A Closed-Form Solution
for the Production-Break Retrograde
Method

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Anderlohr's Retrograde Method

What Production Breaks Cost

G. Anderlohr, J. of Ind. Eng. 34, 103 (1969)

GAO "Best Practice"

Two parts

- Calculation of the loss of learning
  - Results in a "percentage of lost learning"
- Impact to the learning curve
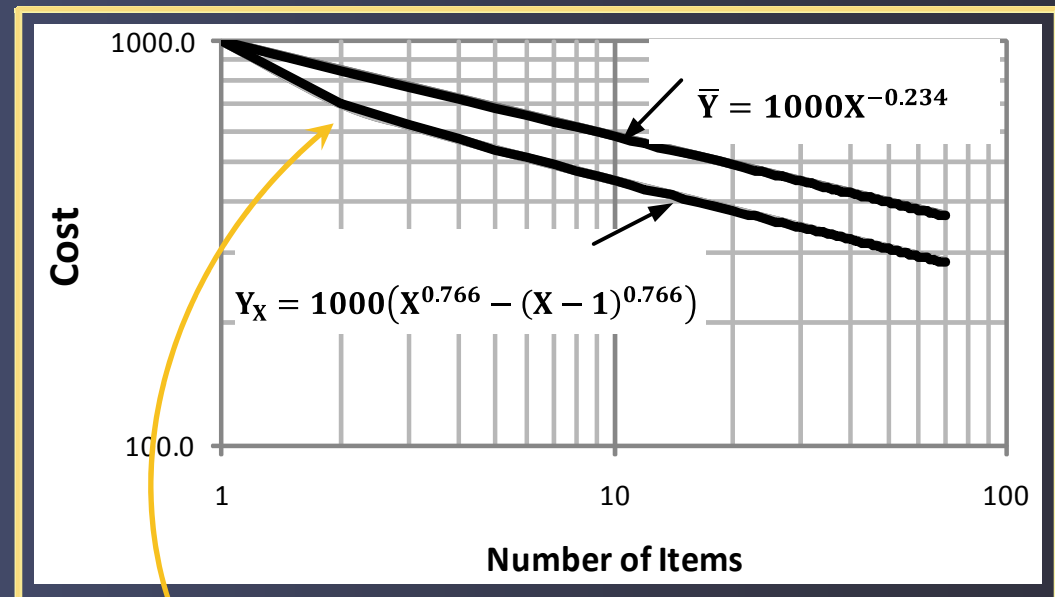 - Back up, or retrograde, the curve

Used Cum Average Theory for his calculation

Complexities of Cum Average Theory

- Item-value curve is non-linear even in Log space

- Finding the value of X when Y is known is complex

  - Can be handled by Excel's Solver™ now

85% Cum Average Curve

$$\overline{Y} = 1000X^{-0.234}$$

$$Y_X = 1000\left(X^{0.766} - (X-1)^{0.766}\right)$$

Cost

Number of Items

Slight Bend in the curve

Differences in Anderlohr & GAO

- GAO uses a different reference point than Anderlohr

- Potential for up to a 1 unit difference in start point

- We consider a moot issue since the Lost Learning factor has less mathematical rigor than that
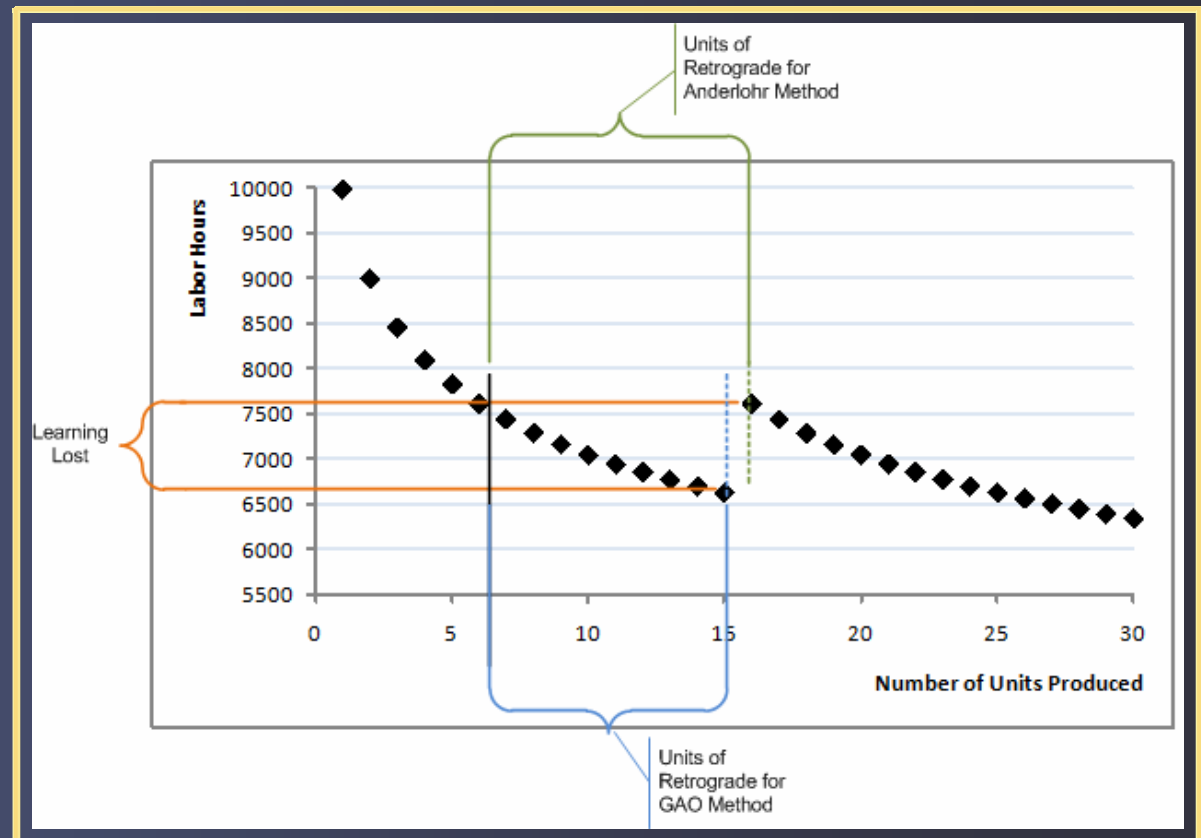
Example: Fictitious component, the X5

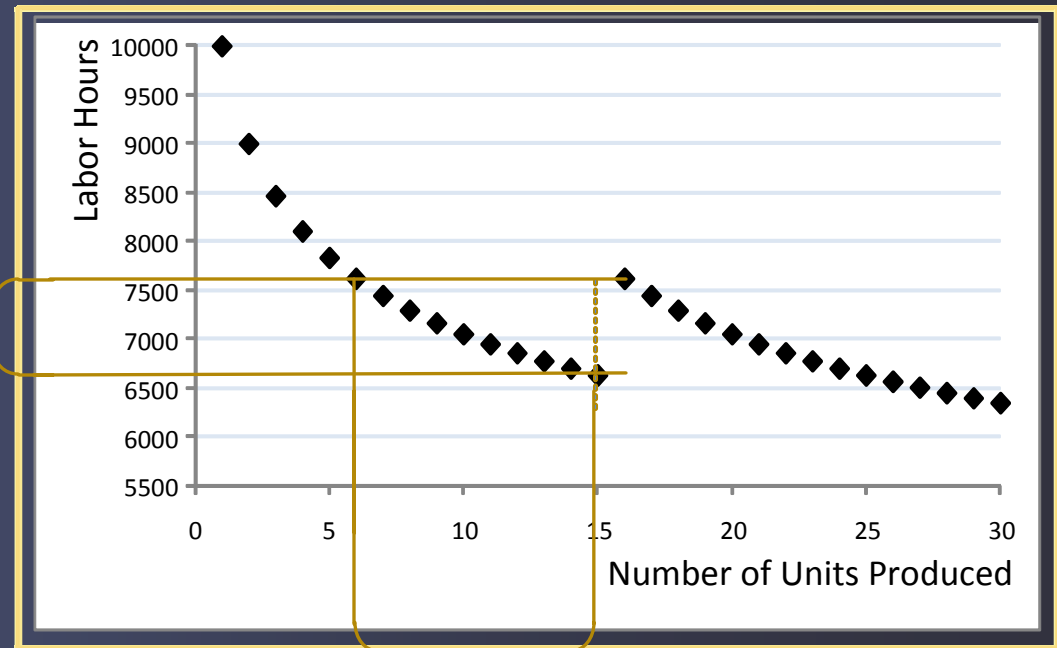First component costs 10,000 labor hours

15 units have so far been produced

Production followed a 90% learning curve trend

Labor dispute resulted in a six-month production break.

Break resulted in an estimated 29% lost learning factor due to management changes and assembly crew losses.

Traditional GAO method applied to Unit Theory

Step 1 - *Find the amount of learning achieved to date*

$$LA = Y_1 - Y_{F-1}$$

$$b = \frac{\ln(slope)}{\ln(2)}$$

$$Y_{F-1} = Y_1 \times (F-1)^b$$

$$LA = Y_1 - Y_{15},$$

$$b = \frac{\ln(0.90)}{\ln(2)} = -0.1520$$

$$Y_{15} = 10,000 \times 15^{-0.1520} = 6,625.7$$

Step 2 - *Estimate the amount of learning lost*

$$LL = LA \times LLF,$$

$$LL = 3,374.3 \times 0.29 = 978.6$$

LA = Learning Achieved

b = slope coefficient

F = First unit after the break

LL = Calculated Learning Lost

Traditional GAO method applied to Unit Theory

Step 3 - *Estimate the cost of the first unit after the break*

$$Y_F = Y_1 \times F^b$$

$$Y_{16} = 10,000 \times 16^{-0.1520} = 6,561.0$$

$$Y'_F = Y_F + LL$$

$$Y'_{16} = Y_{16} + LL,$$

$$Y'_{16} = 6,561.0 + 978.6 = 7,539.6$$

Step 4 - *Find the unit whose cost is equal to Y'_F*

$$X = \left(\frac{Y_X}{Y_1}\right)^{1/b}$$

$$X = \left(\frac{7539.6}{10,000}\right)^{1/-0.1520} = 6.4 \sim 6$$

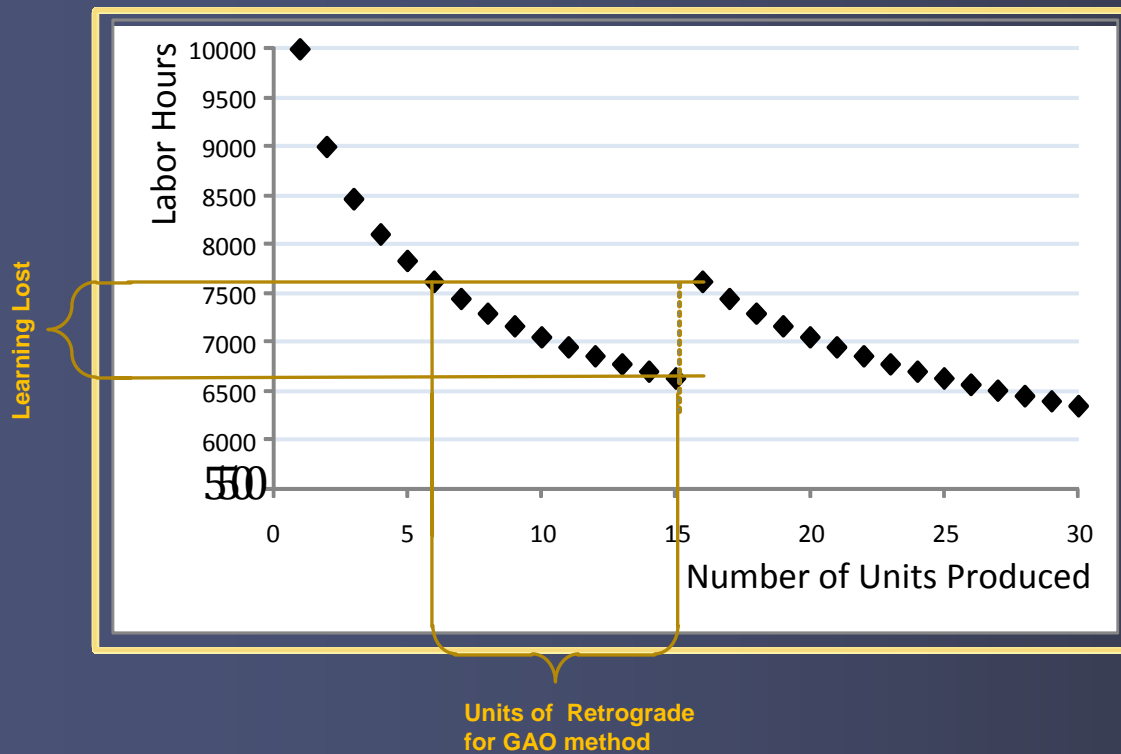F = First unit after the break

X = Calculated new start point

LL = Calculated Learning Lost

Traditional GAO method applied to Unit Theory

Step 5 - *Find the number of units of retrograde*

$$m = F - X \qquad\qquad m = 16 - 6 = 10$$

Learning Lost

Units of Retrograde
for GAO method

F = First unit after the break

X = Calculated new start point

m = Units of retrograde

Derivation of a closed form solution
for Unit Theory

It can be observed that with the application of a bit of algebra, many of the values in the 5-step process cancel out, leaving a single 1-step formula for the same process. The resulting equation is dependent only on the unit number of the first unit after the production break (F), the lost learning factor (LLF), and the learning curve slope (b).

The derivation starts with the observation that: $Y_F' = Y_1 \cdot (F - m)^b$

Plugging in the equation from step 3: $Y_1 \cdot (F - m)^b = Y_1 \cdot F^b + LL$

Now re-express LL in terms of the LLF, F and b: $LL = LA \cdot LLF$

$$LA = Y_1 - Y_1 \cdot (F - 1)^b$$

$$LL = LLF \cdot Y_1 \cdot \left(1 - (F - 1)^b\right)$$

Substituting in gives: $Y_1 \cdot (F - m)^b = Y_1 \cdot F^b + LLF \cdot Y_1 \cdot \left(1 - (F - 1)^b\right)$

Apply to previous problem

$$Y_1 \cdot (F - m)^b = Y_1 \cdot F^b + LLF \cdot Y_1 \cdot \left(1 - (F - 1)^b\right)$$

Divide by *Y* and solve for *m* to obtain the closed-form solution:

$$m = F - \left(F^b + LLF \cdot \left(1 - (F - 1)^b\right)\right)^{\frac{1}{b}}$$

For the previous example of Component X5 this is:

$$m = 16 - \left(16^{-0.1520} + 0.29 \cdot \left(1 - (16 - 1)^{-0.1520}\right)\right)^{\frac{1}{-0.1520}}$$

$$m = 16 - \left(0.6561 + 0.29 \cdot (1 - 0.6626)\right)^{-6.5789}$$

$$m = 16 - \left(0.6561 + 0.0978\right)^{-6.5789}$$

$$m = 16 - 6.4 \approx 10$$

It is interesting to note that finding the cost of any unit (including the first) is not necessary in order to find the number of units of retrograde.

This method provides the exact same answer as the 5-step retrograde method in a single step. The usefulness of this method as opposed to the traditional method will most likely be based on the application and tools being used