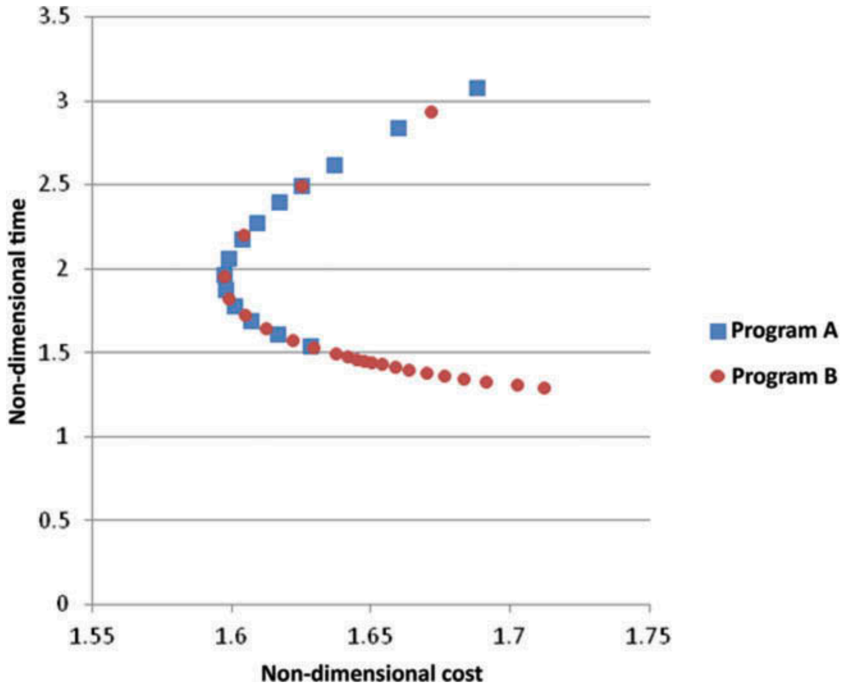

FIGURE 12 Peak phase of Program A and Program B, expanded scales.

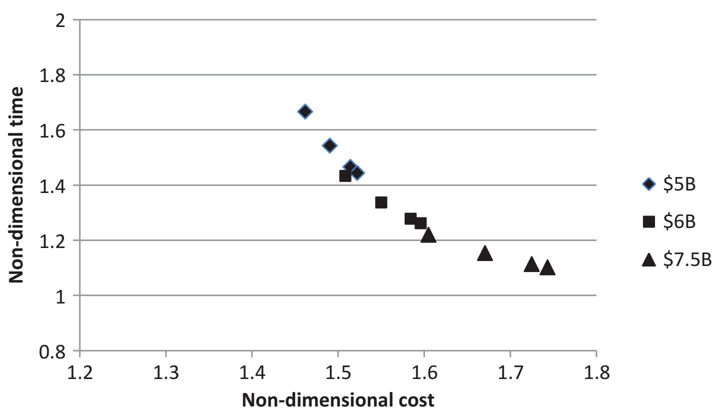

FIGURE 13 Variation of operating points with total cost.

largest executing time exceeds one year by only a few percent, but for a value of $7.5B that excess approaches 15%. The former values might be achieved, but the latter well might not.

Summary

We present a parsimonious model of the cost and completion time (schedule) of a development program, whose key feature is the explicit distinction between a program's value (what it delivers) and its cost (the money paid to acquire the value), and explore the model's application to analyzing impacts of reducing funding for an executing program, and to assess the realism of cost-schedule plans for proposed programs. The application to impacts of reduced funding shows that relative changes in total base-year outlays vary as the square of relative changes in funding rate, for small changes in funding rate. This provides analytic support for the assumption that base-year costs do not increase when a program's funding is varied moderately, while its content is unchanged. The result also suggests that causes for any substantial cost increases associated with moderate funding reductions are exogenous to the model; we believe that exploring those causes is an opportunity for further research. We developed a strategy for applying the model to assessing the realism of cost-schedule plans even with the limited data available to us, and found indications that in this context the model does offer an ability to distinguish between plans characteristic of well-managed and not well-managed programs.

Acknowledgments

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Note

1. To keep the example simple, we assume that each year's outlays go to a single task, and that the task for each year must be completed before work can begin on the following year's task. Modifications of the example's procedure can enable relaxing both assumptions.

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Appendix: Value of Non-Dimensional Staffing for Minimum Non-Dimensional Cost

The non-dimensional cost γ corresponding to the non-dimensional staffing ρ is given by (11). The value of ρ for minimum γ will make the derivative of the right side of (11) equal to zero. Differentiating the right side of (11) term-by-term and collecting the result as a fraction with denominator $(2\rho - \rho^2)^2$ gives the numerator $F(\rho)$, where

$$F(\rho) = \rho^4 - 4\rho^3 + (4 + \frac{2}{\hat{a}})\,\rho^2 + \frac{4\hat{D}}{\hat{a}}\rho - \frac{4\hat{D}}{\hat{a}}. \tag{A1}$$

Thus, the value of ρ for minimum γ will be a zero of $F(\rho)$. Since $F(0) = -\frac{4\hat{D}}{\hat{a}} < 0$ and $F(1) = 1 + \frac{2}{\hat{a}} > 0$, F has at least one zero in (0, 1). We wish to show that there is only one zero in that interval. Consider $G(\rho) \equiv F'(\rho)$. Differentiating F shows that

$$G(\rho) = 4\rho^3 - 12\,\rho^2 + 2\,(4 + \frac{2}{\hat{a}})\,\rho + \frac{4\hat{D}}{\hat{a}}. \tag{A2}$$

Since $G(0) = \frac{4\hat{D}}{\hat{a}} > 0$ and $G(1) = \frac{4+4\hat{D}}{\hat{a}} > 0$, G is positive at both ends of (0, 1). If G is positive throughout (0, 1), F is monotone increasing on that interval, and thus can have only one zero there. We will show that this is the case.

Differentiating G shows that

$$\begin{aligned} G'(\rho) &= 12\rho^2 - 24\rho + 2(4 + \frac{2}{\hat{a}}) = 12\left[\rho^2 - 2\rho + 1 - \frac{1}{3}(1 - \frac{1}{\hat{a}})\right] \\ &= 12\left[(\rho - 1)^2 - \frac{1}{3}(1 - \frac{1}{\hat{a}})\right]. \end{aligned} \tag{A3}$$

Please note that $G'(0) = 2(4 + \frac{2}{\hat{a}}) > 0$. If G' does not change sign in (0, 1), G is monotone increasing from a positive value in that interval, so G remains positive on (0, 1) and our result is established. The zeroes of $G'(\rho)$ are at:

$$\rho = 1 \pm \frac{1}{\sqrt{3}}\sqrt{1 - \frac{1}{\hat{a}}}. \tag{A4}$$

Three cases arise, depending on the value of $\hat{a}$: Case A: $\hat{a} < 1$; Case B: $\hat{a} = 1$; and Case C: $\hat{a} > 1$. In Case A, $G'(\rho)$ has no real zeroes, so does not change sign, and our result is established. In Case B, $G'(\rho)$ does not change sign in the open interval (0, 1), and, as in Case A, the result is established. In Case C, $G'(\rho)$ changes sign precisely once in (0, 1). Since $G'(0) > 0$, the sign change is from positive to negative; and G has a maximum in (0, 1), but no other stationary point in that interval. But G itself cannot change sign in (0, 1), because both G(1) and G(0) are positive, and were G to have any negative values in (0, 1), there would have to be a minimum in that interval (G's graph would have to "turn around" from negative values to reach positive values). Consequently, G is positive on (0, 1), and there is only one zero of F in (0, 1).