## Analogies: Techniques for Adjusting Them

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ASC/Industry Cost \& Schedule Workshop Spring 2004 - Destin FL 21 April 2004

SCEA 2004 - Manhattan Beach, CA 17 June 2004
$72^{\text {nd }}$ MORSS- Monterey, CA 22 June 2004


- Background
- The current method
- Two new methods
- Borrowed slope
- Relational correlation
- Conclusion


## Background

- Considerable attention is devoted to techniques in the development of Cost Estimating Relationships (CERs) for parametric estimating
- Research on CERs
- Methods for calibrating
- Considerable expertise is to be found in buildup techniques
- Many Original Equipment Manufacturers (OEMs) have large cost shops which practice buildup
- Analogy, on the other hand, has been given little attention
- Next, some basic definitions ...


## Definitions

- Parametric Estimates: Estimates made by developing statistical "Cost Estimating Relationships" (CERs) based on one or more parameter and cost
- Estimates involving parameters but not based on statistical analysis are more properly called either "adjusted analogies" or "adjusted buildups"
- Analogies: Estimation by assuming that the costs of a new system will be equal to (or similar to) the costs of a system that is similar
- "Adjustments" are almost always made
- Buildups: Physical Bill of Materials (BOMs) and CAD-generated material lists and the like
- We do not mean "buildups" consisting entirely of Staffing levels*Duration. Such estimating techniques are little more than "engineering judgment" in fine detail
- Buildups often include "adjustments" to allow for size differences
- Composite methods: A method that involves at least two of the three other types
- Adjustments: Scaling of a cost by some physical, performance, or other such attribute
- Scaling is usually directly proportional to the attribute
- Scaling parameters are usually countable or measurable and intuitively tied to cost


## The Current Method

- Adjustments, in the analogy or buildup method, typically rely on an "obvious" characteristic
- The characteristic is most often weight
- Sometimes weight of the new system is not known, and so another characteristic is used (often as a proxy for weight)
- Sometimes a characteristic such as bore diameter of a gun is used
- Usually the ratio of the values of the characteristic in the new system to the value in the old system is multiplied by the cost of the old system
- Sometimes called "j-ing up the estimate"
- Sometimes the characteristic is transformed in a way that is thought to make it proportional to weight
- E.g., the bore diameter of a gun, is cubed
- In these cases, there may be a presumed relationship to weight,


## Implications of the Current Method

- An example adjustment by ratio is:
- The analogy weighs 300 tons and costs $\$ 100 \mathrm{M}$
- The new system weighs 500 tons and so is assumed to cost $(500 / 300)^{*} \$ 100 \mathrm{M}=\$ 166.67 \mathrm{M}$
- This is a typical and familiar adjustment
- What is its implication?
- Should we be inclined to believe it?
- Is it in accord with what we believe?
... let's look at a graph to see what it implies ... there is a surprise there for most of us ... but first, force yourself to predict what the line between the analogy and the prediction looks like ... where does it cross the y axis?


## Adjustment by Ratio - The Graph

- The below graph shows the previous adjustment
- The analogy weighs 300 tons and costs \$100M
- The new system weighs 500 tons and is assumed to cost $(500 / 300) *$ * $100 \mathrm{M}=\$ 166.67 \mathrm{M}$
- Note that the line through the 2 points passes through the origin

Important Observation:
Straight adjustments by ratios always pass through the origin! Most observers fail to predict that, even though it is straightforward to show that it must.
Important Question:
Adjustment by Weight Ratio


Is this reasonable?

## Adjustment by Ratio - Reasonable?

- The y-intercept is a litmus test among cost estimators. There are about three schools of thought:

1. CERs "should" pass through the origin
2. CERs which do not pass through the origin must have an explicable y-intercept
3. CERs must be statistically derived, and if done properly, the y-intercept is just "what it is"

- We'll discuss each briefly and then assume you are of school 2 or 3


Warnings:
1- Almost anyone is from one of these schools of
thought at heart. The writers are no exception.
2- The gulf between these schools is wide.

## - Typical arguments

- "If I spend no money, I get no product"
- Pros:
- Sounds good
- Cons:
- Doesn't seem to match the data. E. g., the price of FlashDrives:



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## - Typical arguments

- "There must be physics-based arguments for CERs"
- Pros:
- Helpful to think about it, within reason
- Cons:
- If practiced to the extreme, good CERs can be rejected just because we do not yet understand them
- Engineers, who hate cost estimation, can usually talk the analyst to a full stop


## - Typical arguments

- We are not trying to predict the y-intercept. We are trying to predict the cost of systems of non-zero size.
- We should take the best advice the data can give us
- We should extrapolate as little as we can
- If the data show that the y-intercept is non-zero, we should not reject a CER just because we do not know why
- Galileo believed the data, even absent a theory of gravity. It took centuries before Isaac Newton knew why - but Isaac Newton wouldn't even have wondered without Galileo showing that there was an explanation missing.
- This approach is what the practice of statistics currently recommends
- Pros:
- Any existing system (i. e., one of the data points underlying the CER) is well-predicted
- Cons
- If the analysis is not well done, there may be a better CER
- Borrowed slope ${ }^{1}$ - a variant of the methods for calibrating CERs
- Adjust a "trusted analogy" by a "trusted slope"
- Relational Correlation ${ }^{2}$ - taking advantage of the geometry of regression
- Adjust a "trusted analogy" by a "best guess slope"

1 A Framework for Costing in a CAIV Environment, R. L. Coleman, TASC; D. Mannarelli, Navy ARO, ASNE 1996, ADoDCAS 1996
2 Relational Correlation, What to do when Functional Correlation is Impossible, ISPA/SCEA 2001, R.L.
Coleman, J.R. Summerville, M.E. Dameron, C.L. Pullen; TASC, Inc., S.S.Gupta, IC CAIG
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- Based on "calibrating a CER"
- A CER is adjusted to "more trusted," or industry, or company specific data by moving the slope to pass through a point or set of points
- Picture follows
- To adjust an analogy, do precisely the same thing
- Instead of believing you are adjusting a CER to specific data, think of it as departing from "the most credible point" via "the most credible slope"


## Calibrating CERs



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## Calibrating CERs



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## The Borrowed Slope Estimate

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- A much more esoteric method is available, which borrows from
- Bivariate normality
- The geometry of regression
- This method is available when there is no "trusted slope" to borrow


## Bivariate Normality

- Suppose $X$ and $Y$ are distributed $N\left(\mu_{x}, \sigma_{x}\right)$ and $N\left(\mu_{y}, \sigma_{y}\right)$
- Suppose $X$ and $Y$ are jointly bivariate normal with correlation $\rho$
$\ldots$.... Then the graph of $X$ and $Y$ will appear as follows


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## The Geometry of Regression

## Regression

- The below facts are known to mathematicians, but obscure, and not remembered in cost analysis ...
- For any two jointly distributed variables, there is a regression line
- The slope is:

$$
m=\rho^{*}(\sigma y / \sigma x)
$$

- The y intercept is:

$$
b=\mu y-\rho(\sigma y / \sigma x)^{*} \mu x
$$

- If the variables are joint bivariate normal, then $\rho$ is the correlation coefficient


## Let's look at the graph...



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This is the equation of one line (passes through the point ( $\mu \mathrm{x}, \mu \mathrm{y}$ ) with slope $\sigma y / \sigma x$ )

$$
y-\mu y=(\sigma y / \sigma x)(x-\mu x)
$$

or
$y=(\sigma y / \sigma x)(x-\mu x)+\mu y$

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The Geometry of Bivariate Normality and the implications for Regression


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The Geometry of Bivariate Normality and the implications for Regression



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- For every regression with apparent slope m, there is an unseen equation
- With steeper slope $\mathrm{m} / \rho$ which is the unseen slope of the two variables
- With an unseen accompanying y intercept
- Once we decide upon the means and the variances of $x$ and $y$, the unseen line is fixed
- Once we pick $\rho$, the regression line is fixed

The Geometry of Bivariate Normality and the implications for Regression


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# Implementing Relational Correlation for Analogies 

## For Single Point Analogies

- Determine a reasonable (preferably historically-based) standard deviation for the $x$ and $y$ variable
- E.g, to estimate ship repair parts as a function of tonnage you'll need:

1. The standard deviation for the analogy ship class repair parts cost
2. The standard deviation for the tonnage within the ship class
3. The standard deviation of repair parts for a single ship of the class

- The ratio of 1 and 2 gives you the unseen slope
- The relationship of 3 and 1 will yield $r^{2}$ (Variance of $y \mid x=\left(1-\rho^{2}\right)$ * $\sigma y^{2}$ )
- For buildups, do as above, but use an analogy for the values, and apply it to your buildup using percents


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## Conclusions

- Adjustments of analogies have received too little attention
- Three methods
- Ratio adjustments
- Current practice
- Overstate above the analogy, and understate below
- Borrowed slope
- Needs a CER
- Relational correlation
- Esoteric
- Does not need a CER
- Hopefully we have convinced you that ratio adjustment is just not good enough!

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## The Problem

- You have two WBS elements
- Warhead cost
- Motor cost
- You know their historic means and standard deviations - for both cost and the driving parameter, say weight
- You know these values from independent data bases
- So, you cannot get correlation
- You do have a CER to predict warhead cost
- You do not have a CER to predict motor cost
- You believe weight is a driver, but a CER cannot be derived
- And, the data you have is too far away from your program, it needs to be adjusted ... but how?
- You do not wish to simply factor the cost by the weight change
- This is a typical problem, and is closely related to the risk problem just described
- We will try to predict motor cost as a function of warhead cost ... a useful equation as well as a helpful CER

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1. Ask the engineer: How much leeway in \% do you typically have for weight (or cost) of the motor if design has not yet begun? (The unconstrained case)

- Note: this may differ from the historic variation, but we will use it only in a relative sense
- We will translate the weight fluctuation into cost fluctuation

2. Ask the engineer: How much leeway in \% do you have for weight (or cost) of the motor, if design of the warhead is complete? (The constrained case)
3. This will give you $\mathrm{r}^{2}$ :

- You already knew the "unseen slope", $\sigma y / \sigma x$, now you know the "seen" slope $\rho(\sigma y / \sigma x)$, and you know $b=\mu y-\rho \sigma y / \sigma x$ * $\mu x$
- The percent reduction in the variance of $y$ is the $r^{2}$, and the square root of that is $r$ (Variance of $\left.y \mid x=\left(1-\rho^{2}\right)^{*} \sigma y^{2}\right)$

4. Implement the result as a CER, by passing the slope through the analogy or average of your data.

> Note: We do not advocate using such a CER in lieu of a standard CER, only if there is no other recourse

## Motor cost



