Self-Organizing Markets And Time

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ABSTRACT:

Markets demonstrate statistically significant self-organization concerning how they respond to changes in prices and the product features offered to them. The nature of these self-organizing activities changes over time. What works for a market now may not work a few years from now. Being able to characterize market self-organization now and in the future is key to optimizing financial success, which this paper examines.

1.0 Market Formation

Many cost organizations take little or no account of the markets in which their products compete. Not recognizing how markets form and self-organize can cause large financial headaches when programs run afoul of barriers they did not recognize in advance.[1] While some markets, such as that for housing, endure cyclical rises and falls, other emerging markets, such as those for personal computers in the 1980s and 1990s and cell phones in 2000s and 2010s experience rapid growth for many years. Mapping emerging markets over time provides a method to predict the market shape in the future. This cartographic necessity requires making maps for demand, value, and cost and seeing their movements over time, to predict where they will be in the future.

2.0 Surface Self-Organization

Before we study how markets organize, perhaps we should see examples in other realms first.

Figure 1: Penguins huddle for warmth

Nature demonstrates seemingly countless ways to arrange itself. As Figure 1 shows,[2] emperor penguins engage in a special form of self-organization in the Antarctic, where they form huddles as temperatures drop. Taking turns on the outer edge of the group to face the cold and wind, they eventually migrate back to the middle of the unit, where temperatures can reach 100° F, which eventually makes them too warm. When that happens, they make their slow relocation to the huddle's edge, where the process recycles itself.

Figure 2: Football players huddle to get information

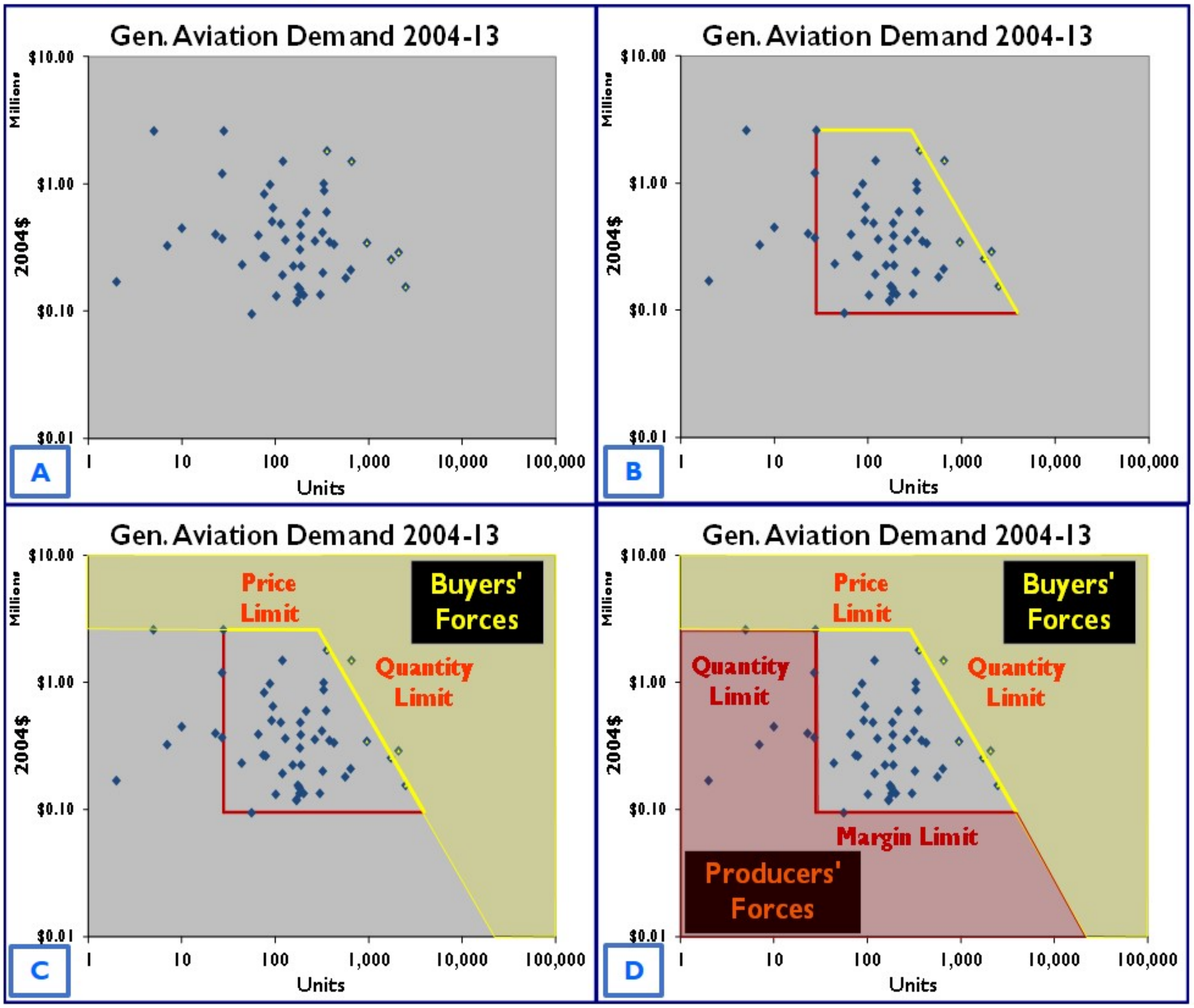
People have their ways of arranging themselves. Some of these methods mimic what we see in nature. In the football huddle in Figure 2[3], players gather to find out what the next play will be, and the agreed-upon starting command. In this case, the human organization reaches back in time before the huddle forms – it takes planning and drilling and repetition to learn football plays. Something as fundamental as the signal starting the play must be agreed upon, taught and repeated until it becomes engrained in each player.

In each instance, the behaviors examined address what happens on a surface, the Antarctic ice shelf for the penguins, a field for the football players. While there is some variation in elevation in both cases, for purposes of analysis, we may consider these be two-dimensional behaviors. Since this paper addresses markets, we may ask ourselves how these huddles relate to markets.

We can see how mature market behavior mimics huddling in Figure 3.[4] With quantity on the horizontal axis and price on the vertical, here we have 52 ordered pairs that describe the quantities sold and prices for the same number of general aviation aircraft models for the decade beginning January 1, 2004 and ending December 31, 2013. To the casual observer, in Figure 3A, without any analysis of the data, the points seem to mimic those found in a loosely grouped huddle. However, a closer examination reveals that less only a few outliers, the bulk of the group fits neatly in the bounded trapezoid described by B. Figure 3B has different meanings for buyers and sellers. In this market, there is an outer boundary defining what a market's customers can absorb, which we call the Demand Frontier, as it reveals a quantity limit on the market as shown in C. At the same time, also in Figure 3C, we find there is an upper limit to what people are willing to pay for this type of product – people do pay more for private transportation by air, but in that case, they move up to business aircraft.

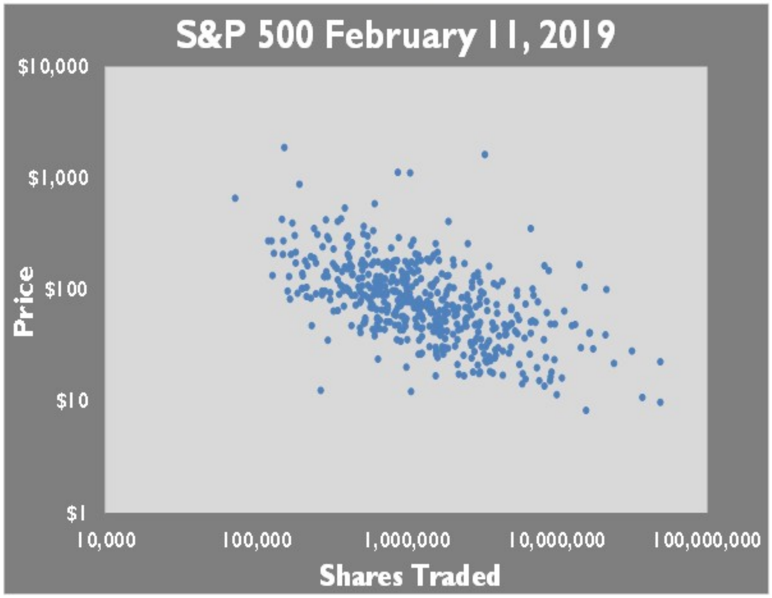
Likewise, manufacturers must abide by market boundaries as well. As Figure 3D shows, there is a lower bound to the market, a margin limit, below which differences between the costs of the models and their sustainable prices are not enough to make a profit. There are less expensive models in adjacent markets, but they form different, less restricted classes of aircraft such as the group collectively known as ultralights. Also, in Figure 3D, it is evident that manufacturers need to build a minimum number of planes each year to their lines working and retain learning, to keep prices adequately low to ensure profitability.

Figure 3: In A, we have the quantities and prices for a decade's worth of 52 models of general aviation aircraft. B shows us that, excepting a few outliers, all of the models huddle together in a region we can outline. C reveals that buyers have upper limits to what they will pay defined by price, and outer limits indicating the quantities the market can collectively absorb. D indicates that producers have margin limits that prevent them from selling at prices lower than their threshold, as well as minimum quantities they need to make on a ongoing basis to keep production efficient. The models below the quantity limit are typically those just entering or just leaving the market.



Note in Figure 3D there are a small number of models that have fewer units sold than the minimum needed to offer a profit to a producer. We may explain this phenomenom by noting some aircraft are just entering the market and do not have a full ten-year complement of sales, thus belying their average annual output. Other aircraft may be leaving the market during the period and have already made their enough profits for their companies; their low sales reflect that. However, any model with fewer than a minimum number of models sold per year will find it hard to keep learning in their production lines, and profits will suffer because of that.

Figure 4: The demand plot for stocks in the S&P 500 is similar in shape to the one for general aviation



The grouping of ordered quantity-price pairs in Figure 3 is common among mature and maturing markets. The S&P 500 shares and prices in Figure 4[5] abide by the same rough shape as the general aviation market did in Figure 3, though an irregular four-sided polygon works better for stocks than the trapezoid did for aircraft.

It is likely that versions of the Figure 4 pattern are repeated throughout stock market histories, which analysts could study. Similar analyses of dissimilar markets reveal these bounded regions in every case studied.

3.0 Spatial Self-Organization

Often surface observations are not adequate to fully describe a given phenomenon. In such cases, we may need to take to the air.

Figure 5: Starlings flock to defend against predators

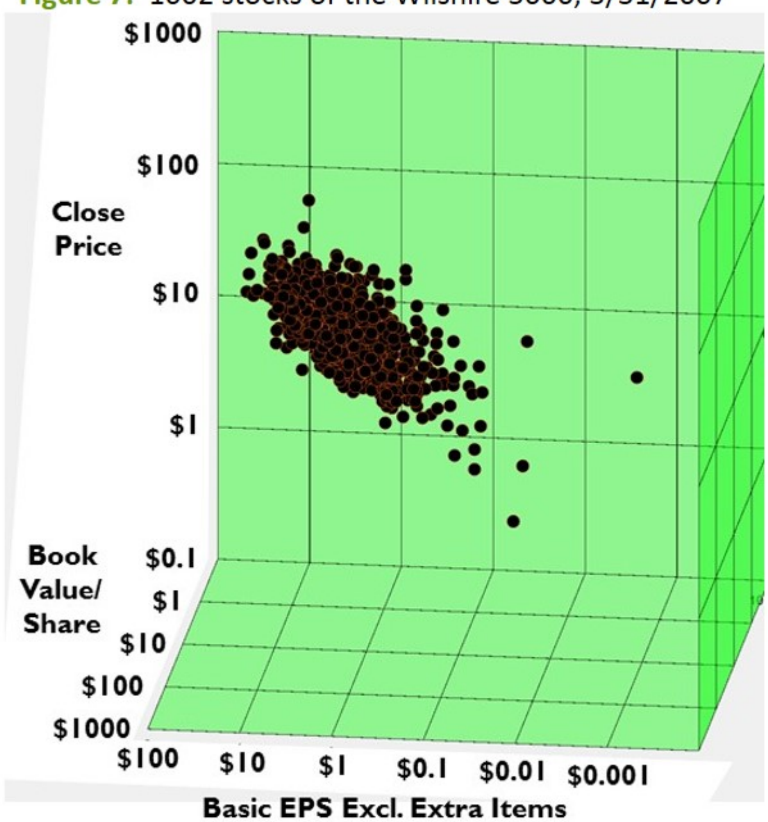
Starlings often go airborne just after sunset, forming large flocks with up to one million birds, in a phenomenon called the Sort Sol ("Black Sun" in Danish), one instance of which we find in Figure 5.[6,7] While they take flight to discover where they will roost for the night, their tight murmuration serves another purpose, that of group defense. If predator birds were to attempt to attack the group, the starlings will collectively fire back from both ends of their alimentary canals, sending down a special kind of rain that soils the feathers of the attackers, making it very unpleasant and hard to fly, or, in the more explosive, fanned-out episodes, sending them crashing into the ground or sea. The individual starlings come together as a group to protect the group. This type of animal activity is of course not limited to birds.

Figure 6: Fighters team up to defend against predators

People have their enemies as well. While coordinated armies and navies protect us on land and sea, we must rely on our air forces to guard us against the hazards that come in from above us. Air forces invariably gather together in groups of varying sizes, such as the squadron of F-117A fighters shown in Figure 6.[8] Fighters take shapes in the sky much like those of Canadian geese, flyng in V-formations or one leg of a vee. Formations of this type are not only easier to defend, but they also enable easier communication and have the added benefit of reducing the total energy required to fly, as following planes encounter less wind resistance. Assembling in this manner also makes accounting for all members of the group virtually effortless.

Behavior in the stock markets across three dimensions uncovers groupings we have already found to be familiar in this section. Sustainable prices for individual stocks tend to cluster in centralized masses based on their performance metrics, or features. As Figure 7 depicts, after performing some filtering on the data,[9] when we compare the book value per share of stocks along with their basic earnings per share (excluding extra items) to their prices on a given day over a decade ago, the resulting three-dimensional points create a stock swarm similar in form to the starling murmuration we found in Figure 5. Unlike the starlings or fighter jets, however, this grouping is not purposefully defending itself. But much like the survival of the fittest, it appears that those stocks that cannot keep up with the swarm get dropped from it.

Figure 7: 1002 stocks of the Wilshire 5000, 3/31/2007



The group actions considered by this paper now address the "what" question of their collective shaping behaviors. Observation reveals what shapes of penguin and football huddles (Figures 1 and 2) take, along with the like outlines demonstrated by general aviation demand (Figure 3) and assemblages of the quantities and prices of stocks (Figure 4). Three-dimensional swarming is more complicated, but we can capture the forms it takes with pictures (Figures 5 and 6) or graphical representations (Figure 7). With the application of science, researchers know the answers to important "why" questions about animal huddling (they do it for heat) and flocking (they do it to fend off predators).

The work at hand seeks to address the time factor involved for group behaviors in markets over time, the "when" question not yet tackled. Some questions in this realm may seem to entirely straightforward, but history proves that this is not always the case. American football, the consensus agrees, began on November 6, 1869, with a game between two college teams, Princeton and Yale.[13] Longtime followers of the sport might, therefore, assume, as indeed the author did himself, the ubiquitous huddle likely began on that same day. Virtually everyone alive today has always seen football games with huddles. However, the formation of the huddle came decades later, as it may be traced back to a 1918 game between Oregon Agricultural College (eventually, Oregon State) and the University of Washington, or even earlier to the 1890s, where Paul Hubbard, the quarterback of Gallaudet College, a school for the deaf, created a huddle to prevent his American Sign Language signals from being stolen. Just because we have seen a pattern forever doesn't mean it has been there forever.

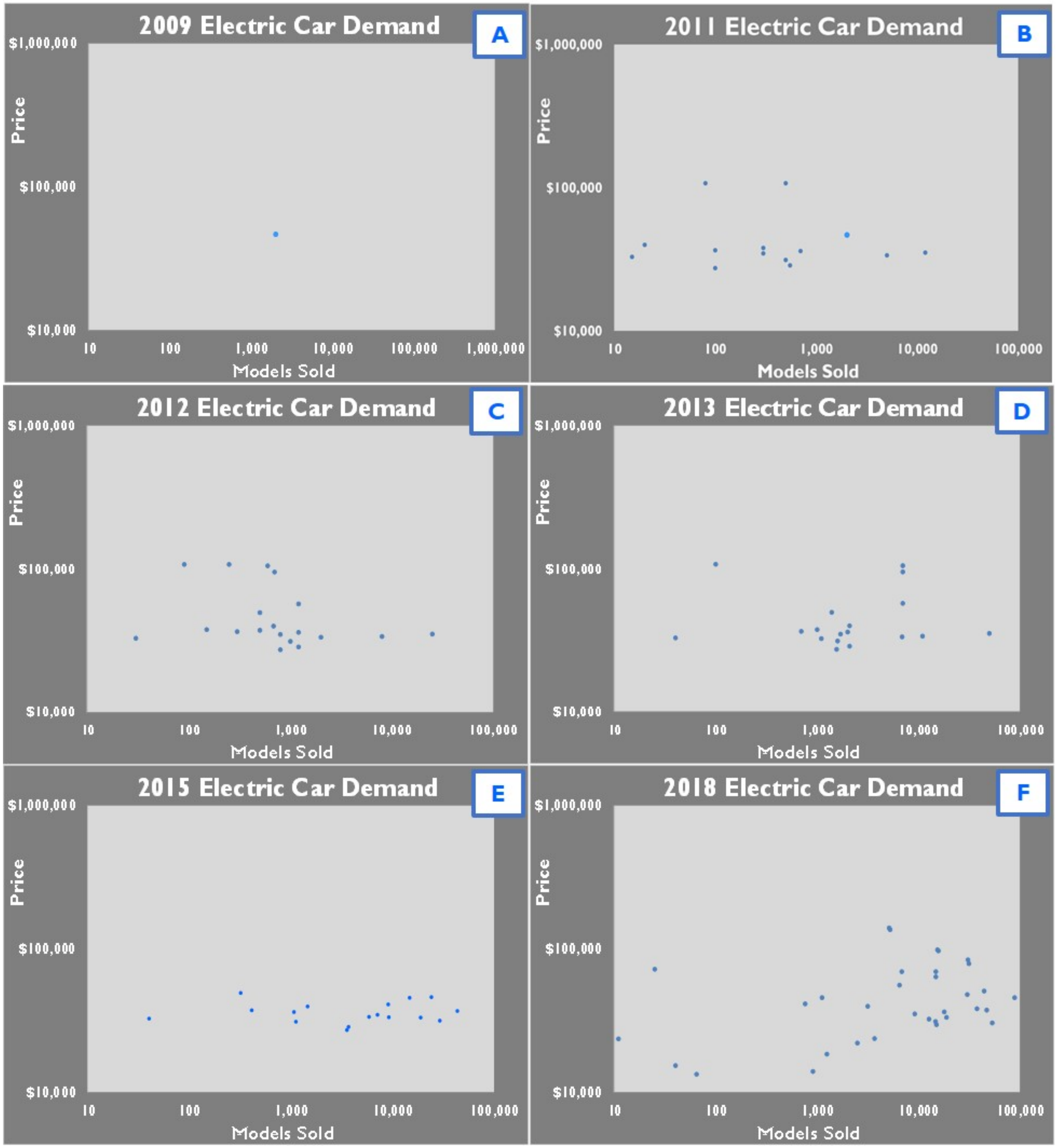
In our modern era, where entirely new markets form constantly, it becomes imperative to gain a sense of how the group behaviors of such entities will evolve. We'll examine that now.

4.0 A Market Begins Anew

After many attempts by countless manufacturers at creating a viable electric car market, a collection of companies new and old began making deliveries newly designed highway-capable models to its customers in 2009. As often happens in capital-intensive endeavors such as this one, the market began with a single competitor, the Mitsubishi i-MiEV, as shown in Figure 8A, which entered the market in 2009 with 2000 units at a price of $47,000.[11,12] At this market's inception, then, there was no huddling behavior, let alone anything like flocking, because we only had the lone entrant. Figuring out what features and price the initial competitor in a market should often is fraught with difficulty, given no current history of this competitive landscape. By 2011, with multiple competitors in the field (Figure 8B), some semblance of order already began to form, as customers bought fewer of the more expensive cars and more of those with less heady price tags. In 2012 (Figure 8C), we had increases in sales in the most popular cars, but only a handful of them sold more than 1,000 of them in a year. The next year, 2013 (Figure 8D), witnessed an increase in the number of models with sales more than 1,000 a year, but fewer models (18) in the market than the prior year (19 models). With 2013 (Figure 8D) we saw a sizeable increase in the number of models with significant sales. The trend to increased sales continued in 2015 (Figure 8E). By the time we made it to 2018 (Figure 8F), we had at least 36 models with sales of ten units or more.

The electric car market, valued at $100 million in 2009, grew to $29 billion by 2018. In the process, the electric car quantities sold, and associated prices market formed loose groupings that defined how the market responded to price changes, and what its sales limits were for a given year.[14] But, how, we should ask, does the electric car market change across its other dimensions?

Figure 8: The modern electric car market began with a single model, the Mitsubishi i-MiEV, in 2009. By 2018, there were at least 36 models for sale. During the same period, total electric car market revenue rose from $100 million to $29 billion per year.



5.0 The Historical Perspective

Figure 9: Ford demonstrated recurring price reductions across millions of Model Ts.

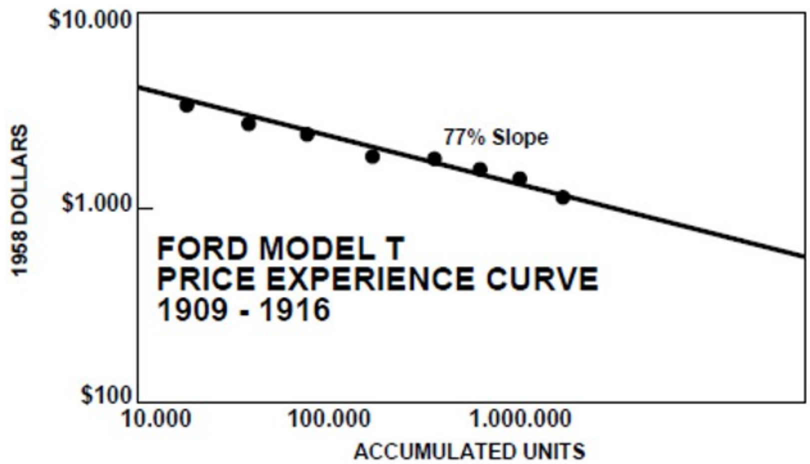
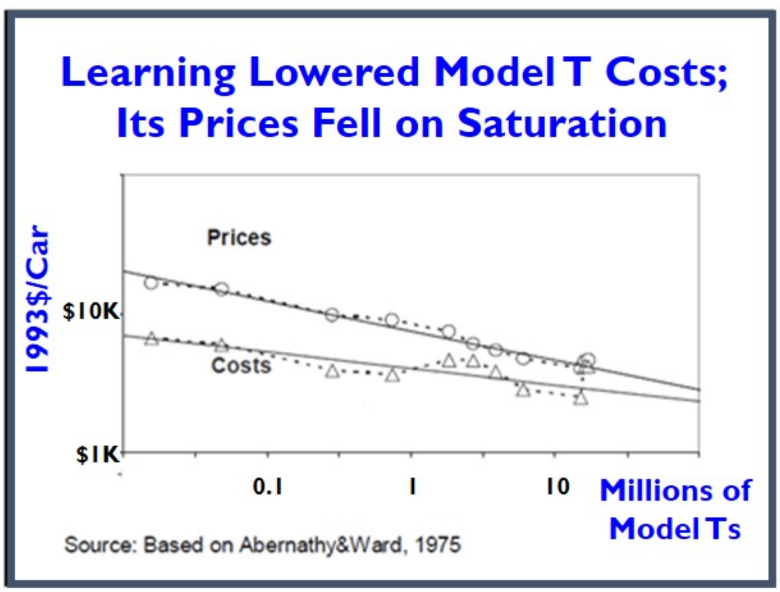


Figure 10: When prices exceeds costs, producers sell at a profit via *sustainable disequilibrium*. Production stops when costs exceed product values.

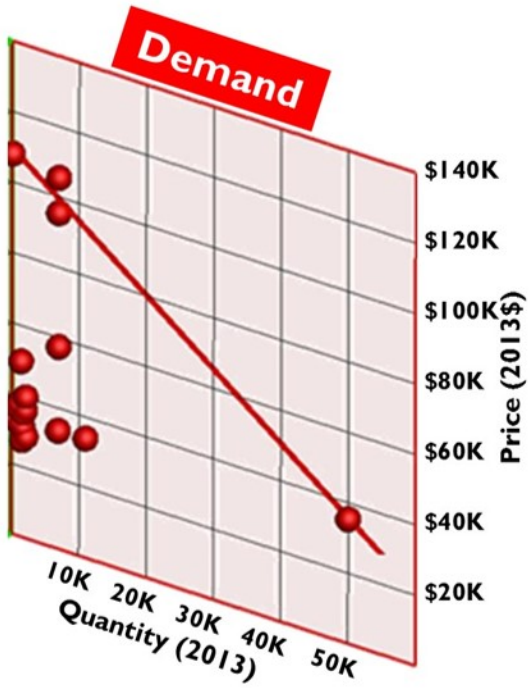


There are a variety of economic forces working in the electric car market, not the least of which is the recurring cost of the models. Current manufacturers do not reveal these figures to the public. If we reach back into history, though, we can get data on one of the most important cars in the industry, albeit one with an internal combustion engine, the Ford Model T. Figure 9 shows how Ford lowered the price of their first best-seller over many years, allowing them to gain millions of customers.[15] Ford managed to make a profit on this line by keeping its costs below its prices. Figure 10 illustrates that the company experienced a learning curve which stayed below their price line for the entire run of the iconic vehicle.[16] Throughout the production run the Model T price, in current dollars, began at $23,005 in 1909 and hit its low point in $3,714 in 1925. Only at the end of its run, after 16.5 million units sold, when costs finally matched prices, did the line come to a halt. Nearly 100 years after the line stopped, it still ranks eighth on the all-time list of the number of cars sold.[17] Generically, we call the price curve for the Model T the product demand curve.

In any production run such as that for the Model T, we can say that when prices exceed costs, that production line is in a state of sustainable disequilibrium, and production can continue if that remains true. Trying to keep any production line going once costs exceed prices is a formula for recurring losses.

6.0 Initial Market Self-Organization

Figure 11: By 2013, the market had a sales limit called a **Demand Frontier**



By degrees and over time, the electric car market began to show signs of self-organization. We get a different view of the 2013 electric car market demand we first saw in Figure 8D in Figure 11 at left in Figure 11. Here, we notice that electric car buyers have already formed a limiting quantity on the number of vehicles they purchased in that year, expressed by the red line, which we call a Demand Frontier. As we will see in our study of the most recent data available, the constant and slope of Demand Frontiers informs producers of the limiting potential of the market not only concerning quantities sold and their prices but also of the amount of revenue available in the various price ranges and settings. By themselves, however, Demand Frontiers do not fully inform the market of the product attributes or features suppliers must offer to be able to gain the prices they hope to fetch. To know that, we need to plot the relationship of those features compared to the vehicle prices.

We can observe how the 2013 electric car features related to their prices in Figure 12. Here we have plotted all the values for range, horsepower and price for all the market entrants in that year as green spheres. When we perform linear regression analysis on that data, we find that the value equation for electric cars in that year was:

(1) 2013 Price = \$6,500 + \$102HP + \$172R + ϵ

Where:

Price = Estimated price in 2013$
HP = Installed horsepower
R = Range in miles
ϵ = Error term

Equation 1 has an adjusted R^2 of 90.4%, a Mean Absolute Percentage

Figure 12: In 2013, electric car value was largely a function of range and horsepower

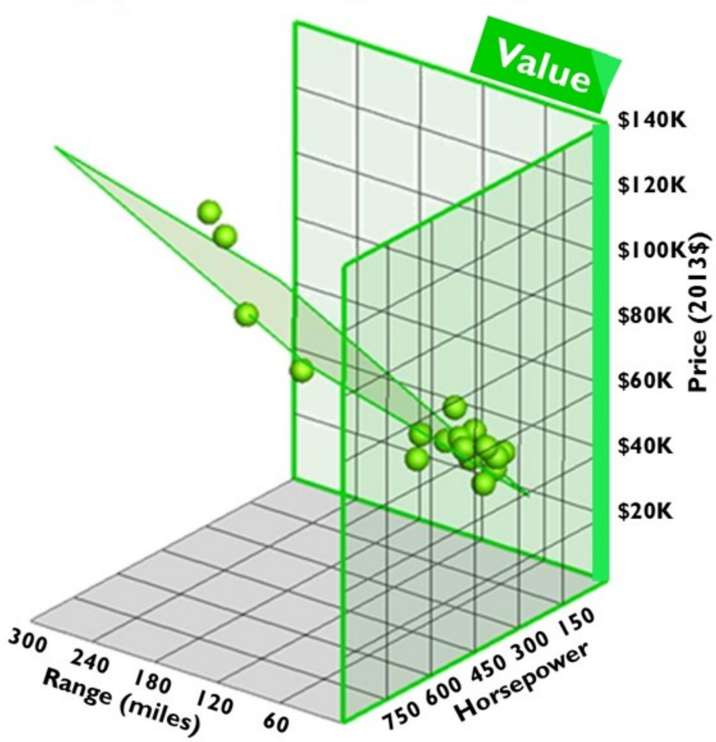
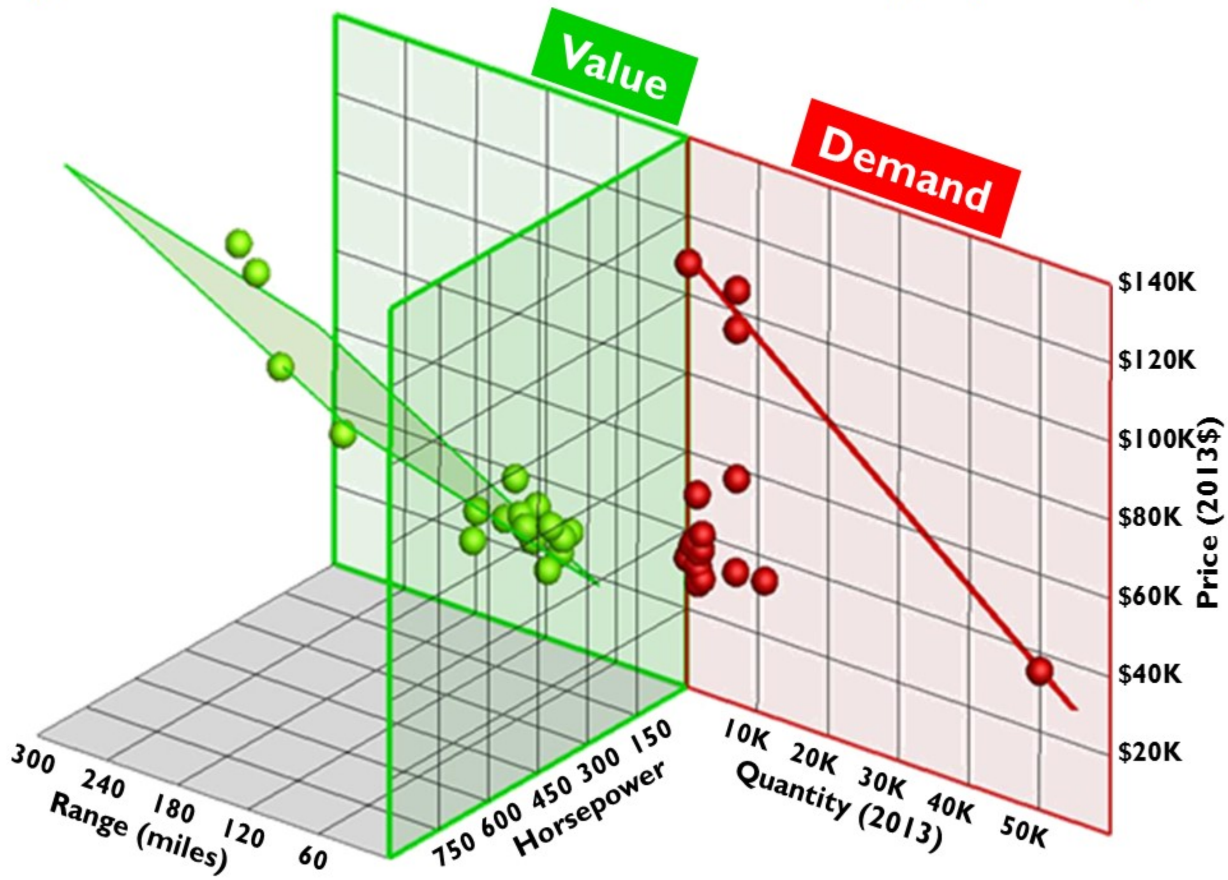


Figure 13: Value and demand elements combine to form ordered quads (Feature 1, Feature 2, Price and Quantity)

Manufacturer	Model	HP	Range miles	2013 Qty	Price
Commuter Cars	Tango T600	805	120	100	$108,000
Tesla	Model S Sig	362	265	7000	$95,400
Tesla	Model S Sig Perf	416	265	7000	$105,400
Nissan	Leaf	110	75	50000	$35,340

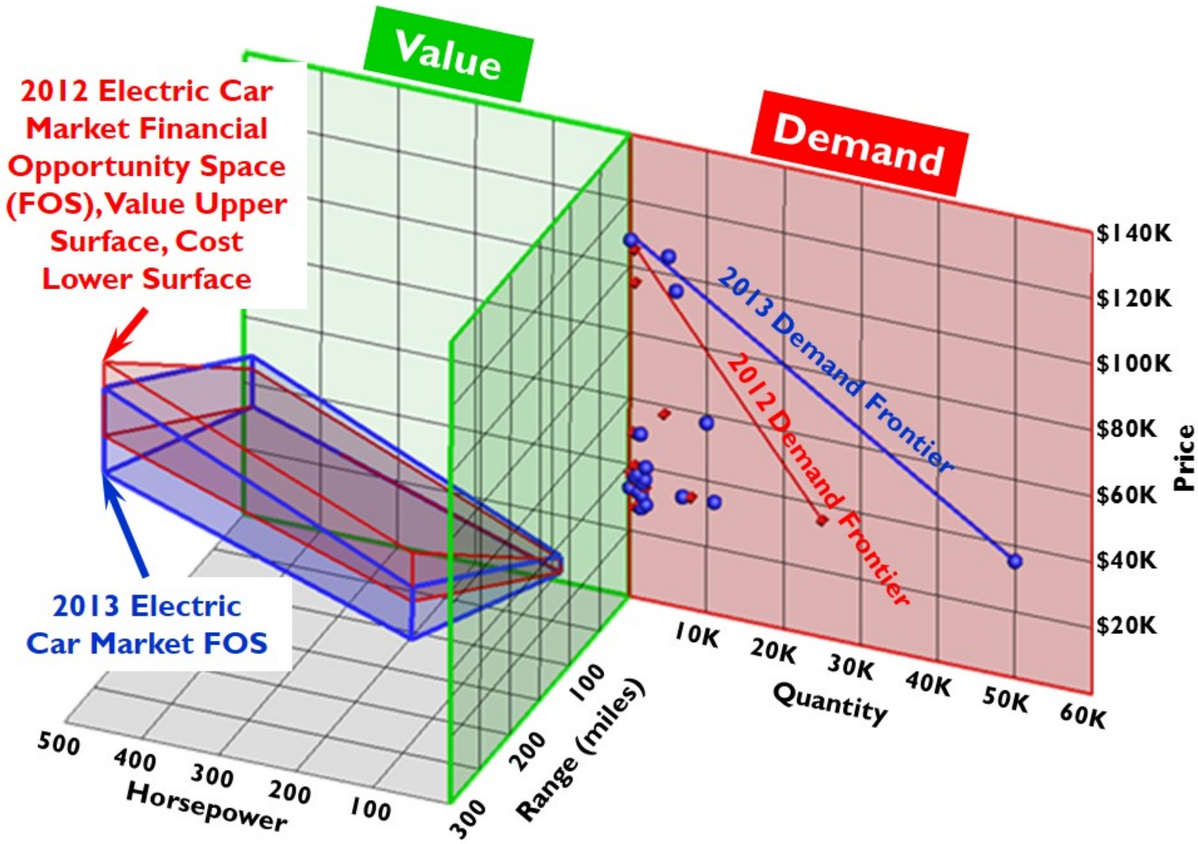
Error (MAPE) of 14.3%, a Standard Error of $8,222 and P-Values of 4.19E-07 and 0.04% for horsepower and range, respectively. If we consider four of the 18 models that made up the simple 2013 electric car database, we get the values in Figure 13. Note horsepower, range and price are the elements addressed in Figure 12, while quantity and price appear in Figure 11. Thus, since Figures 11 and 12 both consider price and are both parts of the same dataset, their individual pieces combine to form Figure 14.

Figure 14: Since 3D Value Spaces and 2D Demand Planes share the price axis, they come together to form 4D structures in which all of the axes are linked, opposing, and nonnegative.



Every point in the left-hand Value Space in Figure 14 has a matching point on the right-hand Demand Plane. All markets work in this way and are influenced by other value factors for which we may account. In this specific case, we have a snapshot in time, showing us what the electric car market looked like at a specific point in time in four dimensions. But, as we witnessed in Figure 8, markets move. We need to add time to our 4D approach and discovered the 5D market actions we have been missing to date.

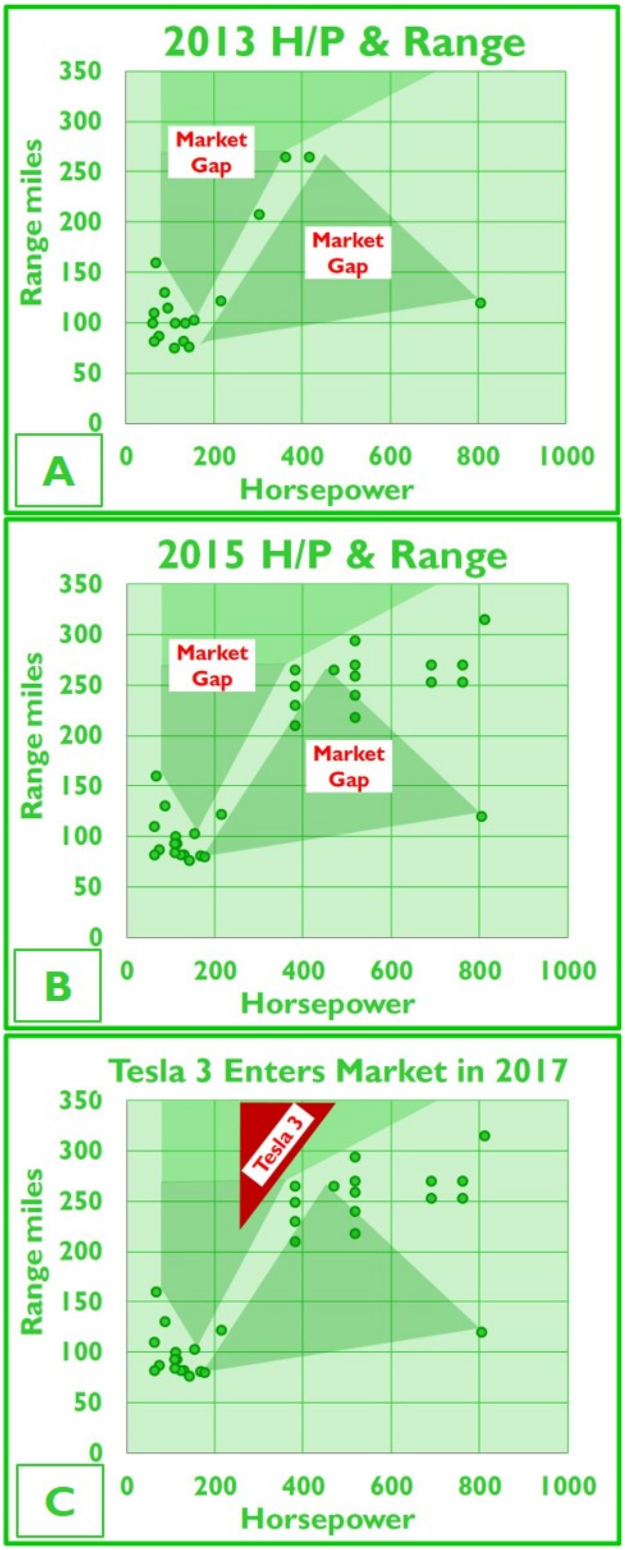
Figure 15: As the Demand Frontier shifts to the right, Value and Cost surfaces move as well. In 2012, there was a Value equation that revealed the sustainable prices for electric cars as a function of their range and horsepower. This is the upper red surface in the left-hand value space. Below that (if producers are making profits) we have the accompanying Cost surface, reflecting how Costs change with added horsepower and range. The space between the upper Value and lower Cost surfaces is called the Financial Opportunity Space (FOS), a region in which manufacturers can make profits. As the market becomes more saturated, the value of horsepower and range falls, along with the costs of producing both. Thus, in 2013, both the Value and Cost Surfaces for electric cars shifted downward, resulting in a new FOS (in blue). Tracking the movement of Financial Opportunity Spaces over time is 5D Economic Trajectory Analysis, or ETA. Knowing where the FOS has been lets us predict where it will be.



Movements such as those depicted in Figure 15 occur across time in all commercial, military and government markets. Open societies, such as those in the United States and Western Bloc countries, routinely post their purchase histories, usually providing details about their buys. Given that analysts can find the specifications of the products purchased, it becomes simply a matter of doing the statistical work to find out how markets move. In many commercial markets, such as the one for electric cars, we can use information available on the internet to create moving maps.

Note on Figure 14 Demand Plane that there are no competitors in the space between $60,000 and $100,000 in 2013. Just as the huddles we previously studied tend to fill up over time, we might wonder if there were space for new competitors in that price range, or in other market gaps.

Figure 16: Market gaps fill up over time



Just as there are market gaps concerning price, the market also creates open spaces about the features on offer to it. Products with the same price can differentiate themselves with difference feature – a two-seat coupe with a lot of horsepower might have the same price as a seven-seat SUV with much less power.

In 2009, the electric car market, with its single entrant, was wide open concerning the open spaces it afforded potential new models. By 2013, though, as shown in Figure 15A, there were many models for sale with a range less than 150 miles and horsepower of less than 200. There then were some competitors with more of each, and in combination with the players in the low end of the market, this created some market gaps relative to those features.

In 2015, Tesla began to offer a wide variety of range and power arrangements, as depicted by the new green dots in Figure 15B that were not there in 2013 with Figure 15A. There were, however, still some significant open regions left, as the shaded areas indicate.

Then, in 2017. Tesla offered three versions of its Model 3, which took up a wide swath of the previously uncontested market space, as we see in Figure 15C. Tesla hit one of the best parts of the electric car market as sales of Model 3 took off. In January 2019, the Model 3 passed the Tesla Model S in sales and became the best selling all-electric car in the process. Its most expensive Model 3, at up to $78,000[18] also fills part of the Figure 14 price gap.

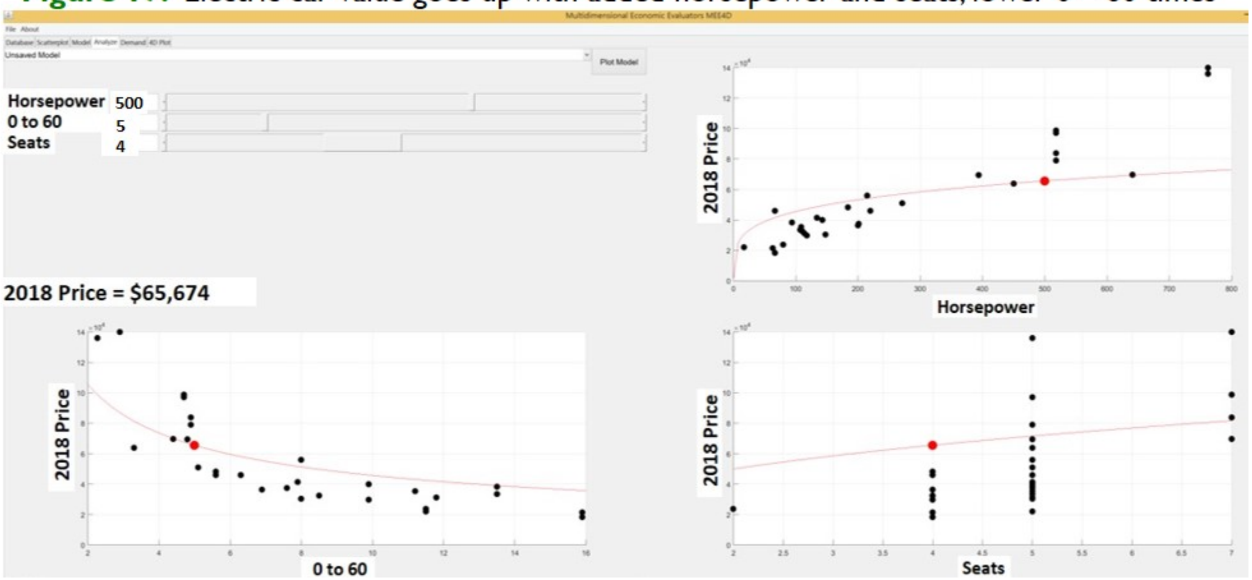
That this new model did so well, while so many new entrants came into the market, it begs the question about market formation. How is the market different than before? Does it respond to the same features? If not, what new features does it value, and in what way?

7.0 The Current Perspective

Looking at the most recent annual data available at the time of this writing, the author created a database of highway-capable electric cars for 2018, included as Appendix B. After paring the data down to remove those models that had less than 100 sales, as well as models priced over $200,000 and as well as those who could not reach 60 miles per hour, the author had a list of 29 viable models (out of 47 researched) from which to analyze their Value and Demand.

Interestingly, the Value proposition changed from 2013 to 2013, as we see in Figure 17.

Figure 17: Electric car value goes up with added horsepower and seats, lower 0 – 60 times



Instead of the linear Value Estimating Relationship (VER) of Equation 1 for 2013, in 2018, we can describe our best VER as this power equation:

$$(2)\ \text{2018 Price} = 21{,}500 * HP^{0.227} * 0to60^{-0.520} * Seats^{0.392} * \epsilon\ \text{(incl Ping Factor)}$$

Where:

Price = Estimated price in 2018$
HP = Installed horsepower
0 to 60 = Best time from 0 to 60 miles per hour, in seconds
Seats = Maximum seating capacity of each model
ϵ = Error term
Ping Factor = Multiplicative factor applied to remove downward bias upon constant[19]

Equation 2 has an adjusted R^2 of 88.2%, a MAPE of 14.5%, and P-values of 0.68%, 0.05% and 3.22% for horsepower, 0 to 60 time and number of seats, respectively, while its standard error is

\$11,584. Note that range was not part of the Value assessment in Equation 2; this is because horsepower largely went up in concert with their range for most of the vehicles in the study.

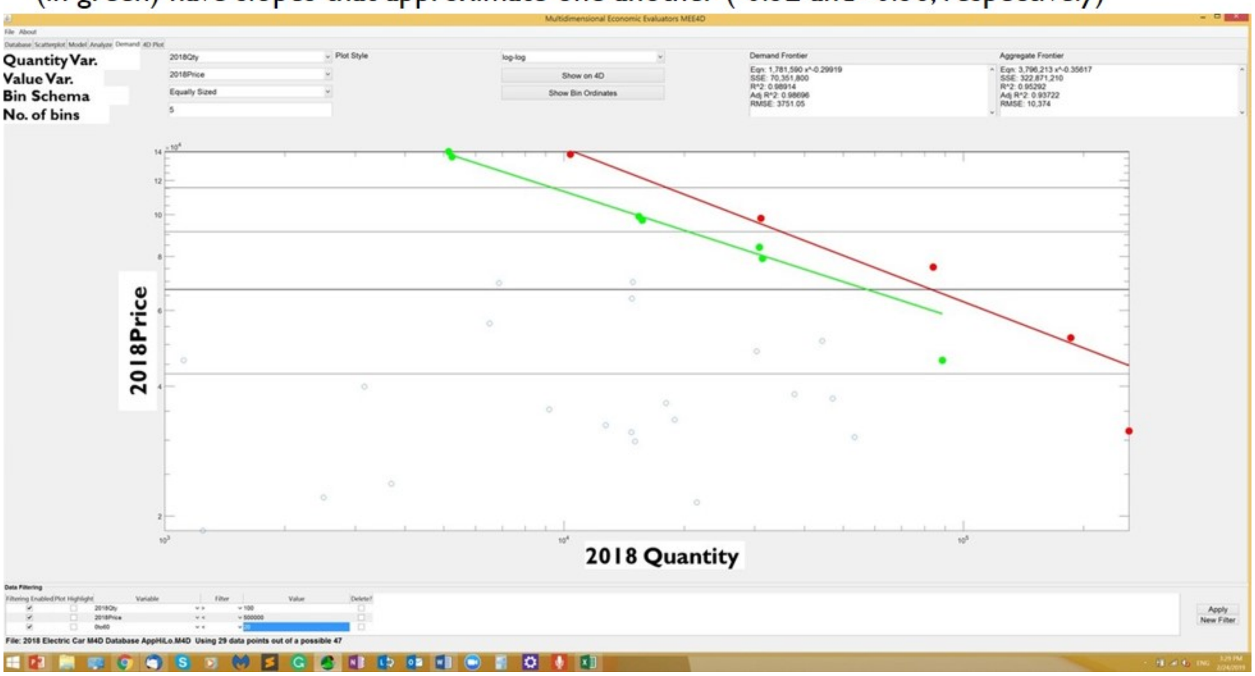
We can dive deeper into Equation 2 to examine its meaning piece by piece. For example, we could isolate the influence of the number of seats on the total value of electric cars. That is, we could take that component value expression, $\text{Seats}^{0.392}$, and find out its multiplicative contribution to value by putting in every whole number possibility in its range, as we do in Figure 18 (note, there is one single seat electric car in the database, the Electra Meccanica Solo[20], but it had less than 100 sales in 2018 and I filtered out of the analysis). Note that going from one seat to two increases value by 31%; moving from one to seven more than doubles the sustainable price. But, at the same time, costs go up for vehicles with more capacity, while demand goes down. Estimators must account for all such forces when studying markets.

Figure 18: Impact of seats on value

Seats	Factor
1	1.00
2	1.31
3	1.54
4	1.72
5	1.88
6	2.02
7	2.14

Opposing the Value proposition is the linked force of Demand, which we can portray in two different ways. We see these methods in Figure 19.

Figure 19: The 2018 electric car Aggregate Demand (in red) and Demand Frontier curves (in green) have slopes that approximate one another (-0.32 and -0.36, respectively)



The market aggregate demand curve, shown as the red line, is given by Equation 3:

(3) $2018\ \text{Price} = 3{,}796{,}000 * \text{Qty}^{-0.356} * \epsilon$ (incl Ping Factor)

Where:
Price = Estimated price in 2018$
Qty = Aggregated quantity for each bin (as explained below)
ϵ = Error term
Ping Factor = Multiplicative factor applied to remove downward bias upon constant

The adjusted R^2 for Equation 3 is 95.3%. The market aggregate demand curve represents the line of best fit through a series of red points determined by binning. It indicates where the bulk of the revenue lies within the market: slopes > -1.0 have more revenue in their lower bins, the opposite is true for slopes < -1.0. Bins may be equally sized concerning price or sequenced by geometric or Fibonacci series such that upper-priced bins have the fewest number of observations and grow accordingly as price ranges fall. Each red bin point represents the total quantity and average price in that bin; users select the number of bins (ranging from 3 to 10).

We use the green line to display the Demand Frontier, described with Equation 4:

$$(4)\ 2018\ \text{Price} = 1{,}781{,}600 * \text{Qty}^{-0.299} * \epsilon\ (\text{incl Ping Factor})$$

Where:
Price = Estimated price in 2018$
Qty = Frontier quantity
ϵ = Error term
Ping Factor = Multiplicative factor applied to remove downward bias upon constant

For Equation 4, the adjusted R^2 is 98.9%. The Demand Frontier represents the outer sales limit of the market in a given period.

Note that slopes of the Aggregate Market Demand (-0.32) and Demand Frontier (-0.30) are very nearly identical, a trait normally found in mature markets. The constituent models that make up the demand in the market in 2018 are found in Figure 21, below.

Figure 20 offers more insight into the workings of Equation 2. In Figure 20A, with the number of seats set to two, the surface describing the Value Estimating Relationship (VER) given by Equation 2 slopes up as horsepower increases, telling us the market is willing to pay suppliers who offer more of a good feature. Note in the same graphic that Value goes up as 0 to 60-time falls; this means buyers additionally value less of a negative feature. No one wants to take forever to get up to speed. Another bad attribute, when considering all sorts of automobiles, is pollution. The added value electric car buyers place on clean-running machines reflects their willingness to pay more for less pollution.

Figure 20B reinforces what we learned in Figure 18. As we move from two-seat machines to those SUVs with seven seats, the value or sustainable price for the models with more seats rises in all market regions.

Figure 20: We use Equation 2 to plot the Value of 2018 Electric cars as a function of 0 to 60 time, horsepower and seats. In A, we set seats to 2, with B, seats = 7.

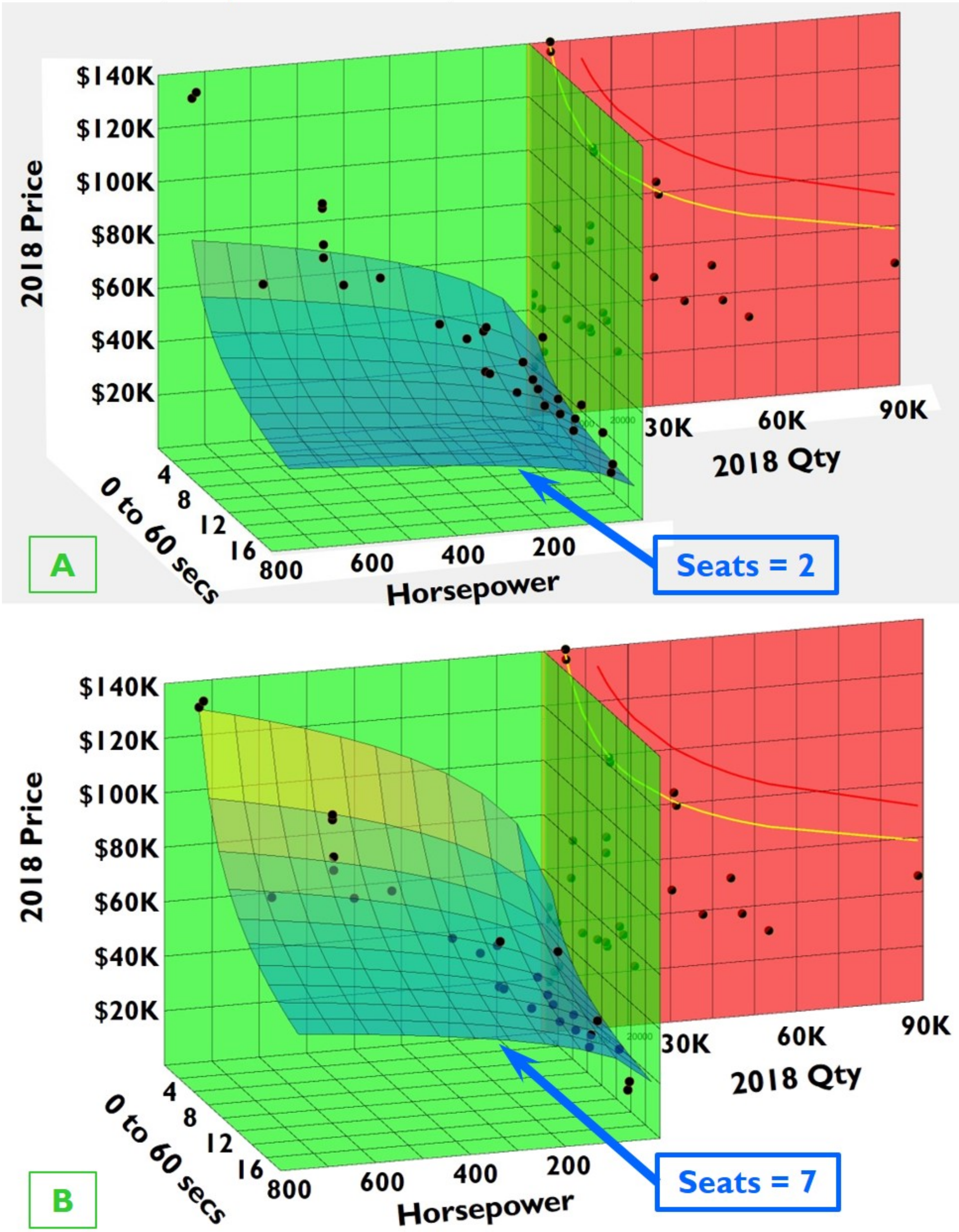


Figure 21: In less than a decade, the electric car market demonstrated self-organizing market behavior with respect to Value (at left) and Demand (at right). The opposing forces of Value and Demand share the price axis, forming systems of order quads, shown generically as (Valued Feature 1, Valued Feature 2, Price, Quantity). The Tesla Model X P100D ordered quad (762, 2.9, $140,000, 5,143) is one example in the database of 47 electric car models.

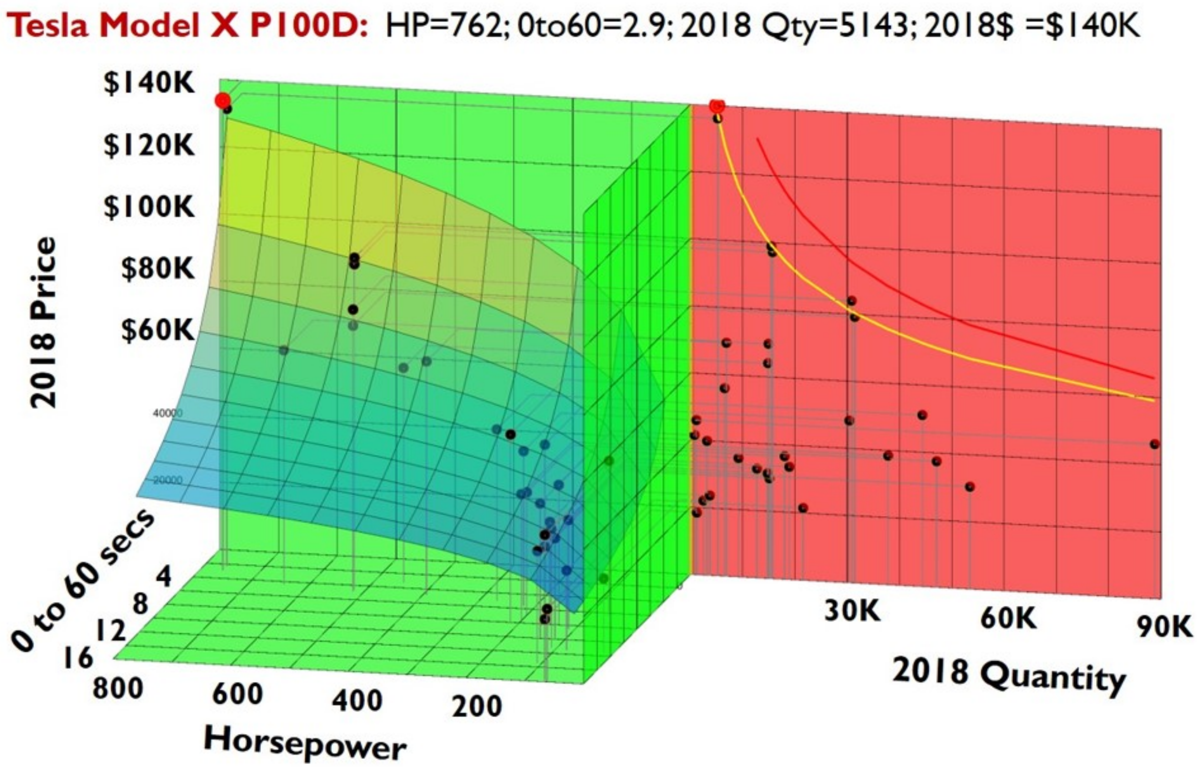


Figure 21 turns the previous chart about the price axis. Note on the right-hand side we can see the Demand Frontier (in yellow), just inside of the red curve that is Aggregate Market Demand. Note that every point on the right-side Demand Plane connects to its counterpart in Value Space through Point Line, which shows how each side is bound to the other for every element in the market. As a specific case, we have the Tesla Model X P100D highlighted as opposing, linked, nonnegative red spheres near the top of the chart: a very powerful seven-seat SUV with a fast 0 to 60 time, it tops the mass-production market. Observing this 4D display, we find Value Space demonstrates swarming behavior and that the Demand Plane has loose, but increasingly tightening huddling.

8.0 Summary

Seemingly primitive avian activities find their reflections in human behavior in games, defensive postures, and markets. Markets demonstrate huddling on Demand Planes and swarming in Value Spaces. Far from being separate activities, these opposing forces operate across linked, nonnegative, coordinate systems at all times in all markets. New markets eventually display these phenomena given enough time. Analysts can characterize and track such movements over time to predict market shapes in the future, which demands the study of Value, Cost and Demand.

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Appendix A: Quantities and prices for 52 general aviation aircraft. Quantity data from the General Aviation Manufacturer's Association, prices from company websites. Current figures will likely vary. Data supplied so that readers might be able to recreate the analysis.

Aircraft	2004-13 Quantity	Price (2013$)	Aircraft	2004-13 Quantity	Price (2013$)
AT-401	79	$265,000	MX-7-180	170	$118,000
AT-402	115	$485,000	M20M TLS	23	$400,000
AT-502A	215	$595,000	M20R OVATION	380	$350,000
AT-602	76	$834,000	M20S EAGLE	184	$305,000
AT-802/A	88	$985,000	EMB-202	190	$225,000
A-1 HUSKY	302	$135,000	PC-6	27	$1,200,000
S-2B/C	119	$192,000	6X	427	$336,000
B-2	2	$170,000	6XT	266	$356,000
172	2481	$155,000	ARCHER II/III (PA-28-181)	565	$182,000
206/T206	956	$345,000	ARROW (ALL PREVIOUS)	44	$232,000
CARAVAN 675	120	$1,500,000	MALIBU MERIDIAN	359	$1,810,000
CARAVAN I 208B	655	$1,500,000	MALIBU MIRAGE PA-46-350P	331	$880,000
SKYLANE 182	1734	$255,000	SARATOGA SP (PA-32-301)	318	$415,000
SR20	640	$210,000	SEMINOLE (PA-44-180)	128	$362,000
SR22	2080	$290,000	SENECA III/IV(PA-34-220T)	92	$505,000
115/115TC	187	$388,000	WARRIOR II/III(PA-28-161)	174	$155,000
TB 10/10GT TOBAGO	202	$134,000	M-18 DROMADER	75	$270,000
TB 20/20 GT TRINIDAD	322	$201,000	PZL-106BT	7	$325,000
TB 200/200 GT	183	$148,000	58 BARON	329	$1,000,000
TB 21/21 GT TRINIDAD TC	155	$225,000	A36	352	$600,000
TB 9/9 GT TAMPICO	102	$132,000	F406	5	$2,600,000
EMB-810D SENECA (CIVIL)	27	$370,000	F406 MARK II	28	$2,600,000
LA 250/270 RENEGADE	66	$393,000	660 TURBO-THRUSH	94	$650,000
M/MT/MX/MXT-7-420	10	$450,000	P.68 (ALL PISTON)	186	$485,000
M/MT-7-260	180	$152,000	ZLIN 143L (CIVIL)	56	$95,000
M-7-235	183	$135,000	ZLIN 242L	169	$120,000

Appendix B: Here is the 2018 Electric Car Database, less those vehicles for which sales quantities could not be discovered. Quantity and price data comes from several sources; specifications come from the manufacturers. Models with a 0 to 60 time of 100 seconds did not post their time for this run, or cannot attain 60 miles per hour. Tesla does not break out the details of sale for each model; the author had to make allocations.

Manufacturer	Model	HP	Rmiles	2018Price	2018Qty	Seats	0to60	TopMPH
BMW	i3	184	114	48300	30369	4	5.600	130
Bolloré	Bluecar	67	160	46000	1116	4	6.300	81
BYD Auto	e6	215	186	56000	6508	5	8.000	87
Chery	QQ3 EV	57	120	9600	10221	4	100.000	62
Chevrolet	Bolt EV	200	238	36600	18019	4	6.900	91
Citroen	C-Zero	67	93	18500	1247	4	15.900	81
Electra Meccanica	Solo	82	100	15400	40	1	8.000	80.6
Fiat	500e	111	87	32500	12729	4	8.500	88
Ford	Focus Electric	143	115	40000	3168	5	9.900	84
Girfalco	Azkarra S	301	124	72200	25	2	2.500	148.8
Hyundai	Ioniq Electric	118	124	29800	15076	4	9.900	115
Hyundai	Kona Electric	201	258	37500	47090	5	7.600	104
JAC Motors	JAC J3 EV	17	105	22100	2500	5	11.500	80.6
Jaguar Land Rover	I-Pace	394	234	69500	6877	5	4.800	124
Kewet	Buddy	17	93	13400	65	3	100.000	50
Kia	Soul E/V	109	93	35400	9200	5	11.200	90
Mahindra	e-Verito	41	112	14000	913	5	100.000	53
Mercedes Benz	B-Class	134	124	41500	764	5	7.900	99
Mitsubishi	i-Miev	66	106	23700	11	4	13.000	81
NIO	ES8	641	222	69800	14885	7	4.400	111.6
Nissan	Leaf	148	170	30500	53455	5	8.000	93
Peugeot	Ion	63	93	21546	1651	4	15.900	81
Renault	Fluence ZE	94	115	38400	37782	5	13.500	84
Renault	Zoe	107	250	33500	18931	5	13.500	84
Rimac	Concept Two	1888	402	2298000	150	2	1.850	258
Smart	ED	80	58	23800	3696	2	11.500	78
Tesla	Model 3 Long Range	271	310	51000	44384	5	5.100	140
Tesla	Model 3 Mid Range	220	264	46000	88768	5	5.600	124
Tesla	Model 3 Performance	450	310	64000	14795	5	3.300	155
Tesla	Model S 100D	518	295	97200	15726	5	4.700	155
Tesla	Model S 75D	518	237	79200	31452	5	4.900	140
Tesla	Model S P100D	762	381	136200	5242	5	2.275	155
Tesla	Model X 100D	518	295	99000	15428	7	4.700	155
Tesla	Model X 75D	518	238	84000	30857	7	4.900	140
Tesla	Model X P100D	762	289	140000	5143	7	2.900	155
Volkswagen	E-Golf	114	186	31300	14753	5	11.800	86