PRT-239

Modern Methods for Budget-Constrained Schedule Analysis

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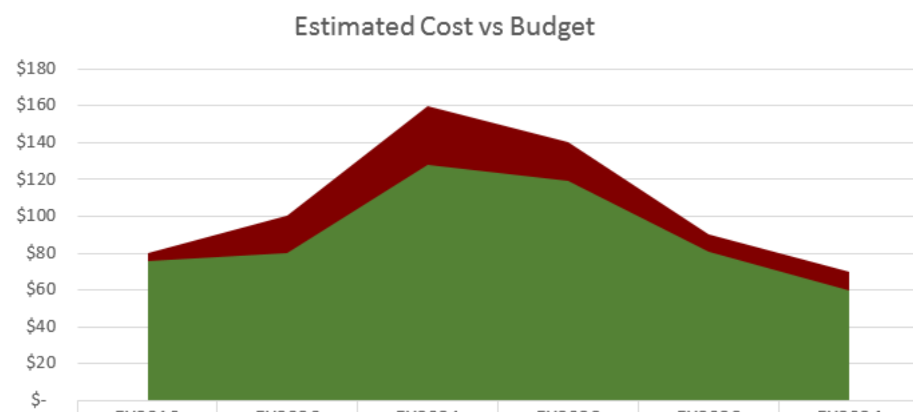
INTRODUCTION

With the push in recent years to merge the cost and schedule analysis disciplines, there is an invigorating effort underway to create the most robust, accurate models possible. This need cuts across the customer base impacting defense, science, construction or any high cost project. There remains the unanswered question that has lingered for years, if not decades: *What effect does the budget have on schedule analysis?*

Schedule analyses typically do not consider disparities among the cost estimate, the budget, and the schedule. This often causes unexplainable discrepancies between cost and schedule assessments (**Figure 1**). If an independent cost estimate results in a cost (i.e. effort required) that is significantly higher than the budget (i.e. resources available) for that item, the overall schedule duration for that item will surely grow unless additional budget (resources) becomes available within the planned duration. This casts a shadow of doubt on all schedule activities linked to that WBS item. A reasonable approach to account for this discrepancy is to calculate the effect of a stretch or delay in the schedule until budget becomes available to complete the work. There are no generally accepted methods to adjust schedule durations in response to a constrained budget.

The community needs a cost and schedule assessment technique that can be used to adjust the schedule based on the assumption that work cannot be performed until funding is available to pay for it. This paper introduces a method to do this.



	FY2019	FY2020	FY2021	FY2022	FY2023	FY2024
Cost	$80	$100	$160	$140	$90	$70
Budget	$76	$80	$128	$119	$81	$60

Figure 1: Visualizing the Discrepancy

WHERE WE BEGIN

Imagine you are developing a schedule model that incorporates cost in some fashion (either a cost-loaded schedule model or a cost estimate with detailed schedule relations). Yearly cost estimate results have been viewed. Potentially, although not necessarily, an uncertainty simulation was completed, generating uncertainty statistics. The report was created and the results of the analysis show the planned finish date and costs. Potentially an s-curve around the finish date and average yearly costs are projected. The program manager is comfortable with the results and then asks:

"Okay, so how will my schedule be affected if the budget profile is a flat $10M across the 5 years of the project?"

Armed with as little as the schedule, the estimated yearly cost and the yearly budget, this paper provides the framework for a budget-analysis model to answer that question and more, such as:

- "Given this budget, will I be able to finish the project with my schedule contingency?"
- "Given this budget, what is the likelihood I can finish this project on schedule?"
- "What budget profile would minimize my likelihood of overrun?"

The ideal scenario is to use the framework presented in this paper to *preemptively* reconcile the discrepancy between the schedule's cost estimate and the realities of the budget to minimize the delay caused by the funding shortfall.

LAYING THE GROUNDWORK

GROUND RULES AND ASSUMPTIONS

Since each project has its own unique challenges, goals, and management approach, the process for understanding how the budget affects the schedule necessarily contains assumptions about the environment of the project. The method described in this paper contains these:

1. Work cannot be performed until funding is available
2. The estimated cost resulting from the model represents the amount of effort needed to be completed; this work can be delayed but not ignored
3. Each year's budget cannot be exceeded, work slows or stops before the budget is exceeded
4. The budget comes from a single source and covers all the activities modeled in the schedule

THE CONCEPT

The estimated cost calculated from the schedule model represents the effort to be completed before the next year's work can start. The budget provided in that year, if less than the estimated cost, limits the amount of work that can be done in that year. If the budget is less than the cost, not all the individual activities can be completed in that time span. This implies that some activities planned in one year are inevitably pushed out into the next year, either by adjusting the schedule ahead of time or unexpectedly due to the lack of funding.

When considering the results of the model, the estimated effort for a given time period, e.g., a year, is easily represented by a rectangle (**Figure 2**). The length of which represents the duration and the height represents the average rate of the effort being performed. While the actual rate of work fluctuates monthly, weekly, even daily, the *average* rate provides an accurate measurement of the project's progression.

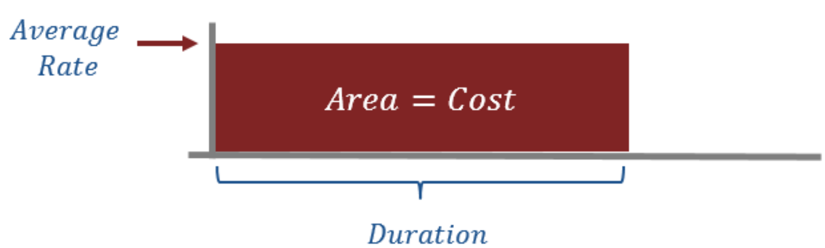


Figure 2: Rectangle of Estimated Cost

The next step is to compare the cost of the project with the budget. There may be a discrepancy staring right at us: we have estimated a certain cost scheduled in a year, yet we do not have the funding to complete all the activities planned. With the assumption that more funding is *not* available, the project will *not* finish on schedule (**Figure 3**). There are a few ways to calculate exactly how the schedule is affected depending on the project environment and management options.

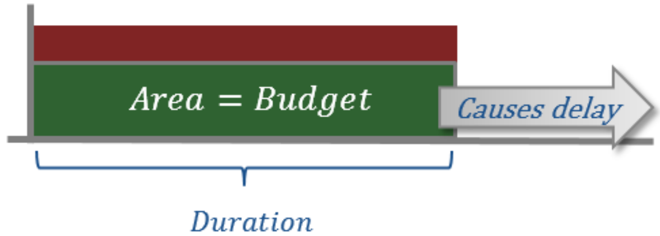


Figure 3: Rectangle of Estimated Cost and Budget

The various methods of adjusting the schedule duration to fit the budget can be solved geometrically. The problem then becomes a matter of creating a *spending profile* where the area is equal to the *budget,* instead of the cost. This new profile can then be used to calculate the extended duration (**Figure 4**).

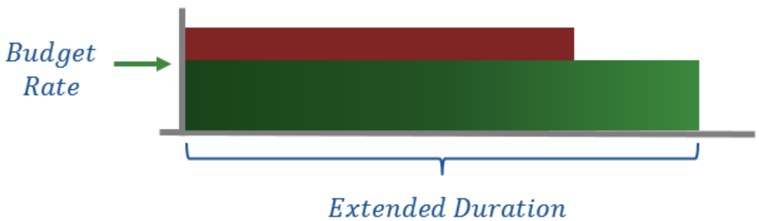


Figure 4: Rectangle of Estimated Cost and Budget with extension

THE SOLUTIONS

This section contains a few different methods for calculating the schedule extension identified in the previous section. Each method provided below satisfies the various requirements of the problem but provides flexibility for various assumptions regarding the individual project. The "best" method is not necessarily the one that provides the shortest delay due to the budget shortfall, but rather the one that conforms to the reality of the project. Determining the amount of time that the schedule is delayed is the goal of the calculations, although the solution is based upon how the rate is adjusted. This rate is used to calculate the extended duration and also provides a tangible foundation for interpreting the calculations.

THE RECTANGLE SOLUTION

The most straightforward solution involves turning the original rectangular profile into a different rectangle with the same area but different parameters (**Figure 5**).

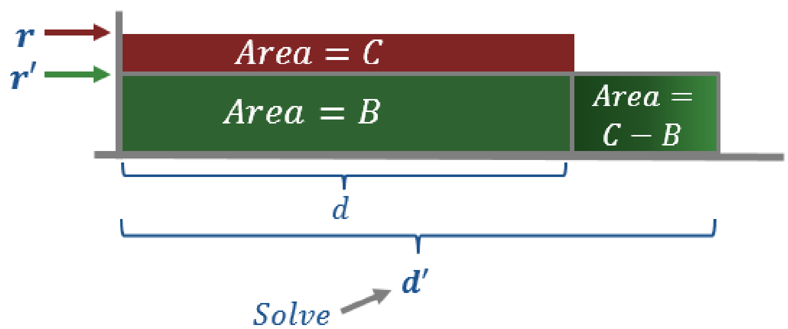


Figure 5: Rectangle of Estimated Cost and Budget with extension and variables

The original cost rectangle is simply flattened and extended, with the requirement that the area within the year equals the budget and the entire area equals the estimated cost.

The calculations below describe the concept at a high level. Cases in which more information is provided, such as the Time Independent (TI)/ Time Dependent (TD) breakout, are discussed later in the paper, along with the full algorithm's steps to implement these solutions.

THE "SHRINK AND EXTEND" CALCULATIONS

$$\textbf{Given Data}:$$

$$C = Estimated\ yearly\ cost$$

$$B = Budget\ for\ given\ year$$

$$Assuming\ C > B$$

$$d = Duration\ of\ work\ (e.g., year)$$

$$Let\ r = \frac{C}{d}\ be\ the\ average\ rate\ over\ the\ duration$$

$$\textbf{\textit{Require}}:$$

Area of rectangles

$$B = dr' \text{and } C = d'r'$$

$$\text{where } d' \text{and } r' \text{are the adjusted duration and rate (resp.)}$$

$$\textbf{\textit{Solution}}:$$

$$\boldsymbol{d' = \frac{C}{r'} = \left(\frac{C}{B}\right) d}$$

$$with\, r' = \frac{B}{d}$$

The requirements stated above are the geometric and mathematical formulation of the assumptions regarding fitting the work into the year at budget while the cost is maintained by pushing out the duration.

It may be necessary to add a user-defined minimum to the adjusted rate to prevent unrealistic results.

The adjusted rate, *r'*, is a flat, average rate of work that results in the budget being met. This adjusted rate implies an explicit reduction in workforce to manage the budget or an implicit slowing down of the completion of activities due to lack of funding for them. Regardless, it is the extended duration that is the goal of the calculations. The important part of the adjusted rate calculation is ensuring it does not drop below an unrealistic level, thus negating the resulting extended duration.

THE TRAPEZOID SOLUTION

Another spending profile solution might be in the shape of a trapezoid. There is no geometric reason the spending profile needs to form a rectangle. The same requirements can be satisfied using other shapes, such as a trapezoid (**Figure 6**). This entails an adjusted rate that decreases over the duration of the year, providing a "ramp down" effect that might be more realistic in certain situations.

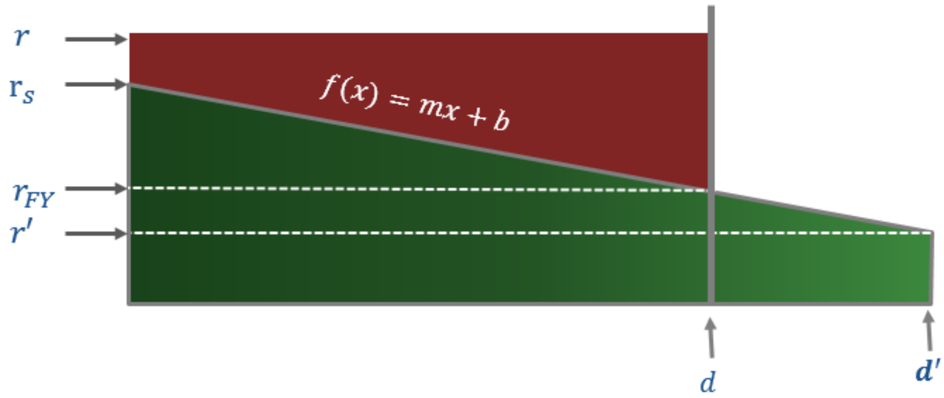


Figure 6: Trapezoid of Cost and Budget with extension and variables

THE "START UP, END DOWN" CALCULATIONS

Given Data:

$$C = Estimated\ yearly\ cost$$

$$B = Budget\ for\ given\ year$$

$$Assuming\ C > B$$

$$d = Duration\ of\ work\ (e.g. a\ year)$$

$$Let\ r = \frac{C}{d}\ be\ the\ average\ rate\ over\ the\ duration$$

Require:

Area of trapezoids

$$B = \left(\frac{1}{2}\right) d(\mathrm{r}_s + r_{FY}) \text{ and } C = (\frac{1}{2})d'(\mathrm{r}_s + r')$$

d' is the adjusted duration

r_s is the rate at the *begining* of the time period, e. g. fiscal year (FY)

r' is the rate at the end of the extension

r_{FY} is the rate at the end of the duration extension

Equation of the line that defines the trapezoid's "top" is needed to help solve the system

$$f(x|C, B, d) = mx + b \text{ with endpoints } f(0) = r_s \text{and } f(d') = r'$$

$f(x)$ is the rate at any given point in time

x is the duration

Solution:

$$\boldsymbol{d'} = \frac{\boldsymbol{2C}}{\mathbf{r}_s + \sqrt{\boldsymbol{r_s}^2 + \mathbf{4C}\frac{(\boldsymbol{B} - \boldsymbol{r_s d})}{\boldsymbol{d}^2}}}$$

The solution for the extended duration, d', is solved for as a function of the starting rate, r_s, because there are multiple dimensions the trapezoid can conform to given the geometric requirements of the two areas. By default, this r_s could simply be set to the same value as the original rate r. This must be done with caution though.

If the starting rate for the trapezoid is set to the original rate (if $r_s = r$), there are situations in which the slope of the top of the trapezoid (i.e. the m in $f(x) = mx + b$) would be so steep it would cause the rate to become negative (or unrealistically small) before the area requirements are met (**Figure 7**). To prevent this, r_s must shrink in order to prevent unrealistic situations.

To do this, a minimum for this end rate can be defined. We also know using the equation of the line that $f(d') = r' > 0$ (or other specified minimum greater than zero). An inequality can be derived that calculates the value that r_s must be less than or equal to in order for this minimum to be maintained. If this inequality is violated, then r_s can be set to the required minimum value calculated by the inequality.

In addition to the geometric restriction on the minimum value the ending rate can be, the analyst will likely desire to set the minimum to be well above the mathematical limitation. This would ensure that extended durations are not calculated based upon unrealistic project rates.

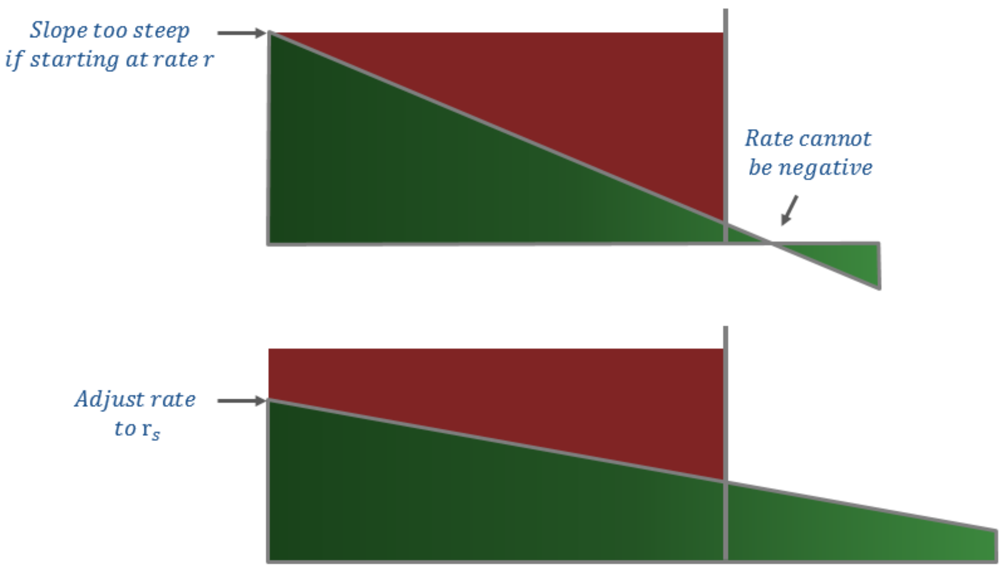


Figure 7: Example of slope too steep

Solution addendum:

$$\text{If } r \leq \frac{2C}{d} - \left(\frac{1}{2}\right)\sqrt{\frac{16}{d^2}(c^2 - CB) + 4(minimum)^2}$$

Then the trapezoid cannot be formed due to the slope being too steep

Then r_s cannot be set to r

Instead set r_s to the value on the right side of the above inequality

THE TRAPEZOID SOLUTION WITH AN UPTICK

Once funding becomes available at the start of the next year, it is feasible that work can pick back up again. Instead of maintaining the rate of the previous year as funds become available again, it is possible to recalculate the trapezoid's shape so that it increases at the start of the next year. This represents the rate slowly increasing once funding becomes available again (**Figure 8**). The calculations are almost the same as the previous section, what changes is the slope of the line at the end of the year, i.e. at d, the slope is flipped from a negative to a positive and the line it forms can be recalculated (**Figure 9**).

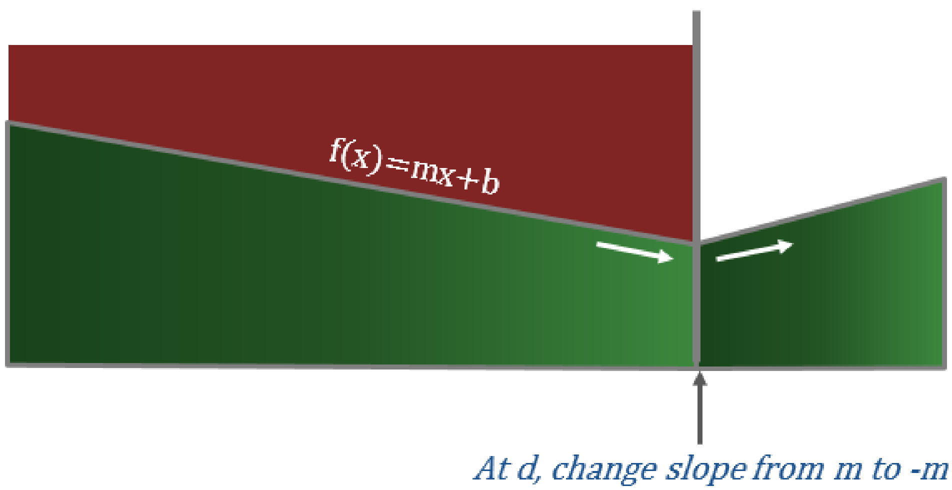


Figure 8: Trapezoid with Uptick

THE "START UP, END UP" CALCULATIONS

$$\boldsymbol{Given\ Data}:$$

$$C = Estimated\ yearly\ cost$$

$$B = Budget\ for\ given\ year$$

$$Assuming\ C > B$$

$$d = Duration\ of\ work\ (e.g., a\ year)$$

$$Let\ r = \frac{C}{d}\ be\ the\ average\ rate\ over\ the\ duration$$

$$\boldsymbol{Require}:$$

Area of trapezoids

$$B = \left(\frac{1}{2}\right) d(\mathrm{r}_s + r_{FY})\ \text{and}\ C = (\frac{1}{2})d'(\mathrm{r}_s + r')$$

With the added requirement of the "ramp up" trapezoid portion

$$C - B = \left(\frac{1}{2}\right)(d' - d)(r_{FY} + r')$$

d' is the adjusted duration

r_s is the rate at the *begining* of the time period, e.g. fiscal year (FY)

r' is the rate at the end of the extension

r_{FY} is the rate at the end of the duration extension

Equation of the line that defines the trapezoid's "top" is needed to help solve the system

$f_1(x|C, B, d) = mx + b_1$ with endpoints $f(0) = r_s$ and $f(d) = r_{FY}$

A second equation for the "ramp up" portion at the end of the year with the opposite slope

$f_2(x|C, B, d) = -mx + b_2$ with endpoints $f(d) = r_{FY}$ and $f(d') = \mathrm{r}'$

$f(x)$ is the rate at any given point in time

x is the duration

Solution:

$$d' = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

with

$$a = -m$$

$$b = r_{FY} + 3md + r_s$$

$$c = 2B - 2C - r_{FY}d - 2md^2 - r_s d$$

and

$$m = \frac{2(B - r_s d)}{d^2}$$

$$r_{FY} = f_2(d) = md + r_s$$

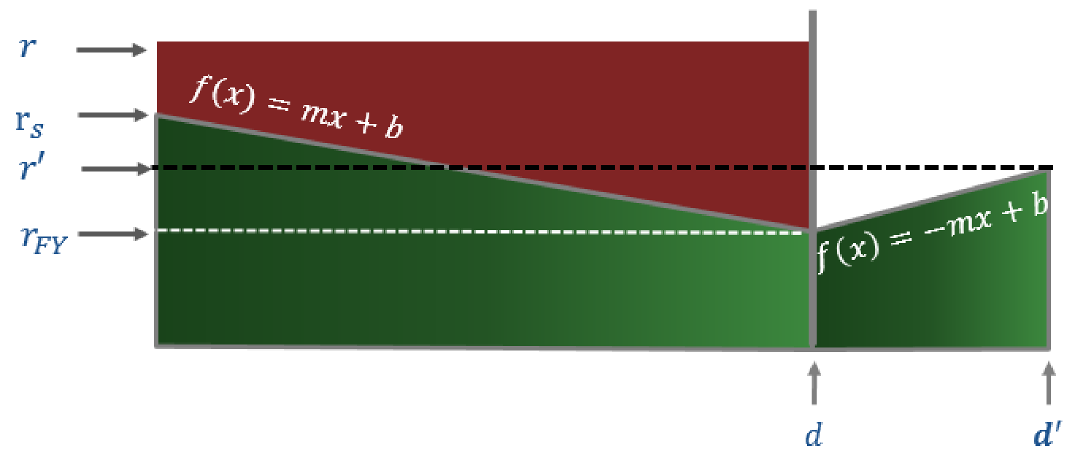


Figure 9: Trapezoid Uptick with variables

GENERALIZED SOLUTION

The solutions provided thus far are some variations of a line. It is possible to generalize the solution so that the rate can take nearly any shape, while still conforming to the geometric requirements of the cost and budget areas. This allows the management team to have full control over this budget analysis process.

The generalized solution uses the same assumptions discussed in previous methods, but does not stipulate the shape of the spending profile. Instead, this solution provides the framework for the analyst to work within the specific environment of their project.

GENERALIZED SOLUTION

$$\int_0^{d} f(x)\, dx = B$$

$$\int_0^{d\prime} f(x)\, dx = C$$

Based upon the number of degrees of freedom left by $f(x)$, another objective function may be utilized to solve for the unknown variables. For example, adding a specific plot point the spending profile needs pass through. **Figure 10** shows how a generalized solution provides a variety of rate curves. Any function is possible as long as the area up to the end of the time period, d, (e.g., a year) equals the budget and the area up to the extension, d', is the estimated cost (d' is the variable being solved)

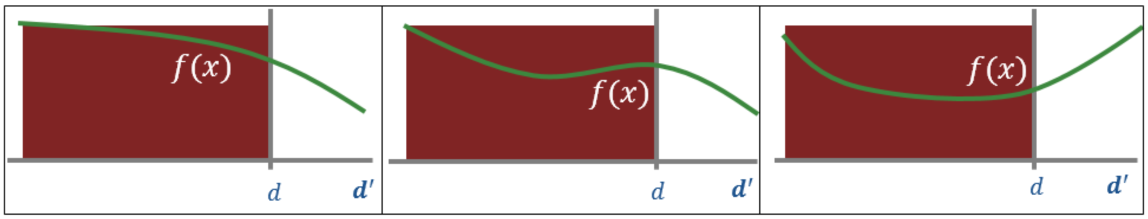


Figure 10: Examples of generic spending curves that satisfy requirements

PROOF OF CONCEPT

To ensure the generalized solution holds, we can take another look at the trapezoid method to adjust the schedule. That method was solved using the geometry for the area of a trapezoid. Now we can take the same problem and use calculus to derive a formula that describes the exact same spending profile. If the generalized solution is valid, it should provide the same results as the trapezoid method (**Figure 11**).

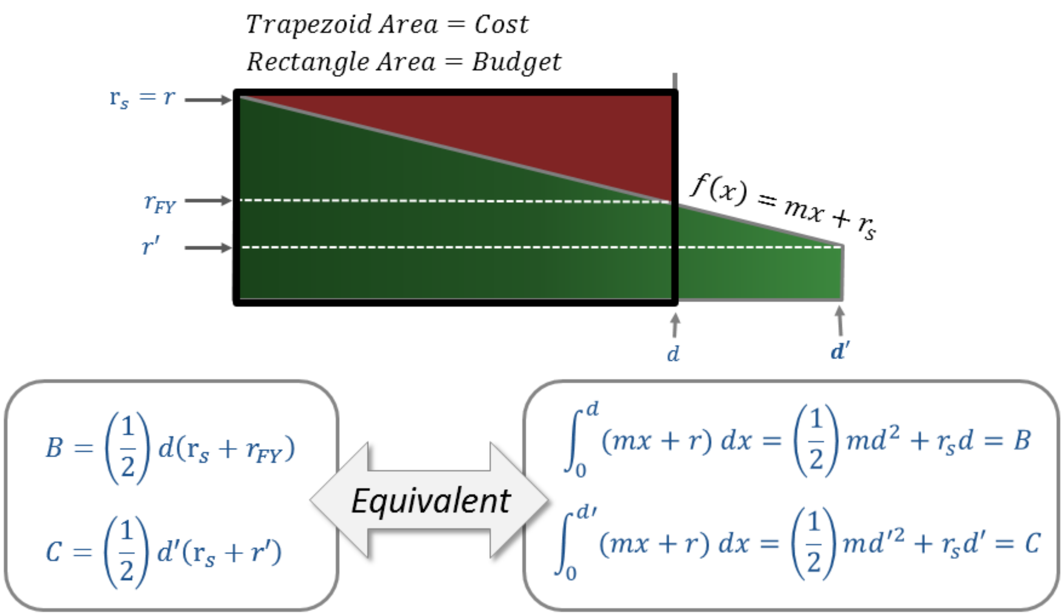


Figure 11: Proving Generalized and Trapezoid Method Equivalence

To recreate the trapezoid setup, we simply use an equation of a line to generate the rate, rather than solving the equations for the area of two trapezoids. Also as with the trapezoid setup, we want to solve for d' as a function of a given starting rate (in the simplest example we assumed this starting rate was the same as the original rate, i.e. $r_s = r$

To keep the calculations simple, we assume this same condition.

Require $f(0) = r_s = r$

Let $f(x) = mx + b$

The $f(0) = r$ requirement forces $b = r$

$$\int_0^d (mx + r)\, dx = \left(\frac{1}{2}\right) md^2 + rd = B$$

$$\int_0^{d'} (mx + r)\, dx = \left(\frac{1}{2}\right) md'^2 + rd' = C$$

Leaving two equations with unknown values for m and d', we are still given $C, B, d,$ and r

Rearranging the first integral to solve for m and then plugging that into the second to solve for d' yields:

$$m = \frac{2(B - rd)}{d^2}$$

$$\left(\frac{1}{2}\right) \frac{2(B - rd)}{d^2} d'^2 + rd' = C$$

$$\frac{(B-rd)}{d^2}d'^2 + rd' - C = 0$$

$$\boldsymbol{d' = \frac{-r \pm \sqrt{r^2 + 4C\frac{(B-rd)}{d^2}}}{2\frac{(B-rd)}{d^2}}}$$

CONCEPT PROVED

The goal of the previous section was to derive the same solution for *d'* by using two different approaches. Indeed the two solutions look different at first glance but with some algebra, they can be shown to be equal to each other, thus proving the generalized solution is the only set of equations necessary to derive any adjusted rate calculations, linear or otherwise.

Trapezoid (geometric) method with $\boldsymbol{r_s = r}$:

$$d' = \frac{2C}{r + \sqrt{r^2 + 4C\frac{(B-rd)}{d^2}}}$$

Generalized (calculus) method using a linear formula:

$$d' = \frac{-r + \sqrt{r^2 + 4C\frac{(B-rd)}{d^2}}}{2\frac{(B-rd)}{d^2}}$$

Setting both equations equal to each other:

$$\frac{2C}{r + \sqrt{r^2 + 4C\frac{(B-rd)}{d^2}}} = \frac{-r + \sqrt{r^2 + 4C\frac{(B-rd)}{d^2}}}{2\frac{(B-rd)}{d^2}}$$

Getting rid of the fractions by moving the denominators of each side to the opposite side:

$$2C\left(2\frac{(B-rd)}{d^2}\right) = \left(-r + \sqrt{r^2 + 4C\frac{(B-rd)}{d^2}}\right)\left(r + \sqrt{r^2 + 4C\frac{(B-rd)}{d^2}}\right)$$

Factoring the left side and distributing the right side:

$$4C\left(\frac{(B-rd)}{d^2}\right) = \left(-r^2 + r^2 + 4C\frac{(B-rd)}{d^2}\right)$$

Leaving this equality and proving both methods lead to the exact same solution:

$$4C\left(\frac{(B-rd)}{d^2}\right) = 4C\left(\frac{(B-rd)}{d^2}\right)$$

EXAMPLE USING A SECOND-DEGREE POLYNOMIAL

If it was desired to use a rate curve rather than a rate line to generate the results, a second-degree polynomial can be used as $f(x)$ in the generalized solution of the rate function.

To keep the calculations simple, we assume that the starting value of the adjusted rate be the same as the original starting rate. Additionally we decide that the rate at the start of the year is at its peak and will start to decrease from there. These requirements can be translated as points on $f(x)$.

Require: $f(0) = r_s$ and $f'(0) = 0$

Let $f(x) = ax^2 + bx + c$

The first requirement forces $c = r_s = C/d$

The second requirement forces $b = 0$

Resulting in $f(x) = ax^2 + r_s$

The first integral is used to solve for a:

$$\int_0^d ax^2 + r_s\,dx = \left(\frac{1}{3}\right)ad^3 + r_s d = B$$

$$a = \frac{3(B-C)}{d^3}$$

Providing us the equation for the rate as a function of *any* duration:

$$f(x) = \frac{3(B-C)}{d^3}x^2 + \frac{C}{d}$$

The second integral is used to solve for the exact extended duration, d'

$$\int_0^{d'} ax^2 + r_s\,dx = \left(\frac{1}{3}\right)ad'^3 + r_s d' = \frac{(B-C)}{d^3}d'^3 + r_s d' = C$$

To find the specific extended duration value d', solve the cubic function for d':

$$\frac{(B-C)}{d^3}\boldsymbol{d}'^3 + \frac{C}{d}\boldsymbol{d}' - C = 0$$

Unfortunately, there is no easy way to solve for a cubic since it depends on the exact structure of the equation, but this method does provide a value for d' as long as the cubic is solvable.

If we did not have one of the restrictions on $f(x)$ as stated above, then there would be one extra variable that would be able to control an aspect of the results. For example, if we did not require the starting adjusted rate $f(0)$ to be the same as the original starting rate r_s, then we could have solved for d' as a function of r_s and maintained more flexibility in the results.

WHAT TO DO WITH THE RATE

Regardless of the chosen method for calculating the extended duration by way of adjusting the rate, the next step is performing this process repeatedly for each year. At each step in this process there are a series of checks and adjustments needed.

ADJUSTING FOR THE TYPE OF COST IN THE MODEL

The solutions discussed consider "cost" as a singular entity. In some models, costs are often split into TI and TD components to account for the different ways costs behave. Often there are level-of-effort (LOE) activities whose costs are separate from individual tasks.

Before the adjusted spending profile is calculated, these types of costs should be considered (**Figure 12**).

TI costs may occur on a fixed date regardless of the progress of the schedule; other times the TI costs may be dependent upon other tasks completing prior to incurring the TI cost. These scenarios should be addressed. One method to incorporate the TI cost into this budget algorithm is to mark its amount and date. When the budget adjustments are made during the year specified, it can subtract the TI cost from the budget prior to the adjustment calculation.

LOE costs may or may not be adjustable. If they are fixed, they can be marked and always taken out of the year's budget prior to the calculation. If the LOE activity is something that may be modified, e.g. a 10% fixed reduction for all LOE for a given year, this can also be calculated and removed from that year's budget prior to the adjustment calculation. For example, this would amount to reducing the project management staff, or reducing overhead costs.

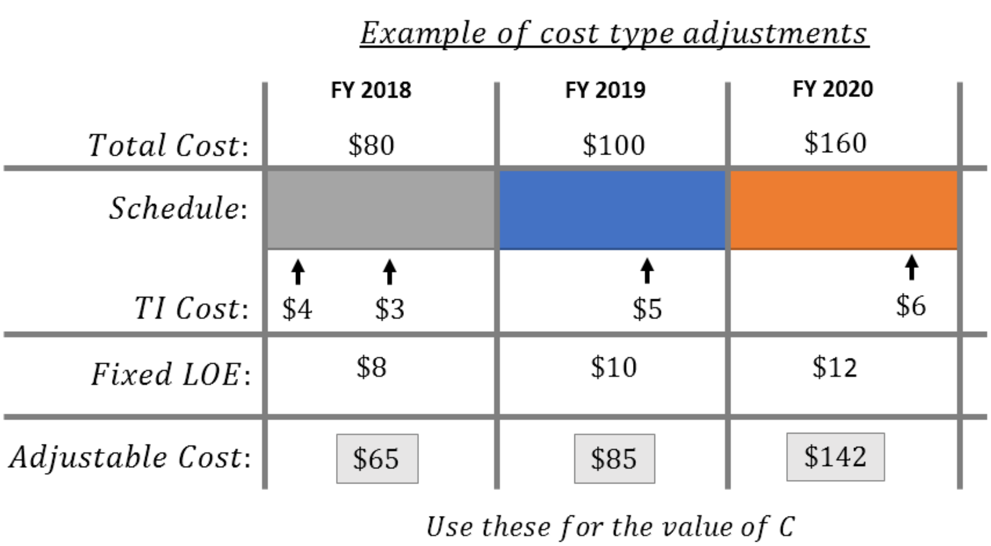


Figure 12: Example Cost Setup Prior to Budget Adjustments

THE BUDGET-ADJUSTMENT ALGORITHM

For the algorithm to initiate, the total cost must be greater than the total budget, for either the deterministic scenario or for a given iteration result. The basic steps of the process are outlined below; a detailed example follows.

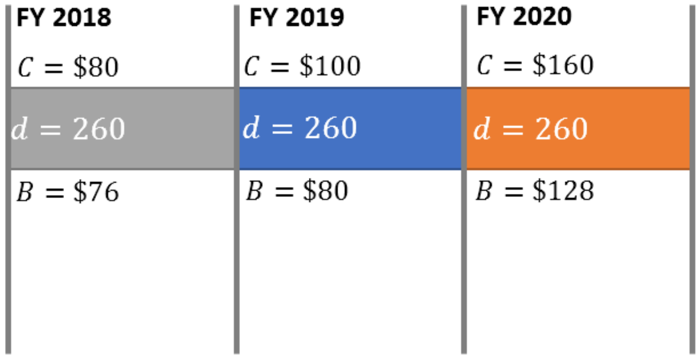
Before step 1, choose a rate-adjustment method, specify any LOE adjustments and mark when TI costs occur.

OUTLINE OF ADJUSTMENT ALGORITHM

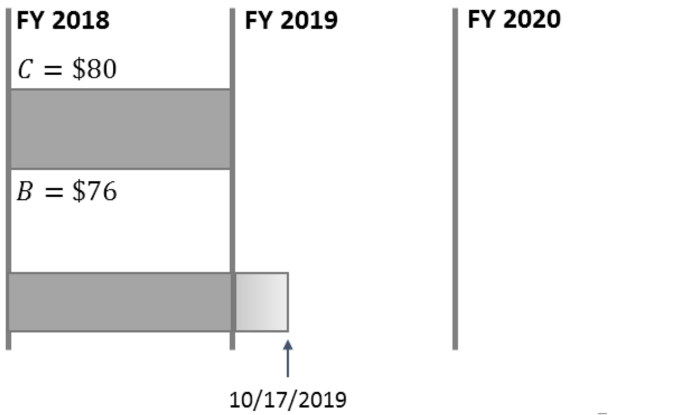
1. Calculate the extended duration
2. Check that there are enough funds in the following year to cover this extension
    a. If there are enough funds, subtract from that next year's budget
    b. If there are not enough funds, re-calculate the rate to reduce it even further so that there are enough funds
3. Move to the next year and start from Step 1

DETAILED EXAMPLE OF ADJUSTMENT ALGORITHM

The Setup



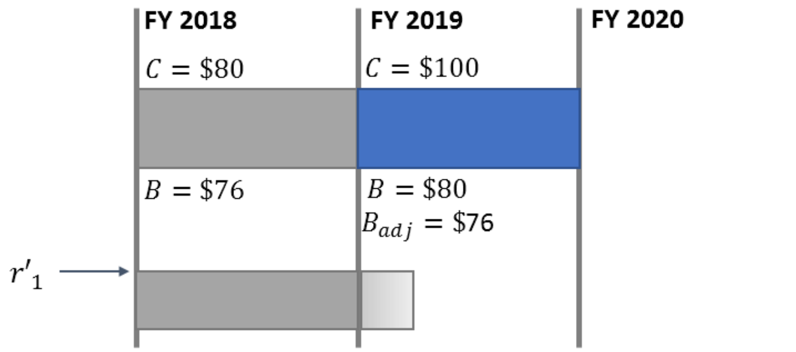
Step 1: Calculate the first year's extension



$$d'_1 = \left(\frac{C}{B}\right) d = \left(\frac{\$80}{\$76}\right) 260 = 274 \text{ workdays}$$

2018 Adjusted Finish Date:
9/30/2019+274=10/17/2019

Step 2: Subtract FY 2018's extended cost from FY 2019's budget



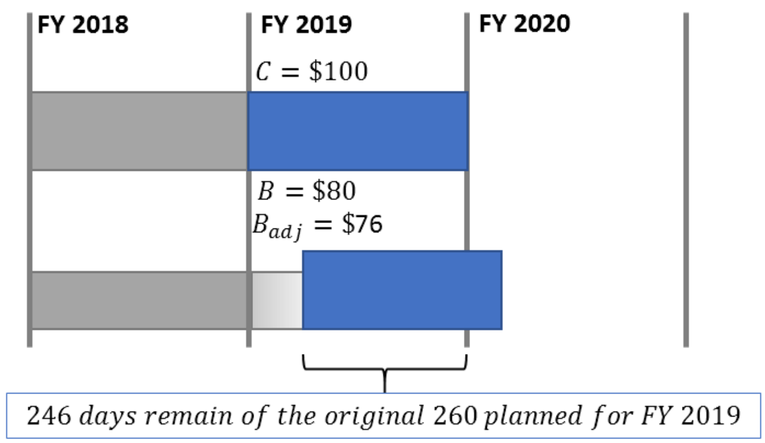
Extended Duration = 274-260 = 14 days

$$r'_1 = \left(\frac{B}{d}\right) = \left(\frac{\$76}{260}\right)$$

FY 2019 Adj. Budget = FY 2019 Original Budget minus the cost of the extension:

$$\$80-14\left(\frac{\$76}{260}\right)=\$76$$

Step 3: Calculate the work expected to complete in 2019 with the new adjusted budget

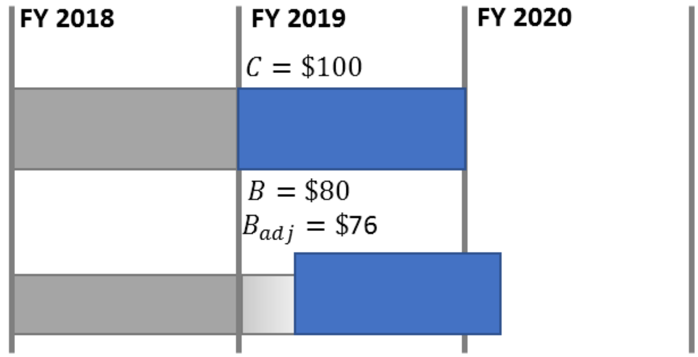


It would be unreasonable to assume/calculate a budget-adjusted rate based upon the expectation of completing all 260 days of work planned for FY 2019 with this new (less) adjusted budget of $76

A fair assumption would be to expect to complete only a proportion of that work with the new adjusted budget

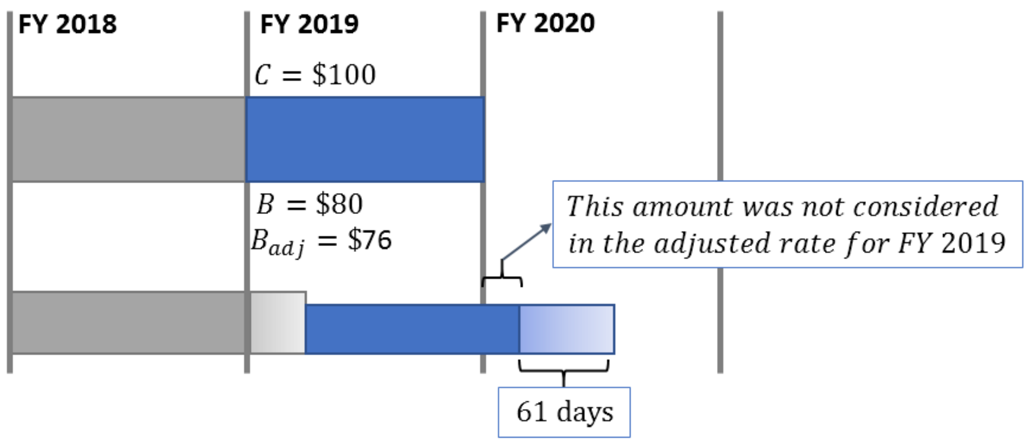
That is 246/260=95% of what was planned for FY 2019 that needs to be covered by $76

Step 4: Adjust the rate of the remaining FY 2019 work to fit the remaining budget



The original FY 2019 rate was $100/260=$0.38
If that rate persisted, the cost of the remaining work in FY 2019 would be
$0.38*246=$95 > $76
Thus FY 2019 remaining work needs to be adjusted using C=$95, B=$76, d=246

Step 5: Extend FY 2019 work into FY 2020



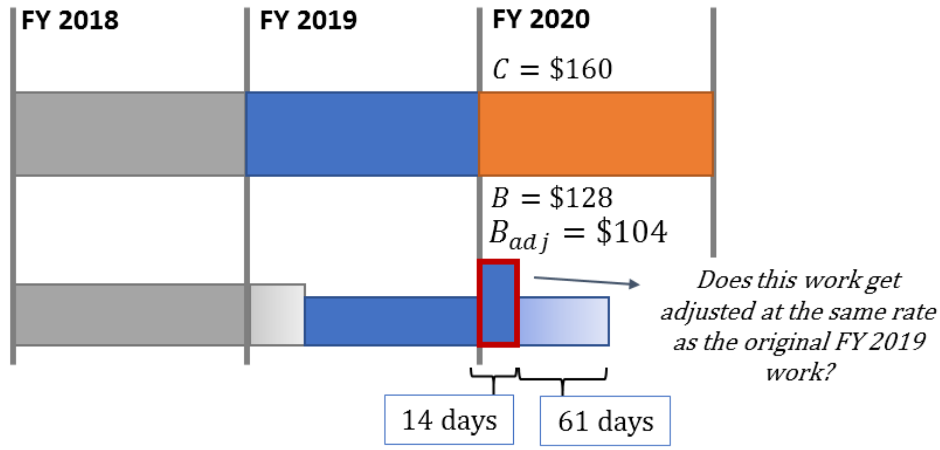
Adjusted rate for FY 2019: $(\frac{\$76}{246})=\$0.31$

Adjusted duration for FY 2019: $(\frac{\$95}{\$76})246=307$ workdays

$307 - 246 = 61$ days extended due to budget-adjustment of original FY 2019 work

But wait! That 61 days only considered the work remaining in FY 2019, it did not consider the work that was pushed out beyond FY 2019 (since we didn't know what funds would be available beyond that until we calculated the adjustments leading into it)

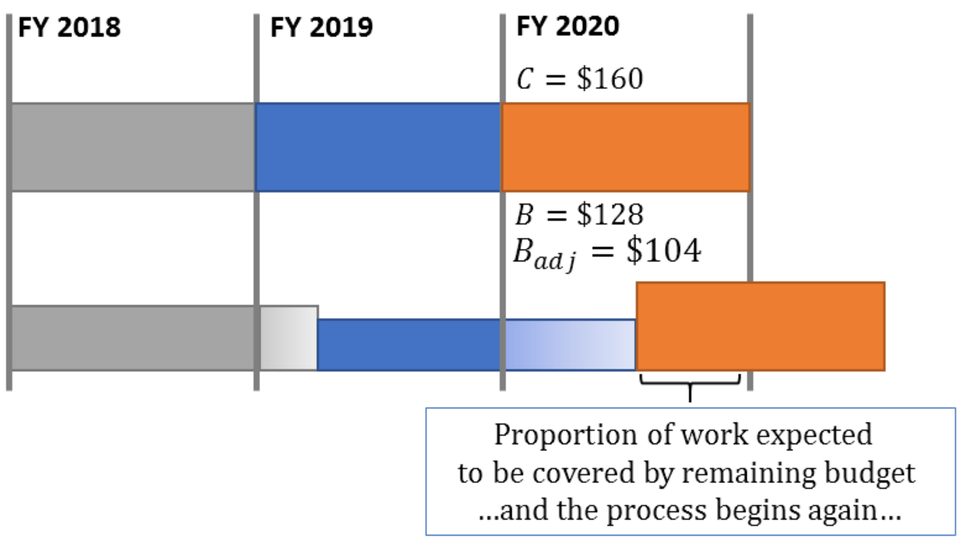
Step 6: Account for the unadjusted work from the previous year



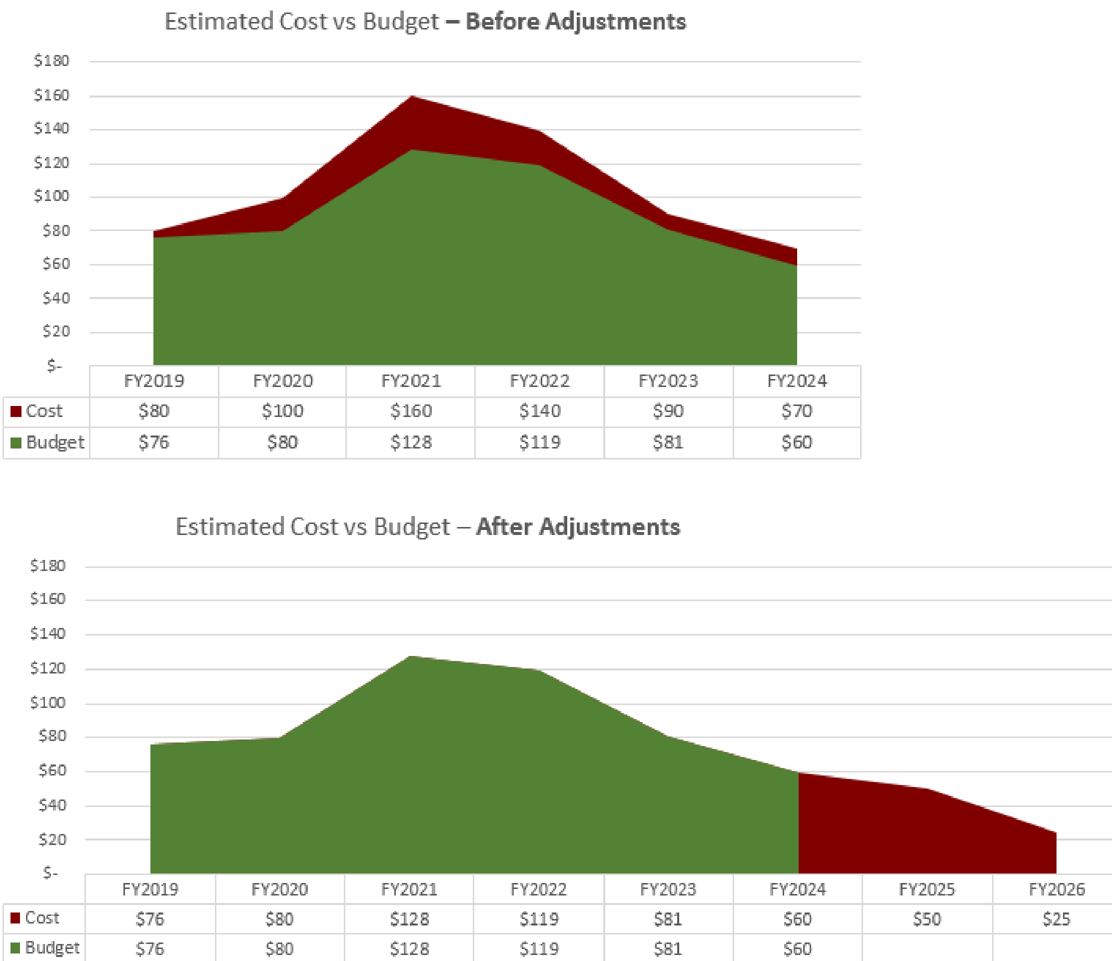
We have 61 days at the adjusted FY 2019 rate of \$0.31
We have the remaining 14 days (decidedly assumed) at the original FY 2019 rate of \$0.38

$$B_{adj} = \$128 - (\$0.31 * 61) - (\$0.38 * 14) = \$104$$

Step 7: Subtract the extension into FY 2020 from it's budget and start over



Once completed, the sum of the yearly estimated cost and given yearly budget match. This also results in an extended duration beyond the original plan with some extra cost associated with the extension (**Figure 13**).



	FY2019	FY2020	FY2021	FY2022	FY2023	FY2024
Cost	$80	$100	$160	$140	$90	$70
Budget	$76	$80	$128	$119	$81	$60

	FY2019	FY2020	FY2021	FY2022	FY2023	FY2024	FY2025	FY2026
Cost	$76	$80	$128	$119	$81	$60	$50	$25
Budget	$76	$80	$128	$119	$81	$60		

Figure 13: Visualizing the Reconciliation

THE SOLUTION IS JUST THE BEGINNING

ACCOUNT FOR UNCERTATAINTY

For a model that provides simulation data, these methods can be applied to each iteration, thus forming a robust view of the uncertainty around the budget adjustments. Once the algorithm is implemented, applying it to thousands of iterations would not be computationally expensive. Each step of the process is a deterministic calculation, as opposed to having to use regression or linear/nonlinear programming method.

The simulation data would look something like the scatter in **Figure 14**, providing an extremely powerful set of statistics to the schedule analysis.

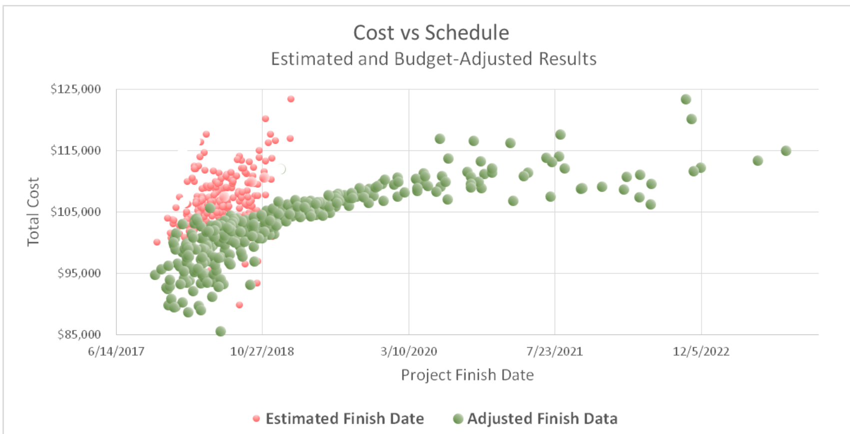


Figure 14: Cost vs Schedule Scatter of Budget-Adjusted Results

A budget uncertainty analysis answers questions such as:

- What is the average delay caused by my budget?
- What is the probability I exceed my schedule reserve due to a funding shortfall?
- What is the probability of my planned finish date in the current model and what is the probability of my planned finish date when I consider the budget?
- How does adding an additional $10M to my FY2019 budget affect the probability of my estimated finish date?

WHAT IF I DON'T HAVE A BUDGET YET?

The formulas laid out in this framework can all be solved for any variable of interest, most notably if the budget is not known but excursions want to be run on the effect of any budget profile.

The current formulation shows the variable d' written as a function of the average starting rate, using the estimated cost, budget, and duration as given inputs to the formula. The same formula can be rearranged to solve for the budget. This would provide information on what the budget needed to be given an acceptable delay.

For example, if there was a known schedule reserve for a given year that was taken out before the simulation, defining *d'* as the year's duration plus reserve would provide the budget needed to ensure the schedule reserve was funded.

This allows answers to questions like:

- Is my schedule reserve covered by the budget provided?
- How many days does my schedule save by increasing the budget by $10M each year?
- If I don't want to be delayed by more than 100 days, what budget profile do I need?
- How can I reallocate the given budget into the years to minimize my delay?

SUMMARY

It does not take a lot of complex data to facilitate a powerful analysis. Although the analysis is made more precise with more information (TI/TD breakout, uncertainty simulation, etc.) these calculations do not require complex models and thus should not be considered beyond the scope of any model used to predict cost and schedule results.

At the heart of the algorithm is the idea that the rate of work over a period of time needs to be adjusted so that the cost of the work performed over that period of time does not exceed a given budget. The adjusted rate provides a new spending profile that is used to calculate the extended duration of the schedule.

This extension can be achieved a number of ways: 1) by doing the work slower, 2) delaying the start of various efforts, 3) reducing the scope (e.g. ordering less units or testing less prototypes), or 4) recalibrating the workforce when funds deplete. The calculations presented here do not assume exactly how the project will cut back when funds become low, it simply provides the mathematical reality that work is not completed as estimated and thus a schedule delay is inevitable.

The example detailing how to use the selected rate-adjustment method is comprehensive, but can be modified based upon the exact capabilities of the management team. In addition, any uncertainty analysis run on the schedule model provides immensely useful capabilities when used in conjunction with the methods discusses in this paper.

CONCLUSION

The need to understand how a budget affects a schedule is a persistent and important facet of the cost/schedule community. As our capability to create robust models evolves, our ability to derive vital, relevant intelligence from those models needs to keep up. The most accurate model to estimate the cost of a project is ineffective if the realities of the budget are omitted. The estimated results hold no weight if the model exists in purely a theoretical state.

The methods introduced in this paper ground the schedule model in reality. The adjustment algorithm and rate formulas presented in this paper provide a framework that the estimating community can use to tackle the next relevant frontier in the evolving cost/schedule universe.

APPENDIX – FORMULA DERIVATIONS

TRAPEZOID – "START UP, END DOWN"

$\boldsymbol{Given\ Data}$:

$C = Estimated\ yearly\ cost$

$B = Budget\ for\ given\ year$

$Assuming\ C > B$

$d = Duration\ of\ work\ (e.g.\ a\ year)$

$Let\ r = \frac{C}{d}\ be\ the\ average\ rate\ over\ the\ duration$

$\boldsymbol{Require}$:

Area of a budget trapezoid:

$$B = \left(\frac{1}{2}\right) d(\mathrm{r}_s + r_{FY})$$

Area of cost trapezoid:

$$C = (\frac{1}{2})d'(\mathrm{r}_s + r')$$

Line defining the rate:

$$f(x|C, B, d) = mx + b = y$$

with endpoints $f(0) = r_S$ and $f(d) = r_{FY}$

$\boldsymbol{Solution\ Steps}$:

1. Solve the area equation of the cost, C, for d'
2. Solve the area equation of the budget, B, for r_{FY}
3. Solve for the slope, m, and intercept, b, for $f(x) = y$
4. Plug the equations for d' and r_{FY} into $f(x)=y$, then solve for r' as a function of r_s
5. Plug the equation for r' into the equation for d' to find the extended duration

Step1:

$$C = (\frac{1}{2})d'(\mathrm{r}_s + r') \Longrightarrow d' = \frac{2C}{\mathrm{r}_s + r'}$$

Step 2:

$$B = \left(\frac{1}{2}\right) d(\mathrm{r}_s + r_{FY}) \Longrightarrow r_{FY} = \frac{2B}{d} - \mathrm{r}_s$$

Step 3:

$f(x) = mx + b$

$f(0) = \ r_s \ \Longrightarrow b = r_s$

$f(d) = r_{FY} \Longrightarrow m = \dfrac{r_{FY} - r_s}{d}$

Thus $f(x) = \left(\frac{r_{FY} - r_s}{d}\right) x + r_s$

Plug in the equation for r_{FY} (derived in step 2) into $f(x)$:

$$f(x) = \left(\frac{\frac{2B}{d} - 2r_s}{d}\right) x + r_s$$

This defines the "top" of the trapezoid, we are interested in the "end", the rate $f(d')$

Step 4:

We are interested in finding the rate $f(d')$ (defined as $f(d') = r'$) to then use to find d'.

We can use the equation for d' from step 1 to solve for r'as a function of the variables we know, namely: (C, B, d, r_s)

$$f(d') = r' = \left(\frac{\frac{2B}{d} - 2r_s}{d}\right) d' + r_s$$

$$r' = \left(\frac{\frac{2B}{d} - 2r_s}{d}\right) \frac{2C}{\mathrm{r}_s + r'} + r_s = \left(\frac{2B - 2\mathrm{d}r_s}{d^2}\right) \frac{2C}{\mathrm{r}_s + r'} + r_s$$

Multiply both sides by $(\mathrm{r}_s + r')$ to clear the denominator:

$$(\mathrm{r}_s + r')r' = \left[\left(\frac{2B - 2\mathrm{d}r_s}{d^2}\right) \frac{2C}{\mathrm{r}_s + r'} + r_s\right] (\mathrm{r}_s + r')$$

$$\mathrm{r}_s r' + r'^2 = 4\left(\frac{BC - C\mathrm{d}r_s}{d^2}\right) + r_s^2 \ + \mathrm{r}_s r'$$

$$r' = \sqrt{4\left(\frac{BC - C\mathrm{d}r_s}{d^2}\right) + r_s^2}$$

This formulation serves three purposes.

1) It leaves the ending rate r' as a function of known variables, as long as we assume the value of $r_s$is the original starting rate $r = \frac{C}{d}$.
2) It allows flexibility for management to choose a lesser rate to being the year if they believe it is possible to reduce the rate upfront.
3) It provides a mathematical minimum for r' to prevent the slope from becoming so steep that the rate becomes negative before the area requirements are met. If the square root is less than 0, it forces $r_s$to be adjusted so that the square root is not violated.

Step 5:

From step 1: $d' = \frac{2C}{\mathrm{r}_s + r'}$

From step 4: $r' = \sqrt{4\left(\frac{BC - C\mathrm{d}r_s}{d^2}\right) + r_s^2}$

Therefore: $\boldsymbol{d'} = \frac{\boldsymbol{2C}}{\mathbf{r}_s + \sqrt{\boldsymbol{4}\left(\frac{\boldsymbol{BC - C\mathrm{d}r_s}}{\boldsymbol{d^2}}\right) + \boldsymbol{r_s^2}}}$

SOLUTION ADDENDUM - STEEP SLOPE VIOLATION

The initial assumption states that the original rate, $r = \frac{C}{d}$, is used to define the initial rate of the trapezoid, r_s. In order to ensure there are no mathematical inconsistencies:

$$r' = \sqrt{4\left(\frac{BC - C\mathrm{d}r}{d^2}\right) + r^2} \geq 0$$

If $r' = 0$ or close to 0, it would imply the project is progressing at an impossibly low unachievable rate, thus a minimum, m, can be defined and solved for as a function of the rate, r, allowing the violation to be checked before any other calculations are performed. This also provides a value for r_s when the value for r cannot (or should not) be used.

Given a user-defined minimum, min for the ending rate r':

$$\sqrt{4\left(\frac{BC - C\mathrm{d}r}{d^2}\right) + r^2} \geq min$$

$$\frac{4BC}{d^2} - \frac{4Cr}{d} + r^2 \geq (min)^2$$

Preparing for the quadratic formula to solve for r_s:

$$r^2 + (-\frac{4C}{d})r + (\frac{4BC}{d^2} - (min)^2) \geq 0$$

Using $r_s = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

With:

$a = 1$

$$b = -\frac{4C}{d}$$

$$c = \frac{4BC}{d^2} - min^2$$

$$r \geq \frac{\frac{4C}{d} \pm \sqrt{\frac{16C^2}{d^2} - \frac{16BC}{d^2} + 4(min)^2}}{2}$$

$$r \geq \frac{2C}{d} - \left(\frac{1}{2}\right)\sqrt{\frac{16}{d^2}(C^2 - BC) + 4(min)^2}$$

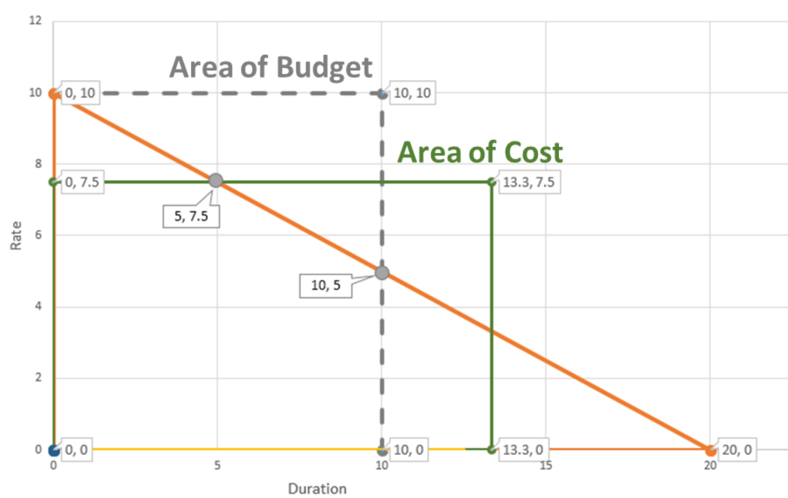
Use the inequality to check the assumption that $r_s = r$ is possible, and if necessary, to adjust, r_s.

If $r \leq \frac{2C}{d} - \left(\frac{1}{2}\right)\sqrt{\frac{16}{d^2}(C^2 - CB) + 4(min)^2}$

Then $r_s = \frac{2C}{d} - \left(\frac{1}{2}\right)\sqrt{\frac{16}{d^2}(C^2 - CB) + 4(min)^2}$

BUDGET-TO-COST RATIO THAT CAUSES STEEP SLOPE VIOLATION

Alternatively, a geometric interpretation can be used that requires only the budget and cost. It is left to the user to convince themselves that when $B/C < (3/4)$ the rate must be adjusted.



TRAPEZOID WITH UPTICK

The same variables from the previous calculations still all apply in this modified version of the trapezoid method.

Require:

Area of trapezoids

$B = \left(\frac{1}{2}\right) d(\mathrm{r}_s + r_{FY})$ and $C = (\frac{1}{2}) d'(\mathrm{r}_s + r')$

With the *added requirement* of the "ramp up" portion at the start of the next year, when funds presumably become available and work can pick up.

$C - B = \left(\frac{1}{2}\right) (d' - d)(r_{FY} + r')$

Equation of the line that defines the trapezoid's "top" is needed to help solve the system

$f_1(x|C, B, d) = mx + b_1$ with endpoints $f(0) = r_s$ and $f(d) = r_{FY}$

A second equation for the "ramp up" portion at the end of the year with the opposite slope

$f_2(x|C, B, d) = -mx + b_2$ with endpoints $f(d) = r_{FY}$ and $f(d') = \mathrm{r}'$

The assumption here being that at the end of the year, the rate that was declining in order to fit into the budget will begin to increase when the next year's funds become available. This is modeled using f_2.

The goal in the solution steps is to solve for the slope and intercept of the function f_2 so it can be used to calculate the extended duration.

Solution Steps:

1. Solve for the intercept b_2 using f_1 and f_2 (we already have the equation for the slope, m)
2. Place $f_2(d') = r'$ in the equation for the extended area $C - B$
3. Use the equation for the extended area $C - B$ to solve for d'

Step 1:

$f_1(0) = b_1 = r_s$

$f_1(d) = md + b_1 = r_{FY}$

$f_2(d) = -md + b_2 = r_{FY}$

$md + b_1 = -md + b_2$

$b_2 = 2md + b_1$

Thus $f_2(x) = -mx + 2md + r_s$

Step 2:

$f_2(d') = -md' + 2md + r_s = r'$

$C - B = \left(\frac{1}{2}\right)(d' - d)(r_{FY} + r')$

$C - B = \left(\frac{1}{2}\right)(d' - d)(r_{FY} + (-md' + 2md + r_s))$

$2(C - B) = r_{FY}d' - md'^2 + 2mdd' + r_s d' - r_{FY}d + mdd' - 2md^2 - r_s d$

Group by the unknown variable d' to arrive at the setup for the quadratic formula:

$-md'^2 + (r_{FY} + 3md + r_s)d' + (2B - 2C - r_{FY}d - 2md^2 - r_s d) = 0$

Solution:

$$d' = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

with

$a = -m$

$b = r_{FY} + 3md + r_s$

$c = 2B - 2C - r_{FY}d - 2md^2 - r_s d$

and

$$m = \frac{2(B - r_s d)}{d^2}$$

$r_{FY} = f_2(d) = md + r_s$