

**DEVELOPMENT AND PRODUCTION COST EQUATIONS DERIVED FROM PRICE-H – TO  
ENABLE RAPID AIRCRAFT (MDO) TRADE STUDIES**

2008 Society Cost Estimating Analysis (SCEA) Conference  
W. Thomas Harwick, Engineering Specialist  
Northrop Grumman Corporation, Integrated Systems  
Industry Hills, California  
24-28 June, 2008

Abstract

One of the challenges in performing engineering design trade studies is to meet the required cycle time or trade study tempo and to have sufficient data to model the trade study alternatives for each architecture concept.

Reduced-form cost equations, derived from industry cost models, provide a convenient way to allow for rapid single as well as multiple architecture aircraft studies. This capability is particularly important in a Multi-discipline Design Optimization (MDO) environment.

This paper will show the method used and the results achieved to develop cost equations for development and production, based upon the PRICE-H model. These Cost Estimating Relationships (CERs) require limited information, and can be rapidly iterated in an integrated multi-discipline engineering model environment.

The presentation will include the following:

1. Identification of major aircraft cost drivers
2. Derivation of the equations for DDT&E and Production for the following aircraft WBS elements (Structure, Engines, Avionics, Subsystems, Software)
3. Presentation of the aircraft cost model

Last, we develop a weight-performance equation that enables the linkage between traditional weights based cost models and top level performance and design requirements. That is, aircraft weight is derived from top level performance requirements which enables concept and early design cost estimating before mass property weight information may be available. The aircraft weight equation is developed using industry performance and weight data.

**DEVELOPMENT AND PRODUCTION COST EQUATIONS DERIVED FROM PRICE-H – TO  
ENABLE RAPID AIRCRAFT (MDO) TRADE STUDIES**

Introduction

A cost equation is needed that can perform early design cost estimating in an MDO environment for both development and production cost. The equation should be capable of using basic inputs from major engineering Disciplines.

It is desired to select a cost model, with its embedded database, that can be used to model the industry cost with limited information which is available during the early design environment.

The PRICE-H model is well suited for developing the development and production CERs due to the ability to generate cost estimates with relatively little data. The DDT&E and production equations can be used to evaluate manned air vehicle trade study alternatives within a given architecture as well as for multiple architectures.

We will use the PRICE-H cost model and the economist's concept of "*ceterus parabus*" to determine which cost driver data to collect and then utilize correlation analysis to determine the relative importance of these cost drivers.

Next, multiple regression methods will be used to determine the best cost drivers statistically. This corresponds to the question: How much of the variation in cost is explained by the variation in the dependent variable. The best statistically performing explanatory variables will be selected as the cost drivers for the cost equation under consideration. Note that the rest of the PRICE-H model's cost variables and parameters are subsumed within the intercept term.

Finally, before starting the paper, we mention the material covered in the Appendices:

Appendix I shows the structural data and development cost derived from PRICE-H. It is an orthogonal data set, which permits construction of the regression equations. The inverse matrix (of the independent variables) is expected to be non-singular, and hence, that statistically unbiased solution exists.<sup>1</sup>

Appendix II shows the modeling baseline and low and high range that are used in the PRICE-H Cost Modeling.

---

<sup>1</sup> This also assumes that none of the independent variables are strongly correlated with the intercept vector. One can check the "t" statistic of the Intercept term to see if there might be a problem. The "t" statistic may have an extremely high value due indicating a strong correlation with an independent variable.

Appendix III presents a parametrically derived weight equation using performance from historical aircraft. In a Multi-discipline Design Optimization (MDO) environment this equation would, in principle, be replaced by a mass properties parametric equation from the mass properties discipline. The weight equation illustrates the possibility of connecting the cost models to aircraft performance and weight through a parametric based performance-weight equation.

### Overview of the Cost Drivers

A first step is the identification of independent variables that “explain” variation in the cost data. In mathematical form:

$$\text{Cost} = f(x_1, x_2, x_3, \dots).$$

Each of the “ $x_i$ ” terms is called a cost driver. A cost model should cover the major cost drivers across types of cost driver classifications.

Cost drivers include management, technical, and market. Schedule is derived from the market assessment. In markets with a few large customers, an individual may drive the market. An example of cost drivers and classification are included in Figure 1.

### Development and Production Cost Drivers

Cost Drivers	Technical	Management	Schedule	Market
<b>Mission</b>				
Operating Environment	x	x	x	x
Mission (speed, range, payload, time-on-station)	x	x	x	x
Technology innovation required (mods, ..., state of art, beyond state of art)	x	x	x	
Unmanned or crewed	x	x	x	
<b>Program</b>				
Team capability & experience		x	x	x
Availability of personnel		x	x	x
Inspection & Procurement standards	x	x	x	
Mass (empty weight)	x	x	x	
Percent of new structure	x	x		
Percent of new electronics	x	x		
Quantity (this customer)			x	x
Schedule constraints			x	x
<b>Market</b>				
Competition and price	x	x	x	x
Quantity (overall market)				x
Schedule constraints	x		x	x
Number of sites	x	x		x
<b>Air vehicle - Hardware</b>				
Structure	x	x	x	x
Material type	x	x		x
Engine power requirements	x		x	x
Engine qualification and test required	x			x
Avionics	x	x	x	x
Subsystems	x		x	x
<b>Air vehicle - Software</b>				
Source Lines of Code	x			x
Software Re-use		x	x	x
Software Integration Complexity	x	x	x	x
<b>Air vehicle - Payload</b>				
Weapons systems	x	x	x	x
Surveillance radar	x	x	x	x

Figure 1: Candidate Cost Drivers and Types from Several Cost Models

For a high level cost model, where the costs per pound vary by major systems, cost equations can be developed at the major subsystem level for the purpose of performing aircraft design trades. These systems include:

1. Structure
2. Propulsion
3. Avionics
4. Sub-systems
5. Software.

The PRICE-H model is used to model the first four categories. Software is modeled using the Constructive Costing Model (COCOMO) with a few changes for intra-complexity (within CSCI) and inter-complexity (across CSCIs).<sup>2</sup>

<sup>2</sup> Since the COCOMO equations have been articulated by Barry Boehm in his Software Engineering Economics, Prentice-Hall, 1981, the *ceterus parabus* and regression methodology is not needed for the software WBS.

The extensive list of cost drivers and types suggest that it is very important to develop Ground Rules and Assumptions. While this paper stresses methods, a short list of ground rules and assumptions are listed as follows:

1. The cost equations are developed for development and production cost
2. Flight Test effort is excluded from our cost equations
3. Operations and Support cost is excluded from our analysis
4. Software effort is included and is modeled using the COCOMO model framework
5. Cost is modeled in 2007 (Constant Year) Million dollars
6. PRICE-H default learning curve rate is used
7. No existing manufacturing process and line is assumed.

### Method to be used

For each WBS element the *ceterus parabus* method of analysis is performed for the PRICE-H model. The architecture trade study “baseline” provides a basis to generate cost sensitivity data. The parameters are selected using *a priori* knowledge about the cost variables appropriate for any cost model. These include mission environment, sizing, complexity, team capability and experience, quantity (learning curve), and schedule constraint cost drivers.

The cost sensitivity for each cost driver candidate is modeled. A correlation matrix is developed for each cost equation which yields the most important cost drivers as measured by the highest correlation (both positive and negative). The data generated for a given WBS is regressed against the appropriate cost data (development or production cost data). The Excel model regression package is used.

For illustration, we will discuss the Structure WBS element. The results for the other WBS elements are shown in the second section, Aircraft Cost Model.

### Structures

The initial baseline used for modeling the Structures WBS is shown in Figure 2. The range utilized for each input parameter is shown in Appendix II.

Structure Development Cost	Inputs
Intercept	1
WS	100,000
ECMPLX	1.4
MCPLXS	7.1
NEWST	0.9
PLTFM	1.8
PROTOS	5

Figure 2: Structure Modeling Baseline

A brief definition of the PRICE-H symbols is as follows:

- WS = Structural weight, in pounds
- ECMPLX = Design complexity and team experience
- MCPLXS = Manufacturing complexity for structure, load-bearing structure
- NEWST = Percent of new structure (avoids duplicate design effort)
- PLTFM = Operation Platform for Structure
- PROTOS = Number of prototypes.

Electronic items, which appear below use the following abbreviations:

- WE = Electronic Weight, in pounds
- MCPLXE = Manufacturing complexity for electronics
- NEWEL = Percent of new electronics (avoids duplicate design effort)

For the Production Cost structure equation, we also add Quantity of production.

QNTY = Quantity of production.

The development cost correlation matrix shows the best development cost drivers as follows:<sup>3</sup>

1. Structure weight (75 % correlation with Development Cost)
2. Manufacturing Complexity (47% correlation with Development Cost)
3. Engineering Complexity (25% correlation with Development Cost)
4. Number of Prototypes (23% correlation with Development Cost)
5. Percent of New Structure (19% correlation with Department cost)

<b>DevCost Correlation Matrix</b>	<i>DevCost (2007M\$)</i>	<i>WS</i>	<i>ECMPLX</i>	<i>MCPLXS</i>	<i>NEWST</i>	<i>PLTFM</i>	<i>PROTOS</i>	<i>QNTY</i>
DevCost (2007M\$)	1							
WS	0.748625455	1						
ECMPLX	0.257491864	-0.015546	1					
MCPLXS	0.473650384	0.004324	0.001216	1				
NEWST	0.191391592	-0.055979	-0.015744	0.004379	1			
PLTFM	0.115058354	0.029901	0.00841	-0.002339	0.030283	1		
PROTOS	0.23093603	-0.052885	-0.014874	0.004137	-0.05356	0.028609	1	
QNTY	-0.079793865	-0.065497	-0.018421	0.005123	-0.066334	0.035432	-0.062667	1

Figure 3: Development Cost Correlation Matrix

The production cost correlation matrix shows that the best production cost drivers are:

1. Quantity (correlation of 85% with Production Cost)
2. Structure weight (correlation of 35% with Production Cost)

<sup>3</sup> Note that the correlation matrix also includes the dependent variable, y, where y is Development Cost in our example. Traditionally, the correlation matrix only included the x, or independent variables.

3. Manufacturing Complexity (correlation of 34% with Production Cost)

	<i>ProdCost (2007M\$)</i>	<i>WS</i>	<i>ECMPLX</i>	<i>MCPLXS</i>	<i>NEWST</i>	<i>PLTFM</i>	<i>PROTOS</i>	<i>QNTY</i>
ProdCost (2007M\$)	1							
WS	0.349075344	1						
ECMPLX	-0.018743726	-0.015546	1					
MCPLXS	0.336706562	0.004324	0.001216	1				
NEWST	-0.074468463	-0.055979	-0.015744	0.004379	1			
PLTFM	0.040623197	0.029901	0.00841	-0.002339	0.030283	1		
PROTOS	-0.074424637	-0.052885	-0.014874	0.004137	-0.05356	0.028609	1	
QNTY	0.84968624	-0.065497	-0.018421	0.005123	-0.066334	0.035432	-0.062667	1

Figure 4: Production Cost Correlation Matrix

Regression Modeling and Results – for Structures

The DDT&E data was used to develop linear and log equations.

The CER equation form is:

$$\text{Model domain: DDT\&E Cost} = A * (x_1)^{B1} * (x_2)^{B2} * \dots * (x_n)^{Bn}$$

The CER uses those cost drivers which are statistically most important and are likely to be available early in the design cycle.

The development cost multiple regression results for Structures are shown in Figure 5.

<i>DevCost - Structures</i>	
Multiple R	0.998757
R Square	0.997515
Adjusted R Square	0.997034
Standard Error	0.023038
Observations	38

  

ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	6	6.60326	1.100543	2073.654119	0.000000
Residual	31	0.01645	0.000531		
Total	37	6.61971			

  

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
Intercept	-13.08631	0.24177	-54.12617	2.91426E-32	-13.57941	-12.59321
WS	0.76435	0.00883	86.52794	1.54898E-38	0.74634	0.78237
ECMPLX	1.21203	0.03926	30.87425	7.62889E-25	1.13197	1.29210
MCPLXS	5.17406	0.09903	52.24860	8.60680E-32	4.97209	5.37603
NEWST	0.47658	0.01712	27.84042	1.69013E-23	0.44167	0.51150
PLTFM	1.47582	0.17688	8.34375	2.00296E-09	1.11508	1.83656
PROTOS	0.33150	0.01043	31.76921	3.22985E-25	0.31022	0.35278

Figure 5: Development Cost CER Coefficients and Statistics for Structures WBS

We note that the F-statistic is much greater than 5. Thus, the overall model is highly significant. The adjusted  $R^2$  is greater than 0.99. This means that more than 99% of the variation in the cost data is explained by the variation in the cost explanatory data. Assuming that there is no correlation between the independent variables (multi-collinearity), this is a very good statistical result. Since the “t” statistic is greater than 2.0 for n greater than 20, we conclude that all the cost drivers are statistically significant.

Next, we show the production cost multiple regression results for Structures.

<b>ProdCost - Structures</b>						
Multiple R	0.99784					
R Square	0.99569					
Adjusted R Square	0.99530					
Standard Error	0.06509					
Observations	38					

  

<b>ANOVA</b>						
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>	
Regression	3	33.239	11.080	2615.366333	2.9532E-40	
Residual	34	0.14404	0.00424			
Total	37	33.383				

  

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
Intercept	-20.55702	0.61806	-33.26038	1.60448E-27	-21.81308	-19.30097
WS	0.89363	0.02491	35.86755	1.32144E-28	0.84300	0.94427
QNTY	0.80199	0.01035	77.46275	7.91467E-40	0.78095	0.82303
MCPLXS	8.20743	0.27978	29.33557	9.96779E-26	7.63886	8.77601

Figure 6: Production Cost CER Coefficients and Statistics for Structures WBS

We note that the F-statistic is much greater than 5. Thus, the overall model is highly significant. The adjusted  $R^2$  is greater than 0.99. Assuming that there is no correlation between the independent variables (multi-collinearity), this is a very good statistical result. Since the “t” statistic is greater than 2.8 for n greater than 20, we conclude that all the cost drivers are statistically significant at the one percent confidence level. The data extracted from PRICE-H is shown in Appendix I.

Also, notice that the exponent with respect to weight is less than zero (0.89). This means that when the aircraft design weight is doubled, costs less than double.

Additionally, the exponent with respect to quantity is less than one (0.80). Since the PRICE-H data sampled is based upon an industry view over many projects, it is a composite learning curve rather than related to a particular factory or program.

### Structure Model Results



The structure modeling equations results for Development and Production cost are as follows:

In symbols,

$$\text{Cost}_{\text{development}} = e^{-13.086} * \text{WS}^{0.764} * \text{Dev\_Cplx}^{1.212} * \text{Mfg\_Cplx}^{5.17} * \text{New\_Stru}^{0.476} * \text{Plat\_Ops}^{1.475} * \text{Proto}^{0.331},$$

Where,

A = Intercept term, where e is defined as 2.718...  
WS = Weight Structure (pounds), (WS in PRICE-H),  
Dev\_Cplx = Design complexity (ECMPLX in PRICE-H),  
Mfg\_Cplx = Manufacturing complexity (MCPLXS in PRICE-H),  
New\_Stru = New Structure percent (NEWST in PRICE-H),  
Plat\_Ops = Operating Platform environment (PLTFM in PRICE-H),  
Proto = Number of Prototypes (PROTOS in PRICE-H).

The associated development exponents (Beta's) above are reflected in the Figure 5 regression results.

In symbols,

$$\text{Cost}_{\text{production}} = A * e^{-20.557} * \text{WS}^{0.894} * \text{Mfg\_Cplx}^{8.207} * \text{Qty}^{0.802}$$

Where,

A = Intercept term, where e is defined as 2.718...  
WS = Weight Structure (pounds), (WS in PRICE-H),  
Mfg\_Cplx = Manufacturing complexity (MCPLXS in PRICE-H),  
Qty = Number of Units Produced (PROTOS in PRICE-H).

The associated production exponents (Beta's) above are reflected in the Figure 6 regression results.

An example calculation is shown below. The inputs are shown by their PRICE-H labels in the "Regression Exponents" column. The Calculation for each term and

overall (Multiplicative) calculation is shown in the “Term Calculations” column. The notional baselines, as well as trade study parameter values are (can be) modeled in the “Inputs” column. See Figures 7 and 8 for sample calculations.

Development Cost	Regression Exponents	Inputs	Results, Terms & Final
Intercept	-13.0863103780	2.718281828	0.000002073422
WS	0.764354978	100000	6,633.991
ECMPLX	1.212030682	1.4	1.504
MCPLXS	5.174059521	7.1	25,378.232
NEWST	0.476584165	0.9	0.951
PLTFM	1.47581886	1.8	2.381
PROTOS	0.331503041	5	1.705
<b>Development Cost, Structure, 2007 M\$</b>			<b>\$ 2,026</b>

Figure 7: Development Concept Baseline and Cost Result

Example calculations are shown for the production cost for structures in Figure 8.

Production Cost, Q=100	Beta's	Inputs	Term Calculations
Intercept	-20.55702224	2.718281828	0.000000001180861
WS	0.893632646	100000	29,387.54
MCPLXS	8.207431275	7.1	9,697,156.28
QNTY	0.801990346	100	40.18
<b>Production Cost, Structure, 2007 M\$</b>			<b>\$ 13,520</b>

Figure 8: Production Concept Baseline and Cost Result

### Aircraft cost model

The aircraft cost model will be presented in the following sections:

1. Estimated coefficients of the model for development and production cost.
2. Presentation of the aircraft cost model (reduced form equations)
  - a. Hardware cost equations
  - b. Software cost equations
  - c. Production equation (hardware).

#### 1. Estimated coefficients of the model

There is one model for development cost and one model for production cost for hardware. There is no production software cost model, as software development costs are book-kept entirely under development and software production and distribution costs are considered relatively small.<sup>4</sup> Figure 9 shows the regression exponents for development cost.

<sup>4</sup> This model excludes software maintenance and upgrade costs. The software maintenance and upgrade costs are often greater than the original development cost over the product life cycle.

Dev - Structure	Regression Exponents
Intercept	-13.0863103780
WS	0.764354978
ECMPLX	1.212030682
MCPLXS	5.174059521
NEWST	0.476584165
PLTFM	1.47581886
PROTOS	0.331503041

Dev - Subsystems	Regression Exponents
Intercept	-12.5132967
WS	0.713461498
ECMPLX	1.172650055
MCPLXS	5.124267357
NEWST	0.413399759
PLTFM	1.287151103
PROTOS	0.248828097

Dev - Propulsion	Regression Exponents
Intercept	-8.080057804
WS	0.228401068
ECMPLX	1.138956421
MCPLXS	5.816041192
NEWST	0.508753547
PROTOS	0.455722349

Dev - Avionics	Regression Exponents
Intercept	-11.42947471
WS	0.092628333
WE	0.542187893
ECMPLX	1.107872576
MCPLXE	4.650874168
MCPLXS	0.64506055
NEWEL	0.567711646
PLTFM	1.538107417
PROTOS	0.366858067

Figure 9: Estimated Regression Coefficients for the Hardware WBS Elements.

The Production Regression Exponents are shown below in Figure 10.

<b>Prod - Structure, Q=100</b>	<b>Regression Exponents</b>
Intercept	-20.55702224
WS	0.893632646
MCPLXS	8.207431275
QNTY	0.801990346

<b>Prod. Subsystems; Q=100</b>	<b>Regression Exponents</b>
Intercept	-19.91861107
WS	0.889075558
MCPLXS	8.073070967
QNTY	0.731413918

<b>Prod. Propulsion; Q=100</b>	<b>Regression Exponents</b>
Intercept	-14.26471837
WS	0.246144264
MCPLXS	8.383832514
QNTY	0.829990947

<b>Prod. Avionics; Q=100</b>	<b>Regression Exponents</b>
Intercept	-21.66365507
WS	0.091730477
WE	0.754885189
MCPLXE	8.143380527
MCPLXS	1.080069962
QNTY	0.73451785

Figure 10: Estimated Regression Coefficients for the Hardware WBS Elements.

## 2. Reduced Form Equations for Engineering Design Trade Study Modeling

Dev - Structure	Regression Exponents	Modeling Assumptions	Terms & Final Results
Intercept	-13.0863103780	2.718281828	0.000002073422
WS	0.764354978	100,000	6,633.991
ECMPLX	1.212030682	1.4	1.504
MCPLXS	5.174059521	7.1	25,378
NEWST	0.476584165	0.9	0.951
PLTFM	1.47581886	1.8	2.381
PROTOS	0.331503041	5	1.705
<b>Development Cost, Structure, 2007 M\$</b>			<b>\$ 2,026</b>

Dev - Subsystems	Regression Exponents	Modeling Assumptions	Terms & Final Results
Intercept	-12.5132967	2.718281828	0.000003677429
WS	0.713461498	25,000	1,373.286
ECMPLX	1.172650055	1.9	2.123
MCPLXS	5.124267357	7.75	36,059
NEWST	0.413399759	1.3	1.115
PLTFM	1.287151103	1.8	2.131
PROTOS	0.248828097	3	1.314
<b>Development Cost, SubSystems, 2007 M\$</b>			<b>\$ 1,207</b>

Dev - Propulsion	Regression Exponents	Modeling Assumptions	Terms & Final Results
Intercept	-8.080057804	2.718281828	0.00030965314
WS	0.228401068	20,000	9.601959
ECMPLX	1.138956421	1.6	1.707984
MCPLXS	5.816041192	7.69	142,097
NEWST	0.508753547	1	1.000000
PROTOS	0.455722349	6	2.262668
<b>Development Cost, Propulsion, 2007 M\$</b>			<b>\$ 1,633</b>

Dev - Avionics	Regression Exponents	Modeling Assumptions	Terms & Final Results
Intercept	-11.42947471	2.718281828	0.000010870317
WS	0.092628333	3,000	2.099
WE	0.542187893	3,000	76.781
ECMPLX	1.107872576	2.300	2.516
MCPLXE	4.650874168	8.92	26,304.336
MCPLXS	0.64506055	8.700	4.037
NEWEL	0.567711646	0.900	0.942
PLTFM	1.538107417	1.800	2.470
PROTOS	0.366858067	2.500	1.400
<b>Development Cost, Avionics, 2007 M\$</b>			<b>\$ 1,524</b>

Figure 11: Example - Development Cost Equations and Calculations

The hardware design equations include Structure, Subsystems, Propulsion, and Avionics. The regression exponent results (in the power equation form) are

shown in the “Regression Exponents” column. The Baseline modeling assumptions are shown in the “Modeling Assumptions” column.

The development cost calculations are estimated for each equation term. Since we have used an exponential cost model, ( $Y = A * X^{b1} * X^{b2} * \dots * X^{bn}$ ), the calculation for each hardware subsystem is the multiplicative product of the intermediate calculations.

The calculation for the Structure, Weight Structure term, WS, equals the following:

$$\text{WS cost (term)} = (100,000 \text{ lbs})^{0.7643}$$

$$\text{WS cost (term)} = 6,633.99.$$

Next, multiply the Terms together to obtain the development cost:

Intercept:  $2.073 \times 10^{-6}$   
 WS: 6,633.99  
 ECMLPX: 1.504  
 MCPLXS: 25,378  
 NEWST: 0.951  
 PLTFM: 2.381  
 PROTOS: 1.705

This yields \$2,026 for development, structure cost, in 2007, million dollars.

Finally the design hardware cost (\$6,390 M), shown in Figure 12, equals the sum of the four design areas: Structure + Subsystems + Propulsion + Avionics, (\$2,026 + \$1,207 + \$1,633 + \$1,524).

Aircraft Development Cost	2007 M\$
Structure	\$ 2,026
Subsystems	\$ 1,207
Propulsion	\$ 1,633
Avionics	\$ 1,524
<b>TOTAL Development Cost</b>	<b>\$ 6,390</b>
Software Development	\$ 3,178
<b>TOTAL Development Cost</b>	<b>\$ 9,568</b>

Figure 12: Example - Development Cost Summary

## 2.b Software Development cost equations

The COCOMO model is employed at a high level for Mission Critical and non-mission critical software:

The major software cost drivers are:

1. Equivalent new SLOC
2. Software Integration Complexity factors (both inter-and intra-CSCI)
3. Environmental factors.

The CONstructive COSting MOdel (COCOMO) model equations are used for the “Embedded” and “Semi-detached” modes. This eliminates the need to apply the ceterus parabus methodology to extract the COCOMO model equations.<sup>5</sup>

The basic software development cost equation is:

$$SW\ DevCost = A * \prod EF_i * (KSLOC/module)^{b1} * (KSLOC/CSCI)^{b2},$$

Where,

EF = Environmental Factors (Analyst Capability, Programmer capability, Product complexity, Security requirements, Schedule constraint),

EF<sub>i</sub> for Mission Critical = 2.0

EF<sub>i</sub> for Non-Mission-Critical = 1.45

KSLOC = Source Lines of Code, in thousands

Equivalent New SLOC = New SLOC equivalents from new + reused code

A = Intercept value = 3.0

b1 = exponent for inter-CSCI complexity = 1.05

b2 = exponent for intra-CSCI complexity (if Mission Critical) = 1.14

b2 = exponent for intra-CSCI complexity (if non-Mission-Critical) = 1.12

The settings for the environmental cost drivers are set at (L, M, H). If the cost drivers are productivity related, cost decreases with a higher COCOMO setting. If the cost drivers involve complexity or impose constraints, then cost increases with a higher COCOMO setting.<sup>6</sup>

The software cost total is shown in Figure 12. The software cost details are not shown as they are not considered the main focus of this paper.

## 2.c Hardware Production Cost

The production cost equations are shown in Figure 14 for a quantity of 100.

---

<sup>5</sup> Boehm, Barry, Software Engineering Economics, Prentice-Hall, New Jersey, 1981, chapter 23.

<sup>6</sup> Ibid, Chapter 23 discussion of the Detailed COCOMO model. To maintain a more workable software cost model within the MDO framework, other Environmental Factors have been omitted. The SEI rating factor has been added to provide a better assignment of the intra-CSCI exponent value. This can be thought of as a bridge between COCOMO I and COCOMO II.

Prod - Structure; Q=100	Regression Exponents	Modeling Assumptions	Terms & Final Results
Intercept	-20.55702224	2.718281828	0.000000001180861
WS	0.893632646	100,000	29,388
MCPLXS	8.207431275	7.1	9,697,156
QNTY	0.801990346	100	40.18
<b>Production Cost, Structure, 2007 M\$</b>			<b>\$ 13,520</b>

Prod. Subsystems; Q=100	Regression Exponents	Modeling Assumptions	Terms & Final Results
Intercept	-19.91861107	2.718281828	0.000000002236
WS	0.889075558	25,000	8,130
MCPLXS	8.073070967	7.75	15,114,569
QNTY	0.731413918	100	29.02872
<b>Production Cost, Subsystems, 2007 M\$</b>			<b>\$ 7,976</b>

Prod. Propulsion; Q=100	Regression Exponents	Modeling Assumptions	Terms & Final Results
Intercept	-14.26471837	2.718281828	0.000000638133474
WS	0.246144264	20,000	11.44653026
MCPLXS	8.383832514	7.3	17296342.04
QNTY	0.829990947	100	45.70691338
<b>Total Production Cost, Propulsion, 2007 M\$</b>			<b>\$ 5,775</b>

Prod. Avionics; Q=100	Regression Exponents	Modeling Assumptions	Terms & Final Results
Intercept	-21.66365507	2.718281828	0.00000000390476
WS	0.091730477	3,000	2.08429
WE	0.754885189	3,000	421.52888
MCPLXE	8.143380527	8.92	54,850,760
MCPLXS	1.080069962	8	9.44932
QNTY	0.73451785	100	29.44664
<b>Production Cost, Avionics, 2007 M\$</b>			<b>\$ 5,236</b>

Figure 14: Hardware Production Cost Model with Example Calculation across WBS Elements

The Hardware Production equations are shown in Figure 14. The Model Coefficients are shown for each of the Systems (Structure, Avionics, Subsystems, Propulsion). Since the cost model is natural log based, the calculation for each hardware subsystem is the multiplicative product of the intermediate calculations.

We show sample calculations for the production cost of the Structure block: This includes calculations for the "Intercept", "WS", "MCPLSX" and "QNTY" terms.

Intercept term:  $e^{-20.5570} = 1.181 \times 10^{-9}$

WS term:  $\text{Structure Wt}^{\text{WS\_Exponent}} = (100,000 \text{ lbs})^{0.8936} = 29,388$



Manuf. Structure Complexity:  $MCPLXS ^ 7.1 = 9,697,156$

And Quantity:  $Qty ^ Qty\_Exponent$   
 $= (100) ^ 0.80199 = 40.18$

The multiplicative product equals \$13,520M  $(= (1.181 * 10^{-9}) * (29,388) * (9.697,156 * 10 ^ 6) * (40.18)$

Finally, the total production cost (\$32,507 M), equals the sum of the system equations: Structure + Subsystems + Propulsion + Avionics (\$13,520 M + \$7,976 M + \$5,775 M + \$5,236 M). The inclusion of the system integration factor yields an overall production cost of \$34,035 M.

## Summary

We have discussed the identification of major aircraft cost drivers and used the correlation matrix to select the most important cost drivers. The equations for design and production have been presented for the following WBS elements (Structure, Engines, Avionics, Subsystems, and Software).

The complete aircraft cost model was presented, including the estimated coefficients, the modeling assumptions. An illustration was presented for the cost model, including the development hardware and software equations, as well as the production hardware equations.

A separate test was done to measure the impact of using the CERs for large, manned aircraft to forecast UAV cost. An unmanned aircraft was selected for testing. Cost results were within 15 per cent for both development and production cost.<sup>7</sup>

It is concluded that one set of cost equations, covering development and production effort, may be employed, in a Multi-discipline Design Optimization (MDO) environment to perform engineering trade studies. Not included in this analysis are the Operations and Support (O&S) Equations. The O&S equations are beyond the scope of this study. Top level O&S equations can be developed, but to model subsystems or components would take extensive effort. There are some industry models which should be looked at in this regard.

Appendix I shows the development costs for the Structures WBS derived from PRICE-H. The constructed dataset is orthogonal, which permits construction of the regression equations. The inverse matrix (of the independent variables) is expected to be non-singular, and hence, that a statistically unbiased solution exists.<sup>8</sup>

<sup>7</sup> Refer to Data Search Associates, U.S. Military Aircraft Data Book, 2006, UAS vehicle section.

<sup>8</sup> This also assumes that none of the independent variables are strongly correlated with the intercept vector. One can check the "t" statistic of the Intercept term to see if there might be a

Appendix II shows the modeling baseline and range (low, high) that was used in the PRICE-H Cost Modeling.

Appendix III presents a parametrically derived weight equation using performance from historical aircraft. The usefulness of the performance based hardware equation depends upon the cycle time for the mass properties group to generate weight estimates. In an MDO environment this equation would, in principle, be replaced by a mass properties parametric equation from that discipline. The weight equation illustrates the possibility of connecting the cost models to aircraft performance and weight via a performance-weight equation.

---

problem. The “t” statistic may have an extremely high value due indicating a strong correlation with an independent variable.

APPENDIX I: PRICE-H Generated Data for Selected Parameters

	DevCost (2007M\$)	WS	ECMPLX	MCPLXS	NEWST	PLTFM	PROTOS	
1	\$	2,610	100,000	1.4	7.057	1.76	1.7	6
2	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
3	\$	3,105	100,000	1.4	7.057	1.76	1.9	6
4	\$	3,337	100,000	1.4	7.057	1.76	2.0	6
5	\$	1,638	100,000	1.4	7.057	1.76	1.8	1
6	\$	1,968	100,000	1.4	7.057	1.76	1.8	2
7	\$	2,222	100,000	1.4	7.057	1.76	1.8	3
8	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
9	\$	3,573	100,000	1.4	7.057	1.76	1.8	10
10	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
11	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
12	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
13	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
14	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
15	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
16	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
17	\$	675	15,000	1.4	7.057	1.76	1.8	6
18	\$	838	20,000	1.4	7.057	1.76	1.8	6
19	\$	1,678	50,000	1.4	7.057	1.76	1.8	6
20	\$	2,285	75,000	1.4	7.057	1.76	1.8	6
21	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
22	\$	4,839	200,000	1.4	7.057	1.76	1.8	6
23	\$	1,749	100,000	0.9	7.057	1.76	1.8	6
24	\$	2,360	100,000	1.2	7.057	1.76	1.8	6
25	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
26	\$	4,629	100,000	2.0	7.057	1.76	1.8	6
27	\$	1,128	100,000	1.4	5.85	1.76	1.8	6
28	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
29	\$	3,305	100,000	1.4	7.25	1.76	1.8	6
30	\$	4,329	100,000	1.4	7.6	1.76	1.8	6
31	\$	5,370	100,000	1.4	7.9	1.76	1.8	6
32	\$	1,612	100,000	1.4	7.057	0.5	1.8	6
33	\$	2,102	100,000	1.4	7.057	1	1.8	6
34	\$	2,591	100,000	1.4	7.057	1.5	1.8	6
35	\$	2,847	100,000	1.4	7.057	1.761	1.8	6
36	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
37	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
38	\$	2,847	100,000	1.4	7.057	1.76	1.8	6
39	\$	2,847	100,000	1.4	7.057	1.76	1.8	6

Figure 1A: Example Development Cost Dataset generated from PRICE-H

APPENDIX II: PRICE-H Input Parameters and Input Ranges

This appendix presents the specification of the PRICE-H parameters and their modeled ranges to be used in a MDO environment. A separate table is constructed for Structures, Avionics, Propulsion, and Sub-systems. The table for Structure cost parameters, modeled baseline, and low to high parameter range follows:

System Name	Parameter	PRICE-H Term	Modeled Baseline	Low Range	High Range
Structure, develop.	Structural Weight (Pounds)	WS	100,000	15,000	200,000
Structure, develop.	Design Complexity	ECMPLX	1.4	0.9	2.0
Structure, develop. & production	Manuf. Complexity, Structure	MCPLXS	7.057	5.85	7.9
Structure, develop.	% of New Structure	NEWST	1.0	0.5	1.0
Structure, develop.	Operating Environment	PLTFM	1.8	1.7	2.0
Structure, develop.	Number of Prototypes	PROTOS	6	1	10
Structure, production	Total Quantity Produced	QNTY	100	1	200

Figure 2A: Development and Production Cost Modeling Data Ranges – Structures

The table for Avionics cost parameters, modeled baseline, and low to high parameter range follows:

System Name	Parameter	PRICE-H Term	Modeled Baseline	Low Range	High Range
Avionics, develop.	Electronic Weight (Pounds)	WE	3,700	100	10,000
Avionics, develop.	Structural Weight (Pounds)	WS	4,300	1,000	20,000
Avionics, develop.	Design Complexity	ECMPLX	1.4	0.7	2.0
Avionics, develop.	Manuf. Complexity, Electronics	MCPLXE	9.112	7.59	10.26
Avionics, develop. & production	Manuf. Complexity, Structure	MCPLXS	6.757	6.3	8.0
Avionics, develop.	Operating Environment	PLTFM	1.8	1.6	2.0
Avionics, develop.	Number of Prototypes	PROTOS	6	3	12
Avionics, develop.	% of New Structure	NEWEL	0.9	0.5	1.0
Avionics, production	Total Quantity Produced	QNTY	100	10	500
Avionics, production	% of New Structure	NEWST	1.0	0.5	1.0

Figure 2B: Development and Production Cost Modeling Data Ranges – Avionics

The table for Propulsion cost parameters, modeled baseline, and low to high parameter range follows:

System Name	Parameter	PRICE-H Term	Modeled Baseline	Low Range	High Range
Propulsion, develop. & production	Structural Weight (Pounds)	WS	35,000	4,000	40,000
Propulsion, develop.	Design Complexity	ECMPLX	1.4	0.9	2.0
Propulsion, develop. & production	Manuf. Complexity, Structure	MCPLXS	7.3	5.85	9.0
Propulsion, develop.	Number of Prototypes	PROTOS	6	1	10
Propulsion, production	Total Quantity Produced	QNTY	100	1	200
Propulsion, production	% of New Structure	NEWST	1.0	0.5	1.0

Figure 2C: Development and Production Cost Modeling Data Ranges – Propulsion

The table for Sub-systems cost parameters, modeled baseline, and low to high parameter range follows:

System Name	Parameter	PRICE-H Term	Modeled Baseline	Low Range	High Range
Subsystems, R&D, Prod.	Structural Weight	WS	20,000	10,000	50,000
Subsystems, R&D, Prod.	Design Complexity	ECMPLX	1.4	0.7	2.0
Subsystems, R&D, Prod.	Manuf. Complexity, Structure	MCPLXS	7.3	5.83	8.54
Subsystems, R&D, Prod.	Number of Prototypes	PROTOS	3	1	10
Subsystems, R&D, Prod.	Total Quantity Produced	QNTY	100	1	500
Subsystems, R&D, Prod.	Operating Platform	PLTFM	1.8	1.7	2.0
Subsystems, R&D, Prod.	% of New Structure	NEWST	1.0	0.1	1.0

Figure 2D: Development and Production Cost Modeling Data Ranges – Subsystems

## APPENDIX III: Weight-Performance Equation

The weight-performance equation is based upon industry data of completed aircraft. The performance data is from Data Search Associates for 63 aircraft.

## LN Weight Equation

<i>Regression Statistics</i>	
Multiple R	0.90382
R Square	0.81689
Adjusted R Square	0.80083
Standard Error	0.39849
Observations	63

## ANOVA

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	5	40.37952947	8.07590589	50.857128	8.854E-20
Residual	57	9.051369158	0.15879595		
Total	62	49.43089862			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
Intercept	3.0419	1.0169	2.9912	0.0041	1.0055	5.0783
LN Payload_Wt	0.5185	0.0745	6.9599	0.0000	0.3693	0.6677
LN Speed	0.4413	0.2346	1.8813	0.0650	-0.0284	0.9110
LN Combat_Rng	0.1895	0.0789	2.4014	0.0196	0.0315	0.3475
LN Crew	0.3293	0.0940	3.5040	0.0009	0.1411	0.5175
LN Ceiling	-0.5659	0.5779	-0.9794	0.3315	-1.7231	0.5912

Figure 4A: Weight-Performance Equation

We note that the F-statistic value of 50.8 is significantly greater than 5. Thus, the overall model is statistically significant. The adjusted R-squared equals 0.80. The Standard Error is less than the other weight equation (model) iterations. The most significant terms are Payload-weight, number of Crew, and Combat Range.

9

<sup>9</sup> For all the parameters (except for the Ceiling term) the "t" statistic is greater than 1.88. For n greater than 60, we conclude that all the cost drivers are statistically significant - except for the Ceiling coefficient. It is observed that aircraft weight decreases with higher ceilings, so this term is accepted in the context of the overall weight regression model.

The weight equation is shown in its more usual form below:

$$AV\_Empty-weight = 20.495 * (PL\_Wt) ^{0.5185} * (Speed) ^{0.4413} * (CRange) ^{0.1895} * (Crew) ^{0.3293} * (Ceiling) ^{-0.5659},$$

Where,

Intercept = (Exp(1)) ^ 3.0419,

PL\_Wt = Payload weight (capability),

Speed = aircraft speed (miles per hour),

CRange = Combat Range (miles),

Crew = number of Crew members,

Ceiling = altitude (thousands of feet).

Sample calculation and initial test of the weight equation

Weight Equation - sample calculation								
Cost Driver	Intercept	LN Payload_Wt	LN Speed	LN Combat_Rng	LN Crew	LN Ceiling	TOTAL	TOTAL
Beta's	3.0419	0.5185	0.4413	0.1895	0.3293	-0.5659	ESTIMATED	ACTUAL
Estimated AV (Baseline)	2.71828	125000	912	7500	4	50		
B-1B	20.9	439.30	20.24	5.42	1.58	0.109	174,260	181,400

Figure 4B: Weight-Performance Equation with Sample Calculation

Is can be seen, using visual inspection, that the total estimated weight is within approximately 5 percent of the total weight for the B-1B (=174,260/181,400).