Journal of Cost Analysis and Parametrics, 4:93–105, 2011
Copyright © 2011 Logistics Management Institute
ISSN: 1941-658X print
DOI: 10.1080/1941658X.2011.627754

ICEAA
www.iceaaonline.com

Estimating Cost and Schedule of Reliability Improvement

DAVID A. LEE and E. ANDREW LONG

Logistics Management Institute, McLean, Virginia

We extend a well-established model of reliability growth in a reliability improvement program, the Army Materiel Systems Analysis Activity Maturity Projection Model (AMPM), to include a model of the program's cost. We show how the extended model may be used to plan cost-optimal or schedule-optimal integrated programs of reliability improvement and testing, from early design through developmental and operational testing, and illustrate the process with an example from an actual program.

Introduction

Reliability, typically measured by mean time between failures (MTBF), has been shown by numerous studies over many years to be a key factor in determining equipments' operating and support costs, which often constitute the greater part of life cycle cost. Failures that cause unscheduled maintenance may bring significant penalties apart from the costs of replacing failed equipment. Consequently, reliability improvement (RI) may offer substantial returns on investment. But until quite recently, cost-estimating relationships (CERs) for calculating the investment required to obtain a specified gain in reliability have been lacking. In fact, it appears that the first usable CER was one by Long and others in 2007. Figure 1 displays this relationship and identifies the military systems on which it was calibrated.

While useful (it is certainly an improvement on the complete absence of quantitative guidance!), this estimating relationship has the limitations of its class, traditional power-law models. There is a good deal of scatter, and only the dispersion relations of its regression indicate where it may be successfully applied.

CERs more helpful than traditional power-law relationships may often be produced by developing models of the processes involved as in Lee (1997). Here we apply this approach to develop a model of the variation of failure rate, which is the reciprocal of MTBF, with investment in RI.

The model takes the form of two parametric equations, in which a certain non-dimensional time is the parameter. One equation gives the variation of failure intensity, the other gives the variation of cost. Taken with well-known equations of reliability testing, the model gives the possibility of planning cost-optimal or schedule-optimal programs of integrated RI and demonstration testing. This application of the model with an example for an actual program is illustrated.

Address correspondence to E. Andrew Long, OSD/DOT&E, 4800 Mark Center Drive, Suite 10E12, Alexandria, VA 22350-2200. E-mail: Andy.Long.ctr@osd.mil; or David A. Lee, Logistics Management Institute, 2000 Corporate Ridge, McLean, VA 22102-7805. E-mail: dlee@lmi.org

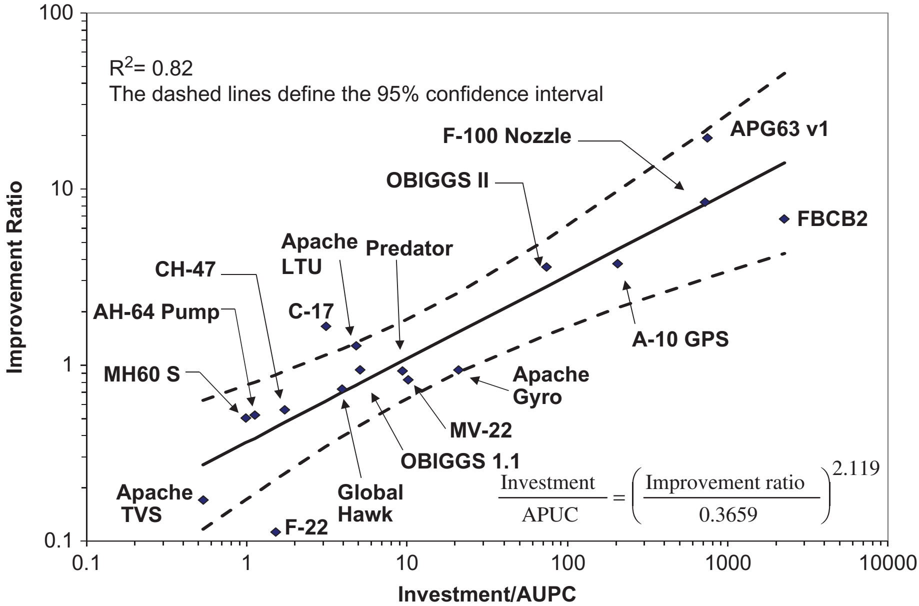

FIGURE 1 Cost-estimating relationship (color figure available online).

Modeling the Variation of Failure Intensity with Investment in RI

Robust RI programs involve a continuous process of identifying modes of failure, with a discrete process of ameliorating the modes that are found, as in Broemm et al. (2000). A RI program usually carries out these processes in two identifiable phases. The first phase deals with the equipment's design. In this phase, the continuous process expends chiefly engineering labor to proactively identify potential failure modes in the design, and the discrete process expends further engineering labor, possibly with some fabrication and testing, to ameliorate or "fix" identified modes. The second phase of a RI project usually follows the design phase with a "test-analyze-and-fix" (TAAF) phase. Here the continuous process is to operate equipments fabricated with the new design in a test environment, observing any failures. The discrete process is to analyze the failure modes and fix them with further engineering and fabricating effort.

In the TAAF phase, certain identified failure modes may be intentionally excluded from fixing, because amelioration may be too expensive or too time consuming. These modes are called "A-modes." The modes that are eventually fixed are known as "B-modes." Clearly, the failure intensity of A-modes must be small compared to the overall failure intensity, because system reliability can be no better than the allowable failure intensity.

Broemm et al. (2000) developed a model of the TAAF process, known as the Army Materiel Systems Analysis Activity (AMSAA) Maturity Projection Model (AMPM). Well-established and useful, AMPM gives a good basis for a process-based CER. As in many discussions of equipment reliability, AMPM assumes that times between failures are exponentially distributed. Among the bases for the exponential distribution of times between failures are the assumptions that the probability of a failure in any "small" time interval δt is equal to $\lambda\delta t + O((\delta t)^2)$ for some constant λ. More precisely, the probability of failure in any small time interval is approximately proportional to the length of the interval, in the specific sense just described where the quantity represented as $O((\delta t)^2)$ is bounded between two constant multiples of $(\delta t)^2$ and, therefore, is dwarfed by $\lambda\delta t$ when δt is less than 1. It is also assumed that the probability of two or more simultaneous failures (i.e., occurring within the same small time interval) is negligibly small. The mean time between failures is $1/\lambda$ and the parameter λ is known as the failure rate.

If a system is made up of any finite number of independent components, and the failures of the *i*th component satisfy the above assumptions with failure rate λ_i, then the system's times-between-failures will be exponentially distributed, and its constant failure rate will be the sum of all λ_i. If an equipment comprises a large number of components whose times-between-failure are independent, and it is also assumed that failed items are immediately replaced with identical units, then Drenick (1960) has shown, under mild assumptions on the not-necessarily-exponential distributions of times-between-failures of the individual components, that for long times of operation the equipment's failure rate tends to a constant value, which in turn forces the distribution of the component's times-between-failures to tend to an exponential distribution.

Following the AMPM discussion, we here assume that interfailure times are exponentially distributed. (In other cases not discussed here, such as cumulative damage failure modes, other failure time statistical models may be appropriate.)

The AMPM discussion starts with a set $\underline{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_K)$ of failure rate parameters. The λ_i are assumed to be realizations of independent, identically distributed Gamma random variables with parameters α and β. That is, each λ_i is assumed to be a realization of a random variable with the probability density function $p(\lambda)$, where

$$p(\lambda) = \frac{\lambda^{\alpha} e^{-\lambda/\beta}}{\alpha!\ \beta^{1+\alpha}}, \tag{1}$$

for $\lambda \geq 0$ and $p(\lambda) = 0$ otherwise. Furthermore, α and β must be positive numbers. The initial B-mode failure rate is the sum of the λ_i.

Then $r(t; \lambda)$, the system failure intensity after fixes have been made to all B-modes that have surfaced by time t, is

$$r(t; \underline{\lambda}) = \lambda_A + \sum_{i=1}^{K} \{1 - d_i I_i(t)\}\lambda_i, \tag{2}$$

where $I_i(t) = 1$ if mode i has occurred by time t, and $I_i(t) = 0$ if it hasn't. $I_i(t)$ is called the indicator function of mode i. The symbol d_i denotes the factor by which mode i's failure rate is reduced by fixing, and λ_A is the rate parameter for A-mode failures.

We can introduce cost c at this point, with the modeling assumption that

$$c(t; \underline{\lambda}) = gt + \sum_{1}^{K} b_i I_i(t). \tag{3}$$

That is, cost is given by a term proportional to test time t, with constant of proportionality g, plus an increment b_i incurred for the *i*th fix operation. Because failure and fix times, as well as other quantities in Equations (2) and (3) are random variables, both $r(t, \underline{\lambda})$ and $c(t, \underline{\lambda})$ are themselves random variables.

Authors of the AMPM development (Broemm et al., 2000) proceed to develop an expression for the failure rate $\lambda(t)$ by calculating the expectation (the "mean" or "expected value") of the random variable $r(t, \underline{\lambda})$, first with respect to time of first occurrence of B-mode i, then with respect to the ensemble of $\underline{\lambda}$, and finally with respect to the relative improvement factors. Using the facts that the expectation of $I_i(t)$ with respect to the first occurrence of failure mode i is $1 - e^{-\lambda_i t}$ and that a generating function for the gamma distribution Equation (1) is $(1 - \beta x)^{\alpha+1}$. They arrive at

$$\lambda(\tau) = \lambda_A + (1 - \mu_d)\lambda_{B,K} + \frac{\mu_d \lambda_{B,K}}{(1+\tau)^{\alpha+2}}, \tag{4}$$

where the non-dimensioned time variable $\tau \equiv \beta t$ and where $\lambda_{B,K}$ is given by

$$\lambda_{B,K} \equiv K\beta(\alpha + 1), \tag{5}$$

that is the expected value of the sum of the λ_i, which will be the expected value of the initial failure rate of B-modes. The parameter μ_d is the mean of the d_i.

Carrying out the same sequence of expectations on Equation (3), we obtain the expected cost $\gamma(t)$, given by

$$\gamma(t) = gt + \mu_b K\left(1 - \frac{1}{(1+\beta t)^{\alpha+1}}\right). \tag{6}$$

In Equation (6), μ_b is the mean of the b_i.

Given our modeling assumption, the result shown in Equation (6) is quite intuitive: the cost at time t is equal to the cost of operating the test-fix-test program for a length of time t, represented by the first term on the right side of Equation (6), plus the average cost of fixing the expected number of B-modes that have surfaced on the interval $(0, t)$, represented by the second term on the right side of Equation (6).

Now

$$K\left(1 - \frac{1}{(1+\tau)^{\alpha+1}}\right) = \frac{\lambda_{B,K}}{\beta}\frac{1}{\alpha+1}\left(1 - \frac{1}{(1+\tau)^{\alpha+1}}\right), \tag{7}$$

in view of Equation (5), so we may write

$$\gamma(t) = gt + \frac{\lambda_{B,K}\mu_b}{\beta}\mu(\beta t; \alpha), \tag{8}$$

where

$$\mu(\beta t; \alpha) \equiv \frac{1}{(\alpha+1)}\left(1 - \frac{1}{(1+\beta t)^{\alpha+1}}\right) = \tau\ {}_2F_1(\alpha+2, 1;\ 2,\ -\beta t). \tag{9}$$

The hypergeometric function ${}_2F_1(\alpha + 2, 1;\ 2;\ -\beta t)$ in Equation (9) arises because

$$(1+\tau)^{-(\alpha+1)} = 1 - (\alpha+1)\tau + (\alpha+1)(\alpha+2)\frac{\tau^2}{2!} + \ldots, \tag{10}$$

so that

$$\begin{aligned}\mu(\tau;\alpha) &= \tau[1 - (\alpha+2)\frac{\tau}{2!} + (\alpha+2)(\alpha+3)\frac{\tau^2}{3!} - \cdots + \cdots \\ &= \tau\sum_0^\infty \frac{(\alpha+2)_k(1)_k}{(2)_k}\frac{(-\tau)^k}{k!} = \tau\ {}_2F_1(\alpha+2,\ 1;\ 2;\ -\tau).\end{aligned} \tag{11}$$

The limit of a large number of B-modes, that is, the large-K limit, has been found useful (Broemm et al., 2000). In that limit, in view of a standard identity for the limiting form of the hypergeometric function (Oberhettinger, 1972), Equations (4) and (8) may be written as

$$\lambda(\tau) = \lambda_A + (1 - \mu_d)\lambda_0^B + \frac{\mu_d \lambda_0^B}{1 + \tau} \tag{12}$$

and

$$\gamma(\tau) = \frac{\lambda_0^B}{\beta}\left[\frac{g}{\lambda_0^B}\tau + \mu_b \ln(1 + \tau)\right]. \tag{13}$$

Now, calculation shows that the ratio λ_0^B/β is the reciprocal of the square of the coefficient of variation of the sum of gamma-distributed rates that defines the initial total failure rates of the B-modes. Accordingly, we write

$$\gamma(\tau) = \frac{1}{cv^2}\left[\frac{g}{\lambda_0^B}\tau + \mu_b \ln(1 + \tau)\right]. \tag{14}$$

Equations (12) and (14) give the relation between RI, measured by the decrease in $\lambda(\tau)$, and the cost $\gamma(\tau)$ incurred to produce it.

We regard the parameter $1/(cv^2)$ as a measure of the "goodness" of the reliability engineering that preceded the RI under consideration. Two arguments support this. First, in a program with well-managed reliability engineering, subsystem managers will be given reliability targets. Powerfully incentivized to meet their targets, the subsystem managers are dis-incentivized to exceed them—that would increase their costs. Consequently, there will be little scatter in the failure rates of systems in a class of well-managed programs, cv^2 will be small, and $1/(cv^2)$ will be large.

This is consistent with a simpler interpretation of $1/(cv^2)$: a well-managed program leaves relatively little "low-hanging fruit" for subsequent RI, so that any given improvement will cost relatively more to make.

The remaining parameters of the model's Equations (12) and (14) have straightforward physical significance: μ_d is mean reduction of failure rates achieved by amelioration; g is the burn rate of process and g/λ_0^B is the cost of operating the continuous process of the improvement program for a time equal to the mean time between B-mode failures at the start of the program; and μ_b is mean cost of a "fix."

Equations (12) and (14) constitute two parametric equations with parameter τ, giving achieved RI as a function of cost. Figure 2 graphs an example of this relationship, showing improvement ratio, $(\mathrm{MTBF}_{\mathrm{final}} - \mathrm{MTBF}_{\mathrm{original}})/\mathrm{MTBF}_{\mathrm{original}}$ as a function of required investment.

The new model offers considerably more, however. The complete model provides schedule information from Equation (12) as well as cost information from Equation (14), as shown in Figure 3.

To use the new model as a CER, μ_d is treated as a parameter specific to each phase of a RI program. g and μ_b are modeled as functions of average program acquisition unit cost; presently lacking more rational grounds for those functions, they have been treated as power laws.

$1/(cv^2)$ has been treated as a function of the reliability assessment score from an AMSAA procedure (U.S. Army Materiel Systems Analysis Activity, 2009) for evaluating

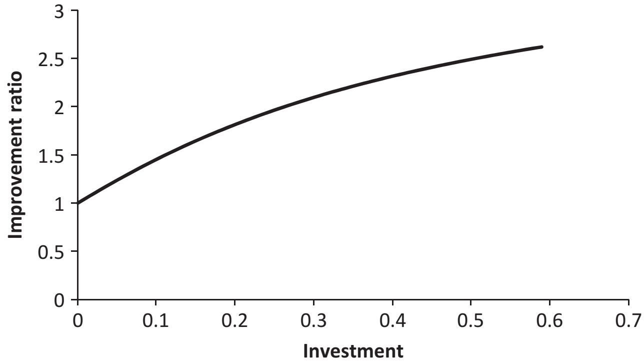

FIGURE 2 Relative improvement in MTBF vs. Investment.

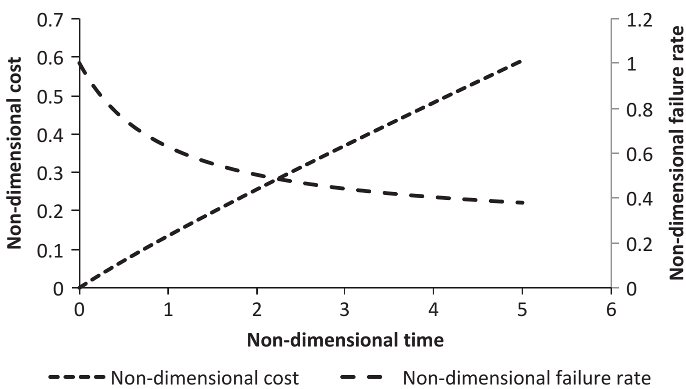

FIGURE 3 Schedule and cost information.

programs' reliability engineering, and, again lacking better grounds, used a power law function.

Figure 4 depicts an example of the calibration results on a set of several military systems.

We employed the bootstrap method (Press, Tuelkolsky, Vetterling, & Flannery, 1996) to analyze goodness-of-fit, and to give indications of the scatter in the model's predictions. Figure 5 shows an example of goodness-of-fit results. The vertical bars extend two bootstrap standard deviations above and below the mean of the bootstrap predictions.

Some features of the calibration are interesting. For one, calibration of the μ_d parameter on TAAF-phase data gave $\mu_d = 0.70$, a value consistent with AMSAA experience. Calibration on design-phase data gave $\mu_d = 0.99$, consistent with comments from reliability engineers that failure modes identified in design reviews would be essentially eliminated.

We packaged bootstrap results into an estimating tool. User inputs are the item's Acquisition Program Unit Cost (APUC), the failure rate of A-modes, starting and ending system failure rates or MTBFs, and a completed AMSAA scorecard worksheet. The estimating tool has a number of outputs:

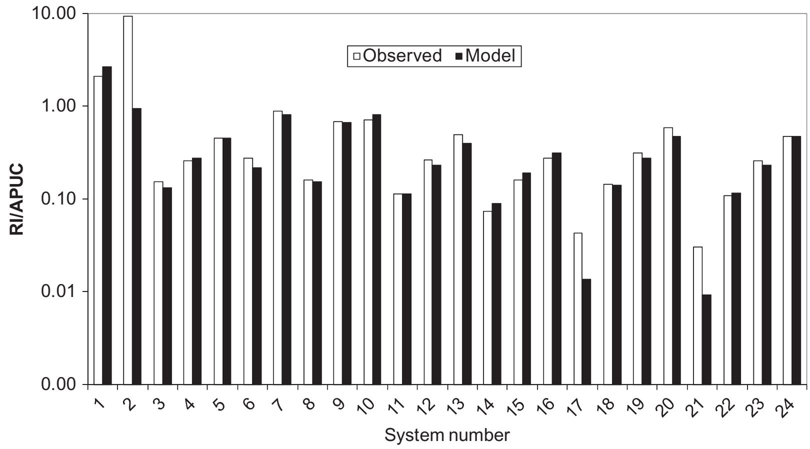

FIGURE 4 Example of calibration results.

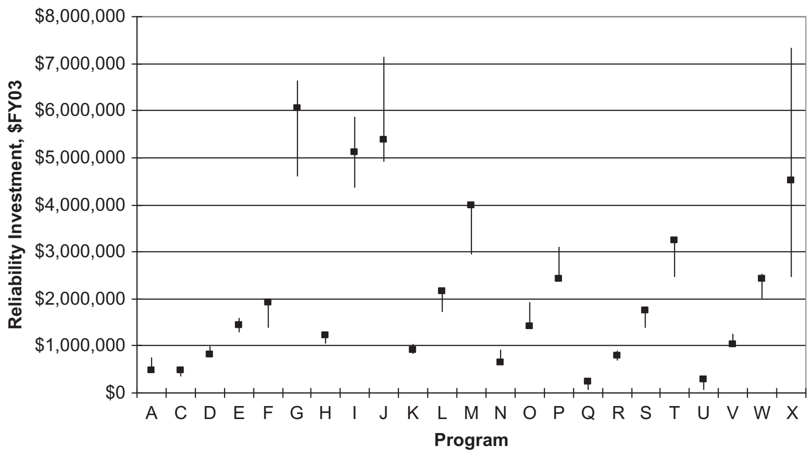

FIGURE 5 Goodness-of-fit results.

- Predicted total RI costs in the phase;
- Standard deviation of prediction;
- Coefficient of variation of prediction;
- Predictions of operating costs (identifying B-modes, management time, overhead, etc.) and of corrective-action costs;
- A sample cumulative distribution function (CDF) of the predicted costs, in scatter-plot format.

Figure 6 shows an example of an application of the estimating tool.

Cost-Optimal and Schedule-Optimal Programs of RI and Testing

The form of the process-based model in Equations (12) and (14) also offers some potential for applications beyond cost estimating. It is possible to integrate schedule information

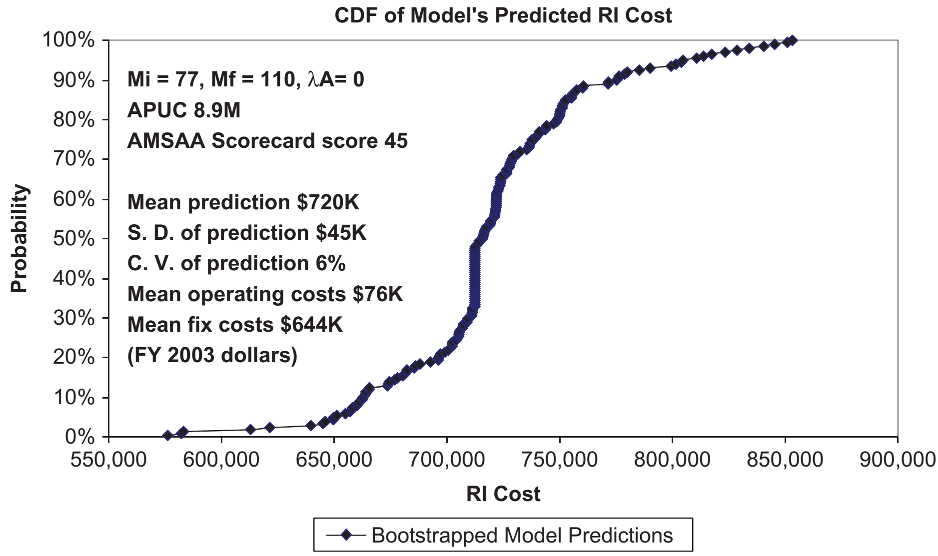

FIGURE 6 Example of estimating tool application (color figure available online).

from Equation (12) with expressions for the time required for certain standard reliability demonstration tests. With the cost equation, Equation (14), one can then develop guidance for optimized integrated programs of reliability growth and demonstration that have minimum time, minimum cost, or, perhaps more realistically for actual programs, minimum cost subject to a maximum-time constraint.

A widely-used reliability demonstration test accepts an article if it exhibits no more than n failures in a specified time T_n. Such a test is designed by specifying a goal MTBF, MG, a consumer's risk R_C (the risk that an unsatisfactory article is accepted), and producer's risk R_P (the risk that a satisfactory article is rejected). These specifications determine a family of tests, indexed by the number n of allowable failures in the duration of the test. Equations for the test design give test duration $T_n(R_C)$ as

$$T_n(R_C) = \frac{MG}{2}\chi^2_{2(n+1),R_C} \tag{15}$$

and minimum achieved reliability $M_n(R_C, R_P)$ as

$$M_n^*(R_C, R_P) = MG\frac{\chi^2_{2(n+1),R_C}}{\chi^2_{2(n+1),1-R_P}}. \tag{16}$$

Equations (15) and (16) or their equivalents are available in many references; a derivation can be found in Long et al. (2010). They imply that for, fixed values of R_C and R_P as the number n of allowable failures increases, the test time T_n increases monotonically and the required minimum MTBF, M_n^*, decreases monotonically. Figure 7 shows an example.

Now let us see how Equations (12), (14), (15), and (16) allow program managers to make schedule-optimal or cost-optimal plans for integrated TAAF and demonstration testing. Turning first to schedule-optimal plans, note that for each value of n, Equation (15) gives the corresponding test length $T_n(R_C)$, and Equation (16) gives the required minimum value of achieved reliability $M_n^*(R_C, R_P)$, which determines a maximum failure rate

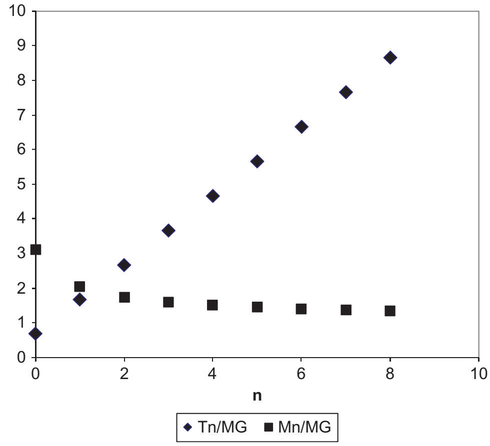

FIGURE 7 Example variation of test time and required MTBF.

$\lambda_n^* = 1/M_n^*$. Then Equation (12) may be solved for the value of τ required to achieve failure rate λ_n^*; dividing this value of τ by the parameter β gives the required time of TAAF, T_{TAAF}. The result is

$$T_{TAAF} = \frac{1}{\beta} \; \frac{\lambda_0^B + \lambda_A - \lambda_n^*}{\lambda_n^* - \lambda_A - \lambda_0^B(1-\mu_d)}. \tag{17}$$

Note that the denominator of Equation (17) becomes zero when λ_n^* falls to the value $\lambda_{n\,\min}^*$ given by

$$\lambda_{n\,\min}^* = \lambda_A + \lambda_0^B(1-\mu_d). \tag{18}$$

Note that the TAAF process cannot achieve a failure rate less than or equal to $\lambda_{n\,\min}^*$, nor an MTBF greater than or equal to $M_{\max} = 1/\lambda_{n\,\min}^*$.

The total time of reliability growth and test is the sum of T_n and T_{TAAF}. As n increases, test time T_n increases, and $M_n^*(R_C, R_P)$ decreases; consequently $\lambda_n^* = 1/M_n^*$ increases, which increases the denominator and reduces the numerator of Equation (17), thereby decreasing the value of T_{TAAF}. Thus, the total time typically exhibits a minimum for some value of n, giving the schedule-optimal plan for coordinated reliability growth and testing.

Now turning to cost-optimal plans, we may introduce cost by considering Equation (14) and a model for test cost C_{test}. Such a model might, for example, be simply the "burn rate" part of Equation (14), since testing doesn't involve fixes. Testing may include significant set-up costs, but since these are always incurred and may not be affected by test length, set-up costs presumably wouldn't be considered in cost-optimal designs. A realistic model of costs in the TAAF-test phases might also need to include costs of program operations apart from TAAF and testing themselves.

For illustration, we'll assume that test costs are simply proportional to test time, with factor G_{test}. Then the total cost C_{tot} of an integrated TAAF-test process will be

$$C_{tot} = \frac{1}{cv^2}\left[\frac{g}{\lambda_0^B}\beta T_{TAAF} + \mu_b \ln(1+\beta T_{TAAF})\right] + G_{test}T_n(R_C), \tag{19}$$

where T_{TAAF} is given by Equation (17).

As noted in the discussion of schedule considerations, increasing n increases T_n and decreases T_{TAAF}. Thus, increasing n decreases the first term on the right side in Equation (19), and increases the second. Again, there may be a cost-minimizing value of n, giving the cost-optimal test plan. The cost-optimal and schedule-optimal values of n are not necessarily the same.

Let us illustrate development of schedule-optimal and cost-optimal plans with data from an actual, but unidentified, program. Figure 8 shows a reliability growth planning curve for a certain Army system preparing to meet the Army threshold requirement.

The system is required to demonstrate with at least 50% confidence that MTBF is not less than $MG = 104$ hours. This example assumes that management wishes to have at least an 80% probability of passing the test. That is, the test is to have consumer's risk R_C of 50% and producer's risk R_P of 20%. Using Equations (15) and (16) and the given values of MG, R_C, and R_P, we readily generated the first three columns of Table 1.

Now let us introduce schedule information. Assuming a mean fix effectiveness factor μ_d of 0.75, the planning curve of Figure 8 is equivalent to a curve of the form Equation (12), with $\beta = 0.00155\ (\text{hr})^{-1}$, $\lambda_A = 0$, and $\lambda_0^B = 0.0125$. With this information, we used Equation (17) to enter values of T_{TAAF} in. The value of M_0^*, 322 hours, is just greater than the value of M_{max}, which is 320 hours. Since M_0^* cannot be achieved, T_{TAAF} is not recorded for $n = 0$. Adding T_{test} and T_{TAAF} gave the total times displayed in the far-right column of Table 1.

Table 1 shows that the minimum total time for reliability growth and test is found for $n = 5$. Accordingly, the schedule-optimal plan for integrated reliability growth and testing,

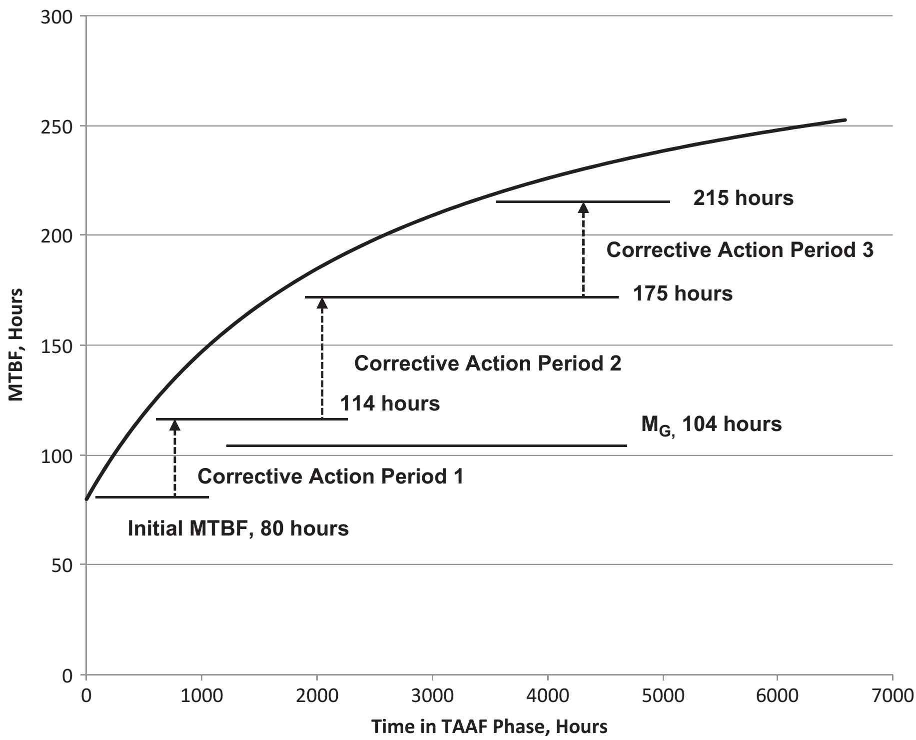

FIGURE 8 Reliability growth planning curve.

TABLE 1 Test times, maximum failures, and achieved reliability to demonstrate $MG = 103.6$ hours with $R_C = 50\%$ and $R_P = 20\%$

Maximum failures, n	$T_n = T_{test}$, hours	M_n^*	T_{TAAF}	Total time
0	72	322		
1	174	211	3,097	3,271
2	277	180	1,858	2,135
3	380	166	1,432	1,812
4	484	157	1,210	1,694
5	587	150	1,073	1,660
6	691	146	978	1,669
7	795	142	908	1,703

for the prescribed goal of 103.6 hours' MTBF and a test with 50% consumer's risk and 20% producer's risk, has a TAAF time of 1,073 hours and a test time of 587 hours; no more than five failures may occur during the test.

Cost-optimal plans can be obtained with Equation (14) and a model for C_{test}. Solely for this illustrative example, we obtained values of $g/(cv^2)$ and $\mu_b/(cv^2)$ from a calibrated TAAF model, to give

$$\frac{Reliability\ \ Investment}{APUC} = 0.0673\,\beta T_{TAAF} + 6.45\ln(1+\beta T_{TAAF}). \tag{20}$$

Also assuming for this example that C_{test} is the same as the "burn rate" part of TAAF cost C_{TAAF}, modeled by the first term on the right side of Equation (20), we calculated the cost values in Table 2.

Data in indicate, perhaps counterintuitively, that total cost decreases steadily with increasing test time. This happens because $C_{TAAF}/APUC$ decreases faster than $C_{test}/APUC$ increases for cases of Table 2. C_{TAAF} will continue to decrease as test time increases, but C_{test} increases steadily. Eventually, total cost will reach a minimum; however, for the calibration we are using this will happen at a very large total time, and clearly other costs, such as operating the program office and contractor staff, would need to be considered in the experimental design for a pragmatic discussion of a minimum cost plan.

Thus, the cost and schedule estimates of Equations (12) and (14), with the test design information of Equations (15) and (16), would lead a manager to operate the TAAF program for 1,073 hours and test for 587 hours, allowing no more than five failures, and to expect a total cost of RI and testing to be roughly 6.6 times the system's $APUC$.

TABLE 2 Cost and schedule implications of test options

Maximum failures	Test times	TAAF time	Total time	TAAF cost/APUC	Test cost/ APUC	Total cost/ APUC
1	174	3,097	3,271	11.7	0.02	11.7
3	380	1,432	1,812	7.8	0.04	7.8
5	587	1,073	1,660	6.5	0.06	6.6
6	691	978	1,669	6.1	0.07	6.2

We think it likely that, as in the present example, programs would often wish to consider minimum-cost programs subject to a maximum-time constraint. It is of course entirely possible that program managers and engineers within the context of robust experimental design could calibrate Equations (12) and (14) for their individual program. They could include relevant costs outside the RI and test processes. With that information they could use Equations (12), (14), (15), and (16) to design cost- or schedule-optimal combined reliability growth and qualification test plans, for assigned values of consumer's risk and producer's risk.

Thus, we have seen how considering models of the processes involved in RI programs leads to a cost- and schedule- estimating method that offers advantages over the traditional power-law model. In particular, it facilitates guidance to program managers for designing optimized, integrated programs of RI and demonstration testing.

Acronyms

AMPM	AMSAA Maturity Projection Model
AMSAA	Army Material Systems Analysis Activity
APUC	Acquisition Program Unit Cost
CER	Cost-Estimating Relationship
MTBF	Mean Time between Failures
RI	Reliability Improvement
TAAF	Test, Analyze, and Fix

Acknowledgments

Material from this article was presented at the First Department of Energy Cost Analysis and Training Symposium, Santa Clara, CA, May 2010. Figures 4 and 5, and the discussion of the bootstrap estimating tool, are from "Developing a Reliability Investment Model, Phase III, Maturing the Model," by E. Andrew Long, Carol DeZwarte, David A. Lee, and Gerald J. Belcher, LMI TR BT901T1, March 2010. The discussion of optimal integrated programs of reliability growth and demonstration testing parallels that of "Using Cost and Schedule Estimates to Plan Efficient, Coordinated Programs of Reliability Growth and Qualification Testing," by David A. Lee and E. Andrew Long, *Journal of the Reliability Information Analysis Center*, Volume 17, No. 4, 2009, pp. 2–8.

References

Broemm, W. J., Ellner, P. M., & Woodworth, W. J. (2000). *AMSAA reliability growth guide*. U.S. Army Material Systems Analysis Activity Technical Report TR-652.

Drenick, R. F. (1960). The failure law of complex equipment. *Journal of the Society for the Industrial Applications of Mathematics*, *8*, 680–689.

Lee, D. A. (1997). *The Cost Analyst's Companion*. McLean, VA: Logistics Management Institute (p. 81 et seq). ISBN: 0-9661916-0-9.

Long, E. A., Forbes, J., Hees, J., & Stouffer, V. (2007). *Empirical relationships between reliability investments and life-cycle support costs*. Logistics Management Institute Report SA701T1.

Long, E. A., DeZwarte, C., Lee, D., & Belcher, G. (2010). *Developing a reliability investment model, Phase III, Maturing the model*. Logistics Management Institute Report TR BT901T1.

Oberhettinger, F. (1972). Hypergeometric functions. In M. Abramowitz & I. Stegun, (Eds.), *Handbook of mathematical functions*, Eighth Dover Edition. New York: Dover Publications (Equation 15.1.3, p. 556).

Press, W. H., Tuelkolsky, S. A., Vetterling, W. T., & Flannery, B. P. (1996). *Numerical recipes in C*, Second Edition, Cambridge, UK: Cambridge University Press (p. 691 et seq).

U.S. Army Material Systems Analysis Activity. (2009). Reliability Program Scorecard, Version 1.2.

About the Authors

Dr. David A. Lee was educated at the University of Missouri, Brown University, and the Illinois Institute of Technology and is an applied mathematician with nearly 50 years' experience. From 1985 through 1993 he was Director of the Operations Research/Procurement Planning Division in the Office of the Secretary of Defense (OSD), under the Assistant Secretary, Program Analysis and Evaluation. In this position he directed operations of the staff of the OSD Cost Analysis Improvement Group (OSD CAIG—now CAPE), which is DoD's chief advisory body on costs of defense programs. Prior to joining OSD, Dr. Lee was Head of the Department of Mathematics and Computer Science at the Air Force Institute of Technology. He is author or co-author of more than 30 archival publications and of "The Cost Analyst's Companion," a widely used reference.

Mr. E. Andrew Long has 30 years of experience in a broad range of systems analysis problems in performance and suitability. He has led or participated in several efforts to help programs address supportability problems during all acquisition and operations and support phases. He has also performed theoretical and applied studies of reliability and availability of systems to assess logistics vulnerabilities and footprint. Recent work includes suitability cost realism analysis for the Coast Guard's Integrated Deepwater System. He performed reliability and logistics footprint modeling and analysis for the Army's Future Combat Systems' Reliability, Availability Maintainability and Testability programs. His current work is directed toward policy, programs and tools needed to assure cost-effective suitability of weapon systems. Mr. Long has an undergraduate degree in Mathematics from New Mexico State University and a graduate degree in Logistics Management from the Air Force Institute of Technology.