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Trade Space, Product Optimization, and Parametric Analysis

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This article shows how to bound, build, and assemble trade spaces for product optimization. The advent of computerized tools that describe available trade spaces has changed not only the nature of optimized product design, but that of parametric cost studies as well. Because these tools allow broader analysis, engineers can produce many more potential designs. Those tools, explained in this article, allow parametricians to analyze trade spaces in a manner that allows them to determine the sets of product attributes that have the best chance for market success. However, rather than trailing such engineering studies, parametricians may be able to lead them. In the process, parametricians may be able to move their organizations toward more economically viable configurations, those that markets will more readily accept.

Introduction

September 16, 1893 was a very big day for a competitive throng in Kansas. At noon, the cannon went off. The roughly 100,000 people, by train and by wagon, on horseback, bicycle, foot, and any other means possible, raced off in search of one of 42,000 parcels that they could claim as their own in a place called the Cherokee Outlet in northern Oklahoma. This was the fourth Oklahoma land rush. Some people, known as "Sooners," jumped the gun up two days earlier and took up some fine parcels before the official start of the race. Those waiting for the gun to go off, referred to as "Boomers," had to wend their way through partially charted territory and find the best plot of land that remained (Doughty, 1998). As shown in Figure 1, the area was rather large and not entirely accessible by modern means (Dickinson Research Center, 1997).

An enlargement of the region as Figure 2 reveals what could have faced that crowd on that hot day in the nineteenth century. "Sooners" jumped the gun and many occupied some prime parcels, notionally colored as red in Figure 2. "Boomers," meanwhile, though faced with a reduced inventory from which to draw, still had some prime pieces left, colored in green in the same figure. Ideally, there must be some place like that marked with the star, the optimal site in view of all other options. In 1893, it was hard to assess such a place, but certainly easy and consistent access to water was necessary, while at the same time a portion of the ground in that parcel should be sufficiently high to stay out of the water should the river rise. If the Boomer were a farmer, and most were, proximity to a town or river port to export goods to market would add to the parcel value as well.

Whether the Boomer had known it or not, he or she had made a value assessment of the property using a series of variables (access to water, elevation, and proximity to markets).

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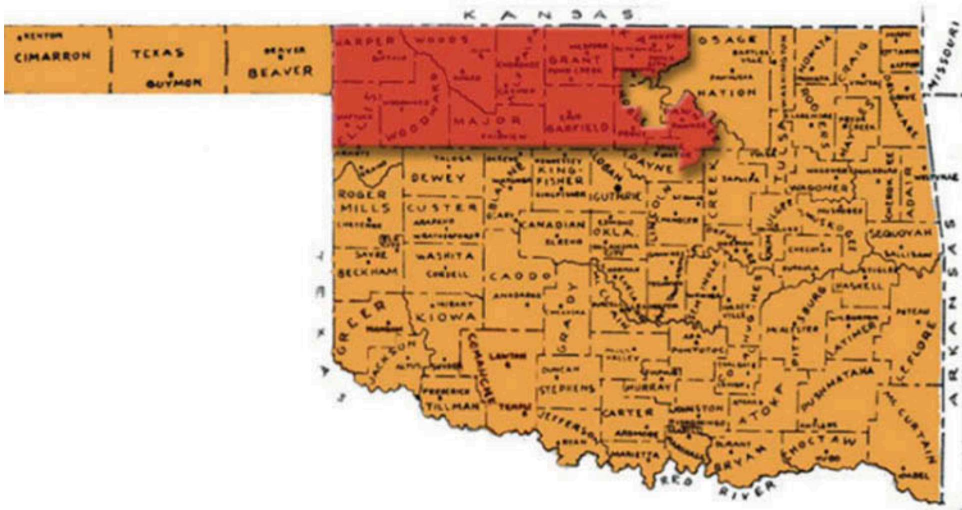

FIGURE 1 The Cherokee Outlet in 1893 (dark area), site of the Fourth Oklahoma Land Rush.

FIGURE 2 Hypothetical map of Cherokee Outlet showing occupied areas (dark areas at left), optimal open spaces in the region (shaded area at right), and theoretical best place left open (star).

This is a formulation of a Value Estimating Relationship, or VER. They used VERs to evaluate the goodness of one parcel over another.

There was a great deal at stake in the properties that they chose. Many settlers, and indeed the federal government, attracted to the area during an unusually wet period, thought, “The rain follows the plow” (National Drought Mitigation Center, 2006).

We now know this to be inaccurate. This semi-arid region typically received less than 20 inches of rain per year. A variety of outdated farming practices from 1893 to 1930 led to massive erosion. When the abnormally wet phase ended in 1930, the combination of less rain and poor attention to the needs of the soil hit a critical point, and the dust storms began. Areas of sustained high winds and less rainfall felt the impact the worst, as shown in Figures 3 and 4. The more eastern part of the Cherokee Outlet did not feel these consequences as severely due, in large measure, to more rainfall and rivers and less winds, the favored attributes that settlers in that region selected. In the end, it was clear that the eastern parcels were more valuable.

In modern times, in any endeavor, contractors have a broad range of options when it comes to the selection of attributes that they choose for their next product offerings.

FIGURE 3 Change came with the wind in the 1930s Dust Bowl.

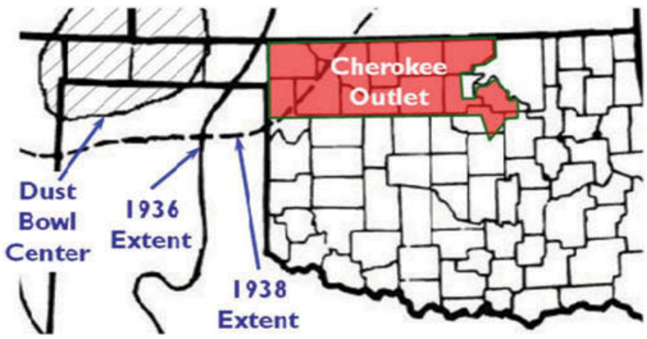

FIGURE 4 Some parts of the 4th Land Rush did better than others.

In many companies, someone gets an idea and starts it in motion, only calling in the parametrician to estimate the cost at the last minute. Engineers assure optimal design with respect to the structure. However, they may ignore optimal design with respect to market needs.

Manufacturers must satisfy the customers' evaluation of value as defined by their Value Estimating Relationship. They need to discover the best position in which to place their new products, much as the Boomers wanted to find the best possible plot of land.

The Oklahoma Land Rush of 1893 was important in other respects as it applies to business analysis. Clear lines divided the land that was up for grabs from that which was not. Laying claim to a piece of Kansas or running all the way to Texas were not options. Similarly, there are limits to what the market may absorb. Staking a business case to a plan that calls for more products than the market demonstrates it absorbs is not impossible, but very likely may be infeasible if there is not a sufficient reason for the market to demand more.

It is important to understand how cost, value, and demand respond with respect to specific influences. If costs go up faster than the value of for a vehicle approaching Mach 1, it will not make sense to go faster, since net profit falls. If the value of adding cabin height exceeds the cost to providing it, it may make sense to make a taller cabin. If the market does not support more than 500 vehicles at an $11 million price point over a decade, proposing to sell 1,000 over the same period may not make sense.

This article addresses these questions of value, cost, and demand simultaneously, using the business aircraft market in 2002 as an example. It was then broad in scope and still growing.

At the bottom of that market, a turboprop carried four passengers and cruised at 164 miles per hour that sold for less than $1 million. Meanwhile, at the top of the market, a converted airliner that carried 26 passengers and cruised at 542 miles per hour sold for nearly $55 million. In between there were over 40 vehicle models. Clearly, in between the least and most expensive planes there was a great deal of space into which the other models fit.

It seems obvious that more speed ought to fetch more money than less speed, and that the same should hold true for passenger capacity. What might not be as obvious is that the market responds in predictable ways with respect to changes in cruise speed and capacity. Indeed, as this article shows, the market reacts predictably with respect to several attributes at the same time. This knowledge, reduced to predictive equations for value when combined with like equations for cost, allows parametricians to lead trade studies by revealing the open regions in the market and describing the attributes to fill those spaces and to maximize profit.

Demand

In order for a parametrician to analyze a trade space completely, he or she must be able to describe the demand in the market. For this article, we will examine the market for Business Aircraft from 2002 to 2011 (Forecast International, 2001).

As shown in Figure 5, there were 46 business aircraft models that had the predicted sales figures in terms of quantity and price. Below $10 million, predicted sales were for 6,153 vehicles at an average price of $5.01 million (this is the lowest and rightmost circular dot on the chart). Between $10 million and $20 million, Forecast International's prediction was for 2,064 units at an average price of $15.6 million (the next highest circular dot). Continuing on, the next bin, with a lower bound of $20 million and an upper limit of $35 million, had 1,335 units at an average price of $27.4 million (the circular dot just below the highest circular dot). Finally, in the highest bin, those planes above $35 million,

FIGURE 5 Business aircraft Aggregate Demand.

the projection was for 1,115 units at an average price of $42.2 million (the highest circular dot). Running a regression through the circular dots yields an Aggregate Demand curve described by Equation (1):

$$\bar{Pr} = 163{,}000 * \text{Quantity}^{-1.2}, \tag{1}$$

where:

$\bar{Pr}$ = Median aircraft clearing price in 2002$;
Quantity = Projected sales from 1-1-2002 to 12-31-2011.

While the Aggregate Demand curve shows the total sold by price group, another useful concept with respect to quantity limitations in the market is that of the Demand Frontier, as revealed in Figure 6. Note that the quantity term in Aggregate Demand curve was several times the largest value for any individual aircraft model in each bin. Because of this phenomenon, the Aggregate Demand curve slope provides insight into price responsiveness in the market, but because no one manufacturer can attain these aggregate figures, the quantity term loses meaning. The Demand Frontier solves this problem by either (1) running the Aggregate Demand curve slope through the rightmost point in the demand array, or (2) running a curve through the two rightmost points in the demand array. In this case, the second option offers a Demand Frontier curve described by Equation (2):

$$\text{DFP} = 123{,}000{,}000 * \text{Quantity}^{-2.61}, \tag{2}$$

where:

DFP = Demand frontier price.

Figure 7 identifies three price gaps, regions in the market in which there are no competitors with respect to price. Since the market has indicated its willingness to support vehicles above and below these price thresholds, it is reasonable to assume that with the proper mix of attributes, new vehicles could be successfully added to the market. The next question before us is this: what is the best mix of features for new aircraft?

FIGURE 6 Aggregate Demand and Demand Frontier.

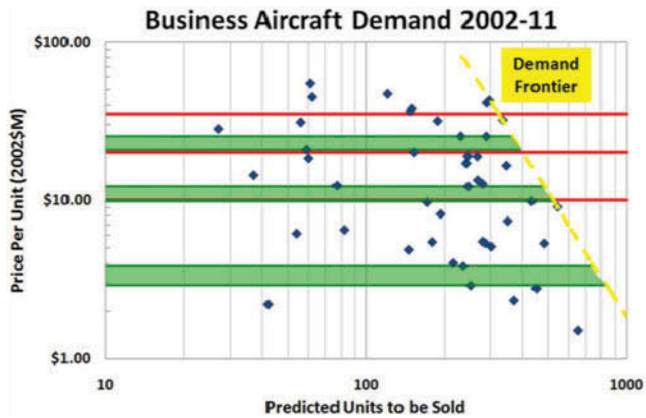

FIGURE 7 Significant market gaps in the shaded areas.

Value

Understanding the importance of how the market reacts to the products put before is central to understanding value analysis. A hypothesis about how the market so reacts, known as the Value Theory of Price Determination, is mathematically described as Equation (3):

$$V_m = A_1 * A_2 * \ldots A_i * e_j, \tag{3}$$

where:

V_m = Market value of aircraft (here, in 2002\$M);
A_i = Contribution of ith attribute;
e_j = Error term of the equation.

In other words, Equation (3) hypothesizes that product value in the business aircraft market is a combination of all factors upon which the market collectively deems important as they vote with the money that they provide to the market. Products priced higher than this collective wisdom will experience little or no sales and producers making such products will find prices for them unsupportable, and will have to lower their sales prices. Products priced too low will experience brisk sales, perhaps so much, so that the manufacturer will not be able to keep pace, and the manufacturer will therefore have to raise prices to avoid shortages. In this formulation there may be several elements contributing to the overall value of a product. While some attributes are more important than others are, ignoring non-primary effects may put the producers in financial peril. That is, according to the hypothesis.

What Does the Data Indicate?

If we consider only a single variable, cruise speed in miles per hour (MPH) as in Figure 8, we get a statistically significant equation, but it is not very well correlated (adjusted R^2 = 64.4%) and has a substantial error term (Mean Absolute Deviation, or MAD of 54%), as shown later in Figure 13. Note that Figure 8 does not well explain the higher-priced vehicles and those aircraft typically have more passengers. In view of this, we add a passenger term and come to Figure 9. Figure 9 indicates that the passenger capacity adds to overall vehicle value, and reduces the contribution from cruise speed alone. The correlation for Figure 9 is much improved (adjusted R^2 of 92.6%) and the MAD is better as well, now down to

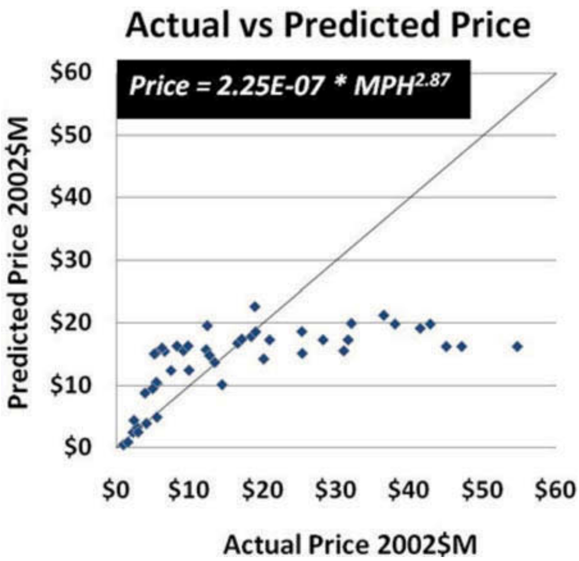

FIGURE 8 Aircraft value as a function of one variable.

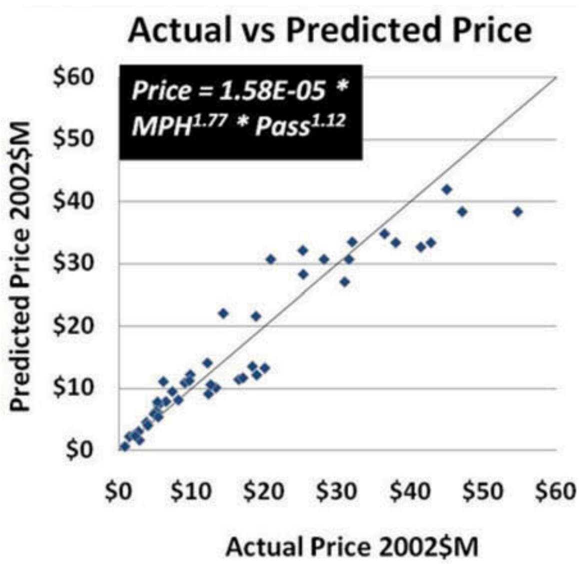

FIGURE 9 Aircraft values as a function of two variables.

22.4%, as shown later in Figure 13. Figure 10 adds the role that cabin height plays in value, which is a variable that accounts for passenger comfort. The cabin height variable offers improvement in both the correlation (adjusted R^2 of 96.0%) and the MAD (which drops to 15.2%). In Figure 11, the vehicle range enters into the mix. The farther a vehicle can go without fueling, the more flexible it is. The range variable continues to refine the analysis, as the adjusted R^2 moves to 97.6% and the MAD goes down to 11.3%. With Figure 12, the number of engines adds to the mix of variables and pushes the adjusted R^2 to 98.1% and the MAD to 10.7%. The number of engines is a safety parameter—some planes with one engine fly faster than vehicles with two engines, but when we consider the contribution of

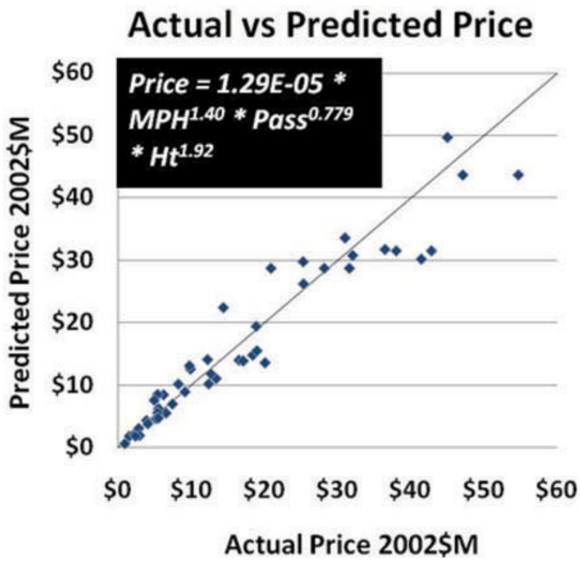

FIGURE 10 Aircraft values as a function of three variables.

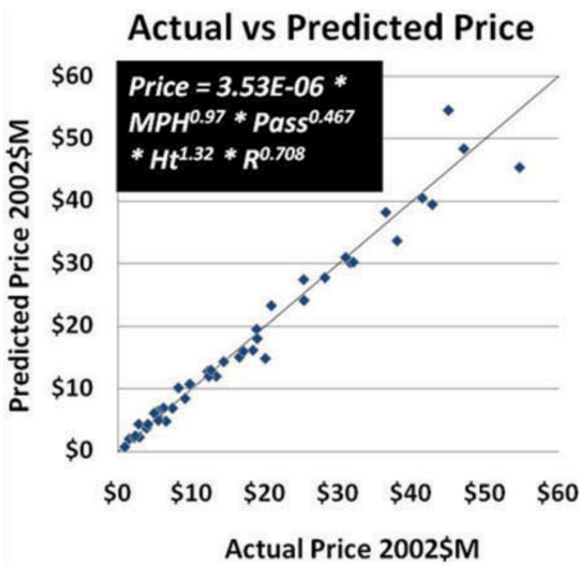

FIGURE 11 Aircraft value as a function of four variables.

the number of engines, the market rewards the added safety of additional engines despite their added cost.

Figure 13 summarizes some of the statistics for Figures 8 through 12 and shows that the usefulness of the equations improved as we added statistically significant terms. Knowing these relationships allows us to determine the value of proposed new configurations and to compare those values to the costs of making those new vehicles.

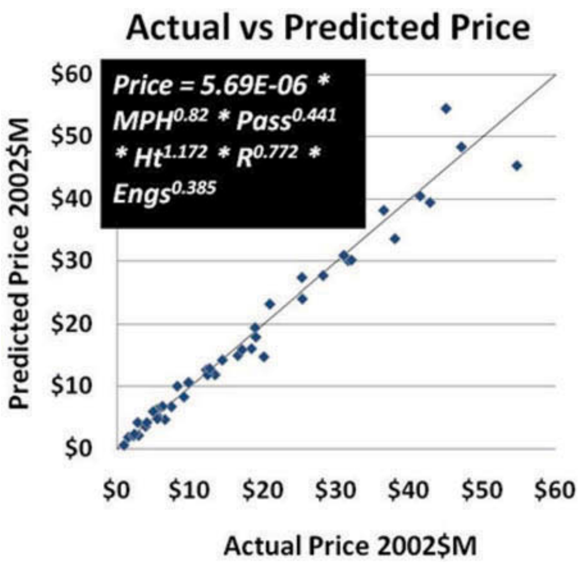

FIGURE 12 Aircraft value as a function of five variables.

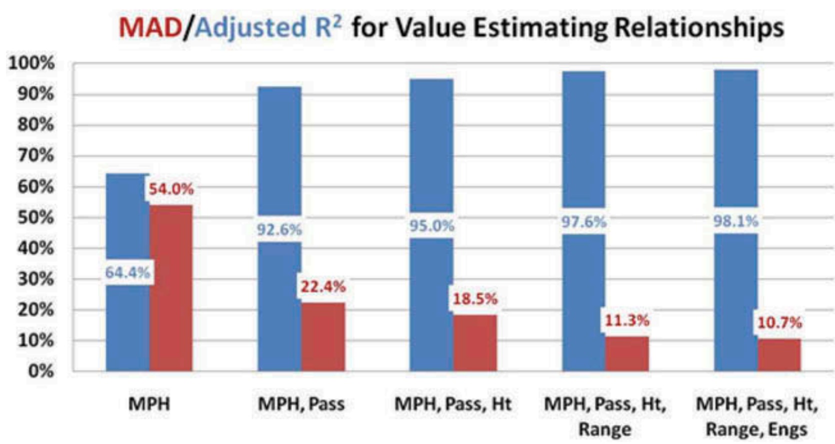

FIGURE 13 Statistics for Figures 8 through 12.

New Product Positioning

In Figure 7, the demand analysis of the business aircraft market revealed at least three significant price gaps. No competitor had a product to offer in these regions. While Figure 7 described market openings with respect to price, Figures 8 through 12 provided equations with progressively more terms and less error that showed how the market reacted to the features offered to it. By the time that we arrived at a value estimating equation with five terms, the error associated with our prediction was down to a Mean Absolute Deviation of just 10.7%. If we use this equation along with maps of the attributes offered to the market simultaneously, we get the views offered in Figures 14 and 15.

In the upper right-hand corner of Figure 14, we find a close-up of the lowest price gap described in Figure 7. The vehicles that form the upper and lower price boundaries of this region, Models 40 and 41, sell for $3.86 million and $2.9 million, respectively, forming a

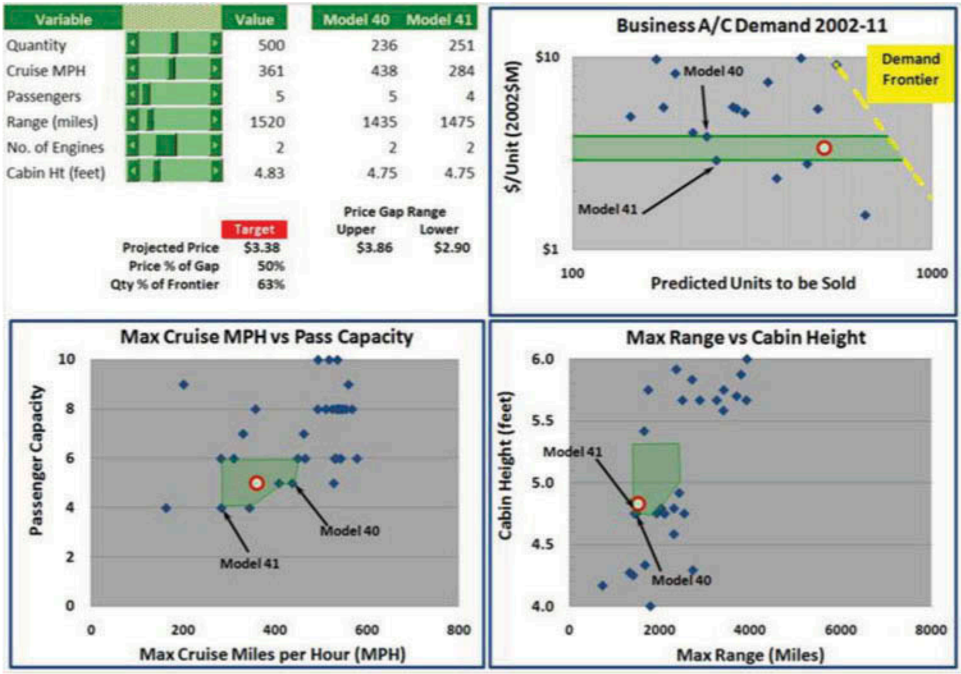

FIGURE 14 The low price gap and attribute maps of nearby competitors.

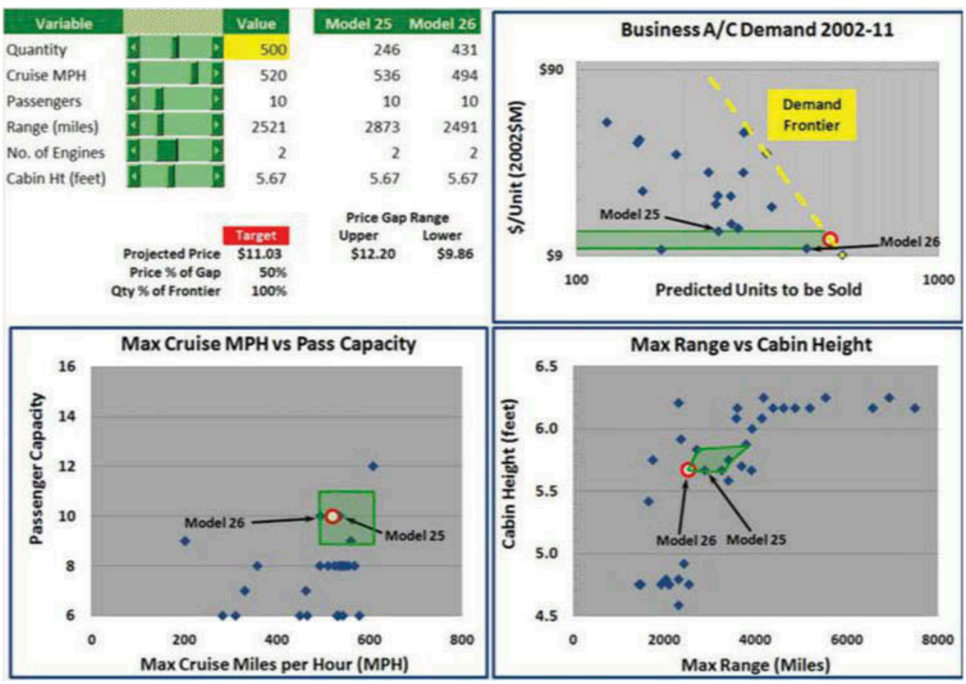

FIGURE 15 The middle price gap and attribute maps of nearby competitors.

price gap of nearly $1 million. In the lower left graph of that figure, the maximum cruise speed and passenger capacity values for Models 40 and 41 bound an attribute region shaded in green in which no competitor offers a vehicle. In the lower right view of the same figure, note that that many manufacturers offer a cabin height of 4.75′, including the makers of Models 40 and 41. In addition, the range for Models 40 and 41 is virtually the same.

A supplier wishing to make a new model directed to this price gap may be inclined to pick a set of attributes listed as "values" in Figure 14. Picking an intermediate target cruise speed of 361 MPH, the difference between Models 40 and 41 splits the speed difference in half.

Selecting the passenger capacity of five matches the more expensive model, and provides more than the less expensive brand. By adding a little more cabin height (4.83′ compared to the 4.75′ offered by the competitors) and range (1,520 miles compared to 1,435 for Model 40 and 1,475 for Model 41), and keeping the number of engines unchanged, the Value Estimating Relationship (VER) in Figure 8 predicts that the value for a vehicle thus configured as $3.38 million. This price is halfway between Model 40 and Model 41. While both Models 40 and 41 had projections for less than 300 units for the period, our hypothetical projection here is for 500 units. While this is many more than the local competition, it does not violate the demand frontier established by the market.

In Figure 15, we move to the middle gap in prices. In the upper right-hand corner of Figure 15, we get a closer look at the local environment. Note the change in scale. Though still logarithmic, this chart now starts at $9.0 million (in order to capture Model 26, priced at $9.86 million) and continues on to Model 25 (selling for $12.2 million) and up to the upper limit of the chart at $100 million. In this instance, the market offers a gap of $2.34 million ($12.2 million–$9.86 million). Just as in the case of Figure 14, we need to understand what the competition offers with respect to the attributes of their individual aircraft models. In the lower left-hand corner of Figure 15, we discover that while both vehicles offer the same passenger capacity (10 passengers each), there is a slight difference between the top cruise speeds of Models 25 and 26. Model 25, the more expensive of the two, can cruise at 536 MPH, while Model 26 tops out at 494 MPH. As an airframe manufacturer considers a new market entrant in this region, that firm might choose to stay with the same passenger capacity, or lower it to 9 passengers or raise it to 11. Additionally, the same types of considerations apply to the selection of the target cruise speeds, but in this particular instance, the manufacturer does not entertain a reduction in speed. In the lower right-hand corner of Figure 15, note that Models 25 and 26 have identical cabin heights but that Model 25 has a range of 2,873 miles, over 15% more than its competitor, Model 26, at 2,491 miles. A new model hoping to compete in the space between Models 25 and 26 might be configured according to the value column in Figure 15. This new model offers more speed than Model 26, and a little bit more range, while keeping the cabin height, number of engines, and number of passengers constant.

The mapping tools in Figures 14 and 15 are only one aid to demarcate possible combinations for a new potential vehicle configuration. In this particular instance for business aircraft, we have identified three large price gaps. Noting that we have a five variable equation that explains value, and that we can vary two of the variables at a time in a three-dimensional model, we can now identify the number of potential studies that need be undertaken, as shown in Figure 16.

Figure 16 takes into account all of the combinations of variable pairs that can vary while the other three variables remain fixed as constants. In the column marked "Engines," cases 1 through 6 address value variations while holding the number of engines fixed at one, while cases 7–12 fix the number of engines at two, and cases 13–18 use three engines in

CASE	Engines	Height	Range	Pass	MPH
1	Fixed-1	Vary	Vary	Fixed	Fixed
2	Fixed-1	Vary	Fixed	Vary	Fixed
3	Fixed-1	Vary	Fixed	Fixed	Vary
4	Fixed-1	Fixed	Vary	Vary	Fixed
5	Fixed-1	Fixed	Vary	Fixed	Vary
6	Fixed-1	Fixed	Fixed	Vary	Vary
7	Fixed-2	Vary	Vary	Fixed	Fixed
8	Fixed-2	Vary	Fixed	Vary	Fixed
9	Fixed-2	Vary	Fixed	Fixed	Vary
10	Fixed-2	Fixed	Vary	Vary	Fixed
11	Fixed-2	Fixed	Vary	Fixed	Vary
12	Fixed-2	Fixed	Fixed	Vary	Vary
13	Fixed-3	Vary	Vary	Fixed	Fixed
14	Fixed-3	Vary	Fixed	Vary	Fixed
15	Fixed-3	Vary	Fixed	Fixed	Vary
16	Fixed-3	Fixed	Vary	Vary	Fixed
17	Fixed-3	Fixed	Vary	Fixed	Vary
18	Fixed-3	Fixed	Fixed	Vary	Vary

GAP	Low $	High $	Gap Size	Gap %
1	$2.90	$3.86	$0.95	46%
2	$9.86	$12.20	$2.34	13%
3	$20.87	$25.27	$4.40	6%

FIGURE 16 Eighteen variable cases over 3 gaps yields 54 studies.

every instance. With this kind of arrangement, we allow every pair of variable combinations to vary while holding the other variables constant.

The number of combinations is a function of both the types of categories from which we take a draw and the number of variables combined. In every case, we have four variables and we combine two variables that we allow to vary. Formula 4 arises from this arrangement:

$$x = n!/(r!(n - r)!), \tag{4}$$

where:

x = Number of combinations;
n = Number of types from which to choose;
r = Number of variables chosen.

For the case at hand,

$$x = 4!/(2!(4-2)!) = 24/4 = 6, \quad (5)$$

since

$4 = n$ (height, range, pass, MPH);
$2 = r$ (two variables combined).

Thus, we have 6 combinations for each of the single, twin, and triple engine combinations offered in the business jet market, which totals to the 18 cases in Figure 12. When we take each case through the identified gaps, we get 54 studies that offer starting points for analysis.

Using Appendix A data, we obtain a predictive equation for vehicle weight as Equation (6):

$$\text{MEW} = 2.70 * \text{Pass}^{0.33} * \text{Cabin Ht}^{1.42} * \text{Range}^{0.68}, \quad (6)$$

where:

MEW = Manufacturer's empty weight;
Pass = Aircraft model passenger capacity;
Cab Ht = Maximum height of the passenger cabin in feet;
Range = Maximum vehicle range in statute miles.

Equation (6) has an adjusted R^2 of 88.6%, a MAD of 18.0%, and its P-values for Passengers, Cabin Height, and Range are 0.01, 0.002, and 0.000014, respectively. Using this equation through the Development and Procurement Cost of Aircraft (DAPCA IV) equations for cost, when we combine those outputs with the respective ones for values, it produces Figures 17 and 18.

In Figure 17's low price gap, we can see a predicted loss position for the program if we look at the case where we have built only 100 aircraft. Note that the upper red surface representing the average cost for those 100 aircraft is far above the predicted value surface for vehicles with varying passenger capacity and top cruise speed with the other three attributes set as in Figure 14 (two engines, range of 1,520 miles and a cabin height of 4.83′). If we increase the number of vehicles built to 500, the model predicts a small profit (predicted value of \$3.38 million—predicted cost of \$3.30 million = per unit profit of \$0.08 million).

Contrast the low gap condition with that of the middle gap portrayed in Figure 18. In that figure, a small profit appears for the 100 aircraft condition (predicted value of \$11.03 million—predicted cost of \$10.71 million) with the Figure 15 attributes (two engines, range of 2,521 miles and cabin height set to 5.67′). A much larger profit appears for the 500 aircraft condition (with costs of \$5.62 million per unit).

However, as shown in the four-dimensional model in Figure 19 (a model in which all four dimensions, quantity sold, cost or price in dollars, passenger capacity, and cruise speed in MPH, are positive), the 500 units predicted for sale pushes the Demand Frontier. Though this condition predicts a better per unit profit than the lower gap, the predicted sales figure should be re-examined.

Indeed, all of the open spaces demand thorough examination. Only by comparing all of the options will producers be able to determine their best option for a new offering to the market.

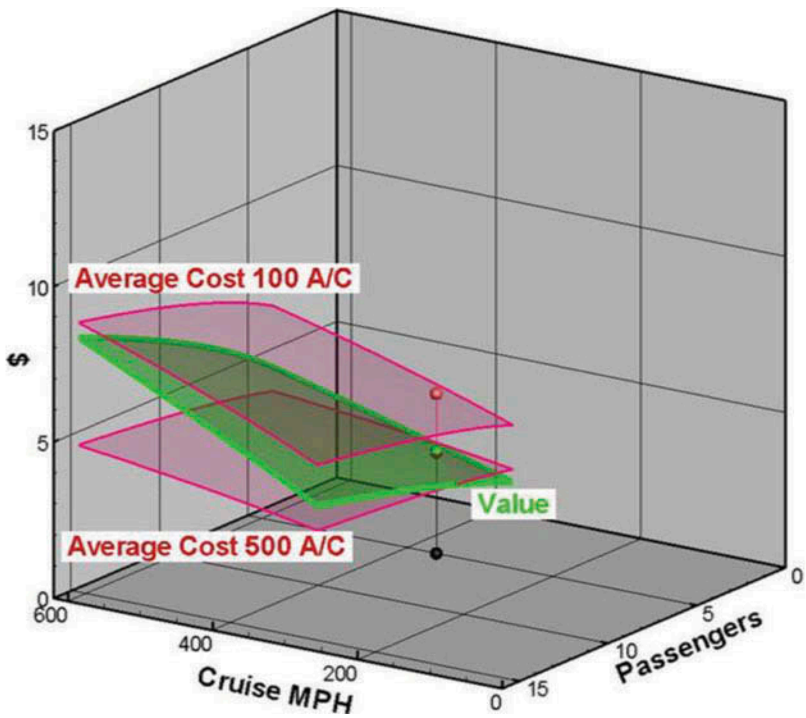

FIGURE 17 The low price gap projected values and costs (case 12).

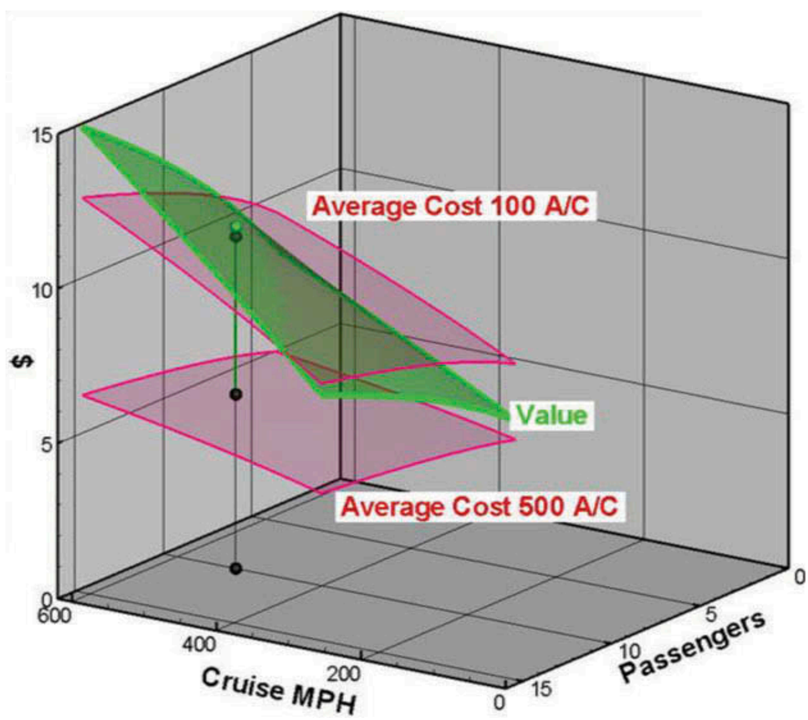

FIGURE 18 The middle price gap projected values and costs (case 12).

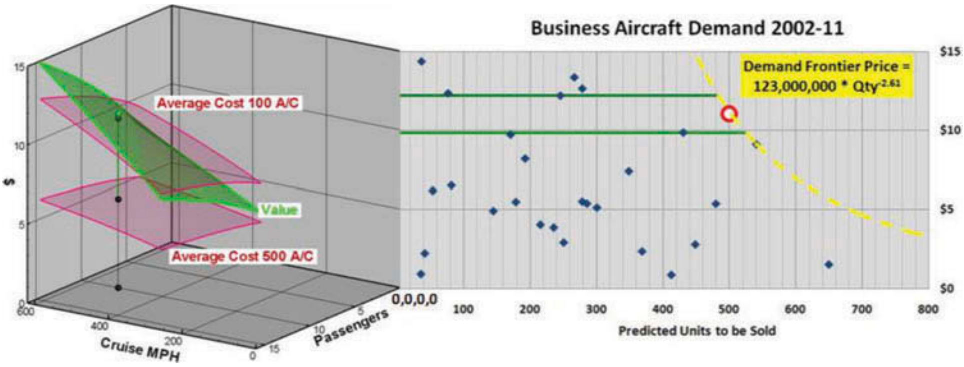

FIGURE 19 The middle gap in this four-dimensional system (passengers, cruise MPH, $, quantity) supports a profit, but the target production of 500 units pushes the Demand Frontier.

Parametricians should perform this multidimensional analysis for any new product proposed for any market. In so doing, they will lead their teams to the best economic solution for their companies.

Summary

A number of economic forces play upon product offerings. Parametricians can model these influences simultaneously. Rather than let engineering take the lead with respect to new product formulation focusing on what new products can do, parametricians can lead the way with analysis describing what new products should do. An exhaustive market study requires discovering all of the variables that affect value, cost, and demand and optimizing their predicted states to ensure maximum profitability.

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About the Author

Douglas K. Howarth founded MEE Inc. in 2011 after working for Lockheed Martin for 31 years. He has a BA in Economics from Washington State University.

Appendix A: Database

The database used for the modeling in this article appears as Figure A-1.

Models	Price 2002$M	Max Crs MPH	Typ No. Pass	Max Ht (ft)	Max Range (Miles)	No of Engs	Empty Wt (lbs)
Model 1	45.00	542	26	7.33	7250	2	88537
Model 2	47.10	542	24	7.08	6882	2	82580
Model 3	54.70	542	24	7.08	6289	2	85910
Model 4	25.33	529	19	6.08	4125	2	20485
Model 5	16.50	548	8	6.08	3567	2	19450
Model 6	38.00	581	19	6.25	5523	2	50350
Model 7	42.81	581	19	6.25	6916	2	50300
Model 8	6.52	533	6	4.33	1665	2	10253
Model 9	9.10	534	8	4.92	2419	2	12780
Model 10	12.20	536	10	5.67	2873	2	14030
Model 11	5.50	463	7	4.79	2301	2	8800
Model 12	7.40	493	8	4.58	2301	2	9977
Model 13	9.86	494	10	5.67	2491	2	12300
Model 14	13.37	511	8	5.67	3245	2	17400
Model 15	18.86	608	12	5.67	3899	2	21625
Model 16	3.86	438	5	4.75	1435	2	6670
Model 17	5.38	466	6	4.79	2025	2	7410
Model 18	8.21	543	6	5.83	2701	2	8160
Model 19	18.37	560	9	5.88	3780	3	21170
Model 20	28.17	554	19	6.17	4603	3	23375
Model 21	31.60	554	19	6.17	5178	3	23875
Model 22	20.87	554	19	6.17	3590	2	20735
Model 23	25.27	568	19	6.17	4373	2	22360
Model 24	36.47	595	19	6.17	6559	3	32700
Model 25	20.06	518	10	5.70	3682	2	29983
Model 26	14.37	460	19	6.21	2300	2	20200
Model 27	31.00	534	18	7.02	4603	2	52769
Model 28	12.35	578	6	5.58	3393	2	12670
Model 29	18.95	568	8	6.25	4163	2	19550
Model 30	32.04	582	19	6.17	4856	2	35500
Model 31	41.44	574	19	6.17	7480	2	39500
Model 32	6.18	539	8	4.75	1920	2	10550
Model 33	12.65	525	8	5.75	3400	2	15497
Model 34	17.07	555	8	6.00	3912	2	21155
Model 35	9.72	543	8	5.92	2353	2	13800
Model 36	5.37	530	6	5.42	1645	2	8120
Model 37	5.13	528	5	4.29	2719	2	7800
Model 38	0.90	164	4	4.17	729	2	3588
Model 39	1.51	202	9	4.25	1404	1	3925
Model 40	2.21	283	6	4.27	1327	2	5033
Model 41	4.90	450	6	5.75	1736	2	7500
Model 42	2.78	311	6	4.75	2529	1	5732
Model 43	2.90	284	4	4.75	1475	2	6052
Model 44	4.04	331	7	4.75	2100	2	7792
Model 45	5.47	358	8	4.75	2086	2	9326
Model 46	2.34	345	4	4.00	1783	1	4575

FIGURE A-1 Business aircraft database.

Appendix B: Five Variable Value Equation

The detailed statistics for the 5-variable value equation appear in Figure B-1.

B1:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	99.14%
R Square	98.29%
Adjusted R Square	98.08%
Standard Error	0.0609
Observations	46

ANOVA

	df	SS	MS	F	Significance F
Regression	5	8.5375252	1.707505	460.81138	3.151E-34
Residual	40	0.1482173	0.0037054		
Total	45	3.6357425			

	Coefficients	Sid Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	−5.2447	0.2315	−22.6577	1.970E-24	−5.7126	−4.7769
MPH	0.8198	0.1222	6.7064	4.819E-08	0.5727	1.0668
Pass	0.4408	0.0759	5.8041	8.912E-07	0.2873	0.5942
Ht	1.1647	0.2731	4.2648	1.188E-04	0.6127	1.7166
R	0.7721	0.0945	8.1676	4.692E-10	0.5810	0.9631
Engs	0.3851	0.1160	3.3187	1.935E-03	0.1506	0.6196

$\text{Price} = 5.69 * \text{MPH}^{0.8198} * \text{Pass}^{0.4408} * \text{Ht}^{1.1647} * \text{R}^{0.7721} * \text{Engs}^{0.3581}$

Appendix C

The Development and Procurement Cost of Aircraft, or DAPCA IV, model as developed by the RAND Corporation (Boren, 1967) and described by Dan Raymer (Raymer, 1989) provides a series of equations that predict the price of producing aircraft, as shown in Figure C-1.

Various combinations of sizes and speeds and quantities pushed through the equations in C-1 provided a series of cost matrices. Regression analysis through those matrices offered a series of predictive equations for average cost for a given number of aircraft, shown in Figure C-2.

In the cases provided, for convenience, the labor rates per hour were cut in half to make the illustrations required. See the Raymer book (Raymer, 1989) for more information on this open source model.

NONRECURRING

NRE =0.0168 * (EW^0.747) * (Vmax^0.8)
NRT =0.01868 * (EW^0.81) * (Vmax^0.579)
DS =0.0563 * (EW ^0.630) * (Vmax^1.3)
FT =1.54 * (EW^0.325) * (Vmax^0.823) * (NTA ^1.21)

Nonrecurring Terms

NRE = Nonrecurring Engineering Hours
NRT = Nonrecurring Tooling Hours
DS = Development Support Cost
FT = Flight Test Cost
NTA = Number of Test Aircraft
EW = Empty Weight

RECURRING (T-100) * (RML100)

RE100 (Hrs) = 0.000306 * (EW^0.88) * (Vmax^1.12)
RT100 (Hrs) = 0.00787 * (EW^ 0.707) * (Vmax^0.813)
RML100 (Hrs) = 0.141 * (EW^0.82) * (Vmax ^0.484)
RMM 100 ($) = 0.54 * (EW^0.921) * (Vmax ^ 0.621)
RQA (Hrs Passenger AC) = 0.076

Recurring Terms

RE = Recurring Engineering
RT = Recurring Tooling
RML = Recurring Manufacturing Labor
RMM = Recuring Manufacturing Material
RQA = Recurring Quality Assurance

FIGURE C-1 DAPCA IV cost model equations.

Aircraft Cost from Cruise Speed & Empty Weight

25 AveACCst = 0.838 * CrsMPH ^ 0.674 * EmptyWt ^ 0.711
50 AveACCst = 0.982 * CrsMPH ^ 0.619 * EmptyWt ^ 0.682
100 AveACCst = 1.53 * CrsMPH ^ 0.551 * EmptyWt ^ 0.639
200 AveACCst = 3.05 * CrsMPH ^ 0.475 * EmptyWt ^ 0.583
300 AveACCst = 4.95 * CrsMPH ^ 0.430 * EmptyWt ^ 0.545
400 AveACCst = 7.13 * CrsMPH ^ 0.399 * EmptyWt ^ 0.517
500 AveACCst = 9.52 * CrsMPH ^ 0.375 * EmptyWt ^ 0.495

FIGURE C-2 Average aircraft cost from DAPCA IV, including all nonrecurring and recurring costs.