To Monte Carlo or not to Monte Carlo….

That is the Question

International Cost Estimating and Analysis Association

Professional Development and Training Workshop

Phoenix, AZ

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Introduction

- Background
- Monte Carlo Overview
- Method of Moments Overview
- Case Study Introduction
- Results
- Risk Spread Methodologies

Background

- Several popular risk / uncertainty analysis tools in use
  - Crystal Ball®
  - @Risk®
  - TruePlanning®
- What are the differences?
- Are the results comparable?

This presentation seeks to answer these questions…AND provide recommendations on how to effectively allocate risk across the program estimate.

Monte Carlo Overview

- Uses randomness to solve a deterministic problem
- Relies on repeated sampling to generate numeric results
- Impossible to execute a program 1,000 times…simulation is smarter

*ESLOC * Productivity * Labor Rate = Estimate*

*10K * 2.3 * $200 = $4.6M*

- Deterministic: we know the information to calculate the answer
- What if
  - ESLOC is unreliable?
  - Productivity could vary based on development team experience?
  - Labor rates are dependent on which contractor performs the work?

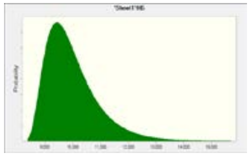
Monte Carlo Overview

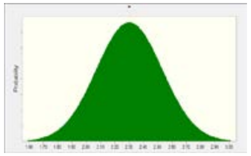
PRICE®

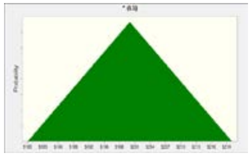
- Monte Carlo allows us to put a RANGE around each input

ESLOC * *Productivity* * *Labor Rate* = *Estimate*

10K * *2.3* * *$200* = *$4.6M*







8K ⟷ 14K

2.0 ⟷ 2.6

$180 ⟷ $220

$3.1M ⟷ $6.5M

- Stochastic: we vary the inputs used in each model iteration
- From this process, we can answer
  - How confident are we in the point estimate?
  - How much in risk dollars should we add to a program to achieve x% confidence?
  - If our budget is X dollars, what is the probability of overrunning?

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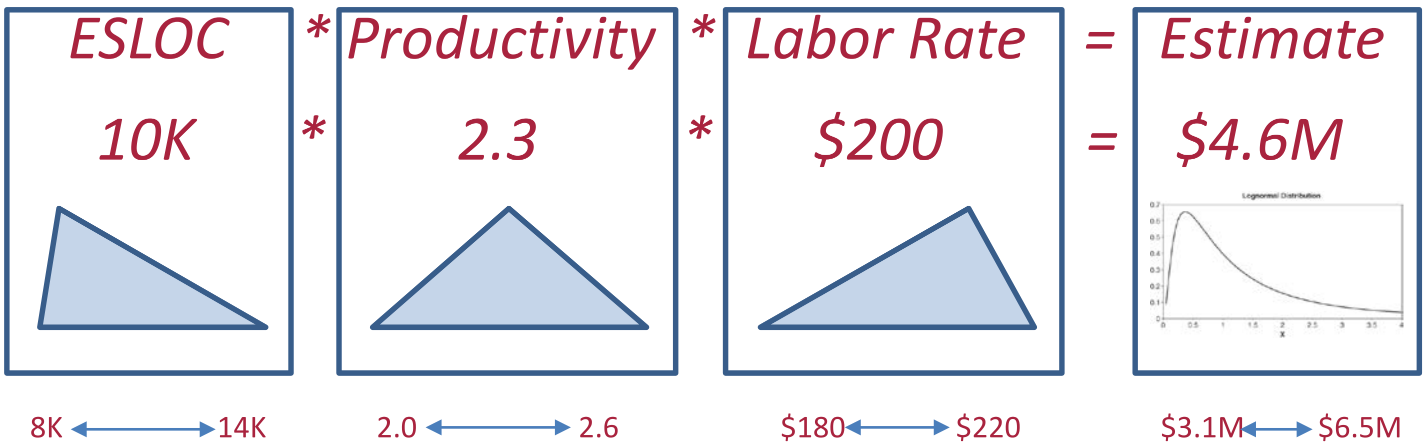
Method of Moments Overview

- "Moments" are characteristics of a distribution
  - Mean ~ Average value in distribution
  - Variance ~ Spread of distribution
  - Skewness ~ Symmetry of distribution
  - Kurtosis ~ Peakedness of distribution
- We can infer moments of an output distribution by evaluating / combining moments of input distributions
- Faster than Monte Carlo; simple math done by a computer
- Similar statistical output provided by Method of Moments

Method of Moments Overview

- Given variability around inputs, we can generate output pdf



$$\mu = \frac{L + M + H}{3}$$

$$\sigma = \sqrt{\frac{L^2 + M^2 + H^2 - LM - LH - MH}{18}}$$

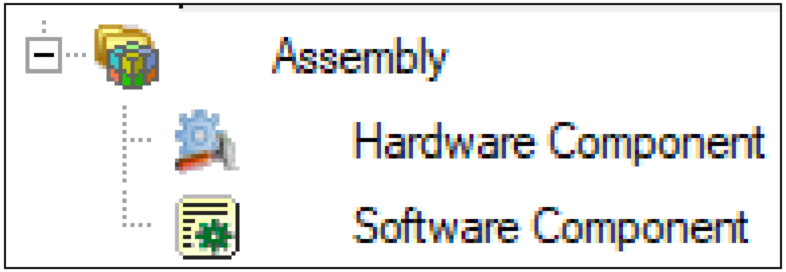
One cost element

$$\mu_s = \sum_{j=1}^{n} \mu_j$$

$$\sigma_s = \sqrt{\sum_{k=1}^{n} \sigma_k^2 + 2\sum_{k=2}^{n} \sum_{j=1}^{k-1} \rho_{jk}\sigma_j\sigma_k}$$

Multiple cost elements

Case Study Introduction



Development and Production estimate involving hardware, software and integration effort
100 units; 20 lbs hw; 15K SLOC

- Ground rules to maintain consistency between MoM and MC
  - Same number of cost elements
  - Same cost estimating relationship
  - Same type of input distributions (triangular)
  - Same range around inputs (+20% / -10%)
  - Same level of correlation (0.0 / 0.2 / 0.5 / 0.7)
- Factors to be evaluated
  - Difference in means
  - Difference in standard deviations
  - Difference at 50% confidence
  - Difference at 80% confidence

Results

	Multivariate Parametric Model Three cost elements Correlation at 0.0			
	MoM	MC	Raw Delta	% Delta
Mean	$7,756,171	$7,200,919	$555,252	7.2%
SD	$ 837,878	$ 581,138	$256,740	30.6%
50%	$7,711,306	$7,109,153	$602,153	7.8%
80%	$8,443,035	$7,667,590	$775,445	9.2%
CV	10.8%	8.1%	2.7%	25.3%

	Multivariate Parametric Model Three cost elements Correlation at 0.2			
	MoM	MC	Raw Delta	% Delta
Mean	$7,756,171	$7,169,694	$586,477	7.6%
SD	$ 904,672	$ 620,266	$284,405	31.4%
50%	$7,703,943	$7,080,520	$623,423	8.1%
80%	$8,495,754	$7,661,599	$834,154	9.8%
CV	11.7%	8.7%	3.0%	25.8%

	Multivariate Parametric Model Three cost elements Correlation at 0.5			
	MoM	MC	Raw Delta	% Delta
Mean	$7,756,171	$7,201,829	$554,342	7.1%
SD	$ 996,503	$ 738,776	$257,726	25.9%
50%	$7,692,938	$7,076,426	$616,511	8.0%
80%	$8,567,626	$7,814,235	$753,391	8.8%
CV	12.8%	10.3%	2.6%	20.2%

	Multivariate Parametric Model One cost element Correlation at 0.7			
	MoM	MC	Raw Delta	% Delta
Mean	$7,756,171	$7,235,478	$520,692	6.7%
SD	$1,053,285	$ 849,101	$204,184	19.4%
50%	$7,685,627	$7,066,739	$618,888	8.1%
80%	$8,611,701	$7,910,361	$701,340	8.1%
CV	13.6%	11.7%	1.8%	13.6%

Results Analysis – Method of Moments

- Mean stays the same
  - Makes sense since same values are used in same formula
- Standard deviation continually increases as correlation increases
  - Makes sense since correlation is a variable in standard deviation equation
- 50% confidence value steadily decreases; 80% steadily increases
  - Due to increased standard deviation "stretching" the distribution
  - More to come on this topic….

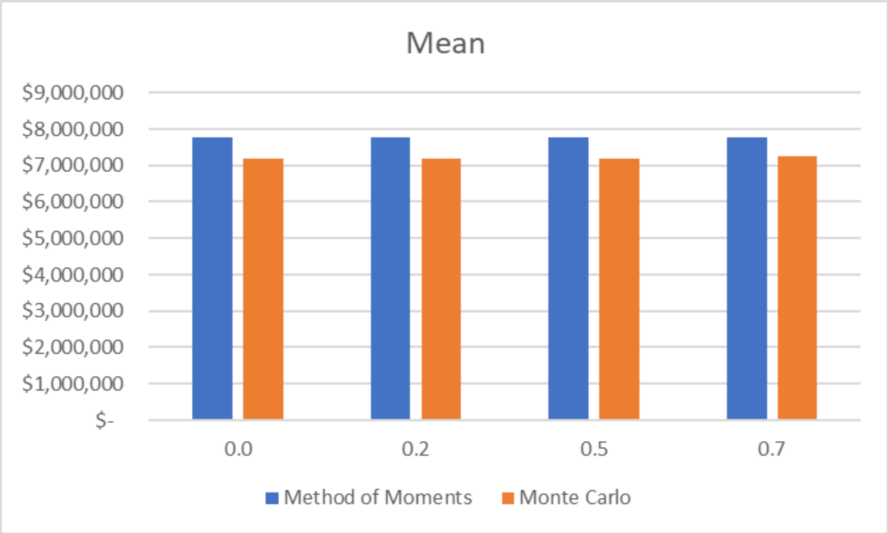
	Method of Moments			
	0.0	0.2	0.5	0.7
Mean	$7,756,171	$7,756,171	$7,756,171	$7,756,171
SD	$ 837,878	$ 904,672	$ 996,503	$1,053,285
50%	$7,711,306	$7,703,943	$7,692,938	$7,685,627
80%	$8,443,035	$8,495,754	$8,567,626	$8,611,701
CV	10.8%	11.7%	12.8%	13.6%

Results Analysis – Monte Carlo

- Mean is variable
  - Makes sense due to random input variable selection
- Standard deviation continually increases as correlation increases
  - As positively correlated values are used together, you'll get higher highs / lower lows
- 50% confidence value steadily decreases; 80% steadily increases
  - Due to increased standard deviation "stretching" the distribution

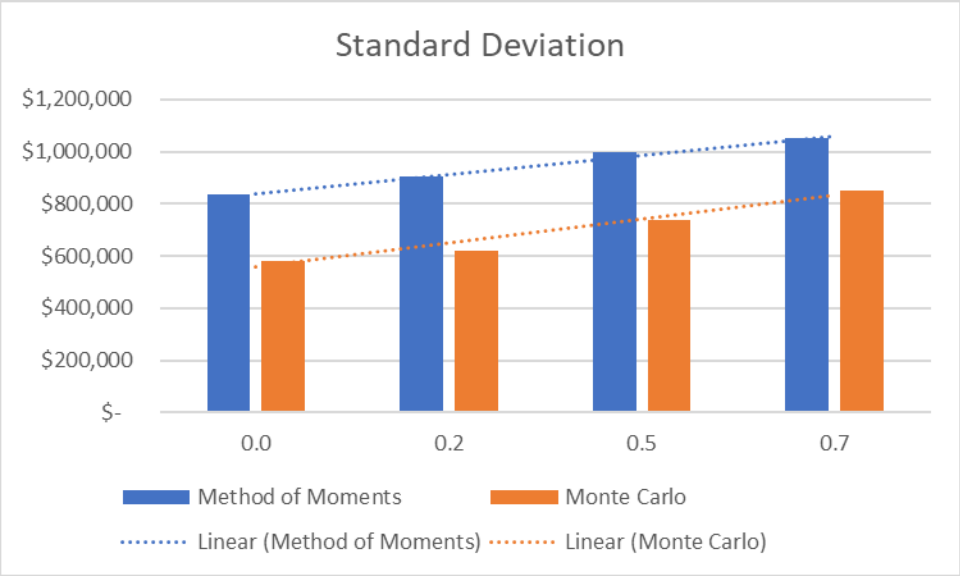
	Monte Carlo			
	0.0	**0.2**	**0.5**	**0.7**
Mean	$7,200,919	$7,169,694	$7,201,829	$7,235,478
SD	$ 581,138	$ 620,266	$ 738,776	$ 849,101
50%	$7,109,153	$7,080,520	$7,076,426	$7,066,739
80%	$7,667,590	$7,661,599	$7,814,235	$7,910,361
CV	8.1%	8.7%	10.3%	11.7%

Results Analysis – MoM and MC Compared

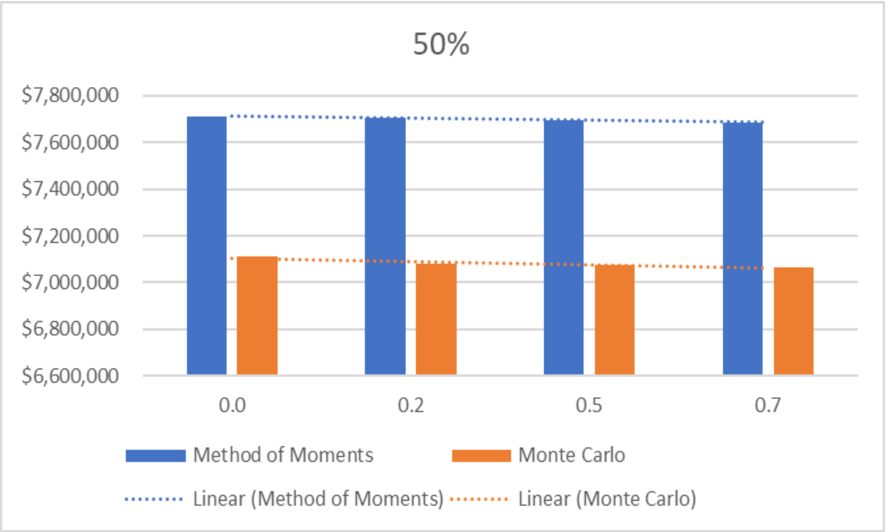


Means for Monte Carlo simulation were lower overall by approximately 7%. This may be by chance. Means for Method of Moments stayed consistent since the same formula and same input values are used each time, regardless of correlation changes.

Standard deviations increases consistently between methodologies as correlation increased. Method of moments consistently led to higher standard deviation than Monte Carlo. This is perhaps the most significant difference between the methodologies and will be explored further.

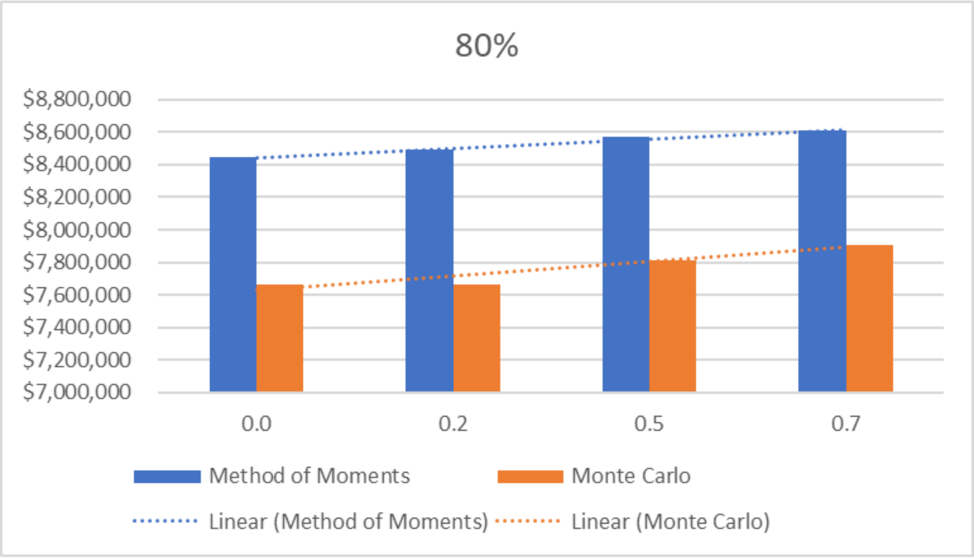


Results Analysis – MoM and MC Compared



The 50th percentile had a consistent, yet slight, decrease across both methodologies. This is due to the aforementioned increase in the standard deviations and will be explored further.

The 80th percentile had a consistent increase across both methodologies. This is due to the aforementioned increase in the standard deviations and will be explored further.

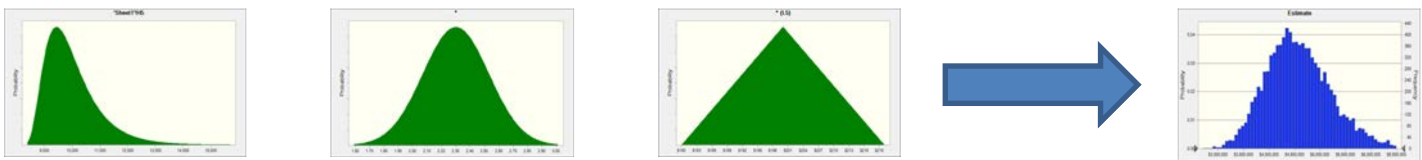


Results Analysis – Standard Deviation Impact

- MoM: mean and standard deviation are calculated from the most optimistic, most likely, and most pessimistic outcomes for each cost element

$$\mu = \frac{L+M+H}{3} \qquad \sigma = \sqrt{\frac{L^2+M^2+H^2-LM-LH-MH}{18}}$$

- MC: mean and standard deviation are statistical results of random selection of input variables….which may NOT be at the extremes



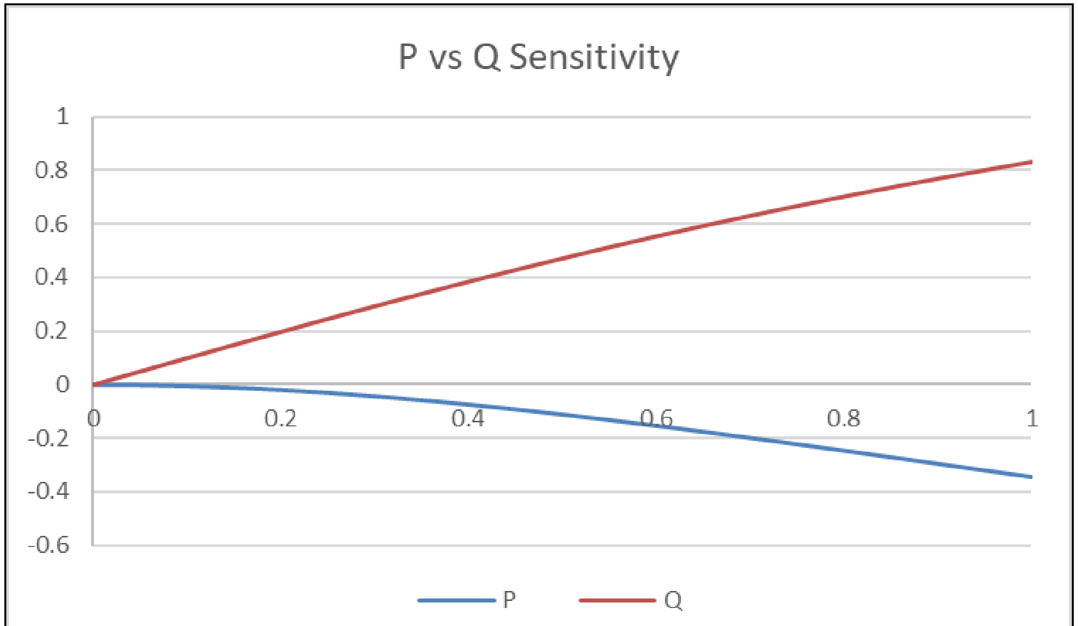
- Method of Moments "guarantees" these extreme highs and lows will be included in standard deviation calculation
- Monte Carlo "provides the opportunity" for extreme high or low input values but relies on random selection, which may be influenced by correlation

Results Analysis – 50% and 80% Impact

- As positive correlation increases
  - Standard deviation increases
  - 50^{th} percentile decreases
  - 80^{th} percentile increases
- MoM: percentile values determined in part by P and Q values
- P and Q values determined by mean and variance

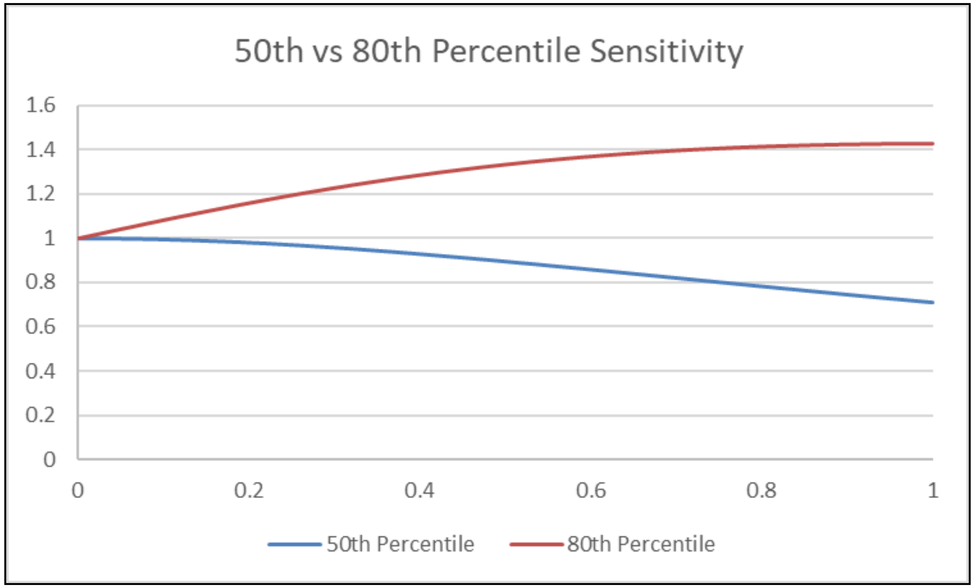
$$P = \tfrac{1}{2} \ln \frac{\mu^4}{\mu^2 + \sigma^2}$$

$$Q = \sqrt{\ln\left(1 + \frac{\sigma^2}{\mu^2}\right)}$$



Results Analysis – 50% and 80% Impact

- MoM: percentile values determined by $e^{P+Z_\alpha Q}$
- Z_a value for 50th percentile is 0.0
- 50th percentile value is solely dependent on exponentiating P
- Z_a value for 80th percentile is 0.84162
- 80th percentile value is dependent on P and Q; Q rise offsets P fall



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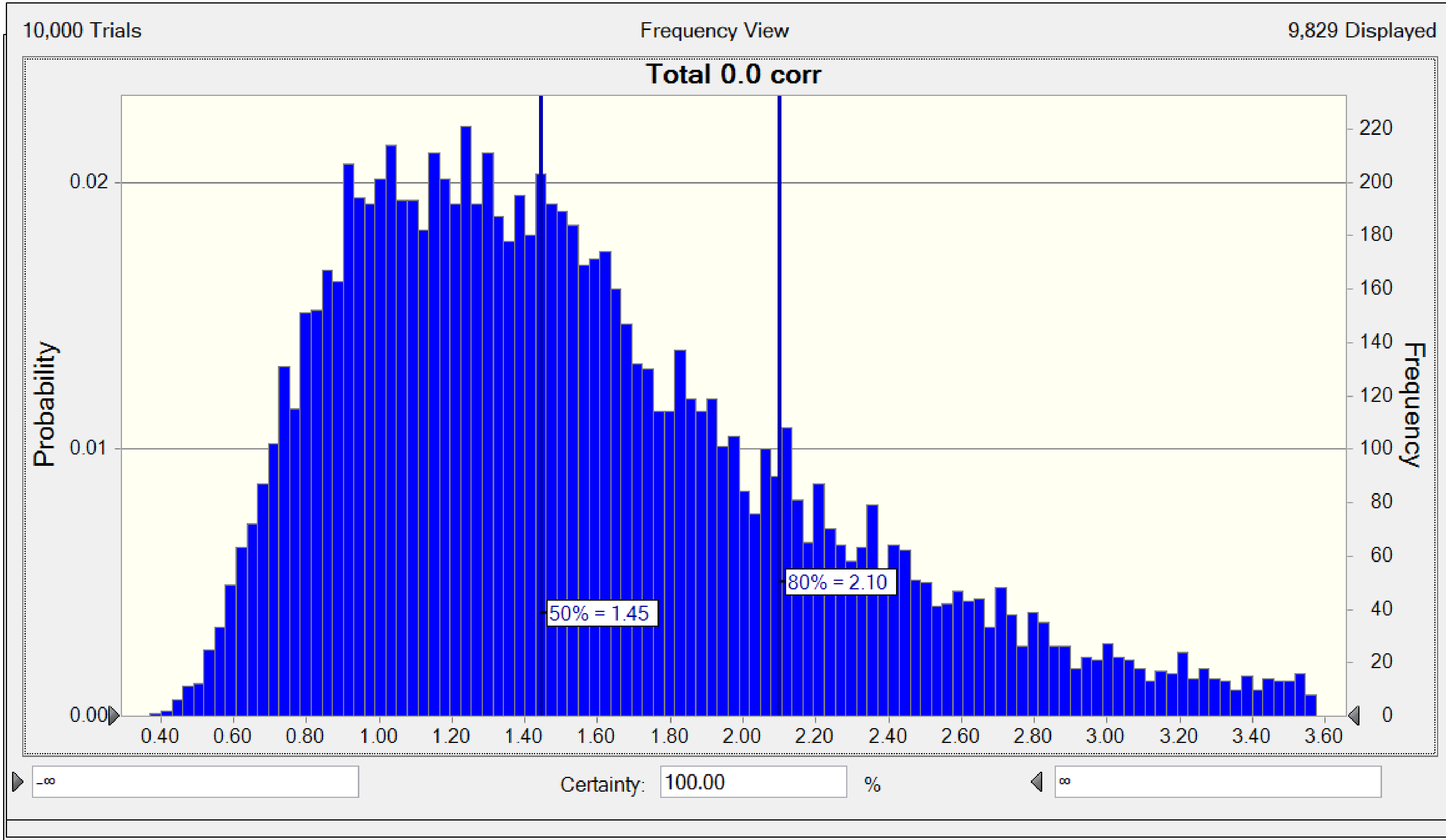
Results Analysis – 50% and 80% Impact

- MC: As distribution is skewed right, percentiles move accordingly



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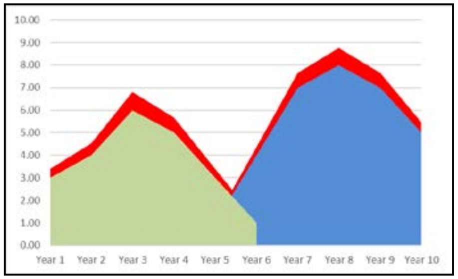
Risk Spread Methodologies

- After risk dollars determined, total must be allocated over program
- Several common methods used:
  - Uniform allocation aka "Peanut Butter Spread"
  - Weighted allocation
  - Event driven allocation
  - Distribution allocation
- Realities
  - Risks "should" be discretely identified through formal risk assessment
  - Allocation of risk dollars "should" be timed to mitigate identified risks
  - Risk timing may differ by program, phase, or milestone…or all the above!
  - Allocating risk dollars is at best an art with some science and sorcery included
  - Ultimate goal is to ensure the program has the right color of money, the right amount of money, and at the right time

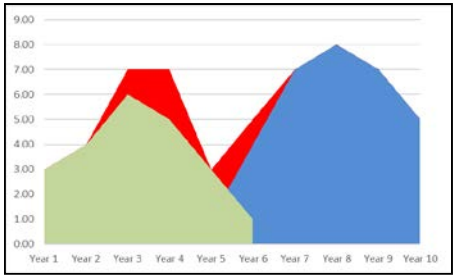
Risk Spread Methodologies

- Uniform spread
  - Risk spread across program evenly, regardless of phase or programmatic considerations
  - Weakest methodology

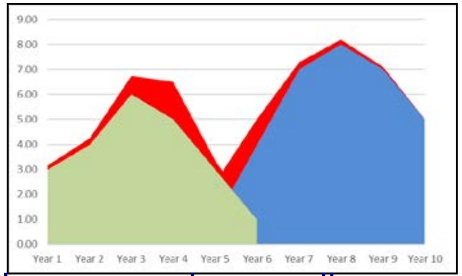
- Weighted spread
  - Allocation based on weighted expenditures
  - Implies more activity = more risk
  - Better than uniform



- Event driven spread
  - Tied to historically risk-prone events by phase
  - Example: development test; initial production
  - Better than weighted



- Distribution allocation
  - Allocate by statistical distribution(s)
  - Combines weighted and event driven methods
  - Easily implemented via spreadsheets



To Monte Carlo or not to Monte Carlo….
That is the Question!

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Mr. Joe Bauer is a Solutions Consultant with PRICE Systems. He is the primary technical focal point for Air Force customers, providing training, mentoring, and consulting. In addition to the Air Force, Joe supports several defense contractors in the US, as well as key government / defense agencies in Canada. Joe joined PRICE Systems after twenty years of service in the US Air Force. Prior to joining PRICE Systems, Joe was the lead hardware estimator for the F-22 Raptor program office. Joe earned a Master of Science degree in Cost Analysis from the Air Force Institute of Technology in 2009. He earned an MBA from the University of Phoenix in 2005. Joe is also a Certified Cost Estimator / Analyst (CCEA) with the International Cost Estimating and Analysis Association (ICEAA). He can be contacted at Joe.Bauer2@pricesystems.com

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