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Modern Methods for Budget-Constrained Schedule Analysis

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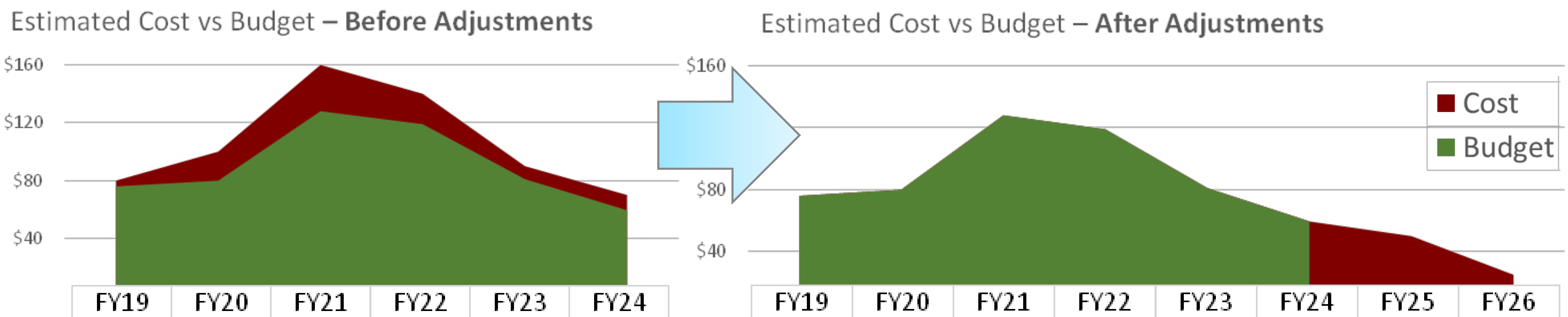
Introduction

- This presentation discusses methods for calculating the impact of a budget profile on a schedule analysis
  - If the yearly budget for a project does not cover the estimated yearly cost, the schedule will inevitably see delays as activities are stretched or delayed until funding becomes available
  - There is no standard approach for how to predict the impact of this common discrepancy between the cost estimate, budget, and schedule



The Path Forward

- An emphasis on the relationship between schedule and cost can be seen right here at ICEAA over the years
- Projects that merge cost and schedule into one model (or at least attempt to analyze the relationship) are heading in the right direction but there is further still to travel
- The next step to ensure maximum return on the effort placed into this analysis is to **reconcile the discrepancy between the results of the analysis with the realities of the budget**

Where We Begin: Ground Rules and Assumptions

1. Work cannot be performed until funding is available
2. The estimated cost resulting from the model represents the amount of effort needed to be completed
3. The budget comes from a single source and covers all the activities modeled in the schedule
4. Delay causes a "smooth slip" domino effect of schedule content
   - As opposed to a re-plan/re-baseline

These are necessary but not exhaustive

Where We End: The Solutions

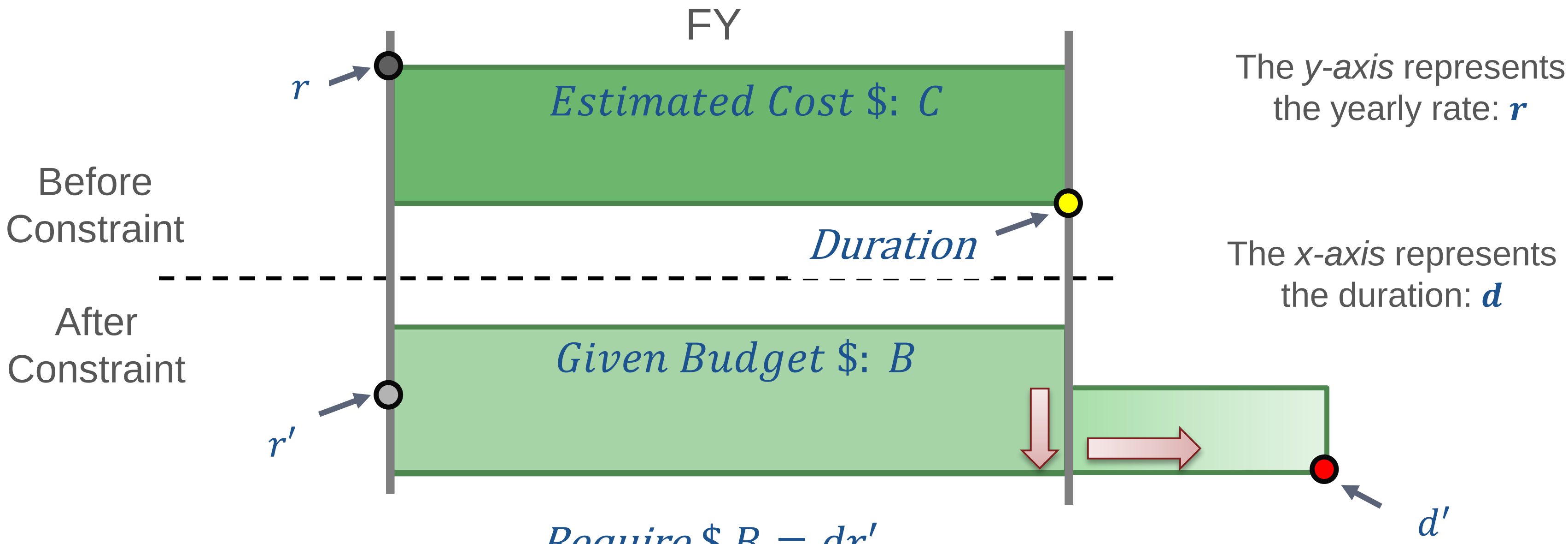
- Create a *spending profile* (expenditure rate curve) where the area is equal to the *budget* instead of the estimated cost for a given year
- This new profile can then be used to calculate the extended duration caused by a year's budget that still maintains the cost (effort) estimated by the model
- After iterating through all the years of the project, the overall delay in the schedule due to the budget constraint is revealed

Solution 1: Rectangles

Solution Concept

Slide animation:

- The general idea: "shrink and extend"
  - *Shrink* the average yearly (*expenditure*) rate so the *budget* is met
  - *Extend* the duration based upon the new rate so the *cost* is met



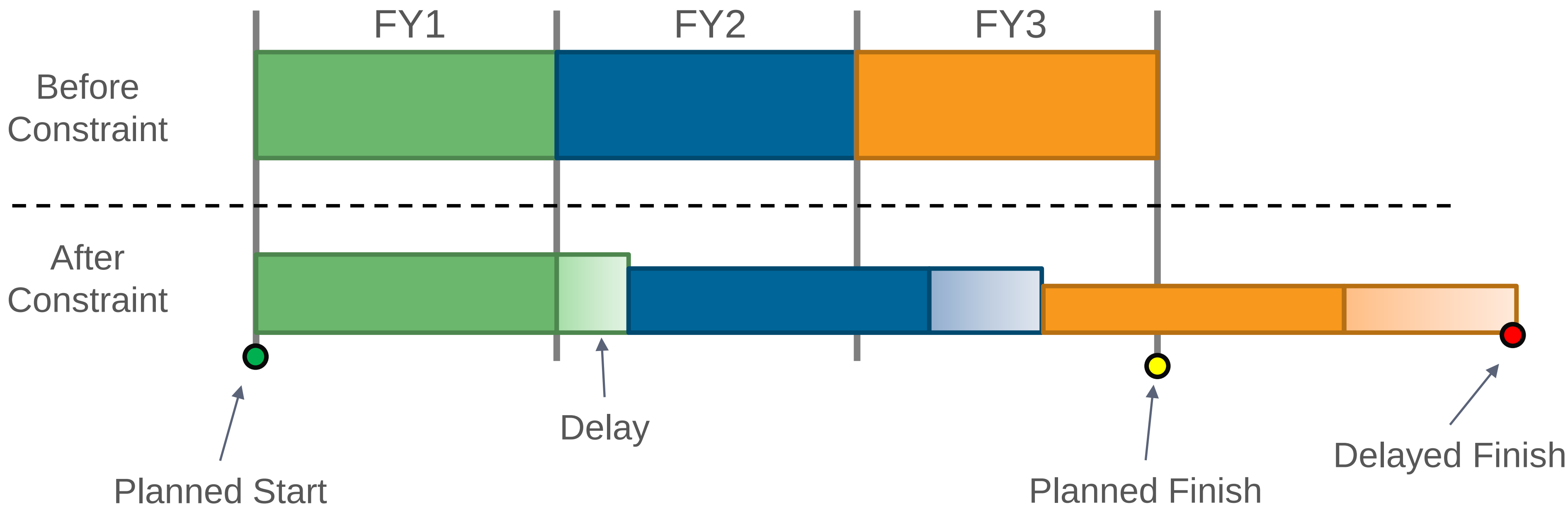
$$\textit{Require}\ \$\ B = dr'$$
$$\textit{With}\ \$\ C = d'r'$$

Solution: $r' = \frac{B}{d} \Rightarrow d' = \frac{C}{r'} = \left(\frac{C}{B}\right) d$

An apostrophe adjacent to a variable symbolizes an adjustment of the original variable

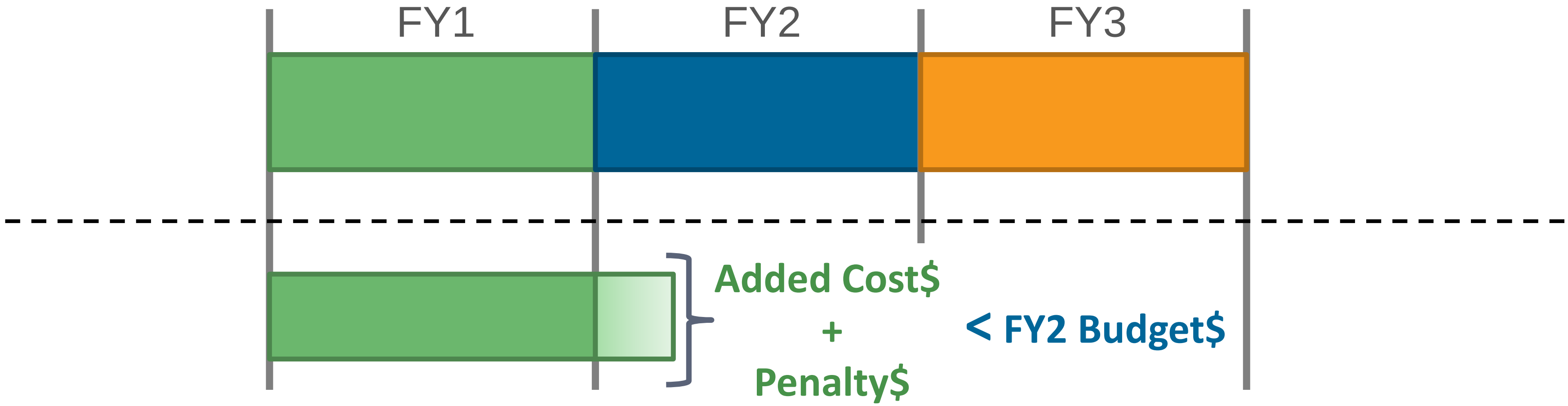
What the Budget-Constraint Looks Like

- We represent the scheduled yearly work before and after the effect of being constrained by the budget
  - Regardless of the calculations of the delay, the effect looks like this:



Solution 1: Flat Rate

- The solution equation is part of an iterative process
- Each subsequent year's adjustment must take into account the work pushed into it from the previous year



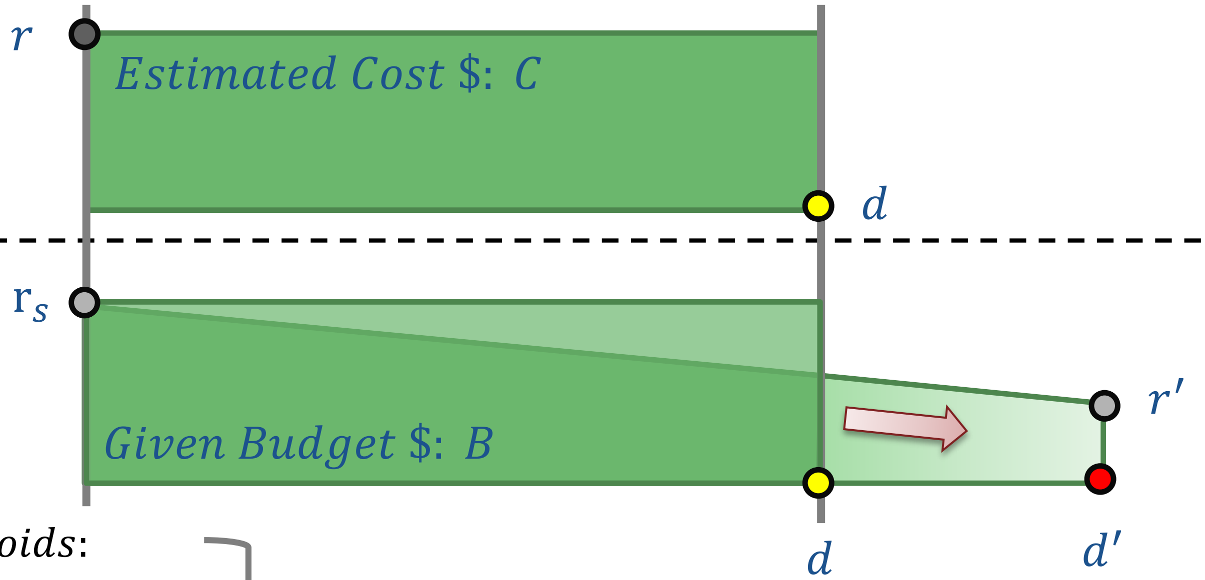
The budget for FY2 must be able to cover the additional cost (and potential penalty) of the work pushed into it before the adjustment calculation is repeated

Details on the algorithm are later in this presentation and the paper

Solution 2: Trapezoids

Solution Concept

- The general idea: “start up, end down”
  - The yearly adjusted rate begins equal to the original, then decreases
  - The same geometric requirements exists
    - Maintain cost by extending duration, conform to the Budget within the year, except this solution uses the area of a trapezoid to solve for the extension, d'



Area of trapezoids:

$$C = (1/2)d'(r_s + r')$$

$$B = (1/2)d(r_s + r_{FY})$$

Linear Cost Rate Function:

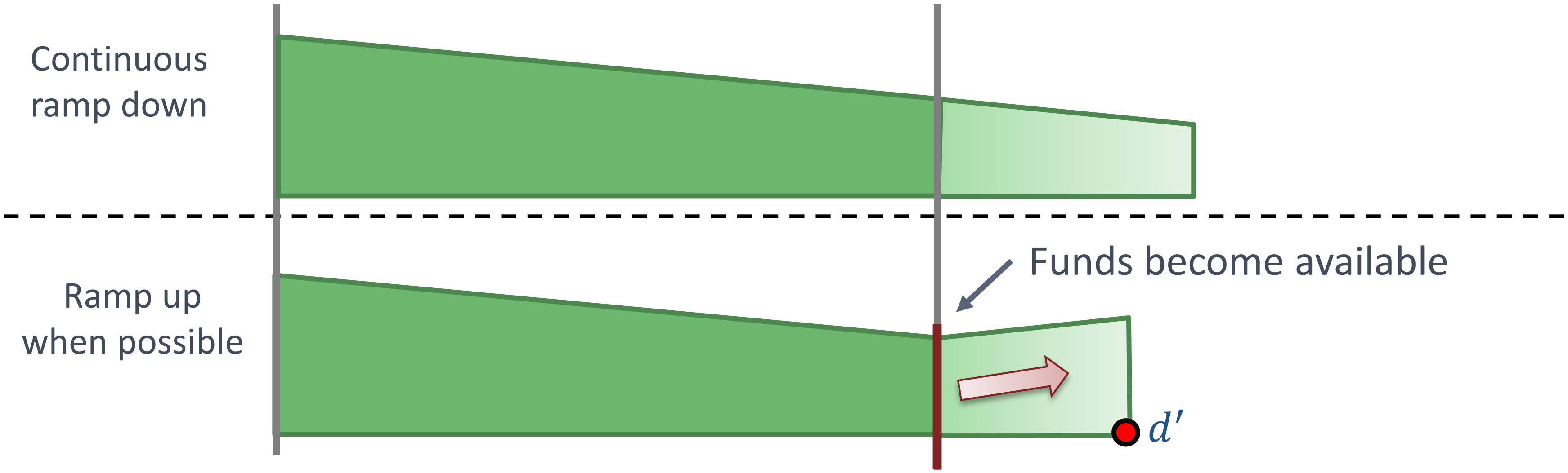
$$f(x) = \frac{f(d) - f(0)}{d}x + r_s$$

$$\boldsymbol{d'} = \frac{2C}{r_s + \sqrt{{r_s}^2 + 4C\frac{(B - r_s d)}{d^2}}}$$

See paper for derivation

Solution Concept

- The general idea: "start up, end down…with uptick"
  - For example, the slope of the extension can be flipped from negative to positive to allow a "ramp up" when funds become available
    - This shortens the extended duration relative to the unaltered trapezoid



The solution is derived from the same geometric setup as before, with another equation for the "ramp-up area" and the line defining the linear rate, leading to the solution:

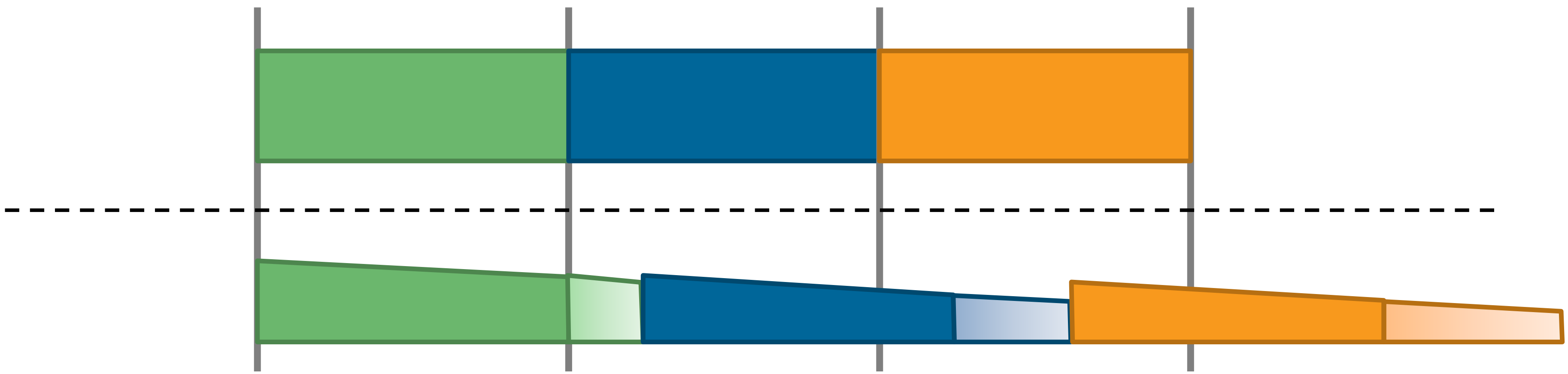
*Use the quadratic formula to solve for **d'**:*

$$-m\mathbf{d'^2} + (r_{FY} + 3md + r_s)\mathbf{d'} + (2B - 2C - r_{FY}d - 2md^2 - r_s d) = 0$$

See paper for derivation

Solution 2: Linear Rate

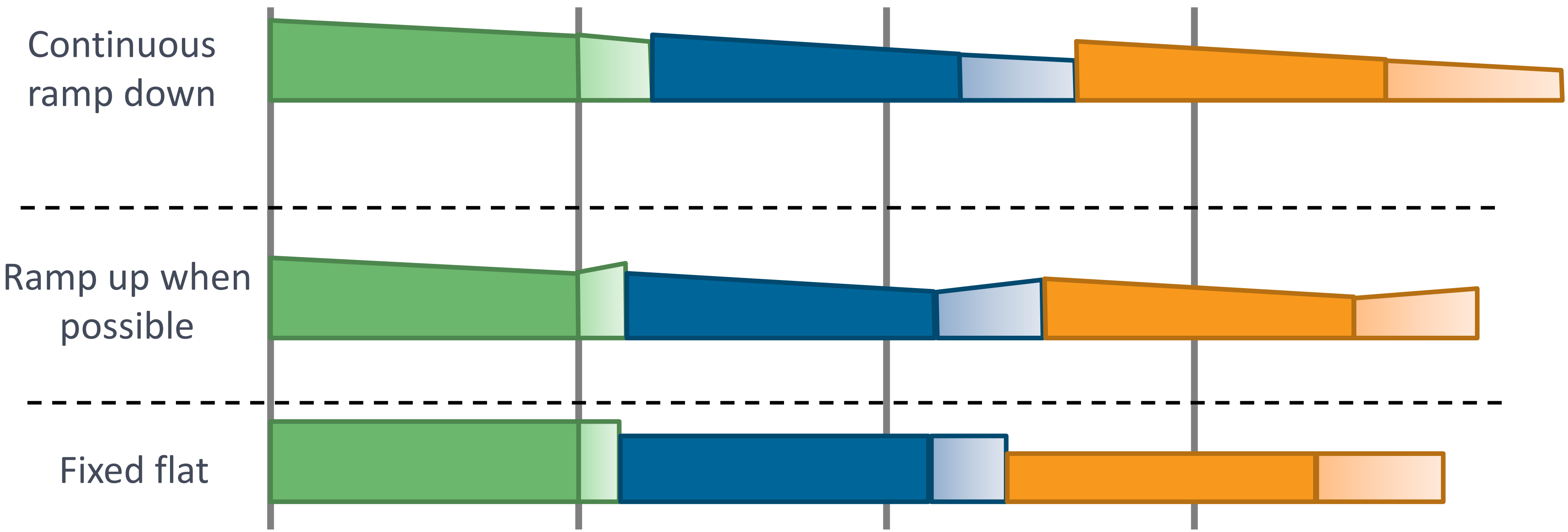
- Rather than transforming from one rectangle to another as with the previous solution, this adjustment uses trapezoids
  - The same geometric requirements exists, i.e. maintain Cost and conform to the Budget



The calculation of the trapezoids needs to take into account the fact that the slope cannot be too steep as to cause the rate to drop lower than a reasonable minimum (user may specify this if desired)

Solution Selection

- Geometric variations provides conformity to natural conditions
  - The selection of the solution method depends on the schedule and the flexibility/freedom of project office



Solution 3: Generalized

Math Warning!

Solution 3: Generalized

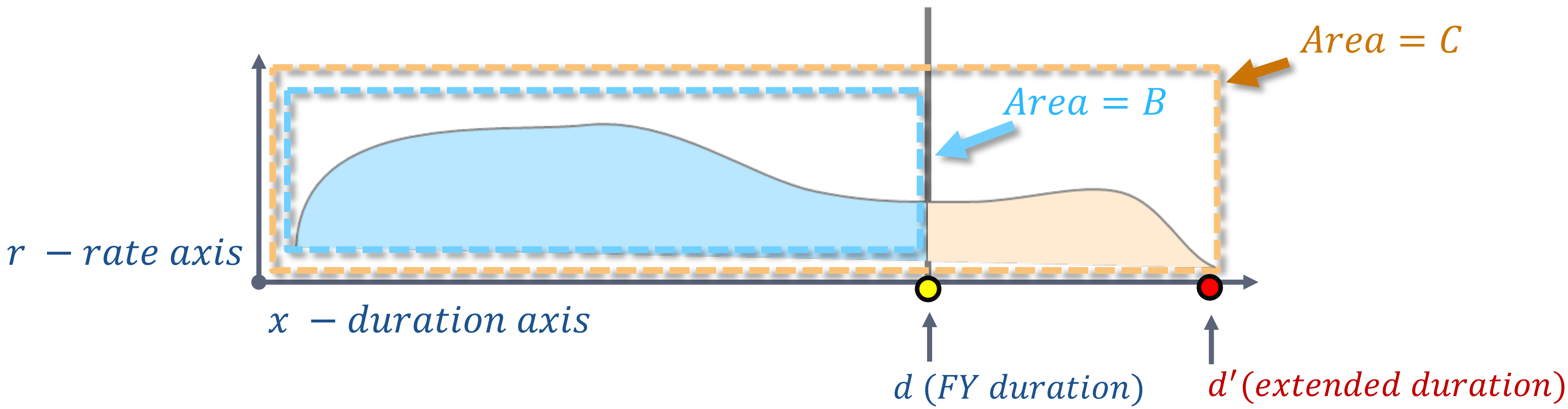
- Solution **1**: single value representing the adjusted rate: 
  $f(x) = x$
- Solution **2**: straight line representing the adjusted rates: 
  $f(x) = mx + b$
- Solution **3**…?
- Solution $\boldsymbol{n}$…?
- In order to provide a flexible and comprehensive management strategy, the next logical step is towards a *generalized* method of calculating the adjusted rate to predict the delay

Framework for a Generalized Method

Solve for this variable

$$\int_0^{d} f(x)\,dx = B \qquad \int_0^{d'} f(x)\,dx = C$$

- The area *within* the given FY must equal the yearly budget
- The area *of* the *entire shape* must equal the FY's estimated cost
- The shape itself, $f(x)$, defines the entire spending profile rate curve, up to the extended duration, d'



Framework for a Generalized Method: Linear Example

- To demonstrate the applicability of this framework, *calculus* is used instead of *geometry* to solve for the adjusted rate, assuming linearity in the rate change
  - Ideally concluding with the exact same results

$$\int_0^{d} (\boldsymbol{mx} + \boldsymbol{b})\, dx = B \quad \& \quad \int_0^{d'} (\boldsymbol{mx} + \boldsymbol{b})\, dx = C$$

Eq. 1 $\rightarrow$ $$\left(\frac{1}{2}\right) md^2 + bd = B \quad \& \quad \left(\frac{1}{2}\right) md'^2 + bd' = C$$ $\leftarrow$ Eq. 2

Three methods to solve this *underdetermined* system of equations for d':

1. Introduce another equation/objective (e.g. rate begins to increase when funding is available)
2. Solve for extended duration d' as a function of another variable (e.g. b, the initial rate)
3. Assume the value of one variable (e.g. $b = C/d$)

Framework for a Generalized Method: Linear Example

- If we prefer control over the rate, we can solve the system as a function b (**method 2**), since b represents the year's starting rate
- If we prefer a deterministic solution, we can use the original rate for b (**method 3**), since we know $b = C/d$

Eq. 1 $\left(\frac{1}{2}\right)md^2 + bd = B$ ⇨ $m = \frac{2(B - bd)}{d^2}$

$m = \frac{2(B - bd)}{d^2}$ ⇨ **Eq. 2** $\left(\frac{1}{2}\right)md'^2 + bd' = C$ ⇨ $\left(\frac{1}{2}\right)\frac{2(B - bd)}{d^2}d'^2 + bd' = C$

Same as "Start up, end down" equation!

Method 2:

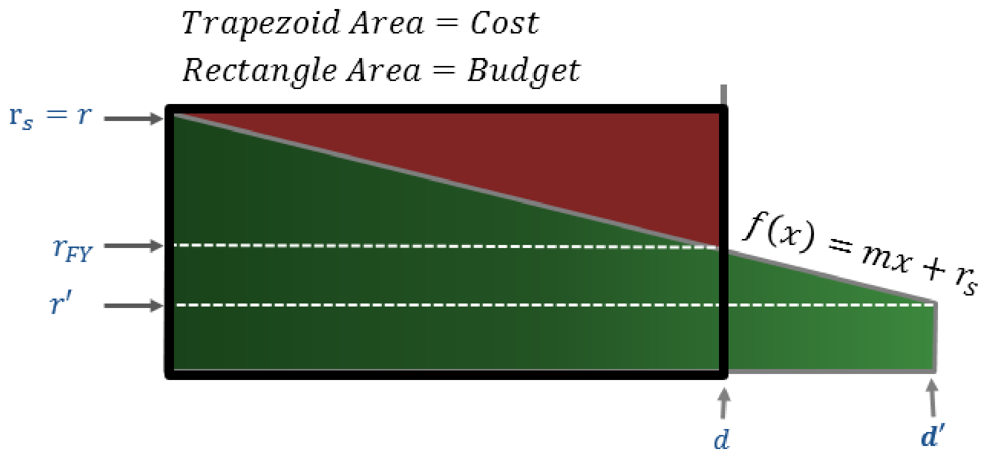
$$d' = \frac{-b \pm \sqrt{b^2 + 4C\frac{(B - bd)}{d^2}}}{2\frac{(B - bd)}{d^2}}$$

Method 3:

$$d'_1 = \frac{-\frac{C}{d} \pm \sqrt{(\frac{C}{d})^2 + 4C\frac{(B - C)}{d^2}}}{2\frac{(B - C)}{d^2}}$$

Framework for a Generalized Method: Linear Proof

- The solution for the extended duration, d', is the *exact same formula* derived either geometrically or using calculus
- Thus showing that the generalized formulation of the budget constrained problem is versatile and accurate



$$B = \left(\frac{1}{2}\right) d(r_s + r_{FY})$$

$$C = \left(\frac{1}{2}\right) d'(r_s + r')$$

Equivalent

$$\int_0^{d} (mx + r)\, dx = \left(\frac{1}{2}\right) md^2 + r_s d = B$$

$$\int_0^{d'} (mx + r)\, dx = \left(\frac{1}{2}\right) md'^2 + r_s d' = C$$

Proof of equivalence in backup slides

Framework for a Generalized Method: Quadratic Example

- A similar process can be used with any polynomial or other nonlinear equation
  - Although there may be little justification to branch far from a linear form

$$\int_0^{d} (ax^2 + c)\, dx = B \qquad \int_0^{d'} (ax^2 + c)\, dx = C$$

$$\left(\frac{1}{3}\right) ad^3 + cd = B \qquad \left(\frac{1}{3}\right) ad'^3 + cd' = C$$

Math happens here

$$d' = The\ solution\ to\ the\ cubic:$$

$$\left(\frac{B-C}{d^3}\right) d'^3 + \frac{C}{d}\, d' - C = 0$$

Note: There may not be a real valued solution in some cases

How to use these solutions

What do you do next?

- Regardless of the method for calculating the extended duration, the next step is performing this process repeatedly for each year
  - At each step there are a series of checks and adjustments needed

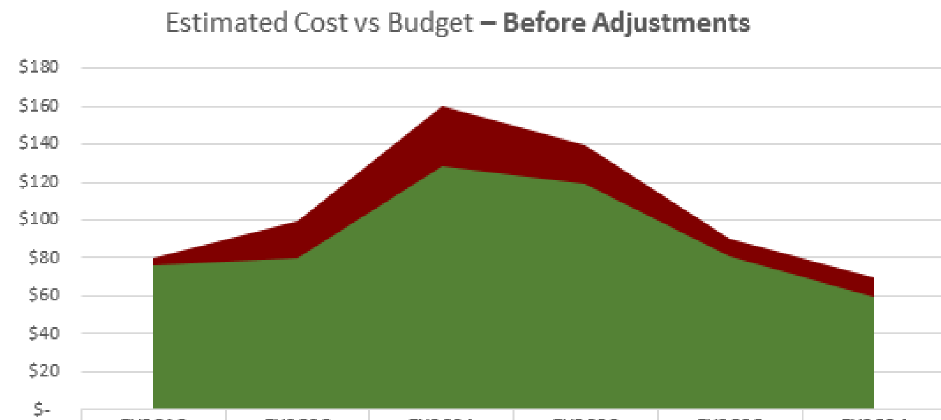
1. Check that the budget is indeed less than the cost
2. Calculate the extended duration
3. Check that there are enough funds in the following year to cover the extension cost and the originally planned work
   - If there are enough funds, subtract from that next year's budget
   - If there are not enough funds, re-calculate the extension that exists in the next year to spread it even thinner, pushing the original work in that year completely out of its planned year
4. Adjust the following year's budget, then calculate the adjustment necessary to account for the work remaining in that year to fit the remaining budget
5. Repeat from step 1

What do you do next?

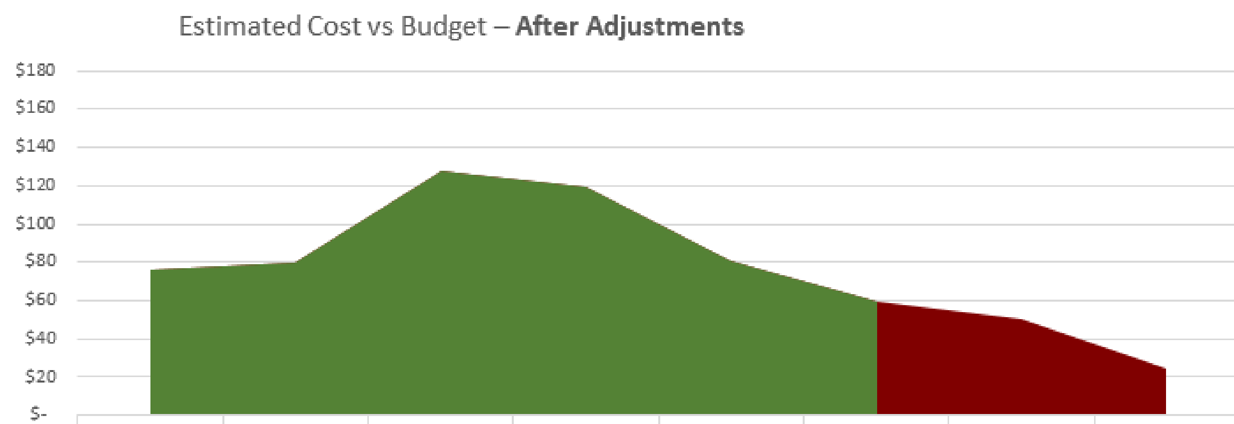
- *See the paper for a detailed description and images of how to run the adjustment algorithm*

Reconciling the Discrepancy

- How the yearly phased results look after the adjustment



	FY2019	FY2020	FY2021	FY2022	FY2023	FY2024
Cost	$80	$100	$160	$140	$90	$70
Budget	$76	$80	$128	$119	$81	$60



	FY2019	FY2020	FY2021	FY2022	FY2023	FY2024	FY2025	FY2026
Cost	$76	$80	$128	$119	$81	$60	$50	$25
Budget	$76	$80	$128	$119	$81	$60		

Accounting for the *type* of costs?

- If the schedule model contains costs broken out into time-independent (TI), time-dependent (TD), level-of-effort (LOE) and/or program management (PM) costs, these need to be accounted for
- Management should decide:
  1. What, if any, control over reducing labor force is possible in order to achieve the predicted results?
  2. Can any of the LOE or PM work can be reduced or is that a fixed rate?
  3. What year do the largest TI costs (material purchases, e.g.) occur? Are they fixed in time or can they be pushed out?

What about the *type* of costs?

- Example of the model setup needed prior to calculation:

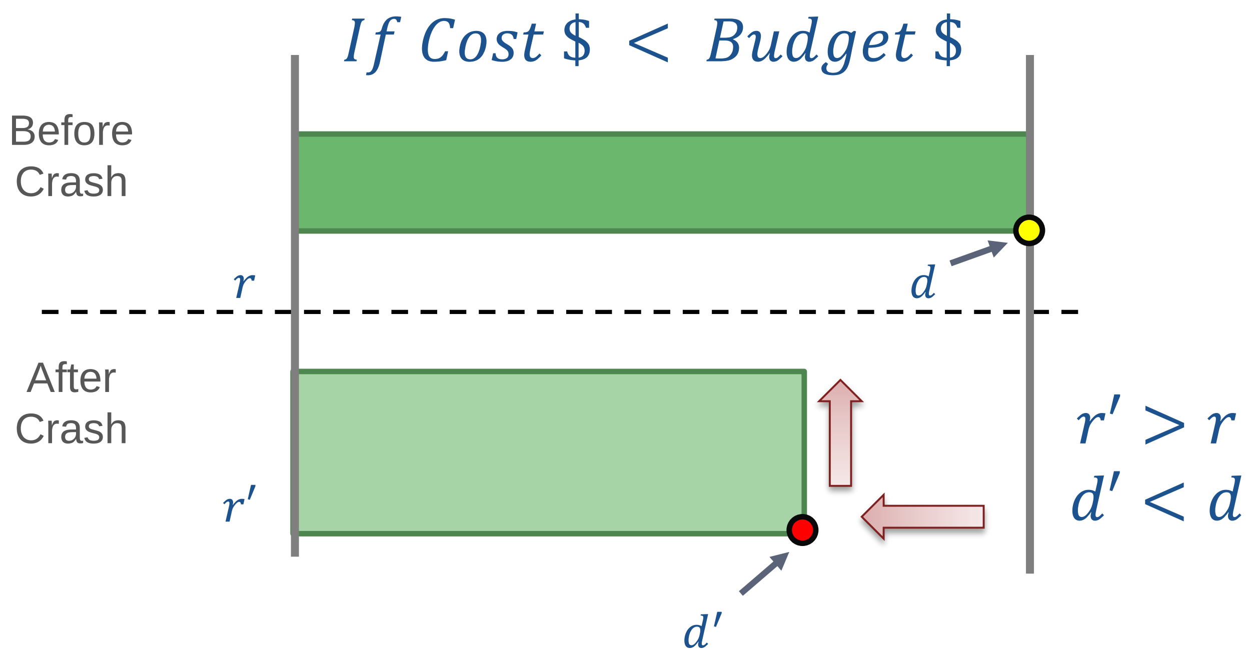
Example of cost type adjustments

	FY 2018	FY 2019	FY 2020
Total Cost:	$80	$100	$160
Schedule:			
TI Cost:	$4 $3	$5	$6
Fixed LOE:	$8	$10	$12
Adjustable Cost:	$65	$85	$142

Use these for the value of C

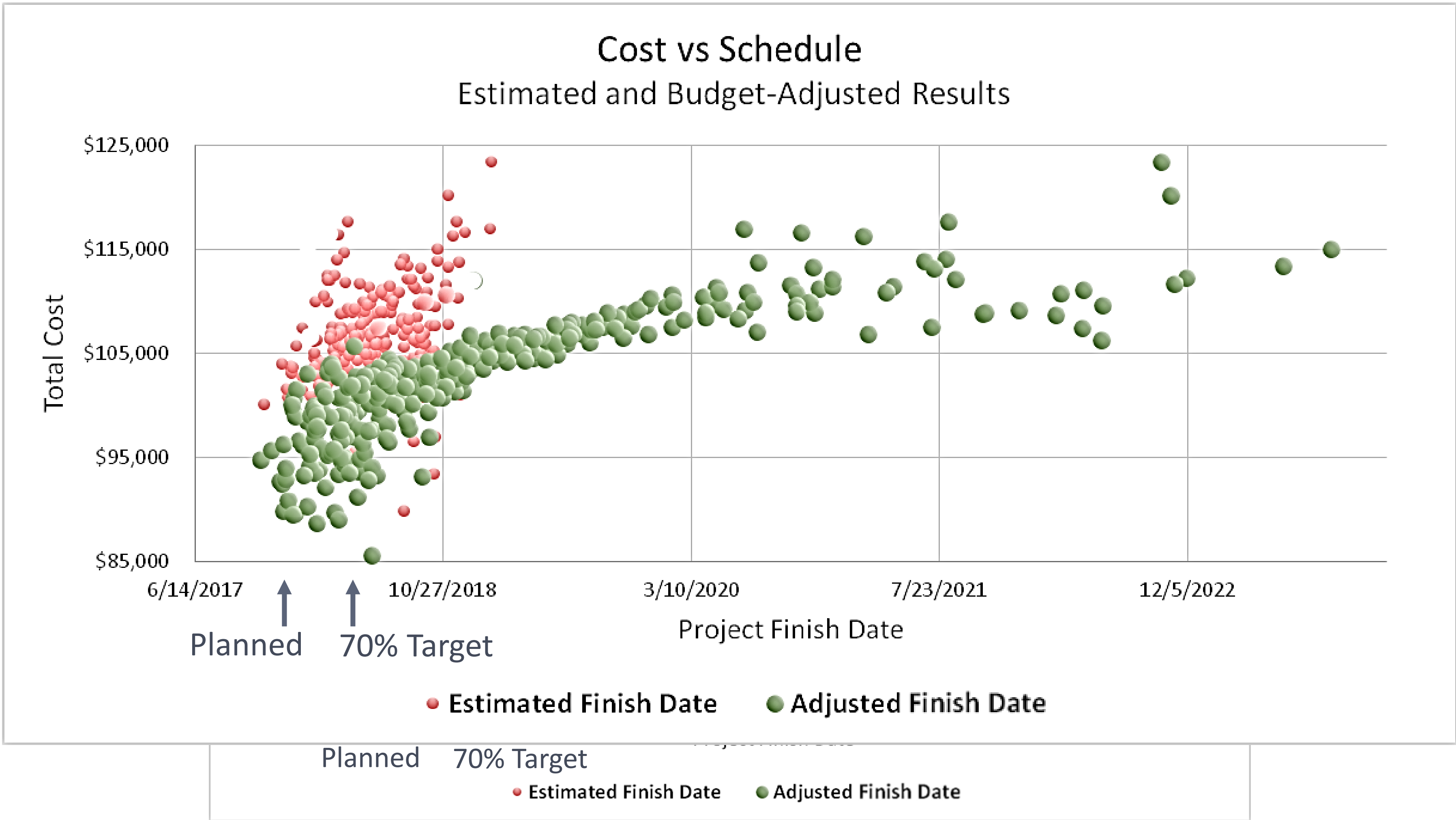
Crash Schedule

- Same concept but with *increasing* rate to *decrease* time!
- The formulas introduced here work just the same if the budget is greater than the estimated cost, thus resulting in a decrease in the amount of time to complete the project
  - It might be the case that there is a combination of overrun and underrun in various years



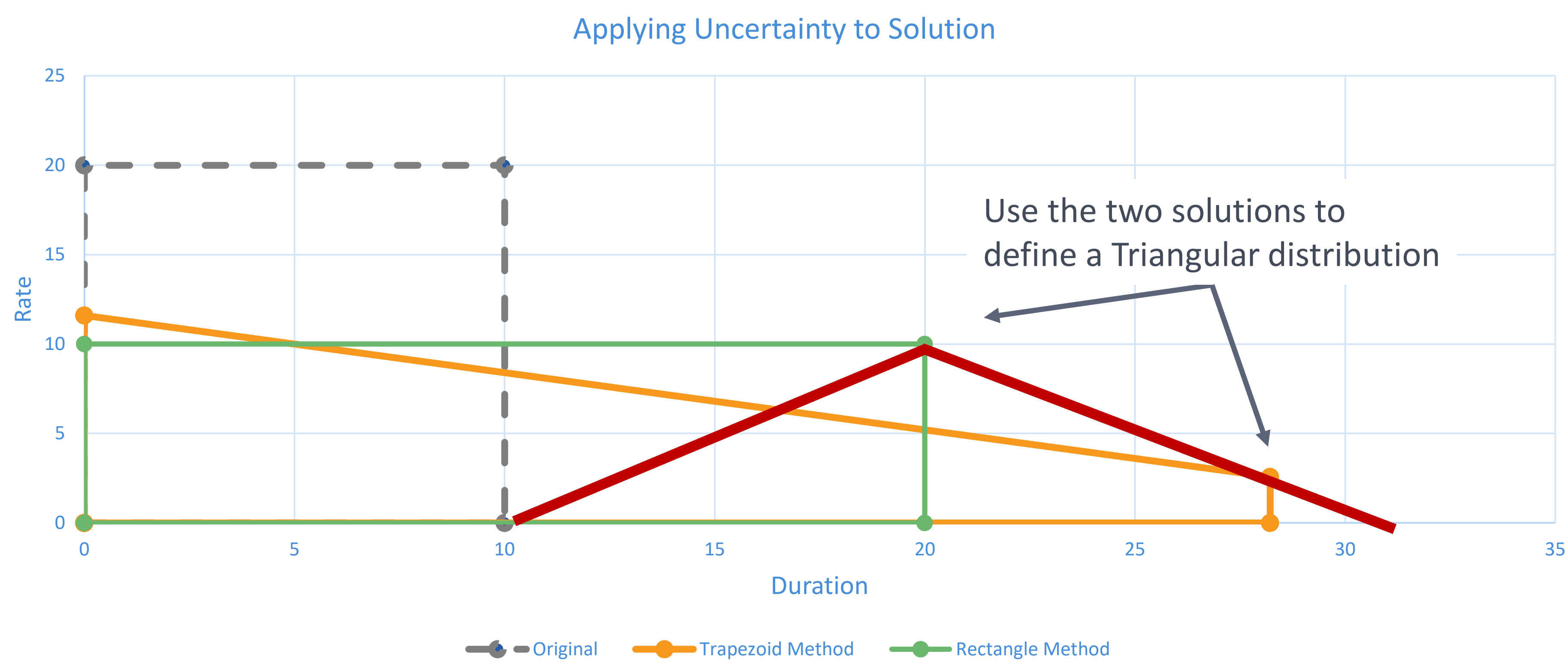
What about the *simulation?*

- Applying this method to each iteration of an uncertainty simulation provides a powerful, new, and exciting analysis



No Simulation? No Problem: Distribution through Solutions

- Instead of deciding which management assumptions result in a specific rate adjustment method, use one or two method solutions as inputs to generate a probability distribution
  - The rectangle and trapezoid methods provide two possible finished dates, these can be used as inputs for an uncertainty distribution



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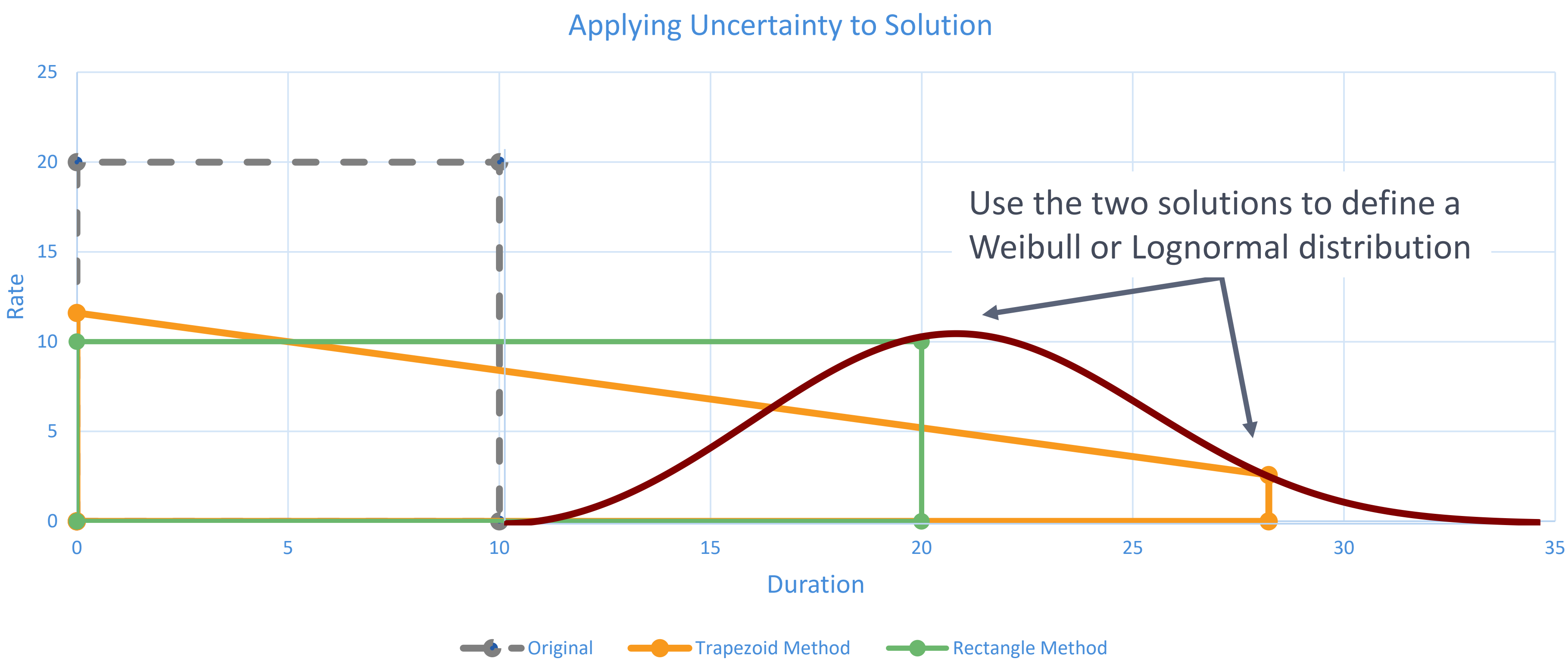

No Budget? No Problem!

- The formulas laid out in this framework can all be solved for any variable of interest, most notably: the budget
- This allows answers to questions like:
  1. If I don't want to be delayed by more than 100 days, what budget profile do I need?
  2. How can I reallocate the budget to minimize my delay?

	FY19	FY20	FY21	FY22	FY23	FY24
Cost	?	?	?	?	?	?
Budget	76	80	128	119	81	60

Thank you!
Questions?

Contact Information

Please feel free to contact me with any questions:

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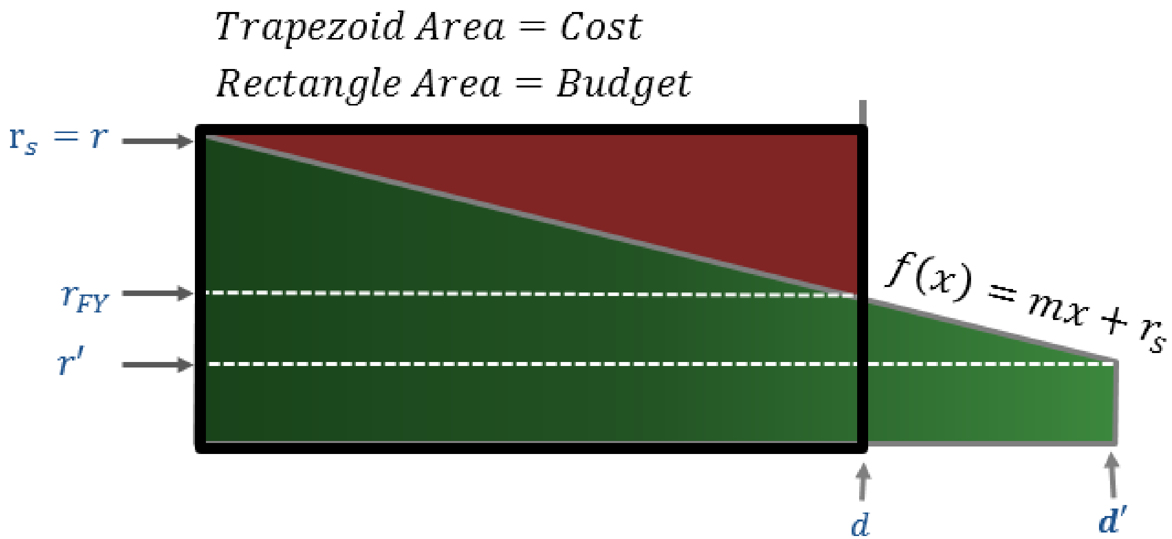
Back Up Slides

Is it really true? Proving Generalized Method



$$B = \left(\frac{1}{2}\right) d(r_s + r_{FY})$$

$$C = \left(\frac{1}{2}\right) d'(r_s + r')$$

Equivalent

$$\int_0^d (mx + r)\,dx = \left(\frac{1}{2}\right) md^2 + r_s d = B$$

$$\int_0^{d'} (mx + r)\,dx = \left(\frac{1}{2}\right) md'^2 + r_s d' = C$$

Yes it is true

Trapezoid (geometric) method with $r_s = r$:

$$d' = \frac{2C}{r + \sqrt{r^2 + 4C\frac{(B - rd)}{d^2}}}$$

Generalized (calculus) method using a linear formula:

$$d' = \frac{-r + \sqrt{r^2 + 4C\frac{(B - rd)}{d^2}}}{2\frac{(B - rd)}{d^2}}$$

Set both equations equal to each other:

$$\frac{2C}{r + \sqrt{r^2 + 4C\frac{(B - rd)}{d^2}}} = \frac{-r + \sqrt{r^2 + 4C\frac{(B - rd)}{d^2}}}{2\frac{(B - rd)}{d^2}}$$

Moving denominators of each side to the opposite side:

$$2C\left(2\frac{(B - rd)}{d^2}\right) = \left(-r + \sqrt{r^2 + 4C\frac{(B - rd)}{d^2}}\right)\left(r + \sqrt{r^2 + 4C\frac{(B - rd)}{d^2}}\right)$$

Factoring the left side and distributing the right side:

$$4C\left(\frac{(B - rd)}{d^2}\right) = \left(-r^2 + r^2 + 4C\frac{(B - rd)}{d^2}\right)$$

Concluding equality:

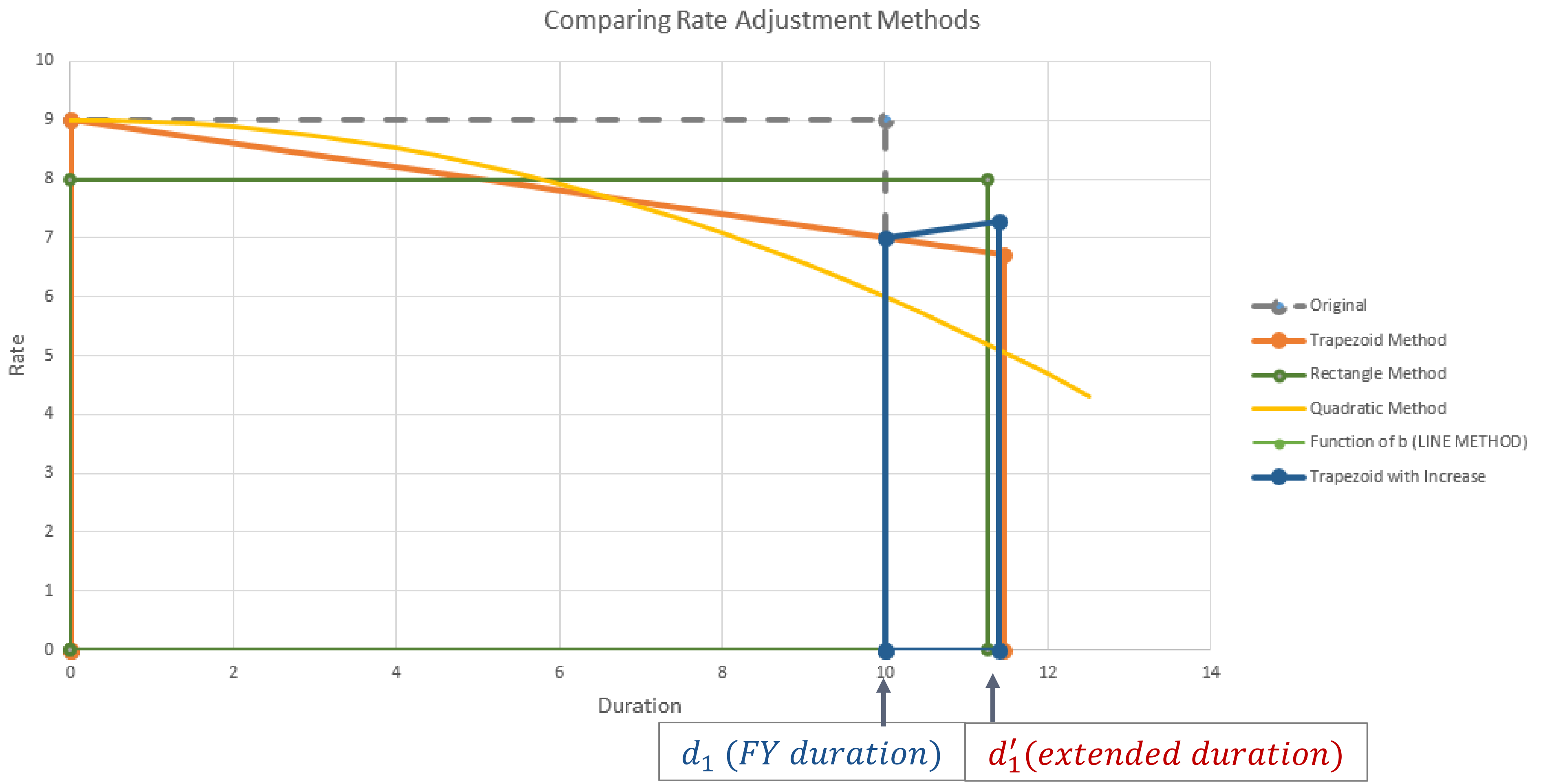
$$4C\left(\frac{(B - rd)}{d^2}\right) = 4C\left(\frac{(B - rd)}{d^2}\right)$$

Where We Begin: What Data Is Needed?

- This process requires a few pieces of information:
  - Project’s estimated cost on a yearly basis
  - Project's budget on a yearly basis
  - Project’s scheduled finish date
- Ideally, and for the most robust results, a few more pieces of information could be well used:
  - Cost types, e.g. time independent/dependent breakout, level-of-effort
  - Iteration results containing yearly costs
  - Any important dates tied to budgetary decisions

How do the solution methods compare?

- Rectangle solution is the shortest and quadratic is the longest
- "Trapezoid with Increase" seems a realistic balance



When is the slope too steep?