Learning Curve analysis with EZ-Quant

An overview

2012 SCEA/ISPA Conference

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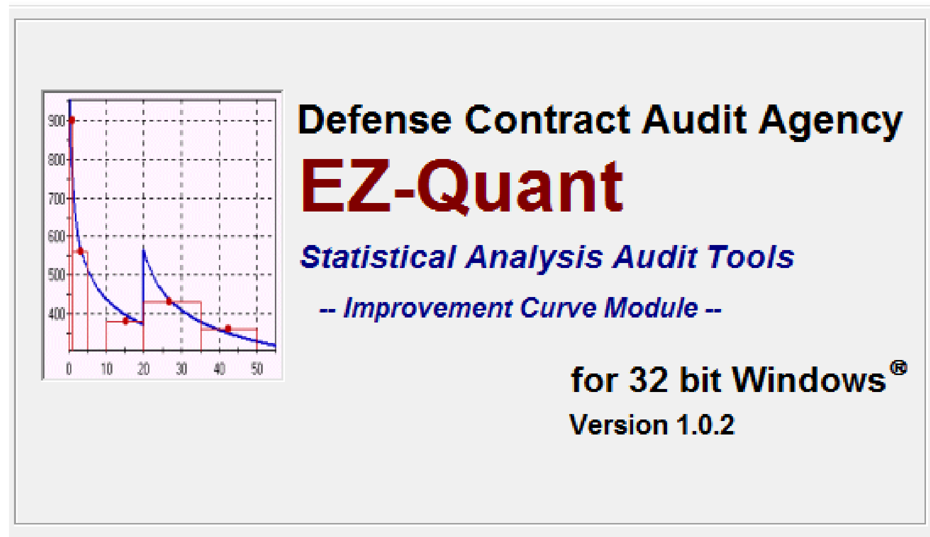
EZ-Quant

- What it is
- Why you want it
- Where to get it
- System considerations
- Outputs
- Examples



What it is

Developed for DCAA auditors, EZ-Quant is improvement curve software that supports curve estimation and projections for:

- **Unit Theory**
- **Cumulative Average Theory**
- **Retained Improvement Unit Theory**
- **Production Break Unit Theory**
- **Fixed Level of Effort Unit Theory**
- **Design Change Unit Theory**

Why you want it

- It's free
- It's thin
- It works
- It has a decent set of help files
- It draws graphs and exports data to Excel
- You can save data files for quick re-use
- Guess what the folks who audit your proposal will be using?

Where to get it

EZ-Quant Statistical Analysis Software

Version 1105

The Defense Contract Audit Agency (DCAA) does not provide technical support on this product outside of DCAA audit activities. This product has been developed and tested by the DCAA for use in its audit processes. It is freeware and its use and copying is unrestricted. While DCAA has made every effort to identify and repair all deficiencies in this product, DCAA does not guarantee that this product is free from defects that might interfere with its operation on the downloader's system. This product is made available "as is", and the use of the software is done at the downloader's own risk. DCAA makes no warranties, expressed or implied, concerning this product.

EZ-Quant is a suite of three stand-alone applications (Statistical Sampling, Regression, and Improvement Curves) and each of these applications may have a different version number depending on the extent and timing of enhancements.

In prior releases, the release version was the same as that of the statistical sampling module. Beginning with this release, the EZ-Quant Installation package will have its own version number disassociated from the version numbers of the three included applications. The EZ-Quant installation version number will consist of the last two digits of the year and the two-digit month of the installation build (in this case, the installer version is 1105 indicating a May 2011 build).

EZ-Quant Installer 1105 contains the following application versions:

- Statistical Sampling (ver. 1.1.2)
- Regression (ver. 1.2.0)
- Improvement Curves (ver. 1.0.2)

Software and Instructions:

- Download EZ-Quant (32.5 MB; May 2011) - Please review the Readme file prior to download and installation.
- Version History Release Notes

Return to Homepage

http://www.dcaa.mil/ezquant.htm

System considerations

- **The download is a zip file (EZ_Quant_Install_1105.zip)**
- **The installation script is a .msi file which requires administrative rights**
- **I've run it on:**
  - **Windows XP**
  - **Windows Vista**
  - **Windows 7**

Outputs

Actual vs. Derived values

Improvement Curve Fit to Production Data

Actual vs. Derived Calculated Dependent Variable Values for Standard Model, Cum Avg Theory
Y = aX^b

Lot No.	Derived Lot Midpoint	Derived Calculated Lot Avg	Actual Lot Average	Difference (Actual-Calc'd)	% Diff (Diff/Actual)
1	1.18	639.3	627.0	-12.3	-2.0
2	4.28	338.8	404.5	65.7	16.2
3	10.97	238.7	219.7	-19.0	-8.7
4	23.89	181.3	172.3	-9.0	-5.2
5	38.29	153.9	166.9	13.0	7.8

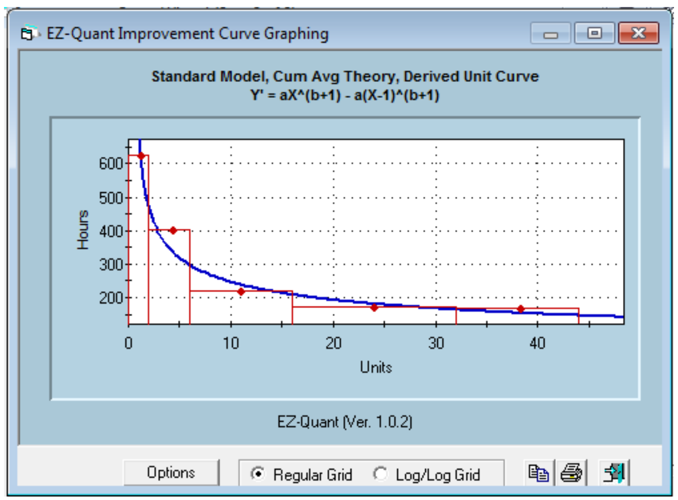
Curve fit results

Fitting an Improvement Curve to Production Data

Standard Model, Cum Avg Theory
Y = aX^b

Data Summary	
Number of production lots input	5
Weighted average hours/unit for all lots	223.38
Estimated Parameter Values	
Computed value of first unit (a)	810.4
Improvement slope coefficient (b)	-0.342164
Improvement percentage (100 x 2^b)	78.89
Statistics	
Coefficient of determination (r-sq)	0.941215
Comparison assurance that:	
Standard improvement curve is better predictor than avg	99.4

Plots: Regular & Log/Log



Projections

Improvement Curve Projection

Projection From an Estimated Curve, Standard Model, Cum Avg Theory
Y = aX^b

Estimated Parameters		
Theoretical first unit value (a)	810.41	
Improvement slope coefficient (b)	-0.342164	
Improvement slope (%)	78.9	
Projected Lot:		
Number of units preceding the lot (min: 0)	50	
Last unit in lot (min: 0)	60	
Number of units in lot	10	
	Cum Avg Hours	Total Hours
Cumulative, through end of projected lot	199.7	11,979.4
Cumulative, preceding projected lot	212.5	10,625.5
Projected lot only	135.4	1,354.0

Graph

Outputs

Results exported to Excel

EZ-Quant (Ver. 1.0.2)

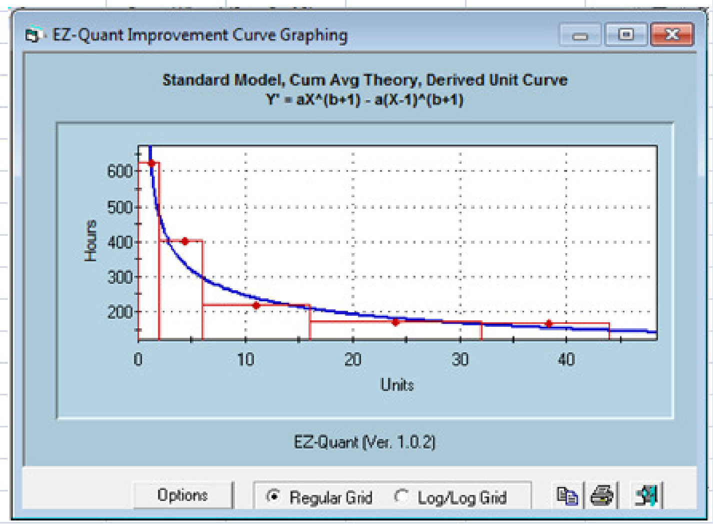
Standard Model, Cum Avg Theory

Y = aX^b

File = C:\Users\mahonem4\Documents\SCEA-7-No3.icv

Data Summary		
Number of production lots input	5	
Weighted average hours/unit for all lots	223.38	
Estimated Parameter Values		
Computed value of first unit (a)	810.4	
Improvement slope coefficient (b)	-0.342164	
Improvement percentage (100 x 2^b)	78.89	
Statistics		
Coefficient of determination (r-sq)	0.941215	
Comparison assurance that:		
Standard improvement curve is better predictor than avg	99.4	
Estimated Parameters		
Theoretical first unit value (a)	810.41	
Improvement slope coefficient (b)	-0.342164	
Improvement slope (%)	78.9	
Projected Lot:		
Number of units preceding the lot (min: 0)	50	
Last unit in lot (min: 0)	60	
Number of units in lot	10	
	Cum Avg Hours	Total Hours
Cumulative, through end of projected lot	199.7	11,979.40
Cumulative, preceding projected lot	212.5	10,625.50
Projected lot only	135.4	1,354.00

Lot No.	Derived Lot Midpoint	Derived Calculated Lot Avg	Actual Lot Average	Difference (Actual-Calc'd)	% Diff (Diff/Actual)
1	1.18	639.3	627	-12.3	-2
2	4.28	338.8	404.5	65.7	16.2
3	10.97	238.7	219.7	-19	-8.7
4	23.89	181.3	172.3	-9	-5.2
5	38.29	153.9	166.9	13	7.8



Example 1

CEBoK Module 7, Question 1

Question 1: Given the unit data at the right, what is the Learning Curve Slope (LCS) using cumulative average (CUMAV) learning curve theory?

Unit	Unit Cost
1	898
2	801
3	734
4	675
5	627
6	580

workspace

Choice D [See slide 17 ff.]

Unit	Unit Cost	CAUC	ln(Unit #)	ln(CAUC)
1	898	898.0	0	6.800170068
2	801	849.5	0.693147181	6.744647941
3	734	811.0	1.098612289	6.698268054
4	675	777.0	1.386294361	6.65544035
5	627	747.0	1.609437912	6.616065185
6	580	719.2	1.791759469	6.578093134

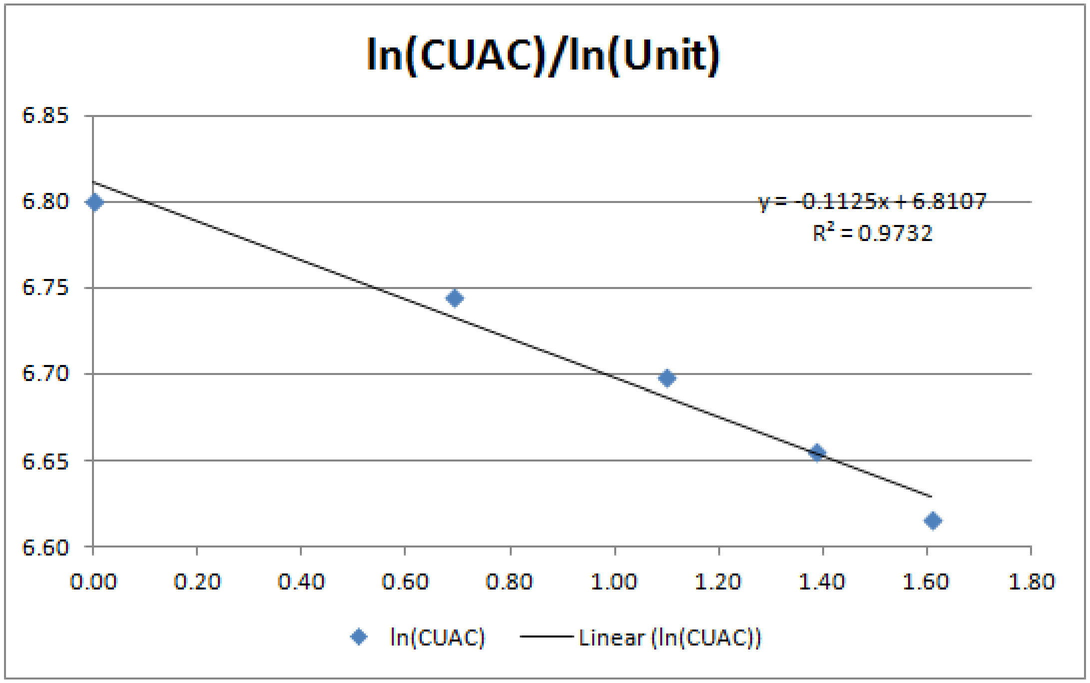
Compute the cumulative number of units and cumulative average unit cost (CAUC) as shown in the table.
Transform into log-log space, fit the regression line to determine the intercept and slope, and raise two to the power of the latter to determine LCS.
This is shown graphically, and b is from the equation of the line: y=b*ln(x)+ln(a).
LCS=2^-0.1223

0.9187218

Example 1

CEBoK Module 7, Question 1

Lot	Qty	Cum Units	Unit Cost	CAUC	ln(Unit)	ln(CUAC)
1	1	1	898.0	898.0	-	6.80017
2	1	2	801.0	849.5	0.69315	6.74465
3	1	3	734.0	811.0	1.09861	6.69827
4	1	4	675.0	777.0	1.38629	6.65544
5	1	5	627.0	747.0	1.60944	6.61607
6	1	6	580.0	719.2	1.79176	6.57809



ln(y) = ln(a) + b*ln(x)

6.57809 = 6.80017+b*ln(6)

6.57809 = 6.80017 + b* 1.79176

-0.22208 = b* 1.79176

-0.12395 = b

Slope = 2^-0.12395 = .918

Example 1

CEBoK Module 7, Question 1

Improvement Curve Wizard (Step 2 of 3)

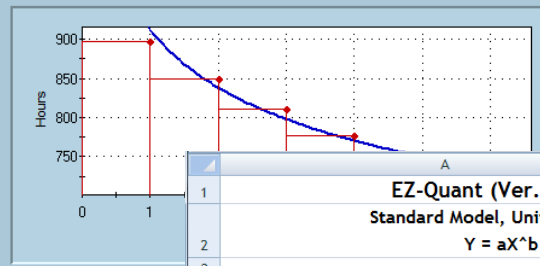
Type the lot dimensions and hours-per-unit data in the spreadsheet below.

Note: Do not leave any completely blank rows within your data!

When you have completed all input, click the next button to proceed with the improvement curve.

	Units Preceding Lot	Lot End	Units in Lot	Hours/Unit for Lot
1	0	1	1	898.0
2	1	2	1	849.5
3	2	3	1	811.0
4	3	4	1	777.0
5	4	5	1	747.0

EZ-Quant Improvement Curve Graphing



Fitting an Improvement Curve to Production Data

Standard Model, Unit Theory,
Y = aX^b

Data Summary	
Number of production lots input	6
Weighted average hours/unit for all lots	800.28
Estimated Parameter Values	
Computed value of first unit (a)	912.6
Improvement slope coefficient (b)	-0.122313
Improvement percentage (100 x 2^b)	91.87
Statistics	
Coefficient	
Comparisor	
Standard	

	A	B
1	EZ-Quant (Ver. 1.0.2)	
2	Standard Model, Unit Theory, Y = aX^b	
3		
4	File = C:\Users\mahonem4\Documents\SCEA-7-No1.icv	
5		
6		
7	Data Summary	
8	Number of production lots input	6
9	Weighted average hours/unit for all lots	800.28
10	Estimated Parameter Values	
11	Computed value of first unit (a)	912.6
12	Improvement slope coefficient (b)	-0.122313
13	Improvement percentage (100 x 2^b)	91.87
14	Statistics	
15	Coefficient of de	
16	Comparison assu	
17	Standard impro	

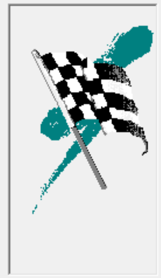
Improvement Curve Fit to Production Data

Actual vs. Calculated Dependent Variable Values for
Standard Model, Unit Theory,
Y = aX^b

Lot No.	Calculated Lot Midpoint	Calculated Lot Avg at Midpoint	Actual Lot Average	Difference (Actual-Calc'd)	% Diff (Diff/Actual)
1	1.00	912.6	898.0	-14.6	-1.6
2	2.00	838.4	849.5	11.1	1.3
3	3.00	797.8	811.0	13.2	1.6
4	4.00	770.2	777.0	6.8	0.9
5	5.00	749.5	747.0	-2.5	-0.3
6	6.00	733.0	719.2	-13.8	-1.9

Improvement Curve Wizard (Step 2 of 2)

Use the buttons below to view reports showing the various statistics and outputs associated with this improvement curve application



Select Curve Theory:
- Standard Curve, Unit Theory
- Standard Curve, Cumulative Average Theory
- Retained Improvement Model, Unit Theory
- Production Break Model, Unit Theory
- Design Change Model, Unit Theory

Show Results and Statistics | Graph Equation and Data | Show Percent Differences | Make Projections

Help | Done | < Back | Next >

Example 2

CEBoK Module 7, Question 2

Question 2: Given the same unit data as in Question 1 (shown again at the right), what is the Learning Curve Slope (LCS) using unit learning curve (ULC) theory?

Unit	Unit Cost
1	898
2	801
3	734
4	675
5	627
6	580

workspace

Choice A [See Slide 31 ff.]

Unit	Unit Cost	ln(Unit #)	ln(Unit Cost)
1	898	0	6.800170068
2	801	0.693147181	6.685860947
3	734	1.098612289	6.598509029
4	675	1.386294361	6.514712691
5	627	1.609437912	6.440946541
6	580	1.791759469	6.363028104

Transform into log-log space, fit the regression line to determine the intercept and slope, and raise two to the power of the latter to determine LCS.

This is shown graphically, and b is from the equation of the line: y=b*ln(x)+ln(a).

LCS=2^-0.2392

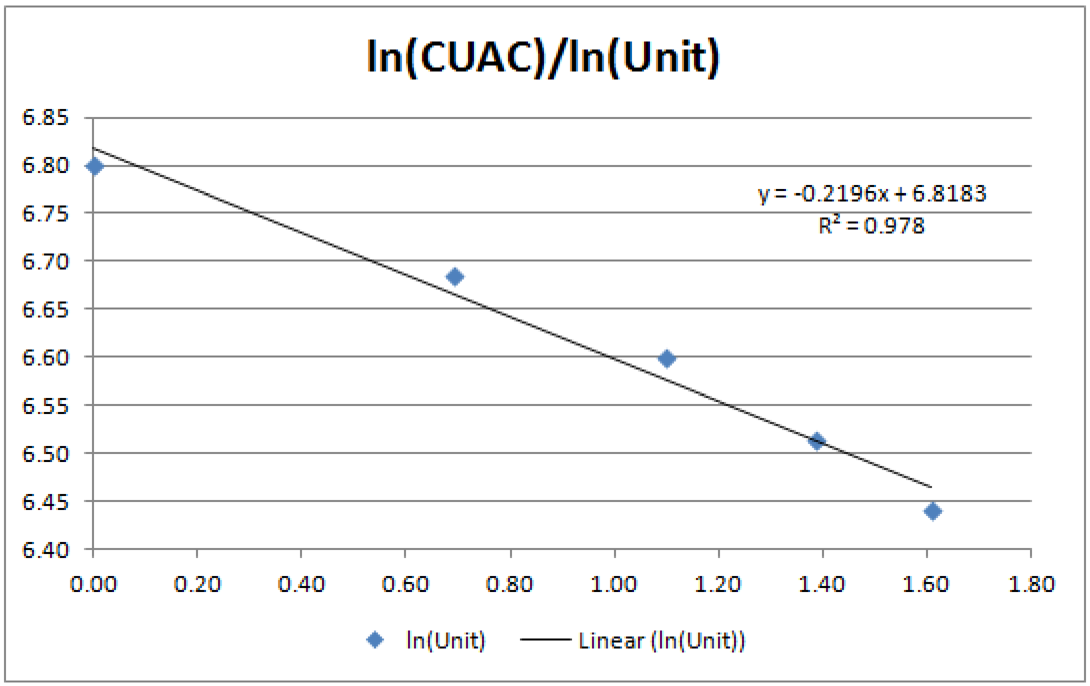
0.847215

Note that it takes an apparently steeper LCS to represent the same cost improvement for ULC as compared to CUMAV.

Example 2

CEBoK Module 7, Question 2

Lot	Qty	Cum Units	Unit Cost	ln(Unit)	ln(Unit)
1	1	1	898.0	-	6.80017
2	1	2	801.0	0.69315	6.68586
3	1	3	734.0	1.09861	6.59851
4	1	4	675.0	1.38629	6.51471
5	1	5	627.0	1.60944	6.44095
6	1	6	580.0	1.79176	6.36303



ln(y) = ln(a) + b*ln(x)

6.36303 = 6.80017+b*ln(6)

6.36303 = 6.80017 + b* 1.79176

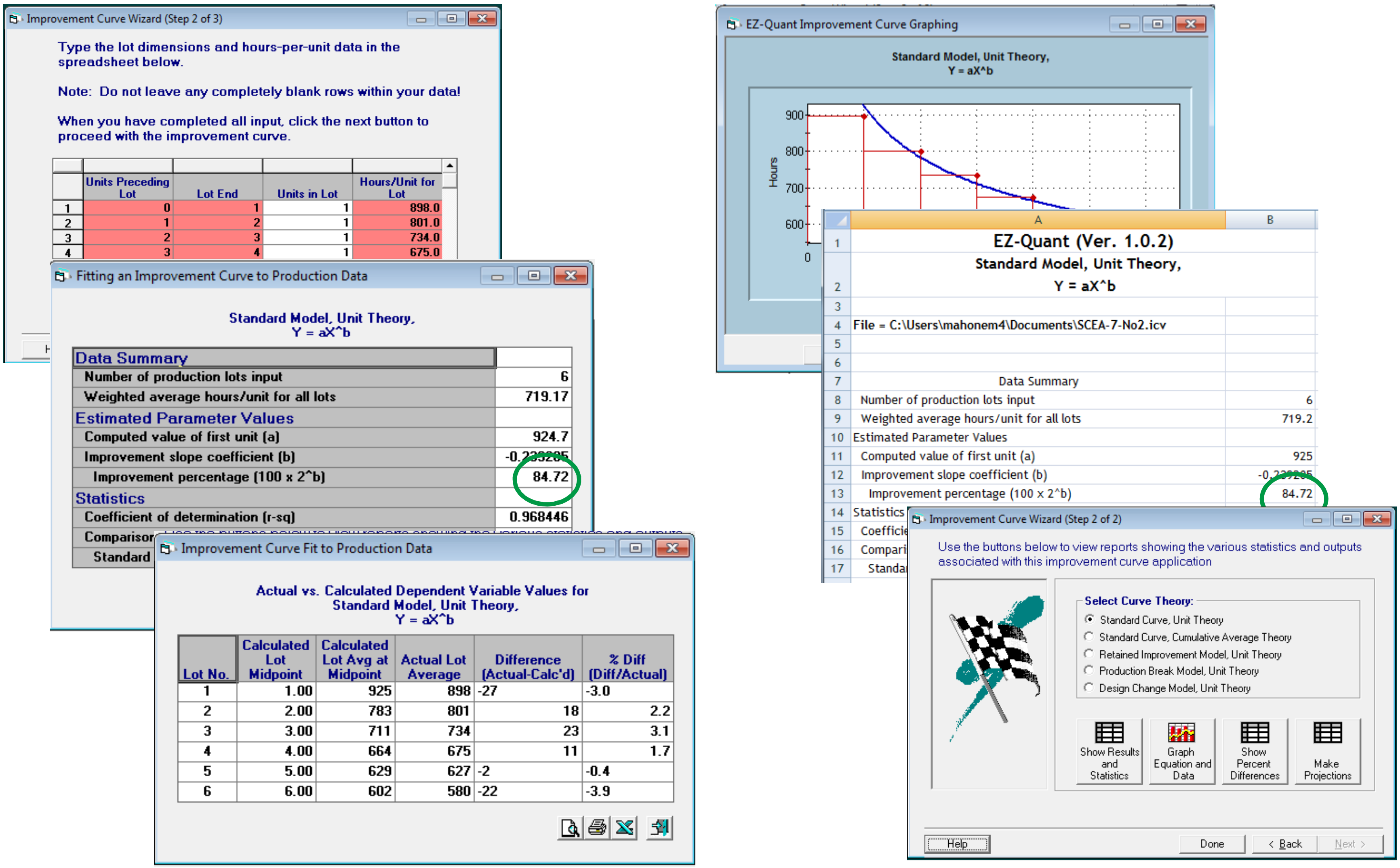
-0.43714 = b* 1.79176

-0.24397 = b

Slope = 2^-0.24397 = .844

Example 2

CEBoK Module 7, Question 2



Example 3

CEBoK Module 7, Question 3

Lot	Number	Lot Cost
1	2	1254
2	4	1618
3	10	2197
4	16	2756
5	12	2003

Question 3: Given the lot data at the right, what is the predicted unit cost of the 50th unit using cumulative average (CUMAV) learning curve theory?

workspace

Choice A [See slide 25 ff.]

Lot	Number	Lot Cost	um Units Prod	Cum Cost	CAUC	ln(Unit)	ln(CAUC)
1	2	1254	2	1254	627.0	0.69315	6.44095
2	4	1618	6	2872	478.7	1.79176	6.17100
3	10	2197	16	5069	316.8	2.77259	5.75831
4	16	2756	32	7825	244.5	3.46574	5.49934
5	12	2003	44	9828	223.4	3.78419	5.40880

ln a =	6.72245
b =	-0.34650
a =	830.847

LCS 78.6%

Compute the cumulative number of units and cumulative average unit cost (CAUC) as shown in the table.
Transform into log-log space, fit the regression line to determine the intercept and slope, and exponentiate the former to give the theoretical first unit cost.
Use the parameters for the learning curve equation, Y = a * X ^ b, to determine the cumulative average and then cumulative total costs through the 50th and 49th units.
The difference between these is the desired 50th-unit cost:

Unit	CAUC	Total Cost	Unit Cost
49	215.71	10569.73	
50	214.20	10710.21	140.47

Example 3

CEBoK Module 7, Question 3

Slope:

$$b = \frac{\sum_{i=1}^{n} X_i Y_i - n\overline{X}\overline{Y}}{\sum_{i=1}^{n} X_i^2 - n\overline{X}^2}$$

X and Y are the raw values of the 2 variables and
$\overline{X}$ and $\overline{Y}$ are the means of the 2 variables

Lot	Qty	Cum Units	Lot Cost	Avg Cost	Cum Cost	CAUC	X ln(Unit)	Y ln(CUAC)	XY	X^2
1	2	2	1,254.0	627.0	1,254.0	627.00	0.69315	6.44095	4.464524	0.480453
2	4	6	1,618.0	404.5	2,872.0	478.67	1.79176	6.17100	11.05696	3.210402
3	10	16	2,197.0	219.7	5,069.0	316.81	2.77259	5.75831	15.96543	7.687248
4	16	32	2,756.0	172.3	7,825.0	244.53	3.46574	5.49934	19.05927	12.01133
5	12	44	2,003.0	166.9	9,828.0	223.36	3.78419	5.40880	20.46793	14.32009
							12.50742	29.27841	71.01411	37.70952
						Average:	2.50148	5.85568		7.541904

b= 71.01411-(5 * 2.50148 * 5.85568) / 37.70952-5 * 2.50148^2

b= 71.01411-73.23947 / 37.70952-31.29

b= -2.22536 / 6.41952

b= -0.346655

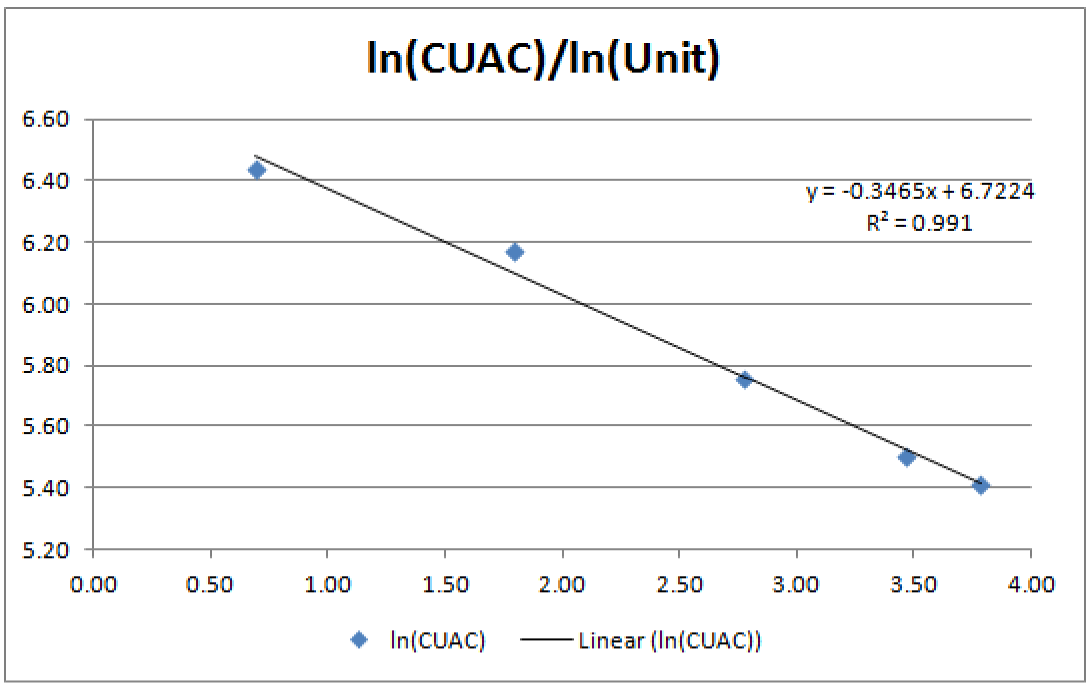
Slope = 2^-0.346655 = 0.78641

Example 3

CEBoK Module 7, Question 3

Lot	Qty	Cum Units	Lot Cost	Avg Cost	Cum Cost	CAUC	ln(Unit)	ln(CUAC)
1	2	2	1,254.0	627.0	1,254.0	627.00	0.69315	6.44095
2	4	6	1,618.0	404.5	2,872.0	478.67	1.79176	6.17100
3	10	16	2,197.0	219.7	5,069.0	316.81	2.77259	5.75831
4	16	32	2,756.0	172.3	7,825.0	244.53	3.46574	5.49934
5	12	44	2,003.0	166.9	9,828.0	223.36	3.78419	5.40880

ln(y) = ln(a) + b*ln(x) [as shown below it is ln(y) = b*ln(x) = ln(a)]

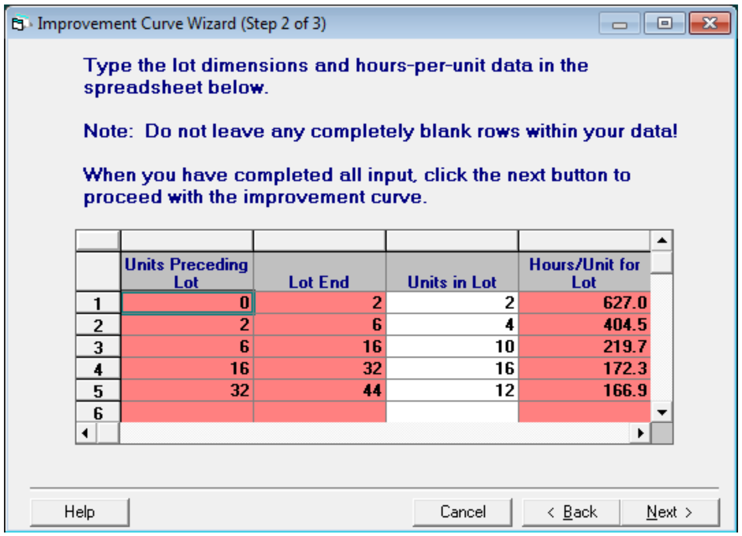


Cum 50th	ln(y) =	6.7224 + -0.3465*ln(50)		
	ln(y) =	6.7224 + -0.3465*3.9120		
	ln(y) =	6.7224 + -1.3555		
	ln(y)=	5.366884		
	y =	214.1944	* 50=	10,709.72
Cum 49th	ln(y) =	6.7224 + -0.3465*ln(49)		
	ln(y) =	6.7224 + -0.3465*3.8918		
	ln(y) =	6.7224 + -1.3485		
	ln(y)=	5.3738843		
	y =	215.69908	* 49=	10,569.25
Unit 50				140.47

Example 3

CEBoK Module 7, Question 3: EZ-Quant Solution

1



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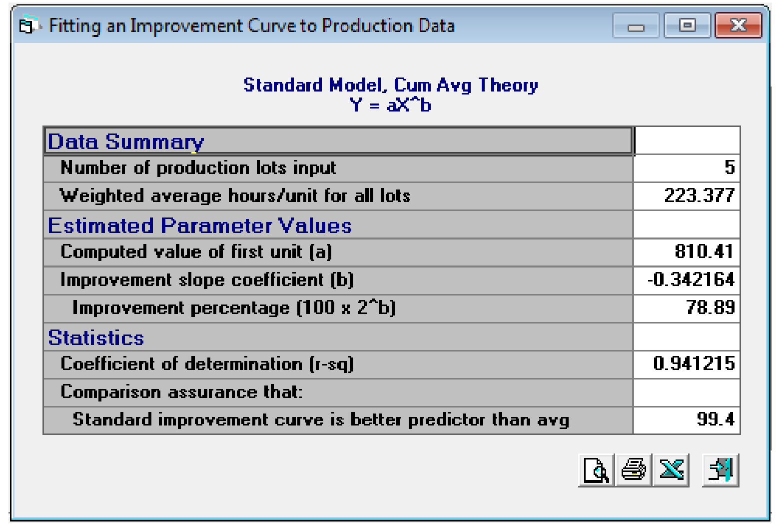

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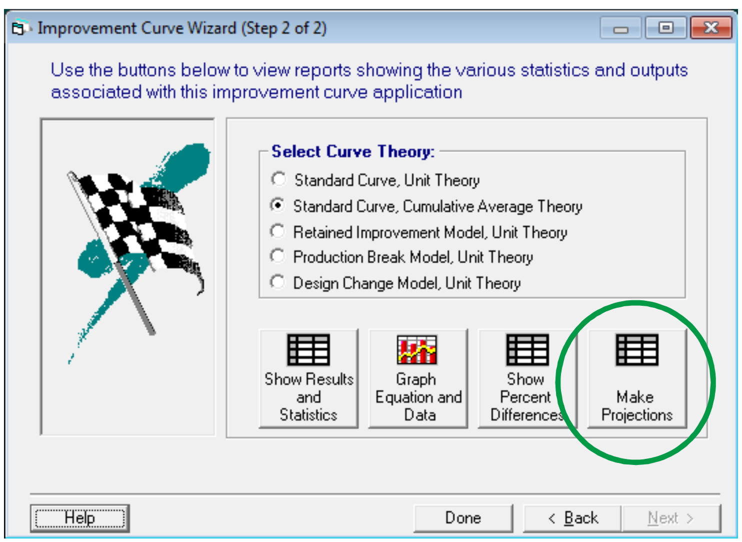

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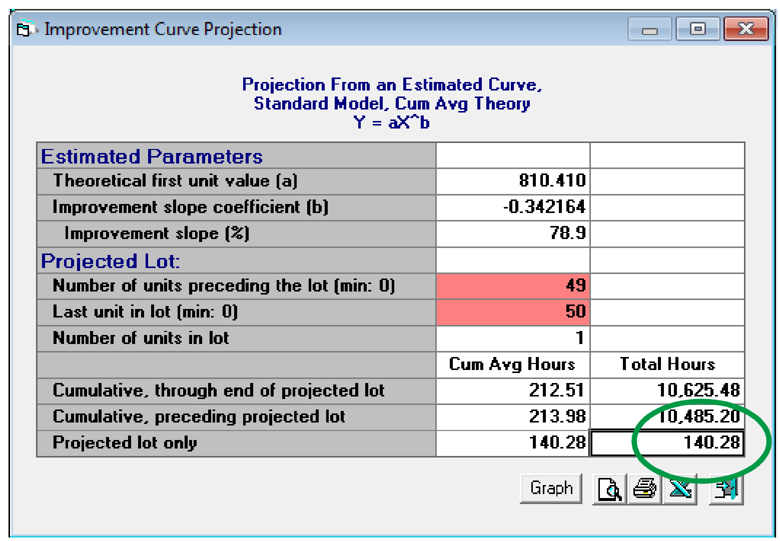


Example 4

CEBoK Module 7, Question 7

Question 7: If a manufacturer's learning curve is 80% and their first widget set took 56,000 hours to complete, how many hours will it take to complete the eighth set? (Assume ULC theory.)

workspace

Choice F [See slide 30]

Using Unit Learning Curve theory, Y=A*X^b, Y=56,000*8^(ln(80%)/ln(2))

56000*8^(ln(.80)/ln(2)) = **28672**

The easier solution, which could be crucial in saving time on the certification exam, is to notice that going from the 1st unit to the 8th unit represents doubling thrice, so that by the definition of learning curve, the first-unit hours will be multiplied by the LCS thrice:

56000*0.8*0.8*0.8= **28672**

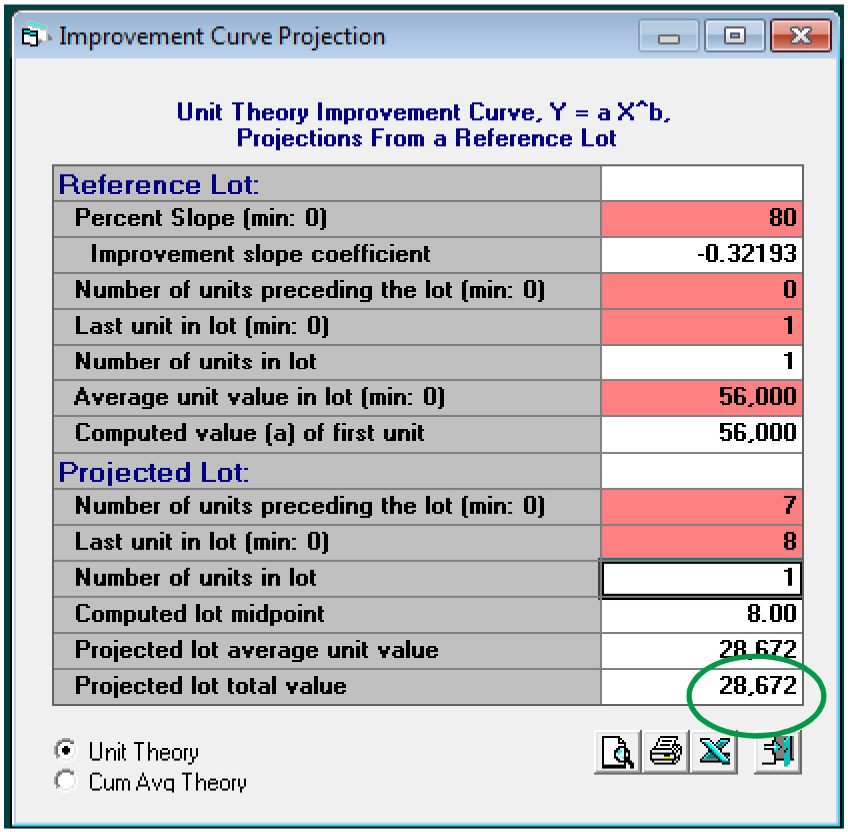
Y = A*X^b

Y = A* X^(ln(b)/ln(2))

Y=	56000 * 8 ^ (ln(.80)/ln(2))
Y=	56000 * 8^ (-.22314/.69315)
Y =	56000 * 8^ -0.32192
Y =	56000 * .51200
Y =	28672

Example 4

CEBoK Module 7, Question 7



Improvement Curve Projection

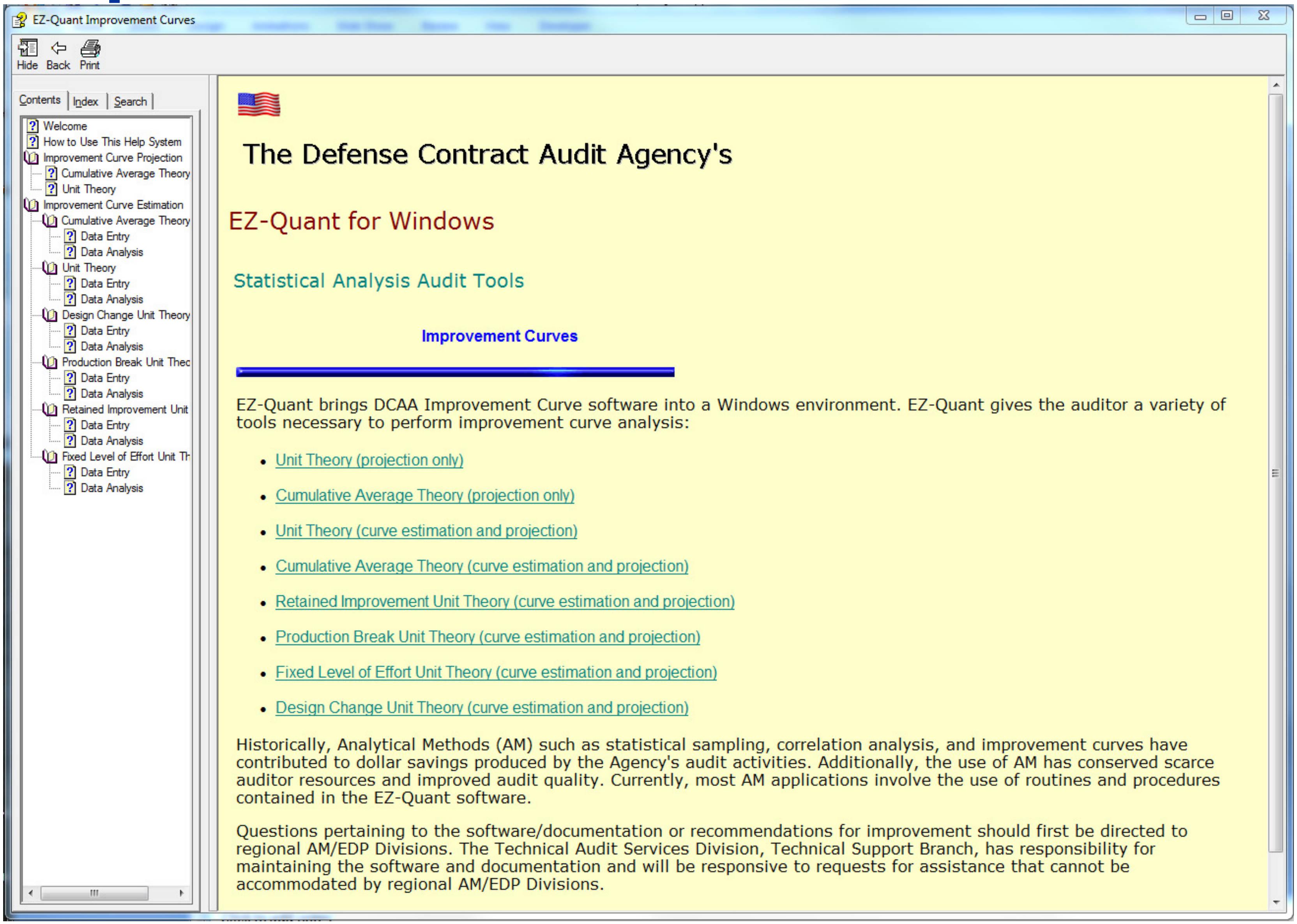
Unit Theory Improvement Curve, Y = a X^b,
Projections From a Reference Lot

Reference Lot:	
Percent Slope (min: 0)	80
Improvement slope coefficient	-0.32193
Number of units preceding the lot (min: 0)	0
Last unit in lot (min: 0)	1
Number of units in lot	1
Average unit value in lot (min: 0)	56,000
Computed value (a) of first unit	56,000
Projected Lot:	
Number of units preceding the lot (min: 0)	7
Last unit in lot (min: 0)	8
Number of units in lot	1
Computed lot midpoint	8.00
Projected lot average unit value	28,672
Projected lot total value	28,672

Unit Theory
Cum Avg Theory

The much easier solution…

Help


EZ-Quant Improvement Curves
Hide Back Print
Contents Index Search
Welcome
How to Use This Help System
Improvement Curve Projection
Cumulative Average Theory
Unit Theory
Improvement Curve Estimation
Cumulative Average Theory
Data Entry
Data Analysis
Unit Theory
Data Entry
Data Analysis
Design Change Unit Theory
Data Entry
Data Analysis
Production Break Unit Thec
Data Entry
Data Analysis
Retained Improvement Unit
Data Entry
Data Analysis
Fixed Level of Effort Unit Th
Data Entry
Data Analysis
The Defense Contract Audit Agency's
EZ-Quant for Windows
Statistical Analysis Audit Tools
Improvement Curves
EZ-Quant brings DCAA Improvement Curve software into a Windows environment. EZ-Quant gives the auditor a variety of tools necessary to perform improvement curve analysis:
Unit Theory (projection only)
Cumulative Average Theory (projection only)
Unit Theory (curve estimation and projection)
Cumulative Average Theory (curve estimation and projection)
Retained Improvement Unit Theory (curve estimation and projection)
Production Break Unit Theory (curve estimation and projection)
Fixed Level of Effort Unit Theory (curve estimation and projection)
Design Change Unit Theory (curve estimation and projection)
Historically, Analytical Methods (AM) such as statistical sampling, correlation analysis, and improvement curves have contributed to dollar savings produced by the Agency's audit activities. Additionally, the use of AM has conserved scarce auditor resources and improved audit quality. Currently, most AM applications involve the use of routines and procedures contained in the EZ-Quant software.
Questions pertaining to the software/documentation or recommendations for improvement should first be directed to regional AM/EDP Divisions. The Technical Audit Services Division, Technical Support Branch, has responsibility for maintaining the software and documentation and will be responsive to requests for assistance that cannot be accommodated by regional AM/EDP Divisions.

EZ-Quant

Questions?