A Second Generation Upgrade to Anderlohr's Retrograde Method for Broken Learning

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1 Abstract

It is not uncommon for cost estimators to confront a situation where particular production efforts will suffer a break in the production line. One of the most well-known methods for estimating the impact of a production break is known as the Anderlohr Retrograde Method. This paper explores the weakness associated with this method and describes the most common solution. It then discusses the weakness of the common solution and proposes a more robust solution.

A fundamental understanding of Learning Curves (Cost Improvement Curves) is a pre-requisite for understanding the arguments offered in this paper. An understanding of the Anderlohr Retrograde Method is also helpful.

2 Introduction

It is not uncommon for cost estimators to confront a situation where particular production efforts suffer a break in the production line. Production processes can be interrupted for a variety of reasons, but in the Department of Defense (DoD) acquisition community, procurement rules can impose such a situation. One such rule is known as the Bona Fide Need Rule. A description of this rule can be found in the Defense Acquisition University (DAU) Acquisition Encyclopedia (Bona Fide Need, 2018). It states:

> *The Bona Fide Need rule (law) requires appropriated funds be used only for goods and services for which a need arises during the period of that appropriation's availability for obligation.*

The DAU Encyclopedia provides several examples, or applications, of this rule that set the stage for the problems addressed in this paper. One such application reads as follows:

> *Supply items: Generally, bona fide need is determined when the government actually requires (i.e., will be able to use) the supplies being acquired.*

In other words, the Bona Fide Need Rule prohibits the procurement of assets in advance of need. In situations where production rates significantly outpace deployment rates, this rule can create challenges for the acquisition strategy. In some cases, the production rate can be slowed to align with deployment rates, but in other cases this is not possible and breaks in the production line become inevitable.

Learning Curves have been the topic of a number of studies. Analysts use Learning Curves to model the efficiencies gained throughout a given production process. It is believed that when a production processes is interrupted, some of the gained efficiencies will be lost. This phenomena is known as Broken Learning.

Over the years, several techniques have been proposed to deal with the problem of Broken Learning. One of the most common techniques used to model the magnitude of this loss was developed by George Anderlohr in an article entitled "What Production Breaks Cost" (Anderlohr, 1969). In his article, Mr. Anderlohr suggested that there are five elements of corporate learning that contribute to the magnitude of lost efficiency due to a production break.

1. Personnel Learning – the amount of learning lost to either personnel actually forgetting work procedures due to the break in production and/or the hiring of untrained replacements as employees transition to other jobs
2. Supervisory Learning – the loss resulting from the transfer of supervisors, limited knowledge of new hires and/or the reduced guidance they furnish because of lost familiarity with the job
3. Continuity of Production – the physical establishment of production lines, the position adjustment for optimal working conditions and work in progress build-up
4. Methods – the rerouting of operations due to in-plant changes since the last production lot. At times entirely different facilities will be used as the original one has been rededicated.
5. Special Tooling – replacement of modified tools and the effect of transition time.

The technique used to incorporate these elements into the loss of learning calculation will not be discussed in this paper. There are, however, theoretical and practical limitations to the Retrograde Method that this paper seeks to resolve. These weaknesses manifest themselves in circumstances where the learning slope prior to a production break cannot be assumed to be equal to the learning slope after said break.

This paper will introduce a technique that is mathematically pure and relieves the analyst of the assumption of equal learning slope prior to and after a production break.

3 Problem Description

The Anderlohr Retrograde Method calculates the efficiencies (or learning) achieved prior to the production break and then estimates the portion of those gained efficiencies that will be lost as a result of the break in production (or lost learning). Again, the intention of this paper is not to explain the Retrograde Method, so a simple illustration should adequately set the stage for the problem this paper seeks to resolve.

Figure 1: Continuous Production, below, illustrates a Learning Curve over 20 units with a First Unit taking 100 hours and a slope of 80 percent.

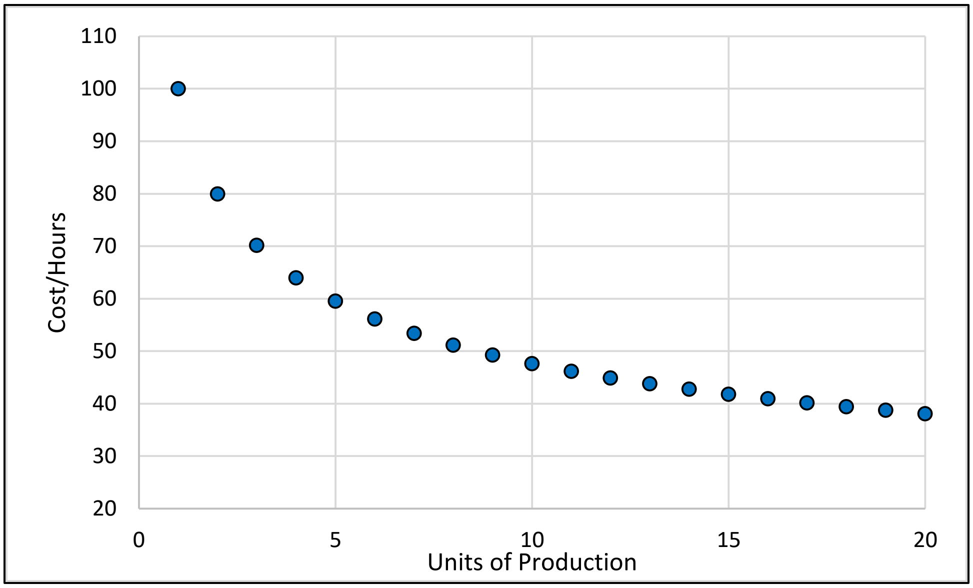

Figure 1: Continuous Production

Without showing the calculations, the Retrograde Method suggests that with a break in the production line at unit 10 and a 45 percent loss of the achieved learning, the first unit following the break (the 11^{th} unit overall) would cost the equivalent of the third unit from the original curve. This is illustrated in Figure 2: Lost Learning, below.

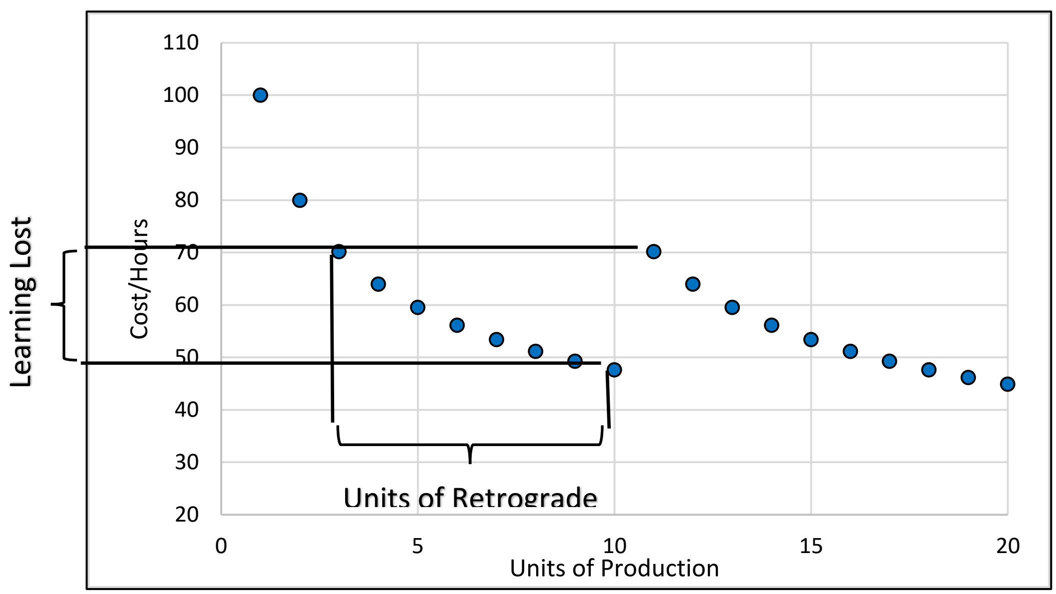

Figure 2: Lost Learning

In this example, the equivalent of seven (7) units' worth of learning are lost. The equation for the units after the break becomes:

$$UC_X = UC_{(X-K)};\ 11 \leq X \leq 20 \qquad (1)$$

Where

UC = Unit Cost
X = X^{th} production unit
K = Number of retrograde units + 1

The right-hand side of equation (1) can be written in the algebraic form

$$UC_X = A(X-K)^b;\ 11 \le X \le 20 \qquad (2)$$

Where

A = Theoretical First Unit
b = ln(slope)/ln(2)

The problem with this method is that it doesn't correctly handle a change in learning slope after the break. Say, for example, that after the break the learning slope changes from 80 percent to 90 percent. In this case the math yields the results illustrated in Figure 3: Retrograde Method with Change in Slope, below, where the cost of the 11^{th} unit does not equal the cost of the 3^{rd} unit from the pre-break curve.

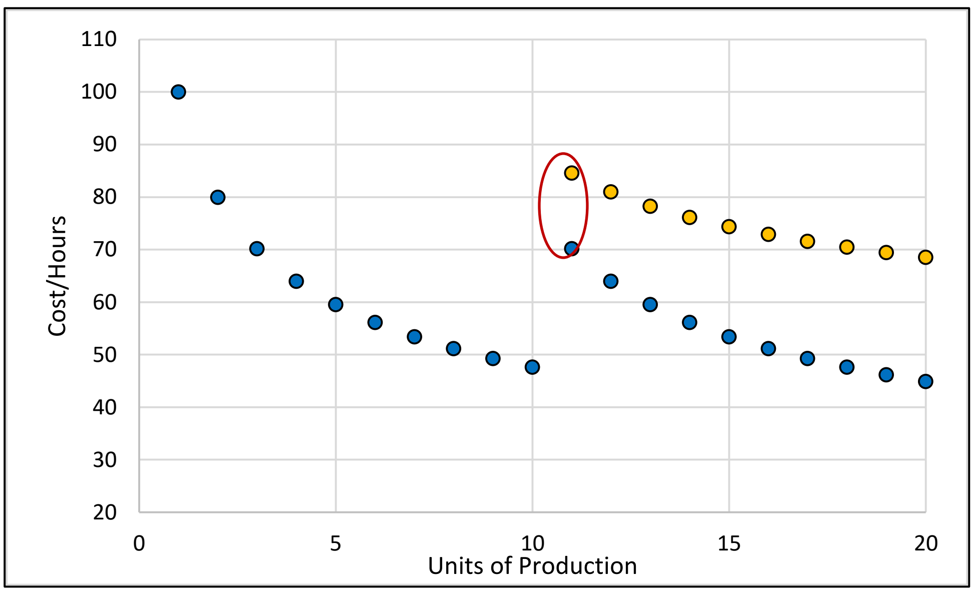

Figure 3: Retrograde Method with Change in Slope

It should be noted that the post-break curve continues down a 90 percent learning slope just as it should, but the cost/hours of each unit are too high because the technique for calculating the starting position is wrong.

The math that accompanies this example is straight forward. Under the original scenario (no change in learning slope) the cost of the 11^{th} unit is calculated as:

$$UC_{11} = 100 * (11 - 8) \wedge (-0.322) = 70.21$$
Where:
$$Ln(0.80) / Ln(2) = -0.322$$

While the calculation given the new scenario (a change in learning slope) is:

$$UC_{11} = 100 * (11 - 8) \wedge (-0.152) = 84.62$$

Where:

$Ln(0.90) / Ln(2) = -0.152$

There are a variety of reasons that an analyst might conclude that the slope of the post-break curve may be different from that of the pre-break curve. For example, some programs may determine to open up the follow-on procurements to a competitive bidding process which can certainly affect the rate of learning. Within the Department of Defense (DoD), procurement will sometimes shift from an acquisition organization to a sustainment organization. Because these different organizations have different relationships with different contractors and because they often have different procurement processes, the resulting cost improvement curves may be different.

The most common solution to this problem is both simple and elegant: release the constraint that the post-break equation share the same TFU as the pre-break equation. Allowing the post-break equation to have a different TFU implies that both equations are treated as distinct curves which share a common value; namely

$$UC_{1,\ F\text{-}K} = UC_{2,\ F} \qquad (3)$$

Where:

F = First unit after the break

K = Number of retrograde units + 1

$UC_{1,\ F\text{-}K}$ *= the Unit cost of the* $(F - K)^{th}$ *unit from the pre-break curve (curve #1)*

$UC_{2,\ F}$ *= The Unit Cost of the* F^{th} *unit from the post-break curve (curve #2)*

In this paper's example, the interpretation is that the unit cost of the 11^{th} unit on post-break curve ($UC_{2,11}$) equals the Unit Cost of the 3^{rd} unit on the pre-break curve ($UC_{1,\ 11-8}$ or $UC_{1,\ 3}$).

$$UC_{1,\ 11\text{-}8} = UC_{2,\ 11} \qquad (4)$$

The general constraint from (3) allows us to solve for the TFU of the post-break equation and thus define the general post-break curve.

By definition, equation (4) can be rewritten as

$$A_1 (F - K)^{b1} = A_2 F^{b2} \qquad (5)$$

Solving for A_2 yields the solution:

$$A_2 = A_1 (F - K)^{b1} / F^{b2} \qquad (6)$$

The equation for the post-break curve becomes:

$$UC_{2,\ X} = (A_1 (F - K)^{b1} / F^{b2})\ X^{B2} \qquad (7)$$

As illustrated in Figure 4: Adjusted TFU on Post-Break Equation, below, the first unit of the post-break curve equals the third unit from the pre-break curve and then continues down a 90 percent learning slope.

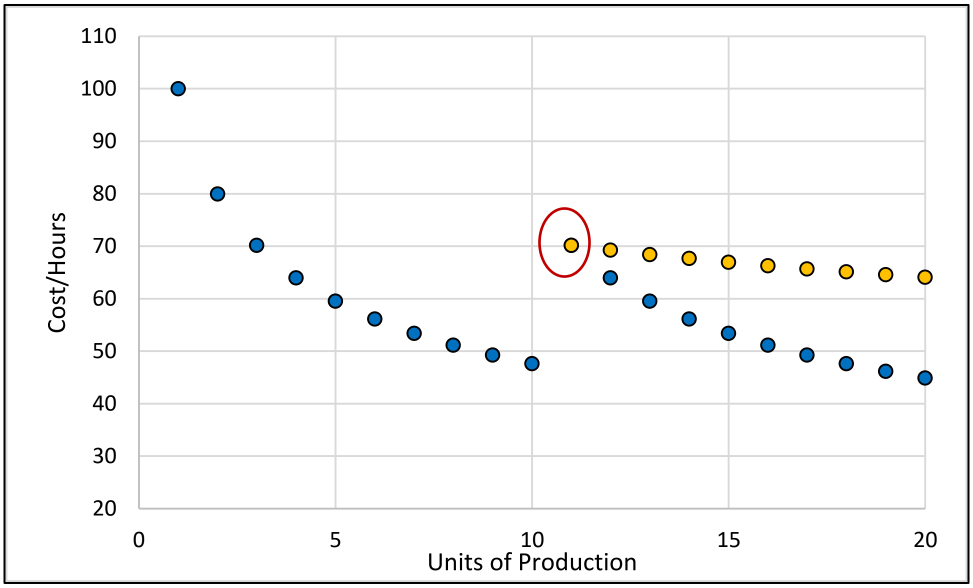

Figure 4: Adjusted TFU on Post-Break Equation

While elegant and simple, this solution suffers from an additional problem for some analysts. To illustrate the problem, let's look more closely at the example. If, in the example, the slope of the post-break curve is set to 80 percent (i.e. no change in the slope prior to, and following, the break) some analysts suggest that the post-break curve ought to mirror/reproduce the Anderlohr Retrograde method. As illustrated in Figure 5: Retrograde vs Solution #1, below, this is clearly not the case.

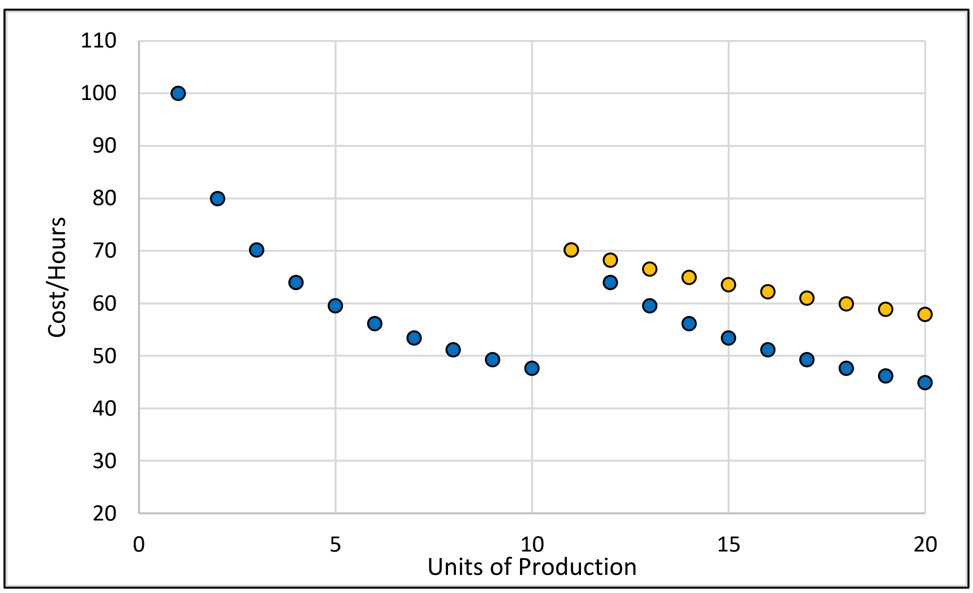

Figure 5: Retrograde vs Solution #1

Because, in our example, the first unit after the break is the 11th unit the learning slope continues from there as an extension from the 11th unit and not from the equivalent of the 3rd unit, as Anderlohr would require.

In general, while

$UC_{1,\,F\text{-}K} = UC_{2,\,F}$ (8)

where:

F = First unit after the break

K = number of retrograde units + 1

the troubling corollary is also true when the learning slopes are not the same:

$UC_{1,\,X\text{-}K} \neq UC_{2,\,X};\ X > F$ (9)

One way to state the problem, then, is that all else being equal, the rate of learning at the first unit after the break does not equal the rate of learning at the corresponding unit prior to the break. For our example, the rate of learning at the 11th unit does not equal the rate of learning at the third unit.

This paper offers a solution to this problem.

4 Solution

The problem is clear: The rate of change at the first unit after the production break, does not equal the rate of change at the equivalent unit prior to the production break.

In mathematics, rate of change translates to derivative. So the problem can be restated to say: The derivative at the first unit after the break does not equal the derivative at the equivalent unit prior to the break.

Two conditions will help us develop the solution to the problem. The first condition is inherited from (3) and sets the unit cost of the first post-break unit equal to that of the equivalent pre-break unit.

$UC_{1,\,F-K} = UC_{2,\,F}$ (10)

The second condition sets the derivative of post break equation at the F^{th} unit equal to the derivative of the pre-break equation at the $(F – K)^{th}$ unit.

$UC'_{1,\,F-K} = UC'_{2,\,F}$ (11)

With these two conditions, A_2 and b_2 can be calculated by solving a series of equations. The calculations are provided in Appendix A and yield the following solutions:

$A_2 = A_1 (F\text{-}K)^{b1} / F^{(b1 * F / (F-K))}$ (12)

$b_2 = b_1 F / (F – K)$ (13)

Figure 6: Retrograde vs Solution #1 and Solution #2, below, illustrates this solution.

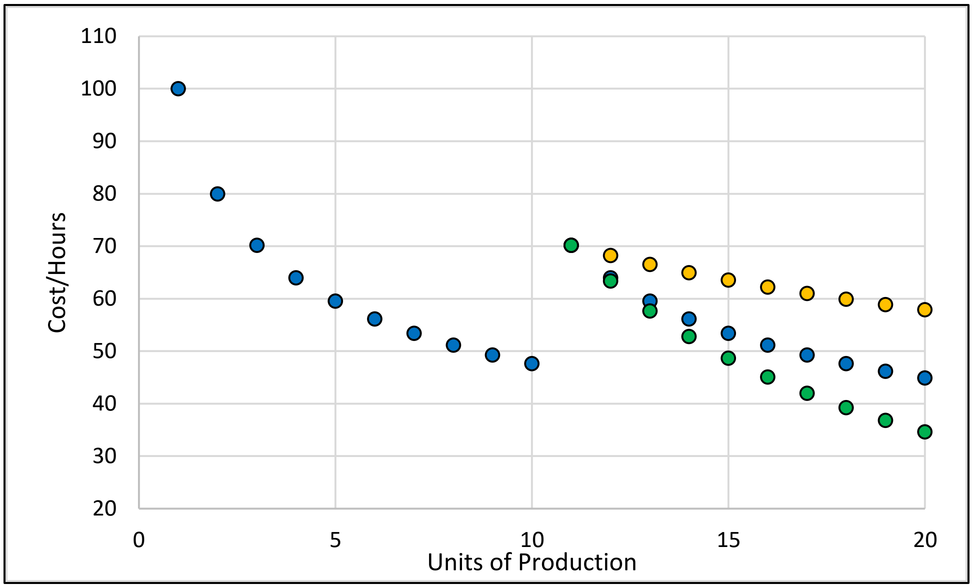

Figure 6: Retrograde vs Solution #1 and Solution #2

From the Initial conditions of the example, it is established that:

$$A_1 = 100$$
$$b_1 = Ln\ (0.80) / Ln\ (2) = -0.3219$$

Substituting A_1 and b_1 into equations (12) and (13) yields

$$A_2 = 100 * (11 - 8)^{-0.3219} / 11^{(-0.3219 * 11 / (11 - 8))} = 1{,}190.32$$
$$b_2 = -0.3219 * 11 / (11 - 8) = -1.1804$$

With these parameters it can be shown that the solution satisfies the condition established in (10)

$$UC_{1,\,3} = 100 * 3^{(-0.3219)} = 70.21$$
$$UC_{2,\,11} = 1{,}190.32 * 11^{(-1.1804)} = 70.21$$

It can also show that the solution satisfies the condition established in (11)

$$UC'_{1,\,3} = A_1 b_1 (F - K)^{(b1 - 1)} = 100 * (-0.3219) * 3^{(-1.3219)} = -7.534$$
$$UC'_{2,\,11} = A_2 b_2 F^{(b2 - 1)} = 1{,}190.32 * (-1.1804) * 11^{(-2.1084)} = -7.534$$

5 Conclusion

The weaknesses with the Anderlohr Retrograde Method is that a change in the post-break learning slope results in a disconnect between the cost/hours of the first unit following the break and the equivalent pre-break unit. The common solution to this problem treats the pre-break and the post-break equations as distinct curves and calculates a TFU for the post-break curve that yields the same cost/hours for the first post-break unit at its pre-break equivalent.

While this method reliably aligns the corresponding pre and post-break unit costs/hours, the resulting slope, or rate of improvement, at the first post-break unit does not reflect that of its pre-break counterpart. The proposed solution to this added complexity requires releasing the assumption that the post-break slope is known. A system of equations can then be solved for the TFU and slope of the post-break curve that aligns the corresponding unit costs as well as the slopes of the curves at the points of interest.

It should be noted that in the problem definition, it was identified that a potential problem lied in the fact that under condition of equal slopes prior to and after the break some analysts suggest that the solution ought to reproduce the Anderlohr Retrograde solution where $UC_{1, X-K} = UC_{2, X}$; X > F. While the proposed solution does guarantee that the rate of improvement at $UC_{2, F}$ equals that of $UC_{1, F-K}$, it, admittedly, does not yield a solution that results $UC_{1, X-K} = UC_{2, X}$ for all X > F.

6 Future Research/Analysis

A condition of broken learning is that the first unit after the break must be greater than zero.

$$F > 0 \tag{14}$$

It is also noted that the number of retrograde units is greater than or equal to zero (zero suggests no loss of learning) and strictly less than F since K = the number of retrograde units + 1 from (3).

$$0 \le K < F \tag{15}$$

Together, (14) and (15) yield the condition:

$$F \ge F - K \text{ and } F / (F - K) \ge 1 \tag{16}$$

It follows from (16) and (13) that that

$$b_2 \le b_1 \tag{17}$$

In the situation where K = 0 (i.e. no loss of learning), equations (12) and (13) yield the conclusion

$$b_2 = b_1$$

and

$$A_2 = A_1$$

Otherwise, the learning slope developed in this study (b_2) will always be less than that of the original curve (b_1).

From (13) it is noted that b_2 is related to b_1 by the ratio F / (F – K). As F / (F – K) increases, b_2 becomes large and negative. Large, negative b yield steep slopes and can become unreasonably steep. The illustration used in this paper, defined F to be 11 since the break is at the tenth unit. The slope of the post-break curve can graphed given the different options for K. This pattern is illustrated in Figure 7: Learning slope for K given F = 11, below.

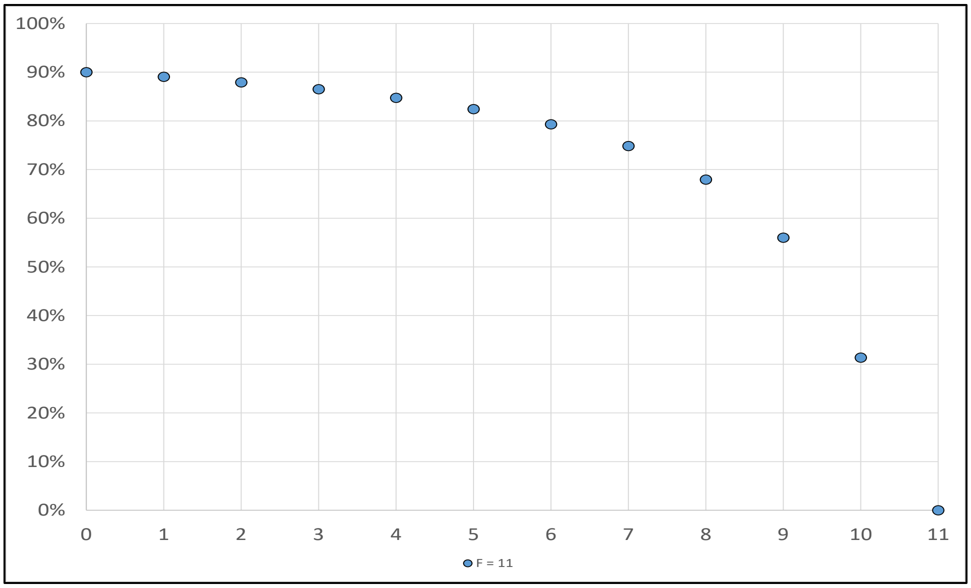

Figure 7: Learning slope for K given F = 11

The chart illustrates that the steepness of the slope changes mildly for small K, but increases dramatically as K approaches F. This pattern is confirmed when experiments are run for larger F as illustrated in Figure 8: Learning slope for K given F, below.

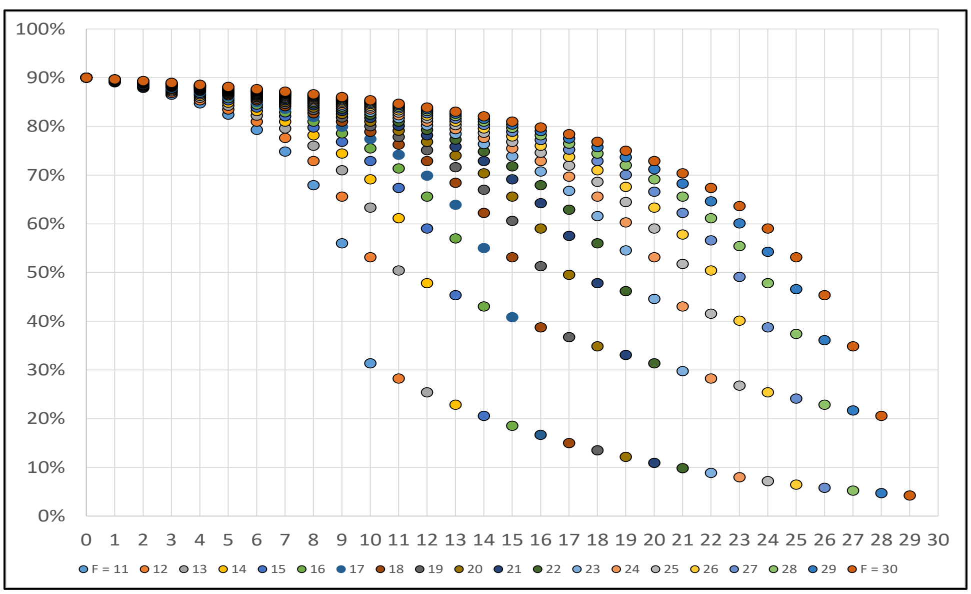

Figure 8: Learning slope for K given F

For each F, there is clearly a “knee” in the curve of the learning slope. It seems likely that the knee could suggest a point where the solution provided in this paper breaks down. Either way, future studies could suggest a standard that identifies appropriate ranges of b_2 based on F and K.

7 Potential Application

A seemingly less common technique for dealing with broken learning is to assume that the lost efficiencies will be "made up" over a specified number of units meaning that the post-break curve will eventually align with original curve as if it had continued without a production break (as illustrated in Figure 9: Potential Application, below). The challenge for the analyst is to define how steep the new slope ought to be and, consequently, the number of units it will take to "make up" for the lost learning.

For example, in a situation where a new production line is established some of the personnel and some of the supervisors from the original line would be asked to help start the new one. With experience from their involvement in the original production process, it could be determined that that rate of learning on the new line would exceed that of the original and that the efficiencies of the new line should, at some point, align with those of the original.

The analyst, then, would need to calculate the slope of the new curve. The technique developed in this study may help the analyst determine an appropriate learning slope for the new production line and calculate the number of units that will have to be built in order to make up the lost efficiencies. The example used in this paper would suggest that the slope of the post-break curve would equal 44% and that the new production line would make up the lost learning after the sixth unit (also illustrated in Figure 9: Potential Application, below).

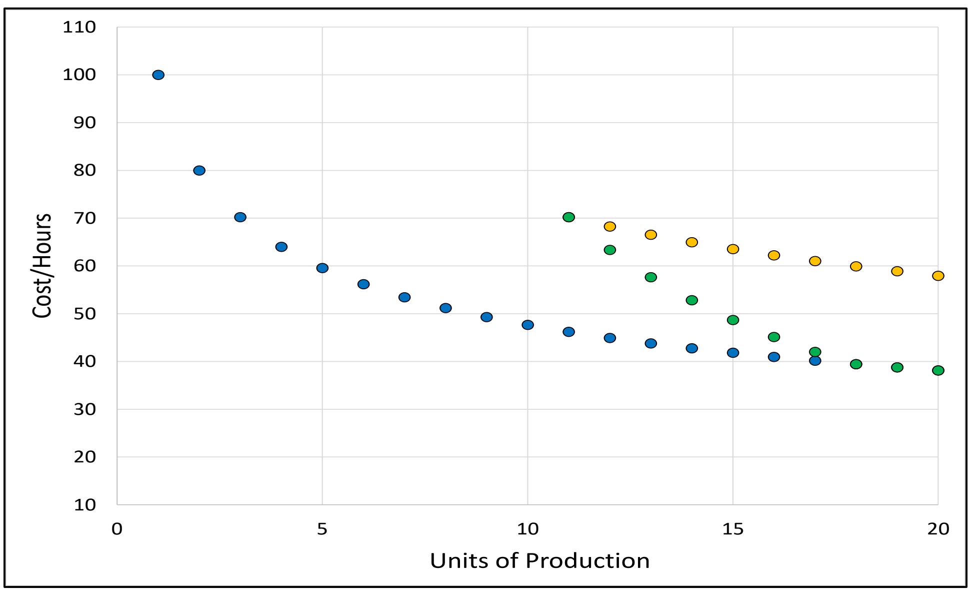

Figure 9: Potential Application

Appendix A

By definition:

$$UC_{1,X} = A_1 X^{b1}$$
$$UC_{2,X} = A_2 X^{b2}$$

The formulas for the derivatives are:

$$UC'_{1,X} = A_1 b_1 X^{(b1-1)} \quad (18)$$
$$UC'_{2,X} = A_2 b_2 X^{(b2-1)} \quad (19)$$

The conditions established for this problem are established in (10) and (11):

$$UC_{1,F-K} = UC_{2,F}$$
$$UC'_{1,X}(F-K) = UC'_{2,X}(F)$$

It follows that (11) can be written:

$$A_1 b_1 (F-K)^{(b1-1)} = A_2 b_2 F^{(b2-1)} \quad (20)$$

From (6), it was established that:

$$A_2 = A_1 (F\text{-}K)^{b1} / F^{b2} \quad (21)$$

Substituting A_2 from (21) into (20) yields:

$$A_1 b_1 (F-K)^{(b1-1)} = [(A_1 (F-K)^{b1} / F^{b2}] * (b_2 F^{(b2-1)})$$

Which simplifies to:

$$A_1 b_1 (F-K)^{(b1-1)} = [A_1 b_2 (F-K)^{b1}] / F$$

Solving for b_2 and simplifying the result yields:

$$b_2 = b_1 F / (F-K)$$

Now that b_2 is defined by known inputs we can substitute back into (21) to get:

$$A_2 = A_1 (F-K)^{b1} / F^{b2}$$

Now becomes

$$A_2 = A_1 (F\text{-}K)^{b1} / F^{(b1 * F / (F-K))}$$

APPENDIX B

Anderlohr, G. (1969). What production breaks cost. Industrial Engineering, September, 34.

U.S. Government Accountability Office (GAO). (2017). GAO cost estimating and assessment guide: Best Practices for Developing and Managing Capital Program Costs. GAO-09-3SP (pp. 369–370)

Bona Fide Need (2018). In Acquipedia. Retrieved from https://www.dau.mil/acquipedia/Pages/ArticleDetails.aspx?aid=b2f52b87-8cc1-4639-87af-b8941f72d965