Risk-Adjusted Contract Price Methodology (RCPM)

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Abstract

There is a wealth of program-level risk and uncertainty benchmarks based on analysis of Selected Acquisition Reports (SARs) (cf. Flynn, et al.), and these benchmarks are often leveraged as either primary risk methodologies, such as the enhanced Scenario-Based Method (eSBM) (cf. Garvey, et al.), or as cross-checks. By contrast, the ability to produce credible risk analyses at the contract level has been hampered by both the lack of a coherent model and the lack of sufficiently granular data. For the first time, the Risk-Adjusted Contract Price Methodology (RCPM) addresses both! It explicitly models both "off-the-shareline" risk and "on-the-shareline" risk to present a complete and accurate distribution of final contract price, building on the previously published Risk-Based Return on Sales (ROS) approach but adding a third dimension for government-directed changes and growth "covered" by contract terms and conditions (Ts & Cs). Drawing from a robust CLIN-level database of cost, fee, and price changes over time, it incorporates historical benchmarks for these dimensions and provides insight into the impact of contract geometry, particularly for incentive-type contracts. RCPM enables governments and other buyers to assess and manage risk at the contract level, from pre-RFP to execution, by better understanding these dimensions and how they are impacted by negotiated incentive structures and Ts & Cs.

Keywords

Cost Management, Data-Driven, Government, Modeling, Risk, Uncertainty

Introduction

The Risk-Adjusted Contract Price Methodology (RCPM) unifies previous research and breaks new ground in modeling the possible final price outcomes of a contract with greater fidelity. It also draws upon the Contracts Database (KDB) as a source of historical benchmarks.

Problem Statement

Life-cycle cost estimates (LCCEs) for major programs typically ignore contract geometry and terms and conditions (Ts & Cs) for major development and procurement contracts. Either these details are not known at the time of the estimate, or there is not the wherewithal to model them correctly. The estimate acknowledges that these contract costs should be *at price* to the Government, but risk and uncertainty are generally applied *at cost*. Fee is not neglected altogether, but often either a flat percentage fee or an uncertainty distribution for fee is applied (in percentage or dollar terms) that may not represent the actual distribution to be experienced.

RCPM remedies this situation by explicitly modeling both "on-the-shareline" and "off-the-shareline" risk.

As with all such cost estimating challenges, where the requisite parameters are not known with certainty at the time of the estimate, they can either be fixed via ground rules and assumptions (GR&A) or themselves including in risk and uncertainty, preferably using historical data.

Previous Research

"Risk-Based Return On Sales (ROS) for Proposals with Mitigating Terms and Conditions" [Braxton, 2009] introduced the notion that contract Ts and Cs introduced to "cover" certain elements of risk, such as escalation, destroy the monotonic nature of the traditional incentive contract shareline. That is, for a given amount of cost growth, if more of that growth "hits" the Ts & Cs, it would result in a higher Price for the Government and a higher Return On Sales (ROS) for Industry than if more of the growth "hits" the shareline. The paper depicted modeling these effects using a traditional Monte Carlo simulation and displayed a scatterplot of ROS vs. Final Cost, revealing a "cloud" of points instead of a continuous function.

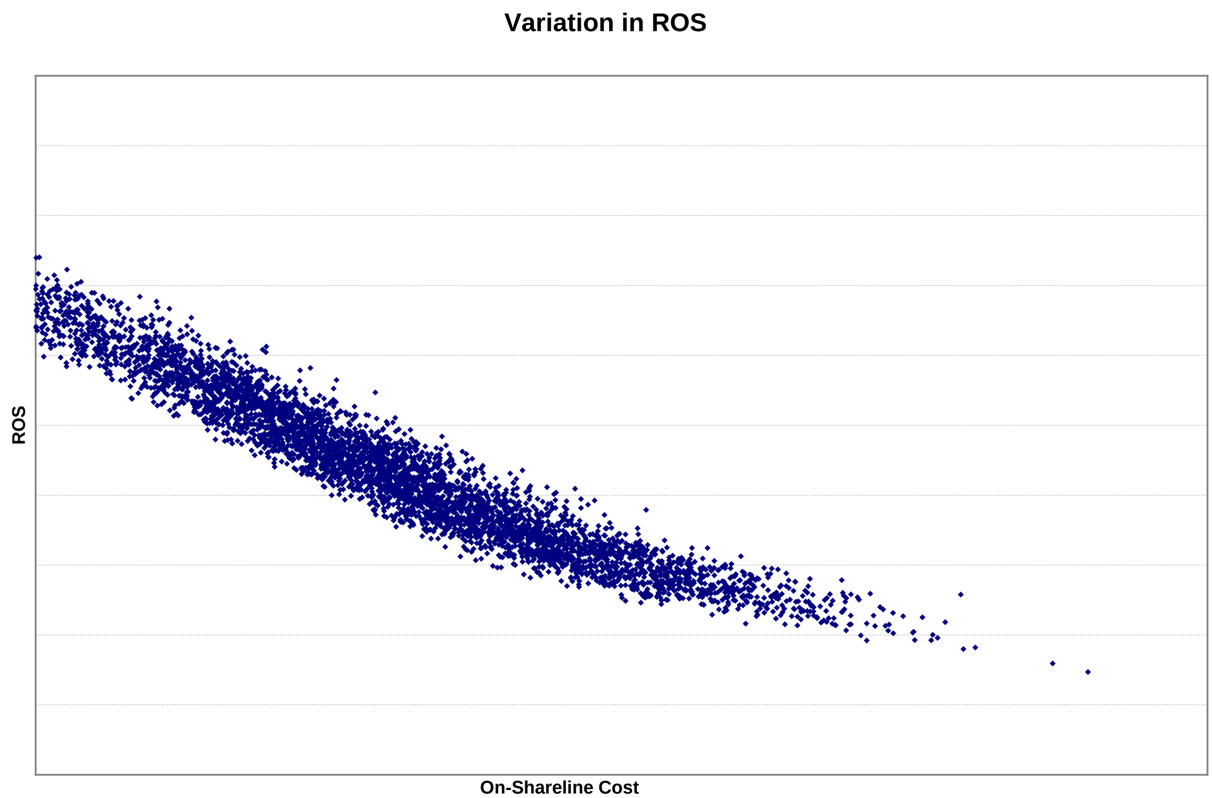

Figure 1. Example of ROS distribution with Ts & Cs

"Risk-Based Return On Sales (ROS) As a Tool For Complex Contract Negotiations" [Braxton, 2010] was written from the Industry perspective and hence focused on the ROS metric. It noted that ROS was variable with Final Cost, even for the Cost Plus Fixed Fee (CPFF) contract type, and derived distributions for ROS for all four major objective contract types (i.e., excluding Award Fee) assuming an underlying Normal distribution. Graphs were presented wherein Monte Carlo simulation runs verified the analytical solutions. It did not treat off-the-shareline risk, nor did it present analytical solutions for cost distributions other than Normal. This paper also introduced the predecessor of the Risk-Adjusted

Contract Price Tool (RCPT), which enabled sensitivity analysis for changes in both cost distribution and contract geometry.

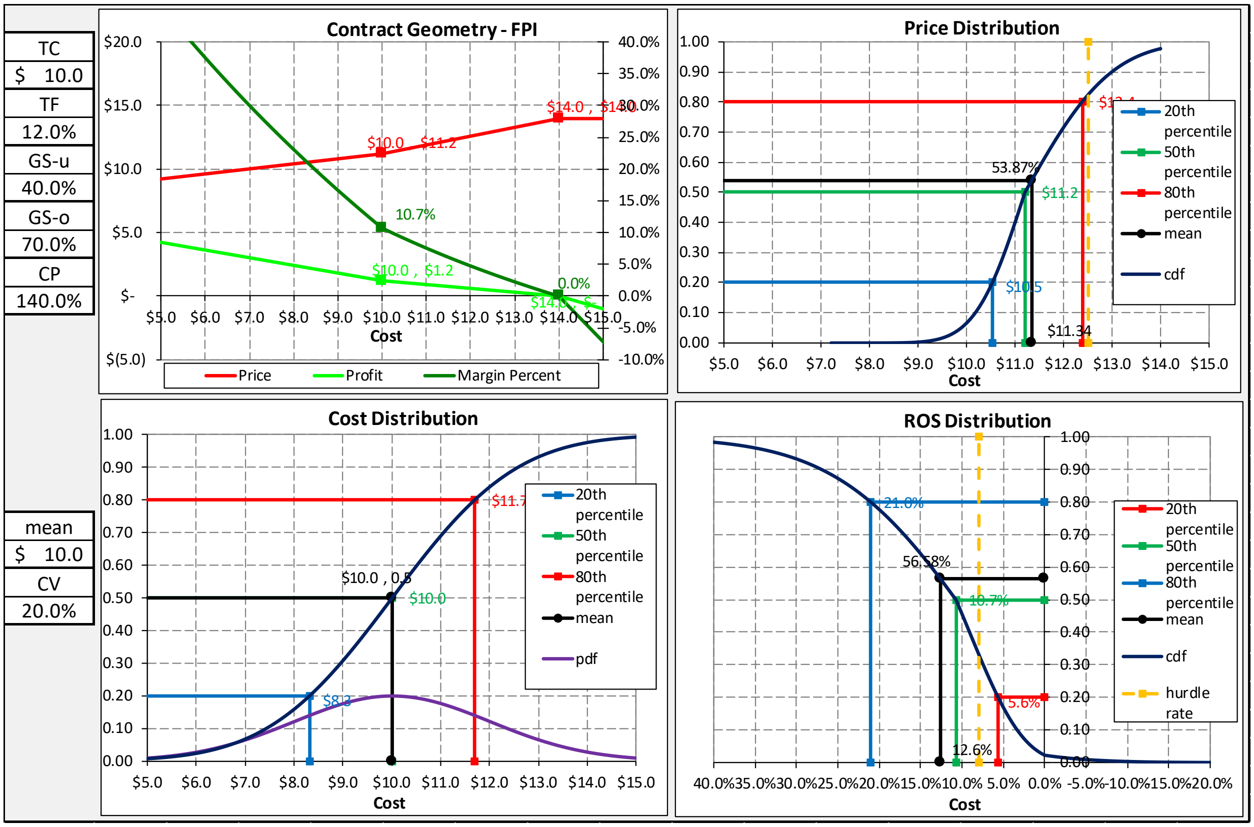

Figure 2. Risk-Adjusted Contract Price Tool (RCPT) dashboard

Several papers, culminating with "Enhanced Scenario-Based Method for Cost Risk Analysis: Theory, Application, and Implementation" [Garvey, 2012] used Selected Acquisition Report (SAR) data to provide Program-level risk and uncertainty benchmarks, primarily cost growth factors (CGFs) and coefficients of variation (CVs), respectively. While SARs include some basic information on Large Active Contracts, they do not specifically address cost growth on those contracts. This work created the publicly-available Naval Center for Cost Analysis (NCCA) S-Curve Tool (https://www.ncca.navy.mil/tools/tools.cfm), which encapsulates these historical benchmarks and can be used to easily generate and annotate S-curves for risk analysis.

"Contract Incentives Under Uncertainty: Data-Driven Contract Geometry Best Practices" [Braxton, 2017] introduced the notion of on- and off-the-shareline risk, which jibes with the contract management corpus on pricing changes, and explored the possibilities of leveraging KDB data to link contract "texture" (number, type, and magnitude of CLINs together with their contract types) to subsequent cost and schedule growth. It proved difficult to designate appropriate and inappropriate contracting arrangements for a given acquisition situation – thought to be a major driver of risk – without significant manual effort on the part of subject matter experts (SMEs) intimately familiar with the historical programs under review.

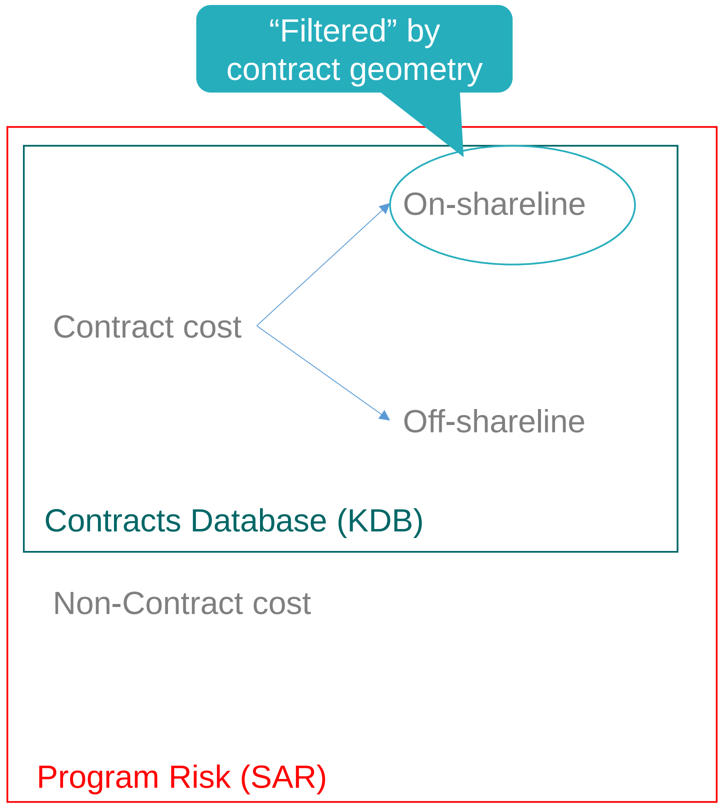

Figure 3. Contract cost growth rubric

Analytical Framework

The analytical framework for RCPM starts with the contract geometry as defined with the four main objective contract types. As shown in [Braxton, 2009], when on-the-shareline risk is assumed to a Normal distribution, the resultant distribution of ROS can be computed, both analytically and via Monte Carlo. RCPM makes this approach more robust by adding an explicit dimension for "off-the-shareline" risk.

Contract Type Functions

For each of the four main objective contract types, Price, Profit/Fee, and ROS can all be defined as piecewise continuous (but not necessarily differentiable!) functions of (Final) Cost. To be legitimate Federal Acquisition Regulation (FAR) contract types, the Price function must be monotonically non-decreasing, and the Profit/Fee and ROS functions must be monotonically non-increasing. That is, as actual cost goes up, price paid cannot possible go down, nor can profit/fee go up.

Firm Fixed-Price (FFP)

Firm Fixed-Price (FFP) arguably has the simplest contract geometry. Price is fixed, which causes Profit to decrease or increase dollar for dollar with any cost overrun or underrun, respectively, essentially a 0/100 shareline.

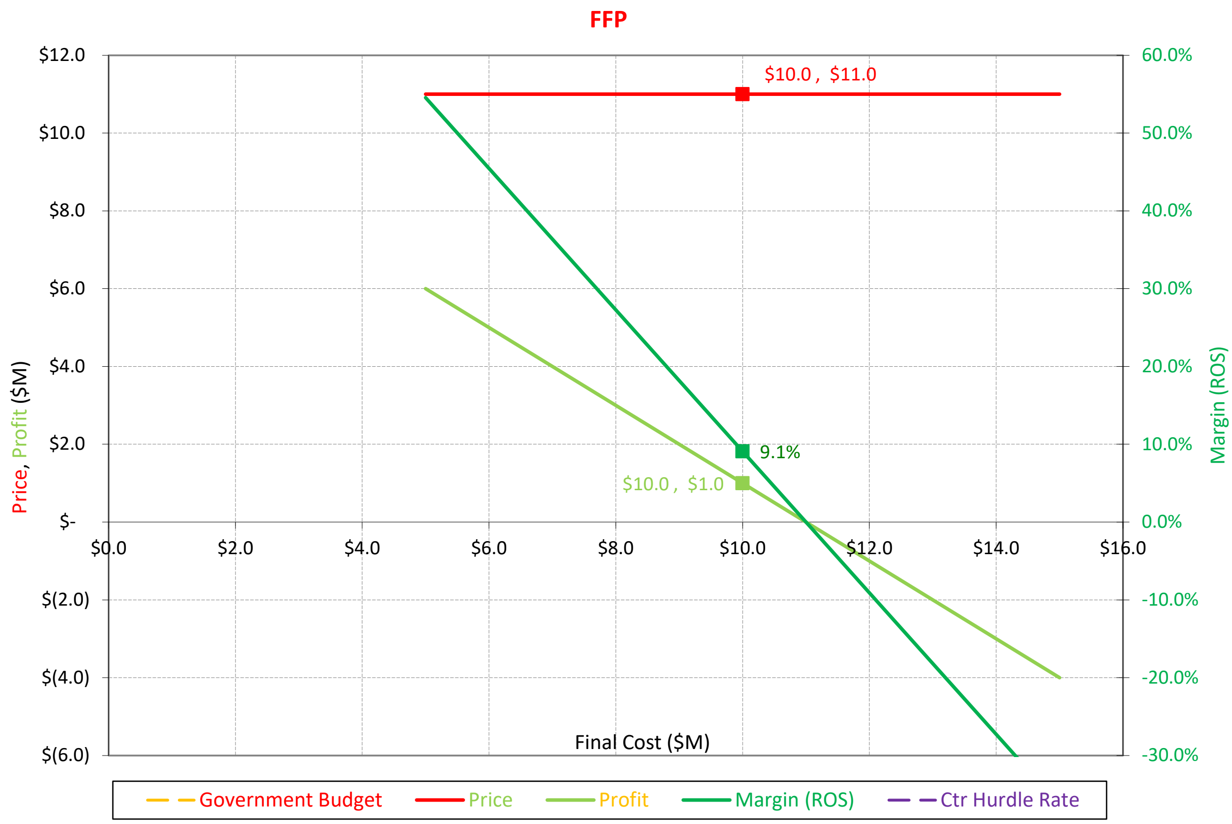

Figure 4. Firm Fixed-Price (FFP) graph from CIIT

Cost Plus Fixed Fee (CPFF)

Similarly, Cost Plus Fixed Fee (CPFF) has a simple contract geometry in which the Fee is a fixed dollar amount. This causes the Price to increase or decrease dollar for dollar with any cost overrun or underrun, respectively, essentially a 100/0 shareline.

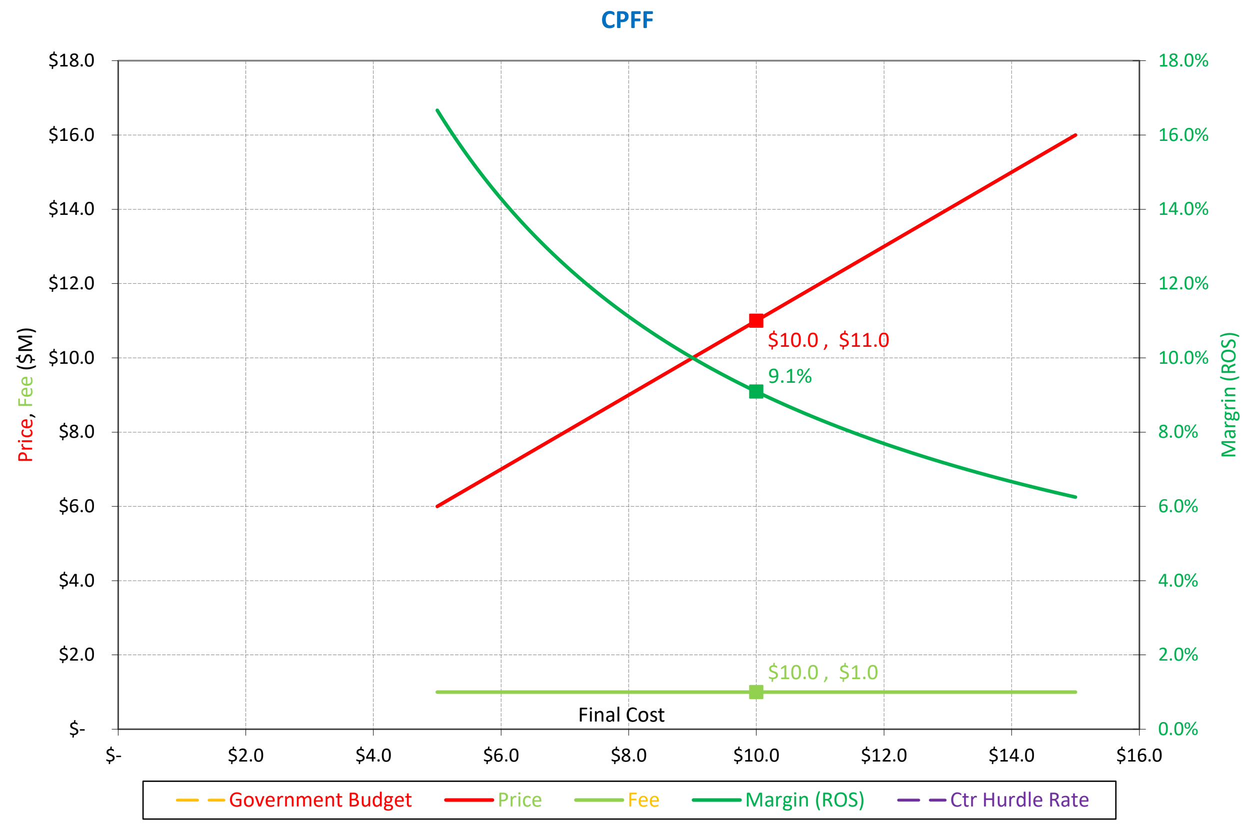

Figure 5. Cost Plus Fixed Fee (CPFF) graph from CIIT

Fixed-Price Incentive (FPI)

For FPI, we focus on Fixed-Price Incentive with Firm targets (FPIF). In FPI, the contract geometry adds a break point at the so-called Point of Total Assumption (PTA), which is where the adjusted price reaches the Ceiling Price. Target Cost itself is generally a break point as well, as we allow different share ratios above (overrun) and below (underrun).

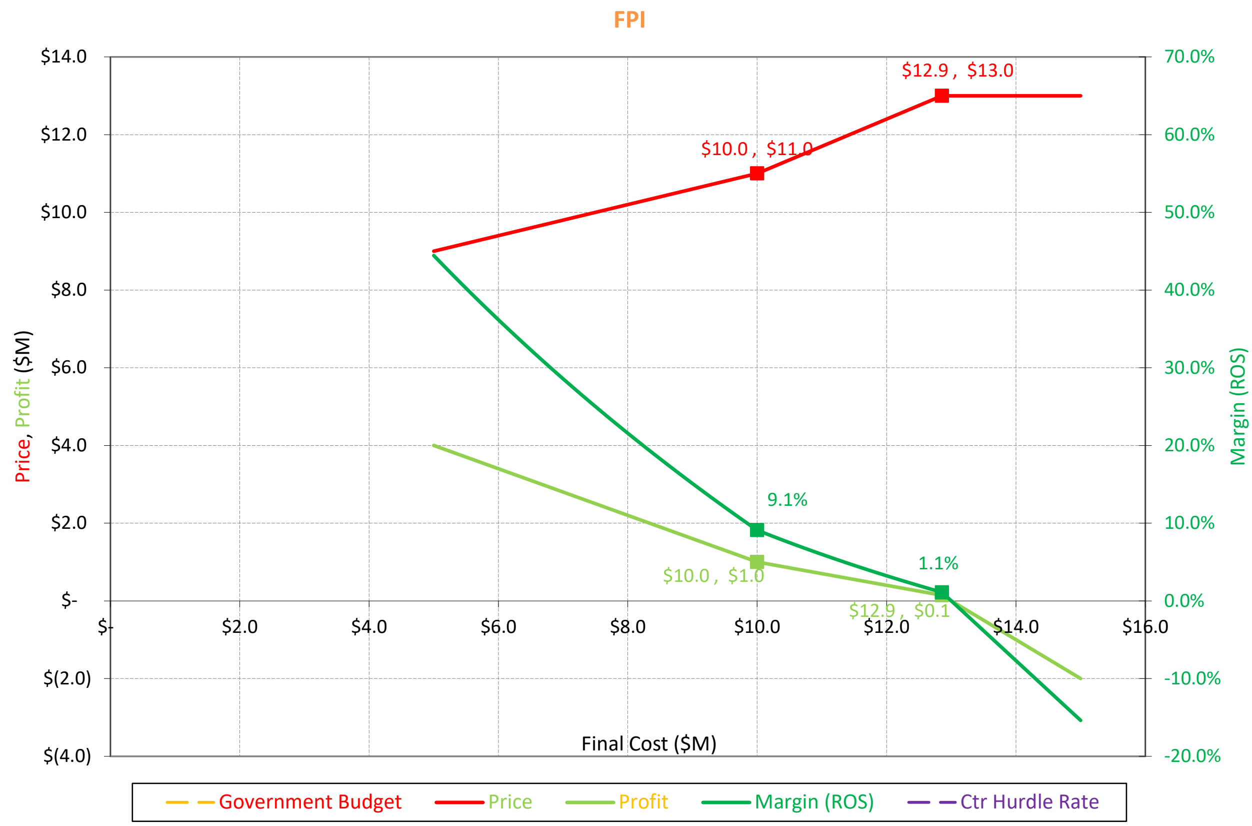

Figure 6. Fixed-Price Incentive (FPI) graph from CIIT

Ceiling Price is often specified as a percentage of Target Cost. [Braxton, 2009] derived an expression for PTA as a function of Ceiling Price and the over-target shareline, which has since been incorporated in CEBoK Module 14.

Cost Plus Incentive Fee (CPIF)

For CPIF, there are three breakpoints: Target Cost, and the left and right endpoint of the so-called Range of Incentive Effectiveness (RIE). These occur where the adjusted fee reaches Max Fee and Min Fee, respectively.

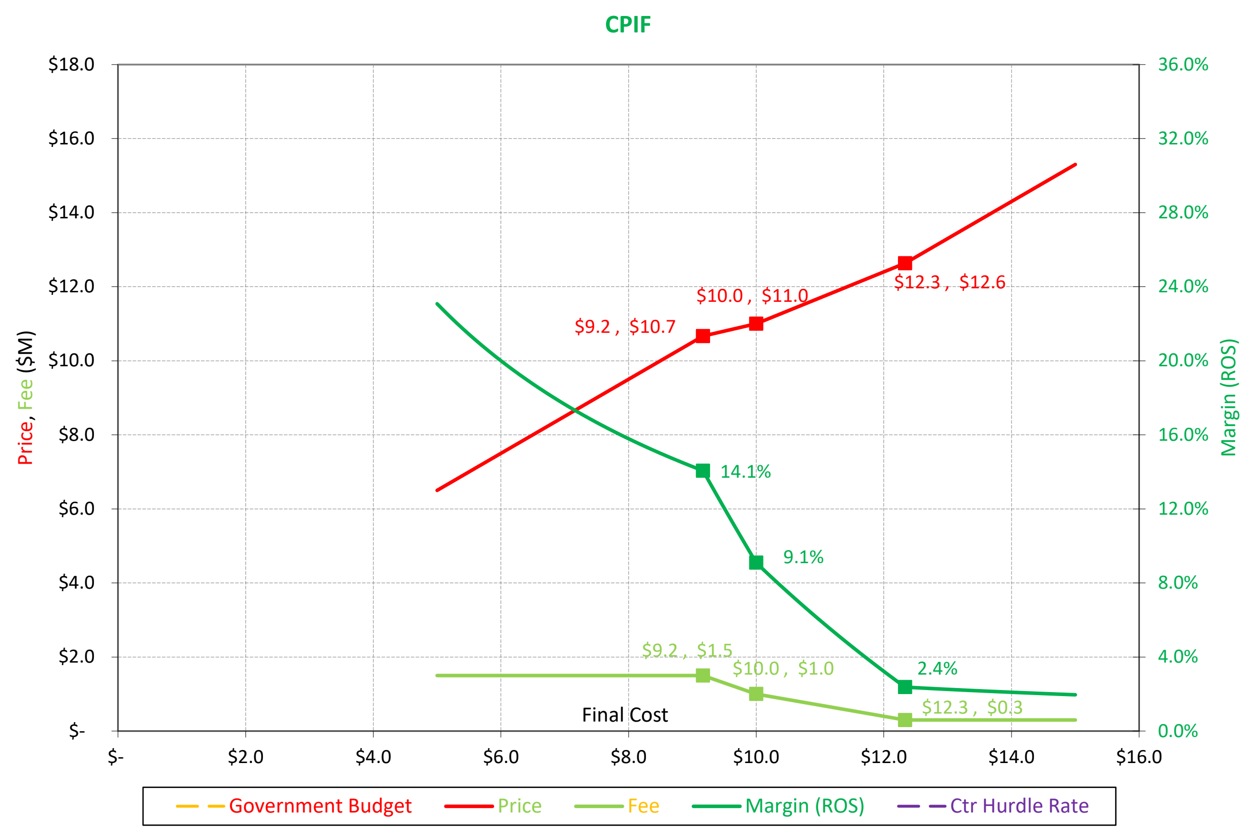

Figure 7. Cost Plus Incentive Fee (CPIF) graph from CIIT

Minimum and Maximum Fee are usually specified as a percentage of Target Cost, but they then become fixed dollar amounts. [Braxton, 2009] derived expressions for the endpoints of RIE as a function of Max Fee, Min Fee, and the under- and over-target sharelines, respectively, which have since been incorporated in CEBoK Module 14.

Distribution of Quantities of Interest

We describe each contract geometry by specifying Profit or Fee as a function of (Final) Cost. In particular, let $X = Cost$ be a random variable representing final on-the-shareline cost. Let $Y = Profit\ (Fee) = f(X)$ be a function of that random variable. This function varies by contract type but is always a piecewise linear function. Furthermore, it is monotonically non-increasing, and in fact monotonically decreasing *except for* CPFF.

Similarly, define $W = Price = X + Y = X + f(X) = h(X)$. This function of cost is piecewise linear, monotonically non-decreasing, and in fact monotonically increasing *except for* FFP.

Finally, define $Z = ROS = \frac{Y}{X+Y} = 1 - \frac{X}{X+Y}$. This function no longer linear and is monotonically decreasing for *all* contract types.

Note that Facilities Capital Cost of Money (FCCM) is a profit-like element included in total price but excluded from Cost for purposes of determining Fee or Profit. For the purposes of this paper, we will omit FCCM from further discussion.

Distribution of Price (Government)

The primary quantity of interest to the Government is (Final) Price, since that is what they will ultimately have to budget for and pay. To derive the distribution of Price, we rely on the fact that it is a monotonically non-decreasing function of Cost, and apply logic to the respective cumulative distribution functions (CDFs). The distribution of Price may be specified by its CDF:

$$F_W(w) = P[X + Y \leq w] = P[X \leq w - f(X)] = P[X \leq h(w)] = F_X\big(h(w)\big)$$

where $h(w)$ can be solved for depending upon the particular $f(X)$ for a given contract type. The probability distribution function (PDF) can then be found by differentiating the CDF and applying the chain rule:

$$p_W(w) = \frac{d}{dw} F_W(w) = F_X'\big(h(w)\big) \cdot h'(w) = p_X\big(h(w)\big) \cdot h'(w)$$

See the Appendix for derivations.

Distribution of ROS (Contractor)

The primary quantity of interest to the Contractor is ROS, since that is the Margin the company earns and can subsequently invest and/or return to its shareholders. Just as the Government must measure Price against available budgets, the Contractor often has "hurdle" rates that must be cleared for certain types of contracts. The Contractor is also interested in Price, which to them is Revenue, but it is generally a secondary consideration.

Similarly, the distribution of ROS may be specified by its CDF:

$$\begin{aligned} F_Z(z) &= P\left[\frac{Y}{X+Y} \leq z\right] = P\left[1 - \frac{X}{X+f(X)} \leq z\right] = P\left[1 - z \leq \frac{X}{X+f(X)}\right] = P\left[X + f(X) \leq \frac{X}{1-z}\right] \\ &= P\left[f(X) \leq X\frac{z}{1-z}\right] \end{aligned}$$

This last expression is tantamount to the probability that the fee is less than or equal to the fee percentage (as a function of ROS) times cost. Now there exists a $g(z)$, which can be solved for depending upon the particular $f(X)$ for a given contract type, for which we can rewrite this expression as:

$$= P[X \geq g(z)] = 1 - P[X \leq g(z)] = 1 - F_X\big(g(z)\big)$$

Once again, differentiating yields:

$$p_Z(z) = \frac{d}{dz} F_Z(z) = -F_X'\big(g(z)\big) \cdot g'(z) = -p_X\big(g(z)\big) \cdot g'(z)$$

See the Appendix for derivations for each of the four main objective contract types.

Off-the-Shareline Risk

The previous distributions assume that all variation in cost – overrun or underrun – "hits" the shareline and thus affects Final Price (in the case of CPFF), Final Profit (in the case of FFP), or both (in the case of FPI and CPIF) according to the established contract geometry. In reality, some variation in final contract cost comes in the form of modifications that are adjudicated off the shareline.

Technical Changes (New Work)

Often new work is added to the contract, either as new CLINs or changes to existing CLINs. In either case, the work is typically added based on estimated cost plus a commensurate fee or profit. Where this fee is the same percentage of (target) cost as in the base work, the mod essentially "moves the goalposts," readjusting the total target cost and target fee as the point of departure for on-the-shareline risk. In this case, the mod is essentially "ROS-neutral." An example is shown below.

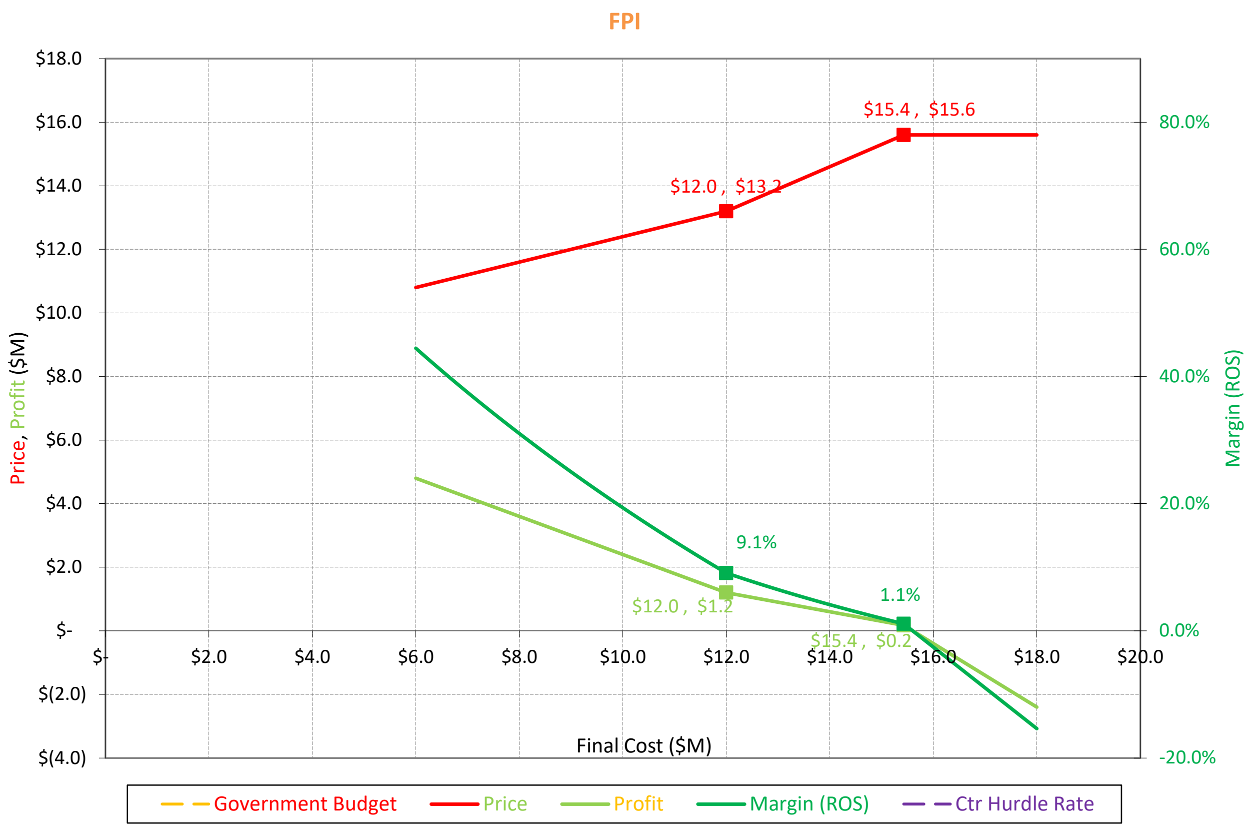

Figure 8. FPI example with ROS-neutral mod

In the graph above, $2M of new work has been added to a base cost of $10M, yielding a new Target Cost of $12M. The 10% profit (at target cost) and 130% ceiling price have been maintained, increasing from $1M to $1.2M and from $13M to $15.6M, respectively. Essentially, the whole graph shifts up and to the right proportionally.

It is important that these changes represent new work on the contract. If they were simply to represent cost growth, they would be violating the FAR's prohibition on contract with constant *percent* fee.

In a Production program, such changes are sometime referred to as Class I changes, since they are directed by the government in a contract mod, and to distinguish them from Class II changes, which are those implemented internally by the Contractor, with no discernable price increase to the Government.

Terms and Conditions (Ts & Cs)

Other cost adjustments may be dictated by contract terms and conditions (Ts & Cs), such as an economic price adjustment (EPA) clause. In this case, the effect is more like a CPFF contract type. Costs are adjusted up or down without a commensurate adjustment in fee or profit, in which case ROS will change based on the changing denominator (total revenue). These can be designated "Profit-neutral" or "Fee-neutral" mods.

If in the above example the $2M were added "at cost," then essentially everything shifts to the right but *not* up. $12M is again the new target cost, but target profit is still $1M, and ceiling price is still $3M above target cost.

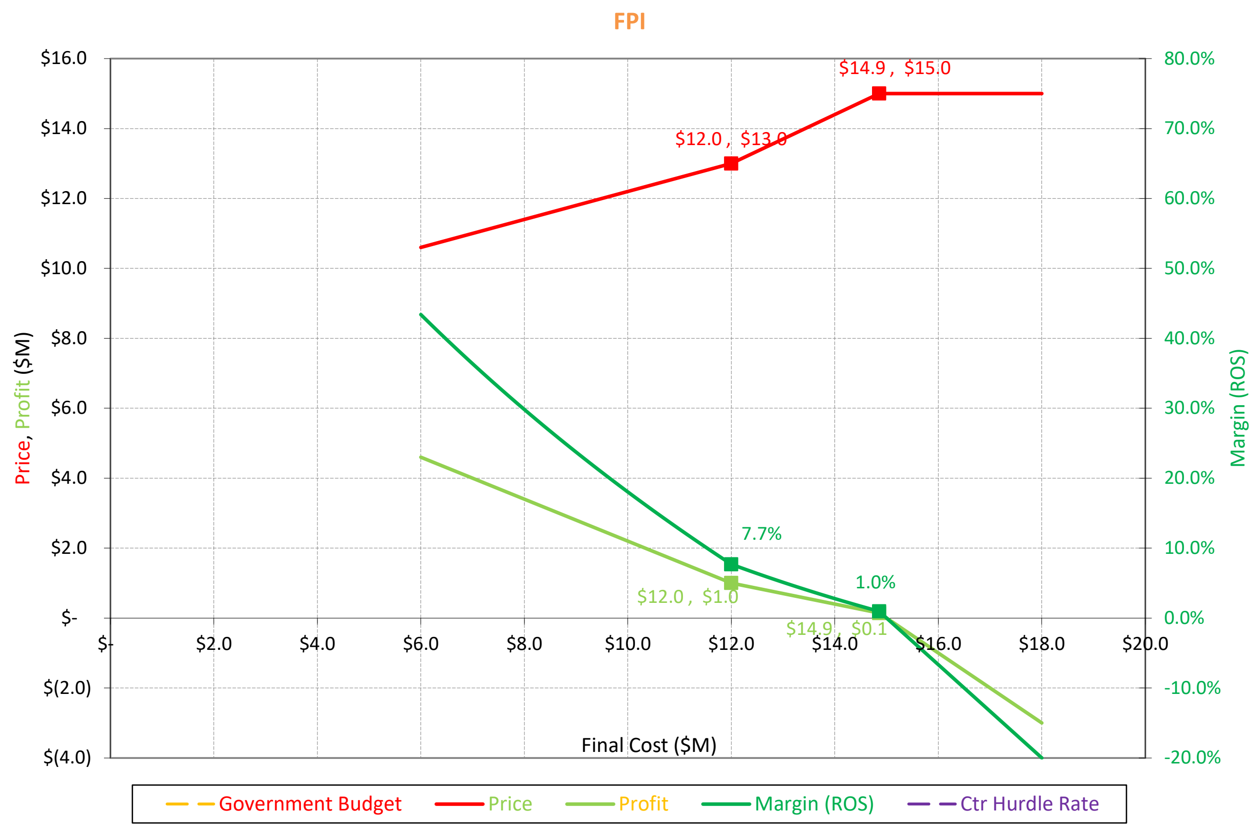

Figure 9. FPI example with Profit-neutral mod

Contracts Database (KDB)

The Contracts Database (KDB) is a robust data source tracking price, quantity, and schedule changes by modification at the contract line item number (CLIN) level. It is particularly important for understanding these off-the-shareline or profit-neutral mods.

Prevalence of Contract Types

The table, updated from [Braxton, 2017], shows the relative prevalence of contract types at the CLIN level within KDB. The vast majority of the “Other” category is Time and Materials (T&M) contracts.

Table 1. Contract types in KDB

Contract Type	Count	Value	By Count	By Value	Average Size
CPAF	1,316	$ 72,240,314,645.87	1.6%	14.5%	$ 54,893,856.11
CPIF	1,364	$ 49,848,101,332.49	1.7%	10.0%	$ 36,545,528.84
FPIF	1,798	$ 82,104,909,935.05	2.2%	16.5%	$ 45,664,577.27
FFP	58,719	$ 231,715,067,446.07	72.0%	46.6%	$ 3,946,168.49
COST & CPFF	10,210	$ 43,560,605,650.77	12.5%	8.8%	$ 4,266,464.80
Other	8,184	$ 17,411,297,203.31	10.0%	3.5%	$ 2,127,480.11
Total	**81,591**	**$ 496,880,296,213.56**	**100.0%**	**100.0%**	**$ 6,089,891.00**

Firm Fixed-Price (FFP) CLINs

FFP CLINs are by far the most common in the database, though they are much smaller on average than incentive-type CLINs. It is instructive to look at the degree of change that occurs even on FFP CLINs. The $231.7B in contract value shown above includes growth relative to a total BASELINE of about $180.9B, or an average growth of about 28.3%. The lion’s share of this represents TECHNICAL growth. See [Braxton, 2017] for a discussion of classification of mods by growth categories in KDB.

Unfortunately, unlike Contractor Cost Data Report (CCDRs), which provide direct insight into profit on FFP contacts, KDB can only provide visibility at the price level, since that is what is reflected in the original contract documentation (BASIC and mods).

Fixed-Price with Economic Price Adjustment (FP-EPA) CLINs

There are a limited number of Fixed-Price with Economic Price Adjustment (FP-EPA) CLINs in KDB (cf. FAR 16.203). These are included in the “Other” category in the table above.

Incentive Contract Examples

It is the so-called Incentive-type contracts, particularly FPI and CPIF, where the distinction between on- and off-the-shareline risk is most clearly evident. For mods classified as COST, the cost increase “runs up” the shareline and the incentive fee is decremented accordingly in the contract.

Historical Benchmarks

KDB can provide a wealth of historical benchmarks, including amount of growth on various groupings of CLINs or contracts, by contract type, commodity, and contractor. It is available to government analysts by download from the Cost Assessment Data Enterprise (CADE) Tools page (go to the My CADE menu within the Data & Analytics application). For more information, browse to http://cade.osd.mil/. KDB includes three Excel tools: the Visual Analysis Tool (VAT), which can automatically generate standard graphs and summary statistics; the Pivot Tool, which allows analysts more flexible access to the essential data from the database; and KDB Contents and Priorities, which provides helpful metadata on which programs and contracts are included in the database, including recent and planned acquisitions.

RCPM Approach

The key to RCPM is to carefully parse sources of risk into how they will manifest relative to the contract structure.

While outside the scope of this paper, sound cost and risk analysis are the foundation of RCPM, as the Risk-Adjusted Contract Cost component. Supporting inputs include Framing Assumptions, Bases of Estimate (BOEs), Risk Register, Independent Technical Assessment (ITA), Independent Cost Estimate (ICE), and Historical Benchmarks.

On- and Off-the-Shareline Risk

In particular, most sources of risk are assumed to manifest as on-the-shareline growth. For new work, estimators often develop an Engineering Change Order (ECO) factor or similar approach to estimate ROS-neutral work that will added to the contract. Profit-neutral work is generally associated with specific Ts & Cs, such as an EPA clause. While such clauses generally results in an upward adjustment, they could result in a downward adjustment, if commodity prices are much lower than projected, for example.

Bivariate Risk Distribution

The simplest conceptual expansion of the Risk-Based ROS model is to add a second risk dimension for ROS-neutral changes or Profit-neutral changes. Then, instead of a two-dimensional graph with a single contract geometry giving Price and ROS as a function of Final Cost, we now have a three-dimensional graph, where the ROS-neutral changes (off-the-shareline cost) axis runs perpendicular to the on-the-shareline cost, creating an infinite family of contract geometry graphs, ever shifting to the right.

The essential computation is the same as in the two-dimensional case, illustrated with the earlier RCPT screen shot. The resultant distributions of Price and ROS are a "mash-up" of the underlying distribution of Cost and the contract geometry imposed thereupon.

Adding both ROS-neutral and Profit-neutral changes simultaneously would essentially take us into four dimensions, with three distinct axes of cost variation (on one the shareline, and two off), and a fourth axis for the resultant responses of Price and ROS. For a conceptual model, the multivariate normal distribution can be used – see [Garvey, 2000] – but in a practical sense, there is no limit to the number of component risks involved in a Monte Carlo simulation.

RCPM Implementation

RCPM implementations may vary. We briefly touch on two key considerations.

Cost Model vs. Risk Model

Ideally, risk and uncertainty are built into the cost model itself. In this case, RCPM can be implemented directly, with any risk impacts modeled appropriately. Often, it is necessary to build an *ex post facto* risk model, with top-level uncertainties based on the output of an unseen cost model. This is often the case with proposal or estimate pass-throughs, such as for government-furnished equipment (GFE). The best way to remedy this situation is to proactively request the submittal of a "live" cost model and/or more detailed risk information.

Analytical vs. Monte Carlo

As with the previous Risk-Based ROS approach, Monte Carlo is generally the computational engine of choice. Analytical solutions are useful for sensitivity analysis and cross-checks.

RCPM Applications

As stated, the goal of RCPM is to model contract risk with the highest possible fidelity without over-complicating the analysis.

RCPM and Budgeting

RCPM plays a key role in Budgeting. By more accurately forecasting a range of likely outcomes for total Price to the Government, it enables decision-makers to budget at the desired confidence level. Considering ancillary contracts, GFE, and Other Government Costs (OGCs) takes us into the realm of Risk-Adjusted Program Cost, which is largely beyond the scope of this paper.

RCPM and Source Selection

While the best forecast of government-directed changes should be included for budgeting purposes, RCPM for source selection should include only those risks allowed by the request for proposal (RFP) and inherent in the respondents' offers. This needs to include both on- and off-the-shareline risk, where the latter is driven by Ts & Cs. In unusual cases where offerors may propose very different Ts & Cs, RCPM is a key enabler of "leveling the playing field" so that all bids are assessed consistently and fairly from a risk perspective.

Future Research

Analytical Solutions

Given the power of modern Monte Carlo simulation, deriving closed-form analytical solutions becomes more of an intellectual curiosity and less of a practical necessity. The piecewise nature of the contract geometry for incentive-type contracts makes these solutions more of a challenge, but it would be helpful to have a library of results for mean Price and ROS, for example. At the very least, this would serve as a cross-check for Monte Carlo results. Note that these analytical solutions may still require the use of the phi function (normal distribution) and other computational techniques such as numerical integration.

Improved Visualization

Often, convincing senior decision-makers of the virtue of data-driven approaches relies on a "killer graphic" or other visual display of information to clearly show what is going on and instill confidence in the process. Since the introduction of off-the-shareline risk essentially takes us into the realm of three (or even four) dimensions, creating and annotating these graphics becomes more challenging and requires more sophisticated software tools than Microsoft Excel.

Running the Gamut of Ts & Cs

The appropriate modeling of off-the-shareline risk is largely dependent upon a knowledge of common Ts & Cs and how they've played out on past contracts. This can be done on an *ad hoc* basis as needed for major procurements, but it would be helpful to do some preliminary research using KDB, buying commands, and program offices as potential resources.

Appendix

Price Distributions

This section uses the same analytical techniques previous applied to ROS in the case of Price. The underlying distribution of Final Cost can be assumed to be Normal or Lognormal. For generality, we use $F_X(x)$ and $p_X(x)$, respectively, for the CDF and PDF of Cost.

Firm-Fixed Price (FFP)

In this case, $f(X) = FP - X$, so we have $F_W(w) = P[FP \leq w]$. Since X vanishes, we have a singular CDF where the cumulative probability is zero to the left of FP and jumps up to 1 at FP. This tells us what we already knew, that there is a 100% discrete chunk of probability at the Fixed Price.

Cost Plus Fixed Fee (CPFF)

In this case, $f(X) = TF$, and $h(w) = w - TF$.

$$P[X + TF \leq w] = P[X \leq w - TF]$$

Thus,

$F_W(w) = F_X(w - TF)$ so that $p_W(w) = p_X(w - TF)$

In other words, the Price distribution is just the Cost distribution shifted to the right by the fixed fee of TF!

ROS Distributions

This section recapitulates the ROS derivations from [Braxton, 2010]. The underlying distribution of Final Cost can be assumed to be Normal or Lognormal. For generality, we use $F_X(x)$ and $p_X(x)$, respectively, for the CDF and PDF of Cost.

Firm-Fixed Price (FFP)

In this case, $f(X) = FP - X$, and $g(z) = TP(1 - z)$.

$$P\left[TP - X \leq X\frac{z}{1-z}\right] = P[X \geq TP(1-z)] = 1 - P[X \leq TP(1-z)]$$

Thus,

$F_Z(z) = 1 - F_X\big(TP(1-z)\big)$ so that $p_Z(z) = TP \cdot p_X\big(TP(1-z)\big)$

Cost Plus Fixed Fee (CPFF)

In this case, $f(X) = TF$, and $g(z) = \frac{(1-z)}{z}TF$.

$$P\left[TF \leq X\frac{z}{1-z}\right] = P\left[X \geq \left(\frac{1-z}{z}\right)TF\right] = 1 - P\left[X \leq \left(\frac{1-z}{z}\right)TF\right]$$

Thus,

$F_Z(z) = 1 - F_X\left(\left(\frac{1-z}{z}\right)TF\right)$ so that $p_Z(z) = \frac{TF}{z^2}p_X\left(\left(\frac{1-z}{z}\right)TF\right)$

Fixed-Price Incentive (FPI)

Now we have a piecewise linear function in three regimes:

$$f(x)=\begin{cases} TF+CS_{under}(TC-X) & X\le TC \\ TF-CS_{over}(X-TC) & TC<X\le PTA \\ CP-X & X>PTA \end{cases}$$

The corresponding break points for ROS are $\frac{TF}{TP}$ at Target Cost and $\frac{CP-PTA}{CP}$ at PTA.

For the three regimes:

$$P\left[TF+CS_{under}(TC-X)\le X\frac{z}{1-z}\right]=1-P\left[X\le\frac{(TF+CS_{under}TC)(1-z)}{CS_{under}+GS_{under}z}\right]$$

$$P\left[TF-CS_{over}(X-TC)\le X\frac{z}{1-z}\right]=1-P\left[X\le\frac{(TF+CS_{over}TC)(1-z)}{CS_{over}+GS_{over}z}\right]$$

$$P\left[CP-X\le X\frac{z}{1-z}\right]=1-P[X\le(1-z)CP]$$

Keep in mind that the three regimes occur in reverse order (i.e., the lowest ROS coincides with the highest Cost, and vice versa). The piecewise CDF for ROS then becomes:

$$F_Z(z)=\begin{cases} 1-F_X\big((1-z)CP\big) & z\le\dfrac{CP-PTA}{PTA} \\ 1-F_X\left(\dfrac{(TF+CS_{over}TC)(1-z)}{CS_{over}+GS_{over}z}\right) & \dfrac{CP-PTA}{PTA}<z\le\dfrac{TF}{TP} \\ 1-F_X\left(\dfrac{(TF+CS_{under}TC)(1-z)}{CS_{under}+GS_{under}z}\right) & z>\dfrac{TF}{TP} \end{cases}$$

Taking the derivative and applying the chain rule yields the piecewise PDF for ROS:

$$p_Z(z)=\begin{cases} CP\cdot p_X\big((1-z)CP\big) & z\le\dfrac{CP-PTA}{PTA} \\ \left(\dfrac{TF+CS_{over}TC}{(CS_{over}+GS_{over}z)^2}\right)p_X\left(\dfrac{(TF+CS_{over}TC)(1-z)}{CS_{over}+GS_{over}z}\right) & \dfrac{CP-PTA}{PTA}<z\le\dfrac{TF}{TP} \\ \left(\dfrac{TF+CS_{under}TC}{(CS_{under}+GS_{under}z)^2}\right)p_X\left(\dfrac{(TF+CS_{under}TC)(1-z)}{CS_{under}+GS_{under}z}\right) & z>\dfrac{TF}{TP} \end{cases}$$

Cost Plus Incentive Fee (CPIF)

Now we have a piecewise linear function in four regimes:

$$f(x)=\begin{cases} MF & X\le RIE_{low} \\ TF+CS_{under}(TC-X) & RIE_{low}<X\le TC \\ TF-CS_{over}(X-TC) & TC<X\le RIE_{high} \\ mF & X>RIE_{high} \end{cases}$$

The corresponding break points for ROS are $\frac{TF}{TP}$ at Target Cost, $\frac{MF}{RIE_{low}+MF}$ at RIE_{low}, and $\frac{mF}{RIE_{high}+mF}$ at RIE_{high}.

For the four regimes:

$$P\left[MF \leq X\frac{z}{1-z}\right] = 1 - P\left[X \leq \left(\frac{1-z}{z}\right)MF\right]$$

$$P\left[TF + CS_{under}(TC - X) \leq X\frac{z}{1-z}\right] = 1 - P\left[X \leq \frac{(TF + CS_{under}TC)(1-z)}{CS_{under} + GS_{under}z}\right]$$

$$P\left[TF - CS_{over}(X - TC) \leq X\frac{z}{1-z}\right] = 1 - P\left[X \leq \frac{(TF + CS_{over}TC)(1-z)}{CS_{over} + GS_{over}z}\right]$$

$$P\left[mF \leq X\frac{z}{1-z}\right] = 1 - P\left[X \leq \left(\frac{1-z}{z}\right)mF\right]$$

Note that the two middle regimes are identical to their counterparts in FPI!

Keep in mind that the four regimes occur in reverse order (i.e., the lowest ROS coincides with the highest Cost, and vice versa). The piecewise CDF for ROS then becomes:

$$F_Z(z) = \begin{cases} 1 - F_X\left(\left(\frac{1-z}{z}\right)mF\right) & z \leq \frac{mF}{RIE_{high} + mF} \\ 1 - F_X\left(\frac{(TF + CS_{over}TC)(1-z)}{CS_{over} + GS_{over}z}\right) & \frac{mF}{RIE_{high} + mF} < z \leq \frac{TF}{TP} \\ 1 - F_X\left(\frac{(TF + CS_{under}TC)(1-z)}{CS_{under} + GS_{under}z}\right) & \frac{TF}{TP} < z \leq \frac{MF}{RIE_{low} + MF} \\ 1 - F_X\left(\left(\frac{1-z}{z}\right)MF\right) & z > \frac{MF}{RIE_{low} + MF} \end{cases}$$

Taking the derivative and applying the chain rule yields the piecewise PDF for ROS:

$$p_Z(z) = \begin{cases} \frac{mF}{z^2}p_X\left(\left(\frac{1-z}{z}\right)mF\right) & z \leq \frac{mF}{RIE_{high} + mF} \\ \left(\frac{TF + CS_{over}TC}{(CS_{over} + GS_{over}z)^2}\right)p_X\left(\frac{(TF + CS_{over}TC)(1-z)}{CS_{over} + GS_{over}z}\right) & \frac{mF}{RIE_{high} + mF} < z \leq \frac{TF}{TP} \\ \left(\frac{TF + CS_{under}TC}{(CS_{under} + GS_{under}z)^2}\right)p_X\left(\frac{(TF + CS_{under}TC)(1-z)}{CS_{under} + GS_{under}z}\right) & \frac{TF}{TP} < z \leq \frac{MF}{RIE_{low} + MF} \\ \frac{MF}{z^2}p_X\left(\left(\frac{1-z}{z}\right)MF\right) & z > \frac{MF}{RIE_{low} + MF} \end{cases}$$

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