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Risk-Based Return On Sales (ROS) As a Tool For Complex Contract Negotiations

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Outline

- Contract Negotiations – Government and Contractor Views
- Risk-Based Return On Sales (ROS)[1] Recap
  - Four main Contract Types as functions that map Cost to Profit, Price, and ROS
  - Analytical and empirical methods for determining distributions
  - Incorporation of terms and conditions (Ts & Cs)
- Contract Negotiations Scenario
- Risk-Based ROS Negotiations Tool
- Exploring the Scenario
- Bottom Line

1. *Risk-Based Return On Sales (ROS) for Proposals with Mitigating Terms and Conditions*, P.J. Braxton, R.L. Coleman, E.R. Druker, B.L. Cullis, C.M. Kanick, A.V. Bapat, J.M. Callahan, B.P. Caccavale, SCEA/ISPA 2009.

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Contract Negotiations – Government View

- Contract Price is of paramount importance
  - Translates directly to cost to the government
    - Measured against budgets
  - Combines with other costs to make up total phase costs
    - Program Management Office (PMO)
    - Government-Furnished Equipment (GFE)
    - Other Government Costs (OGCs)
- <u>Final Contract Price</u> ultimately matters, but budget constraints may drive <u>Target Price</u>
  - This is the price the government fools themselves into believing they might actually pay
  - Critical issue for commodities requiring “full funding”
- Shareline and Ceiling Price are viewed as devices to magically control cost
- Fee is viewed as a necessary evil of capitalism
  - Various degrees of appreciation for the health of the industrial base – generally reactive more than proactive

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Contract Negotiations – Contractor View

- (1) Target Cost
  - If you don't get Target Cost right, you're "mis-calibrated"
  - "You can't manage your way out of a bad deal"
- (2) Target Fee
  - This is what makes the company profitable and makes the owners / shareholders happy
  - Needs to be enough to be sufficient *after* erosion
    - ROS (expected not bid!) measured against corporate hurdle rate
- (3) Shareline
  - Determines how quickly things get worse from a profit perspective
- (4) Ts & Cs
  - Provide protection against factors "out of our control"
  - Not going to make you well if you got #1-3 wrong!

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Contract Negotiations – Cost / Risk View

- (1) Cost Estimate
  - Mean cost = first-order moment
    - Beware: Proposal may very well be below the mean!
  - Point of departure for cost
  - Estimate vs. Target Cost identifies gap
- (2) Cost Estimating Variability
  - Standard deviation = second-order moment
    - Often expressed as Coefficient of variation (CV) = std dev / mean
  - Indicates how quickly you'll run up the shareline
  - Sanity-check against PTA/Ceiling Price or RIE
- (3) Continuous Risks
  - Inflation, learning curve, weight growth, SLOC growth, warranty
  - Often implicit in #2 unless broken out for Ts & Cs coverage
- (4) Discrete Risks
  - May or may not be addressed by Ts & Cs

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Risk-Based Return On Sales (ROS) Recap

- Four main Contract Types
  - Firm-Fixed-Price (FFP) [FAR 16.202]
  - Fixed-Price Incentive (FPI) [FAR 16.204]
  - Cost-Plus-Incentive-Fee (CPIF) [FAR 16.304]
  - Cost-Plus-Fixed-Fee (CPFF) [FAR 16.306]
- Each Contract Type determines functions that map Cost (X) to:
  - Profit (Y)=f(X), Price = X+Y, and ROS = Y/(X+Y)
- Given a distribution of Cost, can determine distribution of Profit, Price, and ROS
  - Analytical method, i.e., calculus
  - Empirical method, i.e., Monte Carlo simulation
- Incorporation of terms and conditions (Ts & Cs)
  - Take some cost risk "off the shareline"

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Overarching Contract Elements

Typical Set of Inputs

Target Cost (TC) = $10.0M

Target Profit (Fee) (TF) = $1.0M

Target Price (TP) = $11.0M [all]

10% Profit (ROC)

9.1% Margin (ROS) [all]

70/30 Over-Target Shareline

40/60 Under-Target Shareline [CPIF/FPI]

Min Fee (mF) = 3%, Max Fee (MF) = 20% [CPIF]

Ceiling Price (CP) = 130% [FPI]

$$RIE_{low} = TC - \frac{(MF - TF)}{CS_{under}}$$

$$RIE_{high} = TC + \frac{(TF - mF)}{CS_{over}}$$

$$PTA = TC + \frac{(CP - TP)}{GS_{over}}$$

Target Cost	$	10.0		
Target Profit	$	1.0	10.0%	Profit Percent
Target Price	$	11.0	9.1%	Margin Percent
Min Fee	$	0.3	3.0%	Min Fee Percent
Max Fee	$	2.0	20.0%	Max Fee Percent
Under Gov Share		40%		
Under Cont Share		60%		
Over Gov Share		70%		
Over Cont Share		30%		
PTA	$	12.9		
Ceiling Price	$	13.0	130.0%	Ceiling Price Percent
RIE Low	$	8.3		
RIE High	$	12.3		

yellow fill = input
blue fill = calculated

Cost Estimating Body of Knowledge (CEBoK), Module 14 Contract Pricing, SCEA, 2009.

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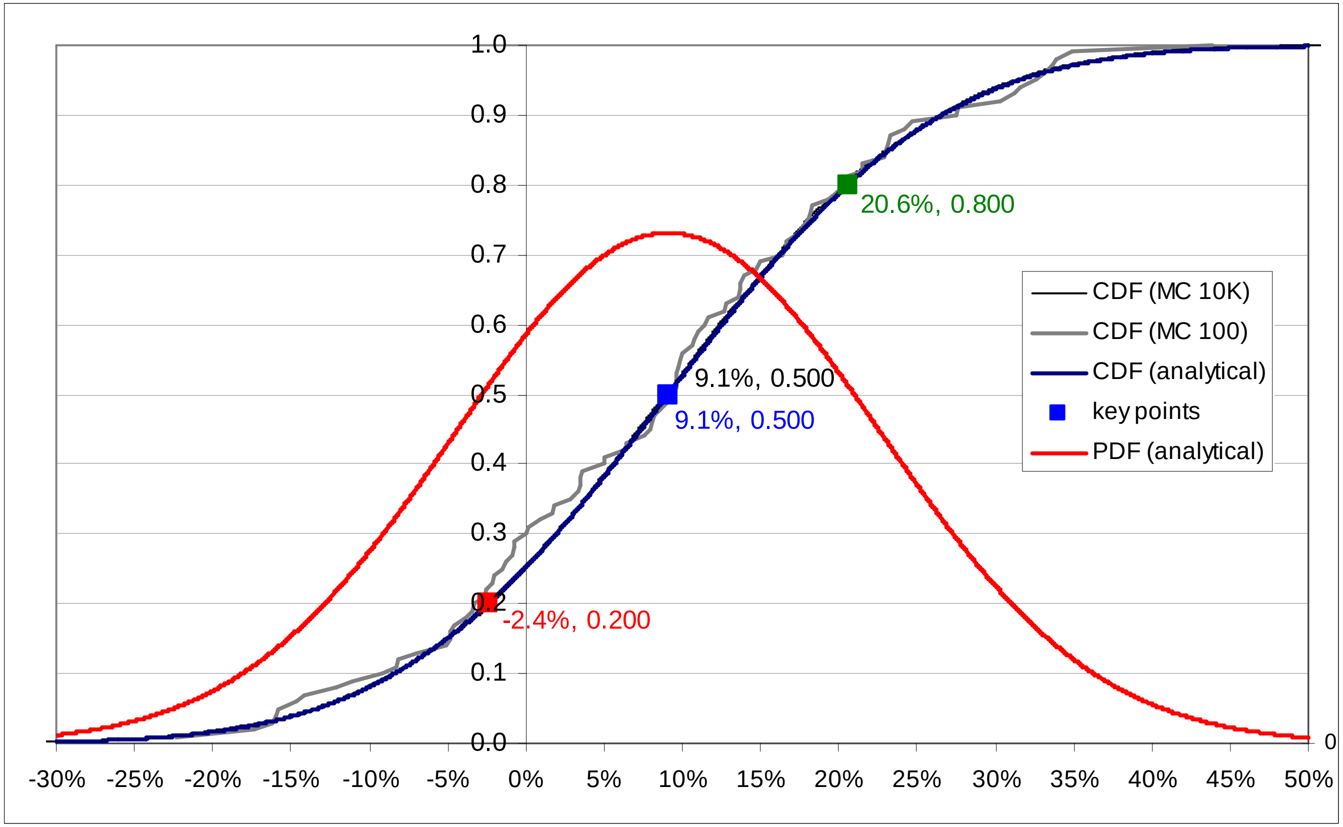
Distribution of ROS Illustrated – FFP

- Percentiles (20/50/80) and mean are shown on graph
  - Symmetric: Mode = Median = Mean



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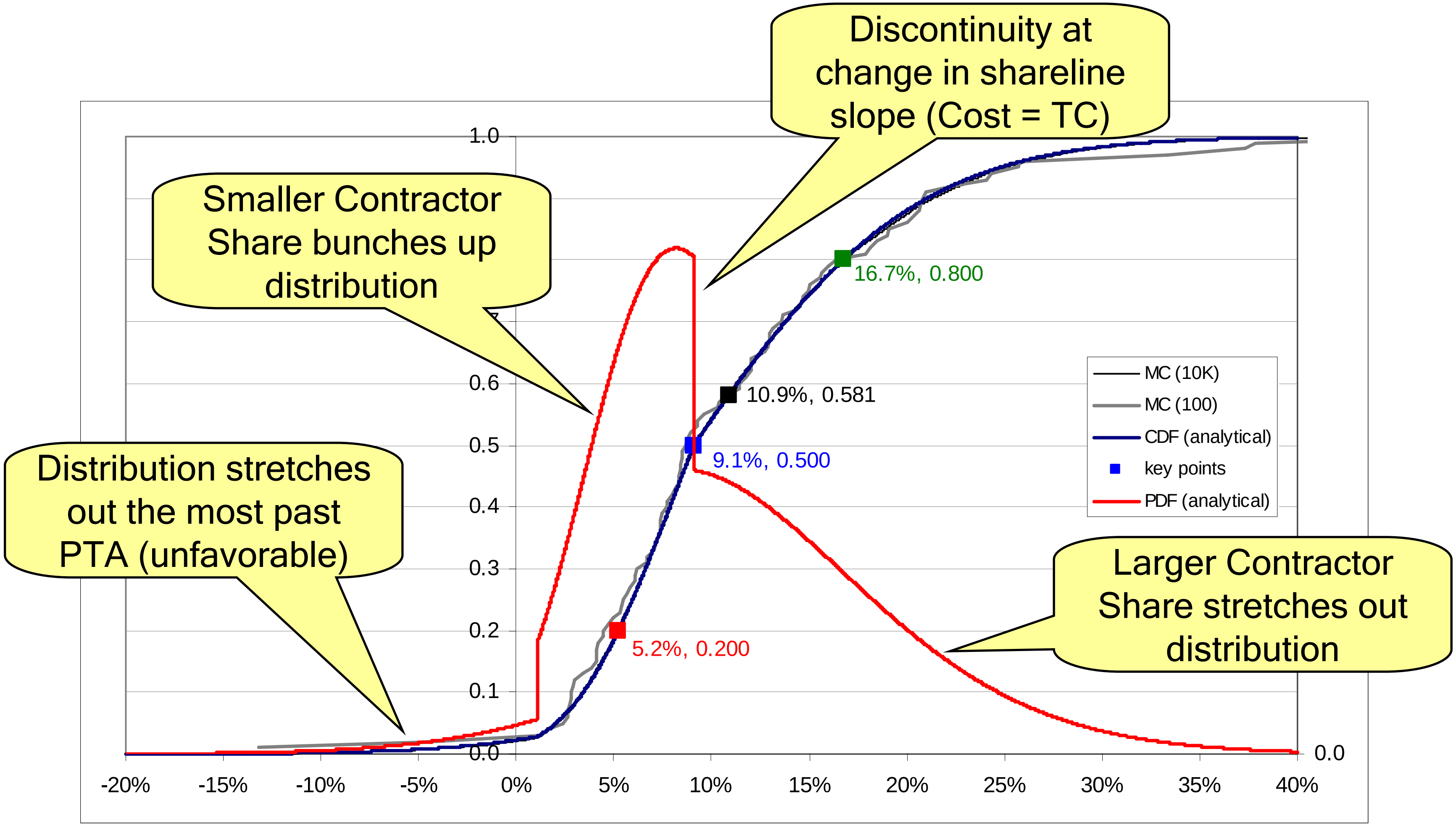
Distribution of ROS Illustrated – FPI

- Percentiles (20/50/80) and mean are shown on graph
  - Skew right: Mode < Median < Mean



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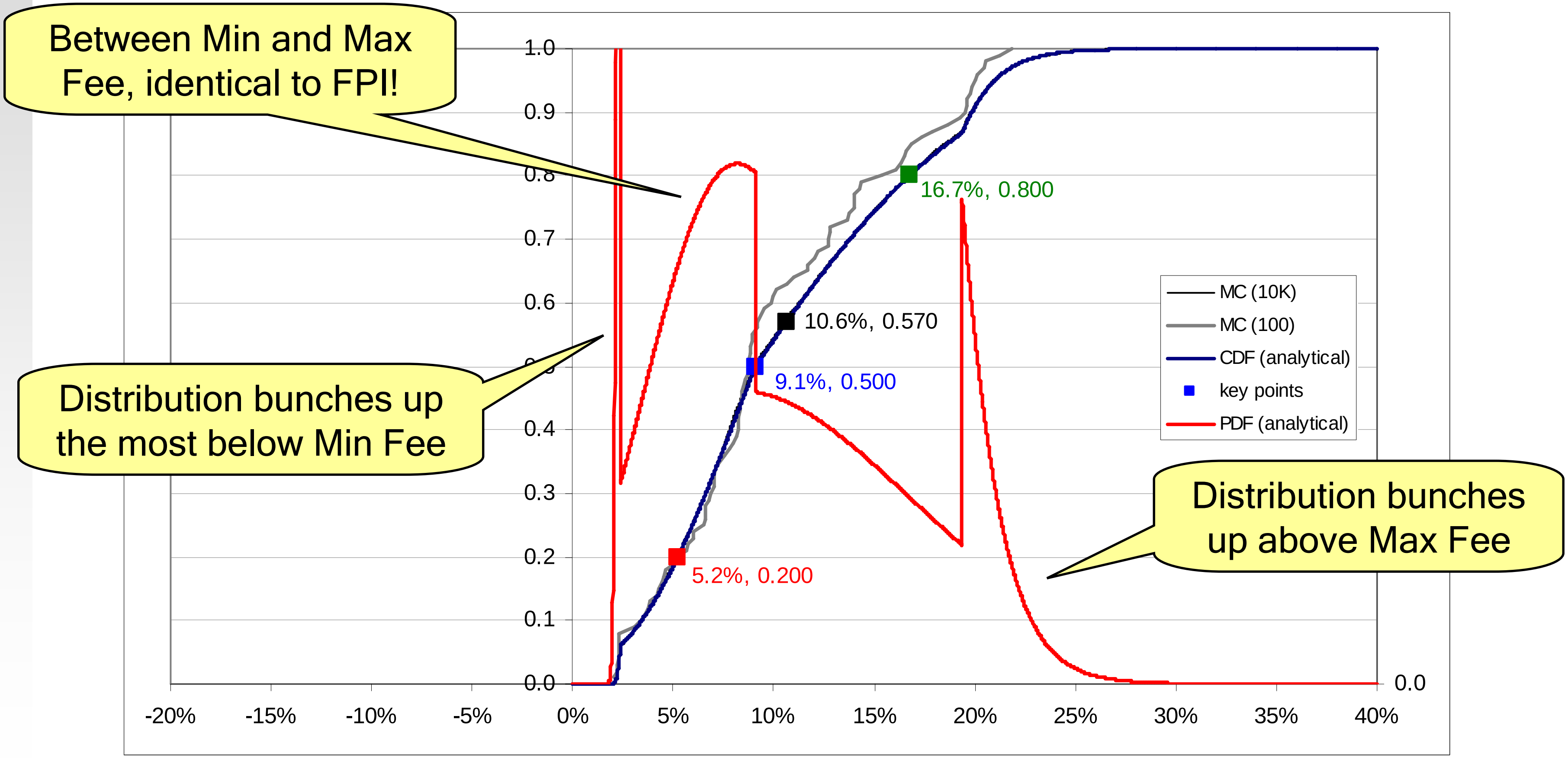
Distribution of ROS Illustrated – CPIF

- Percentiles (20/50/80) and mean are shown on graph
  - Skew right: Mode < Median < Mean



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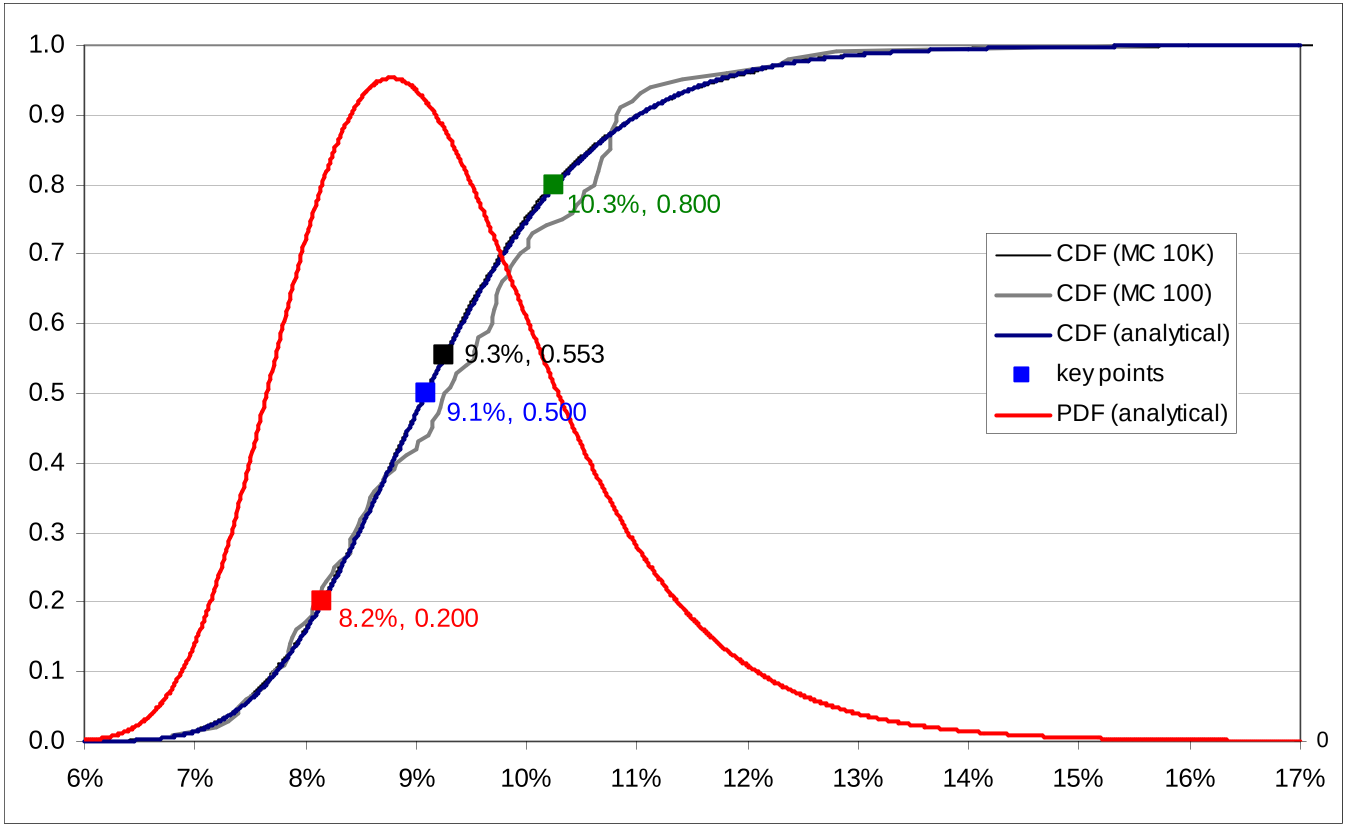
Distribution of ROS Illustrated – CPFF

- Percentiles (20/50/80) and mean are shown on graph
  - Skew right: Mode < Median < Mean



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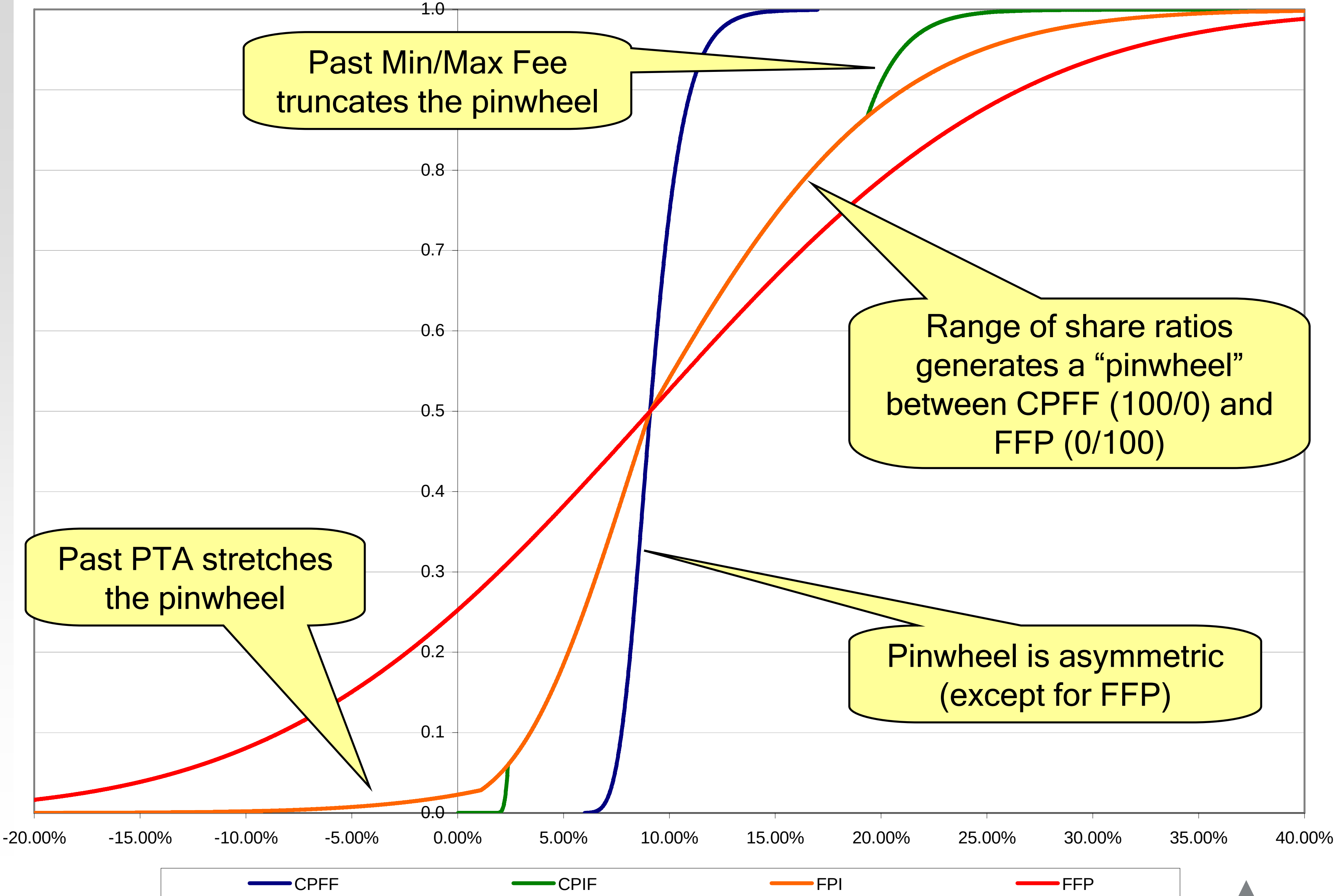
Distribution of ROS – Contract Types Comparison



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Contract Negotiations Scenario

- Sole-source negotiation
- FPI contract type
  - CPIF would behave similarly within the RIE
- Government and contractor agree to disagree on distribution of cost
  - Mean and standard deviation

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Contract Negotiations Scenario

- Fixed Price Incentive Firm (FPIF)
- Target Cost (TC) = $10.0M
  Target Profit (Fee) (TF) = $1.0M
  Target Price (TP) = $11.0M
- 10% Profit (ROC)
  9.1% Margin (ROS) [all]
- 70/30 Over-Target Shareline
  40/60 Under-Target Shareline
- Ceiling Price (CP) = 130% [FPI]

$$PTA = TC + \frac{(CP - TP)}{GS_{over}}$$

Target Cost	$ 10.0		
Target Profit	$ 1.0	10.0%	Profit Percent
Target Price	$ 11.0	9.1%	Margin Percent
Under Gov Share	40%		
Under Cont Share	60%		
Over Gov Share	70%		
Over Cont Share	30%		
PTA	$ 12.9		
Ceiling Price	$ 13.0	130.0%	Ceiling Price Percent

yellow fill = input
blue fill = calculated

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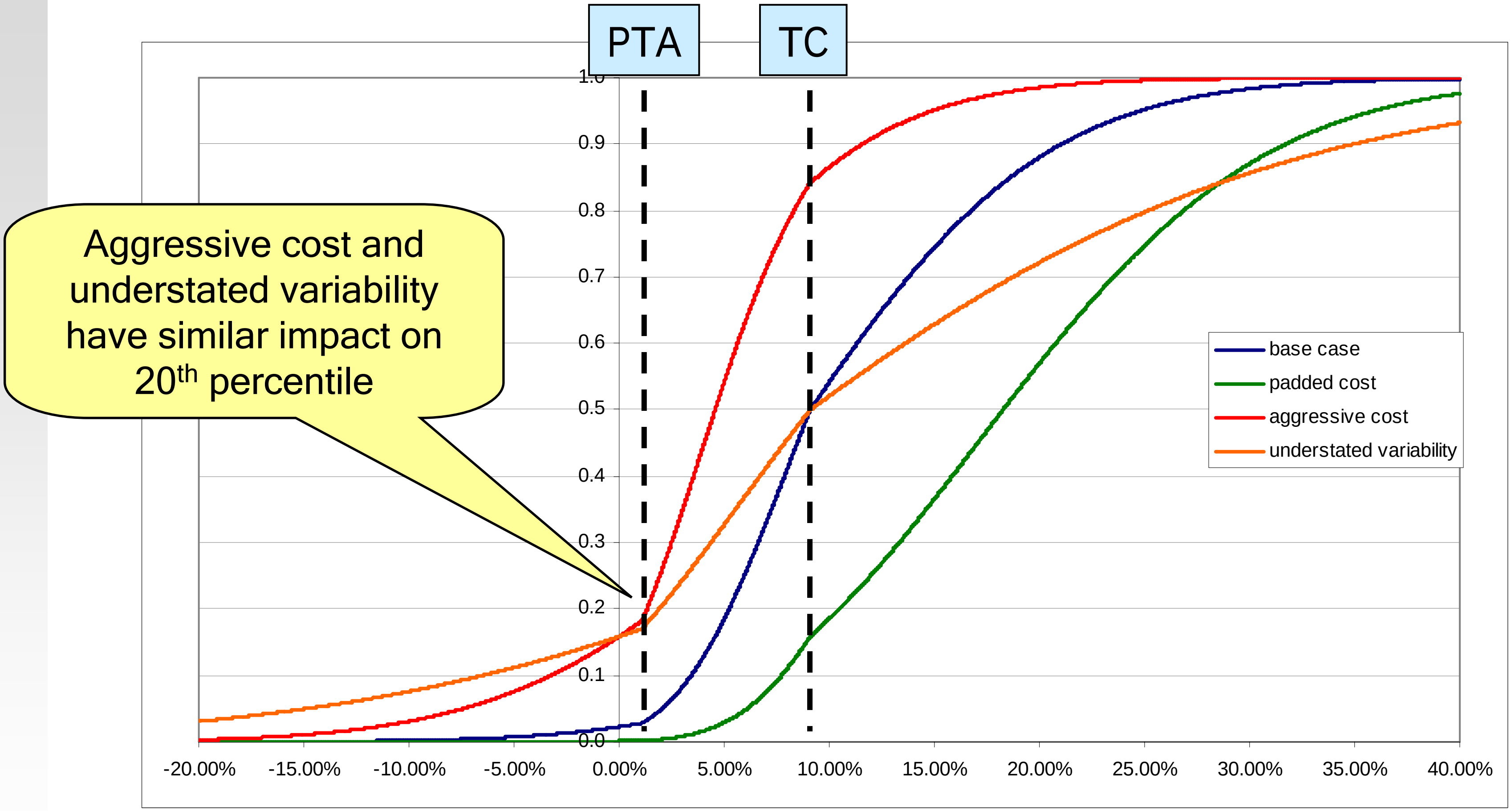
FPI – Pathological Cases



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Risk-Based ROS Negotiations Tool

- Quad chart dashboard
  - Upper left: Contract Geometry
    - Key points highlighted (Target Cost, PTA)
    - The function which enables mapping of Cost
  - Lower left: Distribution of Cost
    - CDF and PDF views
    - Output of cost estimating process (proposal/ICE and POE/ICE)
  - Upper right: Distribution of Price
    - What the government cares about – compare with Budget
  - Lower right: Distribution of ROS
    - What the contractor cares about – compare with hurdle rate
- Enables common view
  - Graphical depiction produces more clear and intuitive results
- Let’s go to the Excel!

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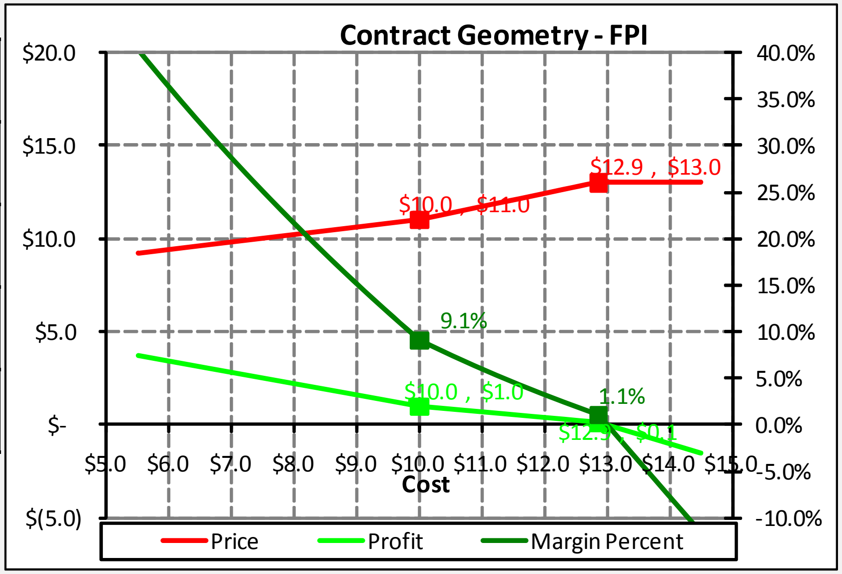
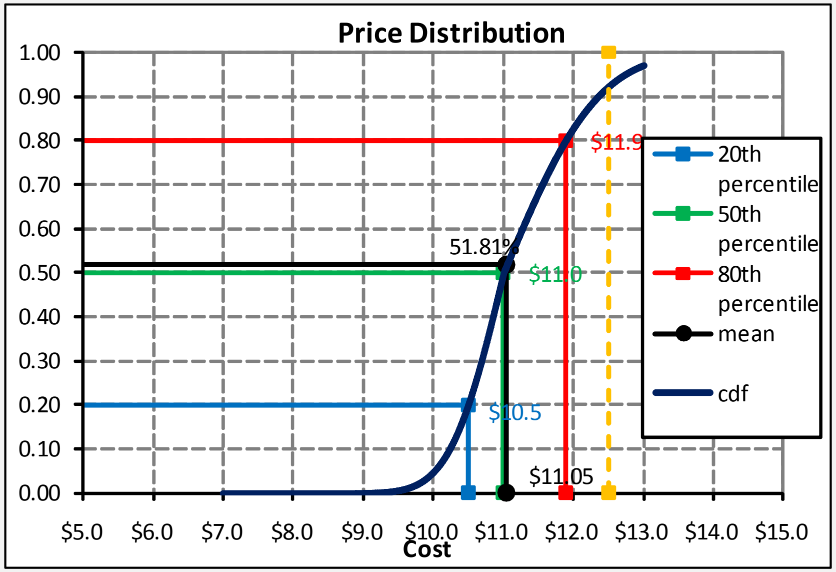
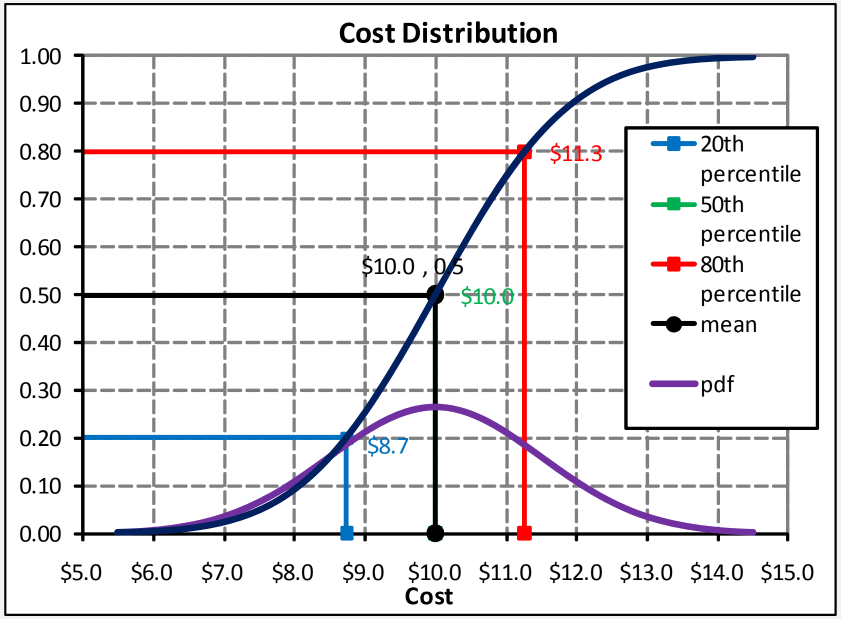
Risk-Based ROS Negotiations Tool

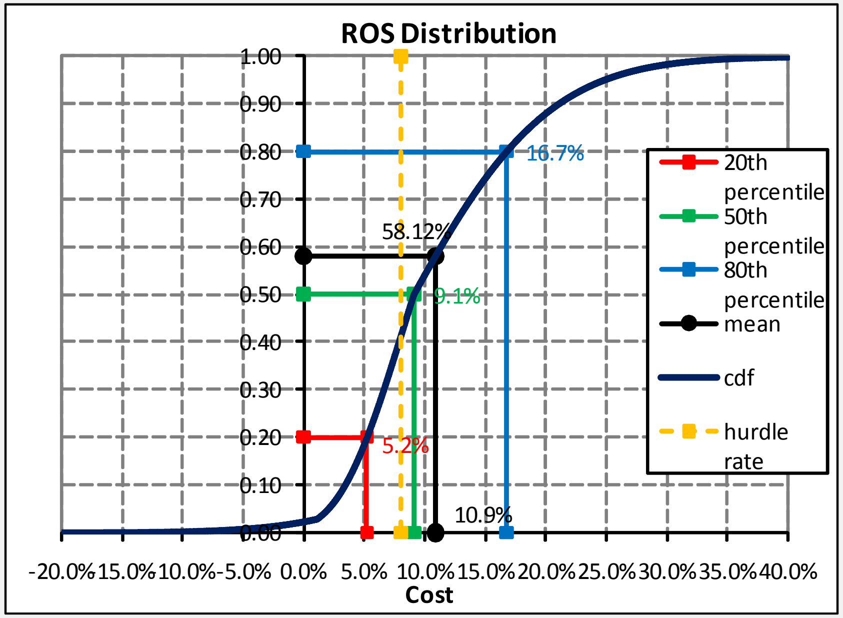
TC	$ 10.0
TF	10.0%
GS-u	40.0%
GS-o	70.0%
CP	130.0%





mean	$ 10.0
CV	15.0%





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Exploring the Scenario

- Vary parameters one at a time
  - Essentially sensitivity analysis
- Two major inputs:
  - Contract geometry
    - This is the subject of the negotiations
  - Probabilistic cost estimate
    - This is the subject of the reconciliation
- Ts & Cs treated offline in Monte Carlo simulation
  - After inputs have been refined using the tool

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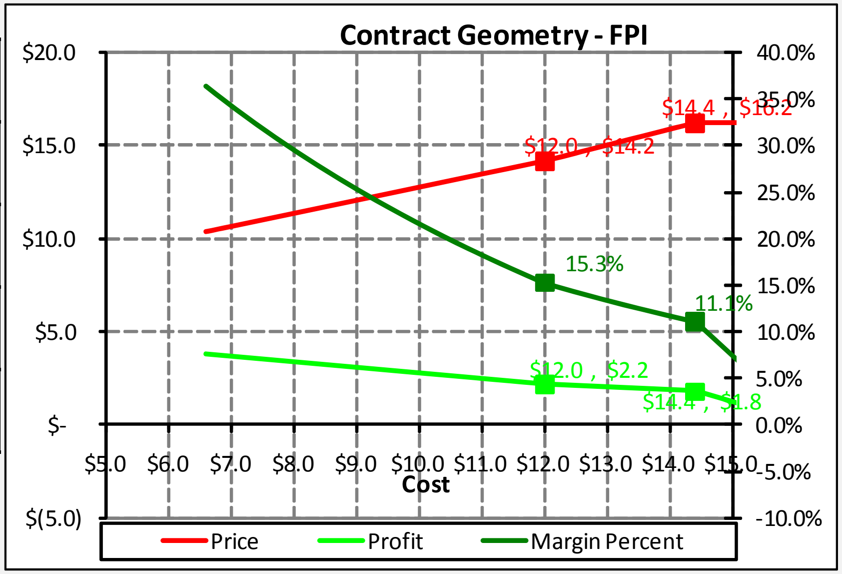
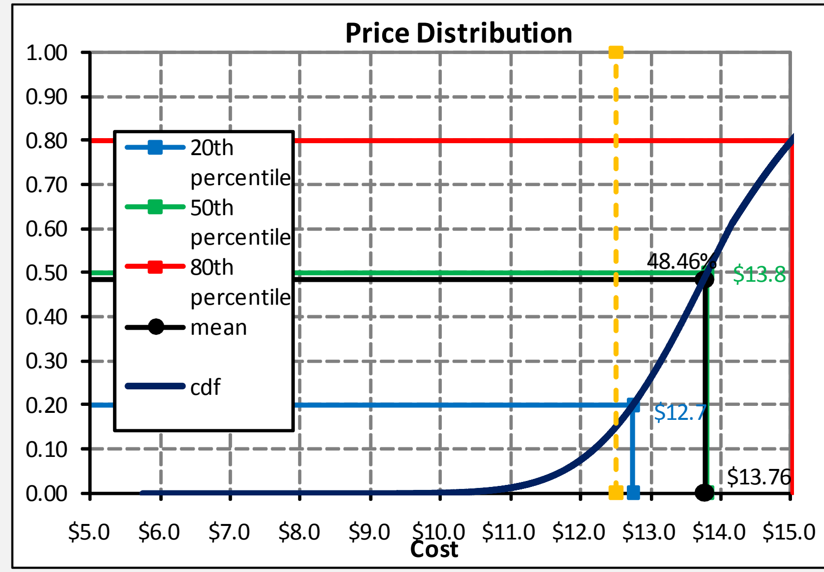
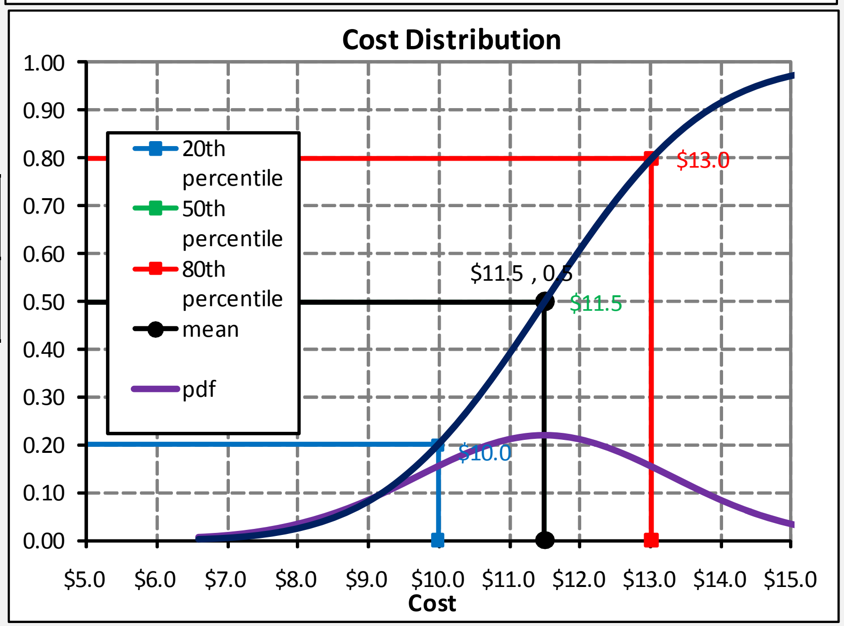
Contractor Initial Position

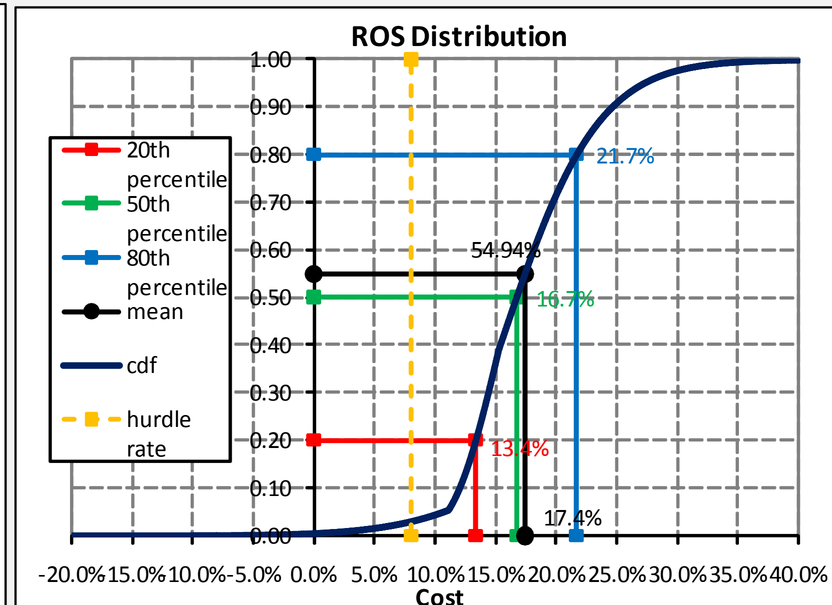
TC
$ 12.0
TF
18.0%
GS-u
70.0%
GS-o
85.0%
CP
135.0%





mean
$ 11.5
CV
15.0%





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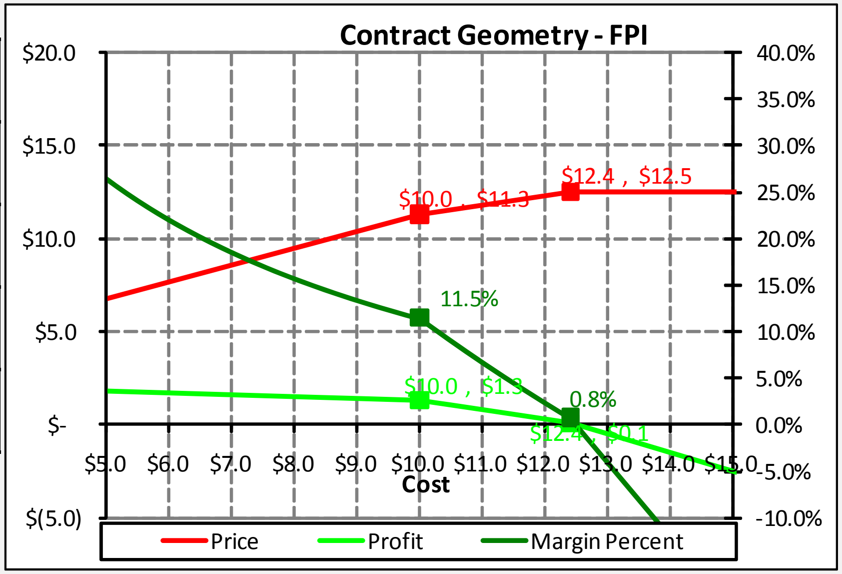
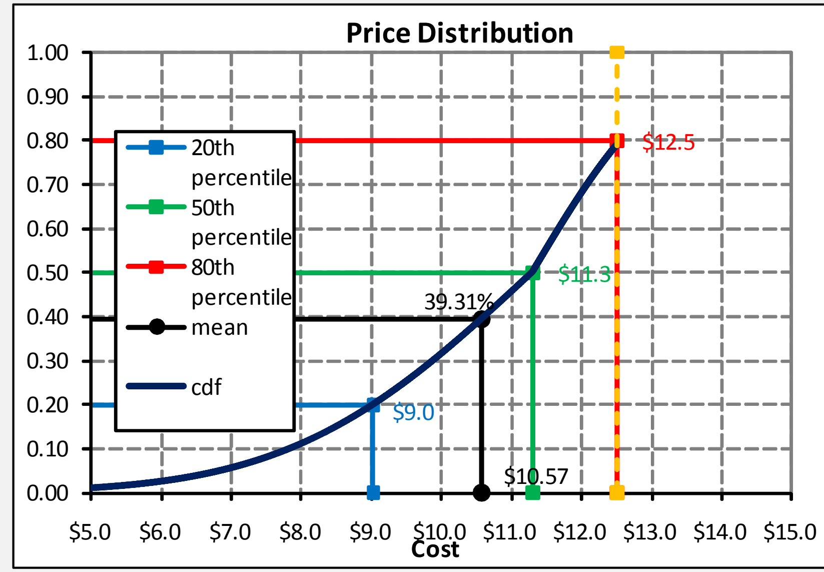
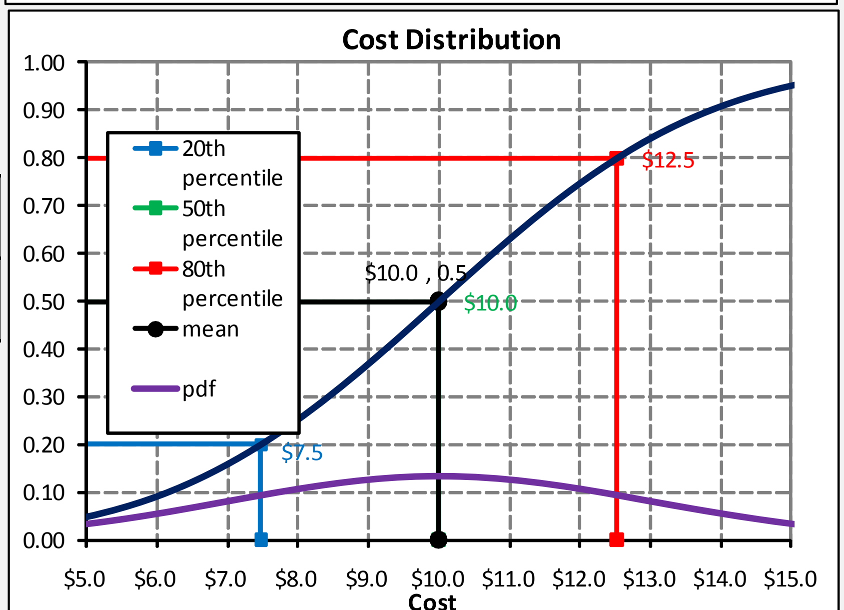
Government Initial Position

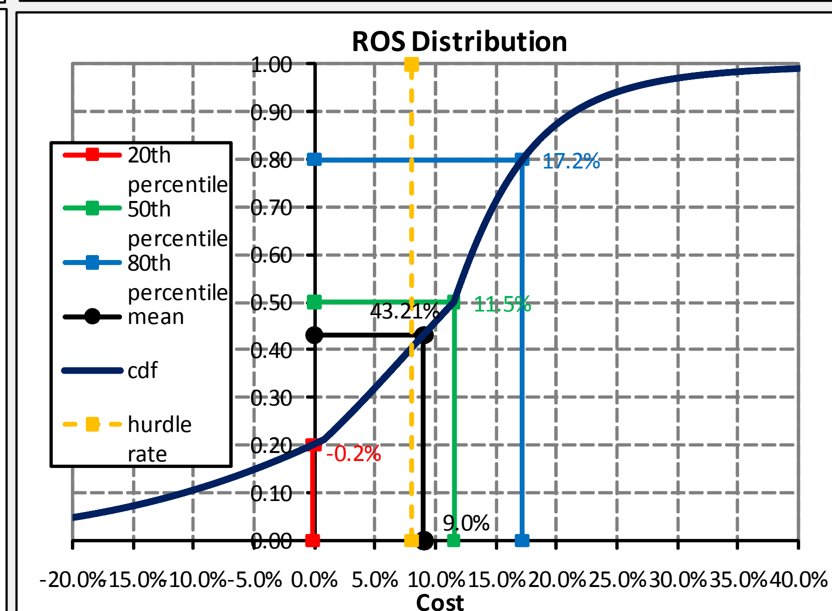
TC	$ 10.0
TF	13.0%
GS-u	90.0%
GS-o	50.0%
CP	125.0%





mean	$ 10.0
CV	30.0%





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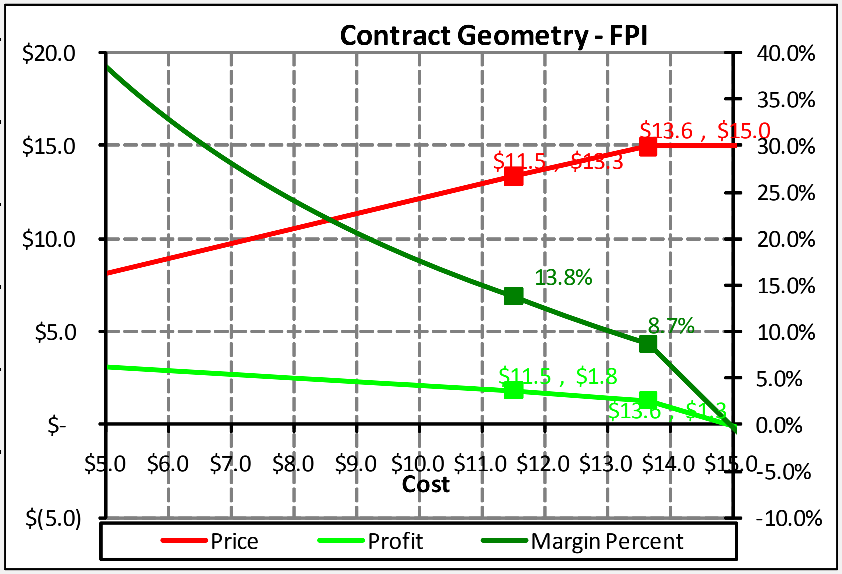
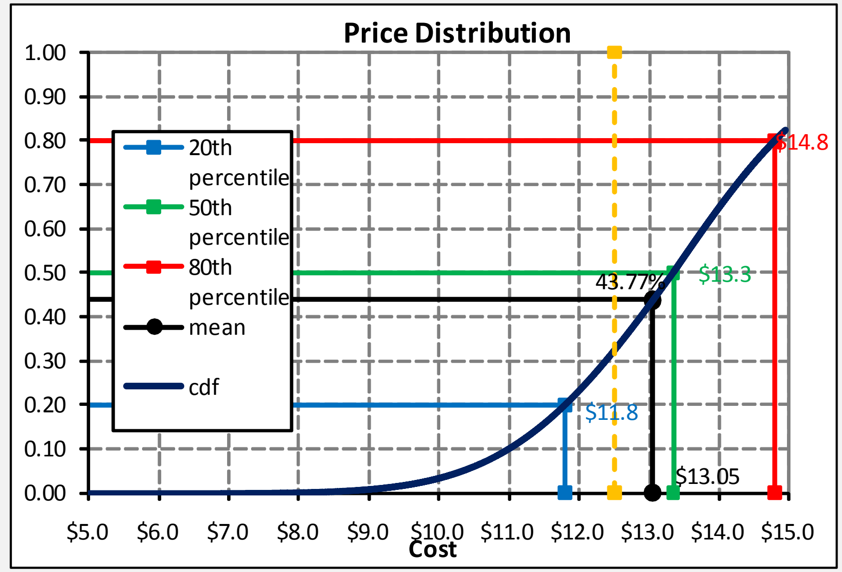
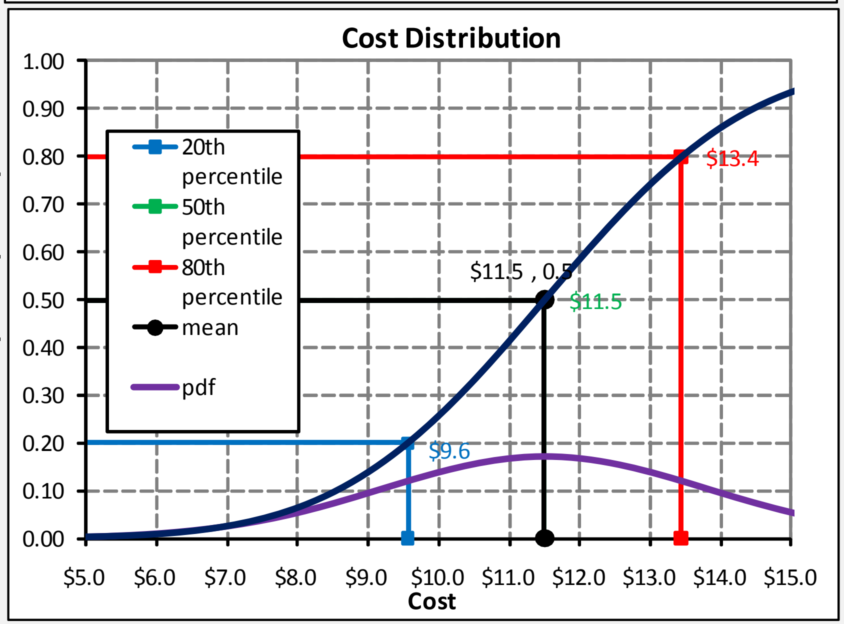
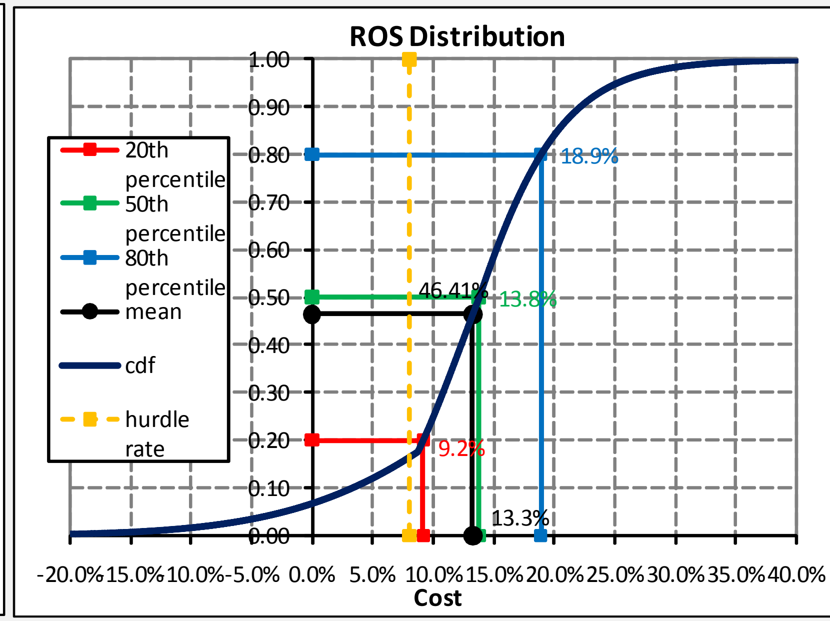
Contractor Counteroffer

TC
$ 11.5
TF
16.0%
GS-u
80.0%
GS-o
75.0%
CP
130.0%









mean
$ 11.5
CV
20.0%

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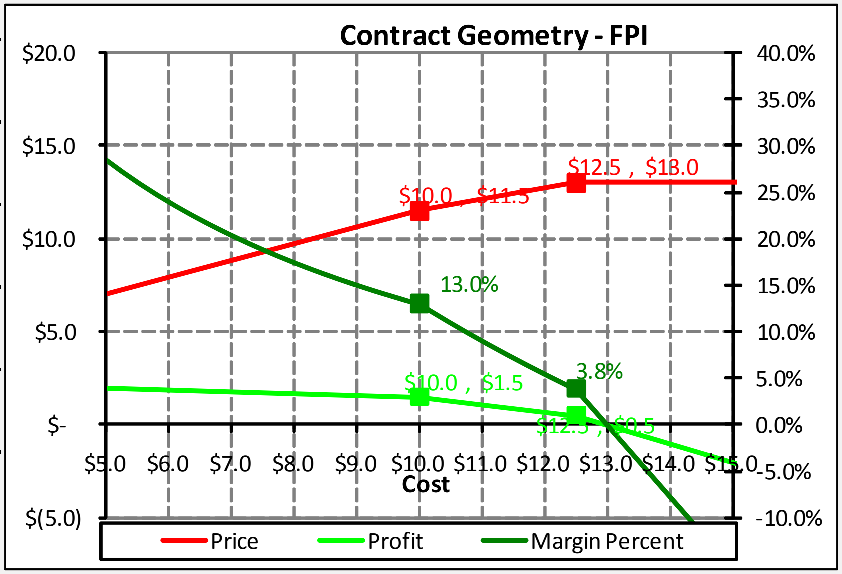
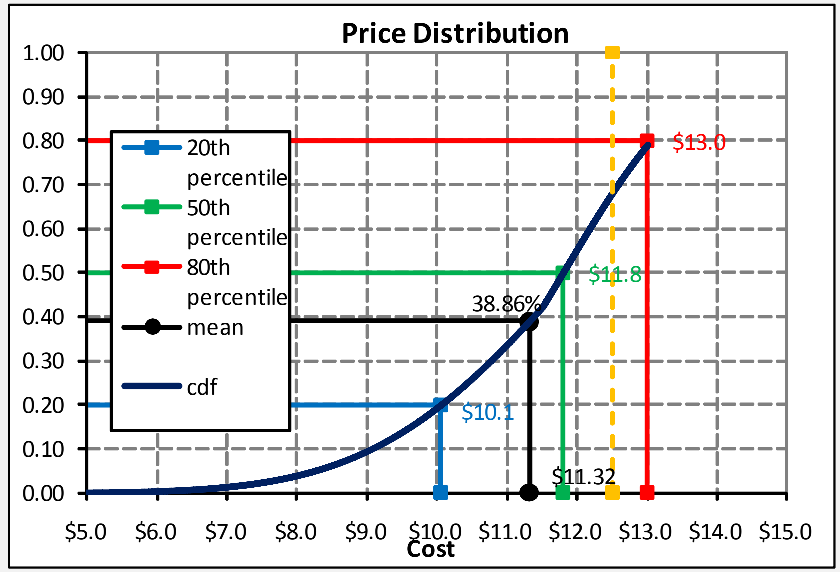
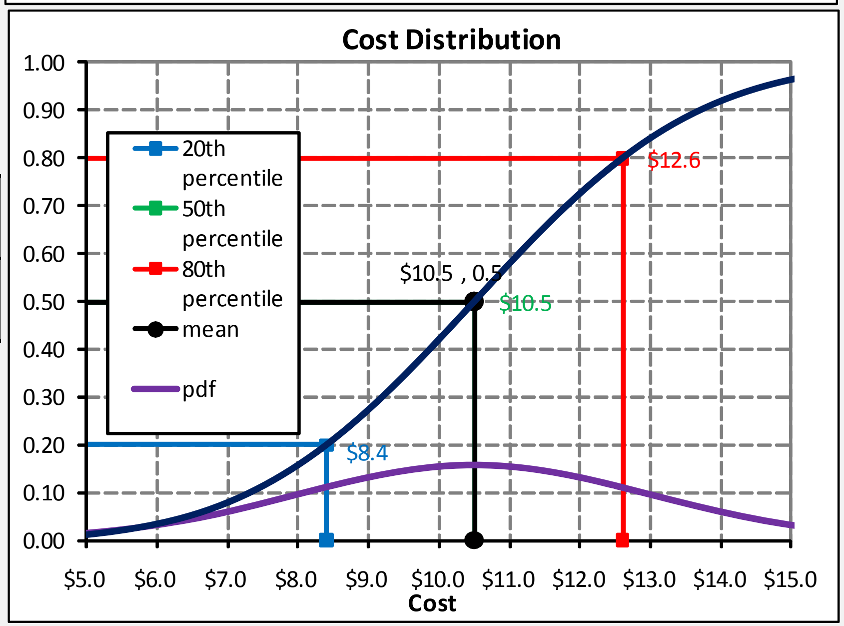
Government Counteroffer

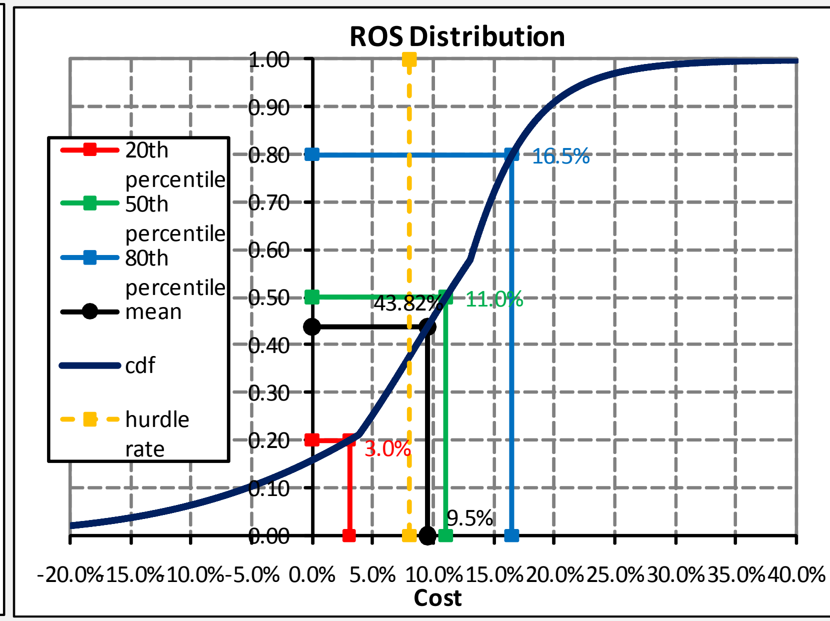
TC	$ 10.0
TF	15.0%
GS-u	90.0%
GS-o	60.0%
CP	130.0%





mean	$ 10.5
CV	25.0%





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Bottom Line

- Consider Risk and ROS in negotiations
  - Rigor and quantitative analysis of Cost applied to Contracts
- Government and contractor need to understand each other's perspectives
  - Primary objectives of affordability and profitability, respectively
  - Acknowledge other party's interests without compromising one's own
- Negotiations are adversarial, but relationship is symbiotic
  - Money paid to contractors gets reinvested in:
    - Economy – via employees, owners/shareholders
    - Industrial base – via corporate training, retention, facilities
    - Government – via taxes!
- Not that many levers!
- Avoid doing something unnatural!
  - Contract type and geometry should be appropriate
  - Use government "weighted guidelines" for fee

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Appendix

- Contract Types Overview
- Profit, Price, and ROS function for four main Contract Types
- Analytical Derivation of ROS distribution
  - General Approach
  - Four main Contract Types
- Analytical Derivation of Price distribution
  - FPI
- Pathological Cases
  - Padded cost
  - Aggressive cost
  - Understated variability

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Contract Types Overview

Price – Cost = Profit

ROS could be negative!

- Fixed-Price
  - Firm-Fixed-Price (FFP) [FAR 16.202]
  - Fixed-Price Incentive (FPI) [FAR 16.204]

Incentive contracts [FAR 16.4]

ROS strictly positive

- Cost-Reimbursement [FAR 16.3]
  - Cost-Plus-Incentive-Fee (CPIF) [FAR 16.304]
  - Cost-Plus-Award-Fee (CPAF) [FAR 16.305]
  - Cost-Plus-Fixed-Fee (CPFF) [FAR 16.306]

Cost + Fee = Price

- Contract Types vary according to
  - Degree and timing of the responsibility assumed by the contractor for the costs
  - ***Amount and nature of the profit incentive*** offered to the contractor for achieving or exceeding specified standards or goals
- We'll omit CPAF because it is by definition subjective

Cost Estimating Body of Knowledge (CEBoK), Module 14 Contract Pricing, SCEA, 2009.

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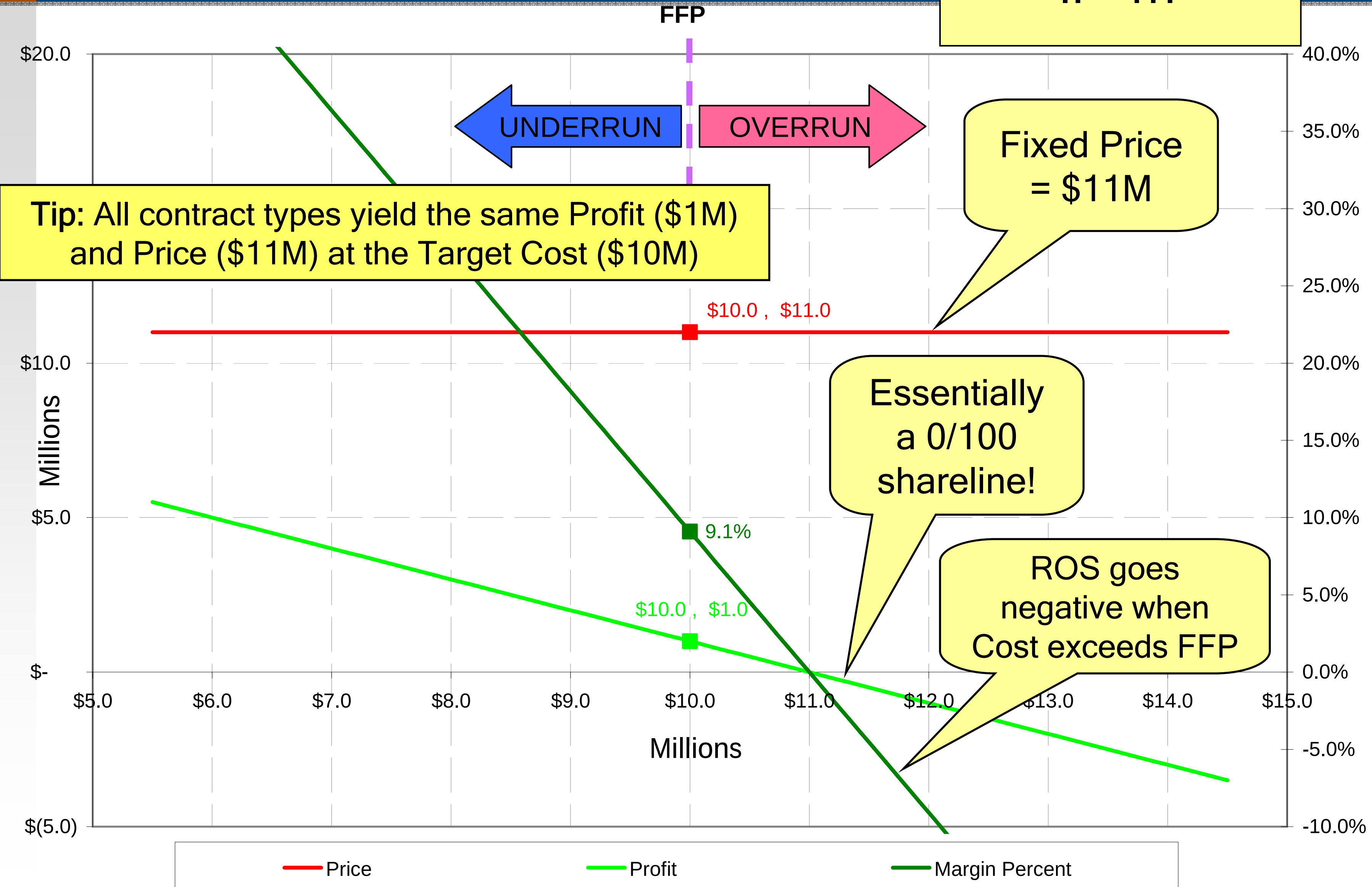
Firm-Fixed-Price (FFP)

<u>Contract Data Elements</u>:
TP = FFP

Tip: All contract types yield the same Profit ($1M) and Price ($11M) at the Target Cost ($10M)



Cost Estimating Body of Knowledge (CEBoK), Module 14 Contract Pricing, SCEA, 2009.

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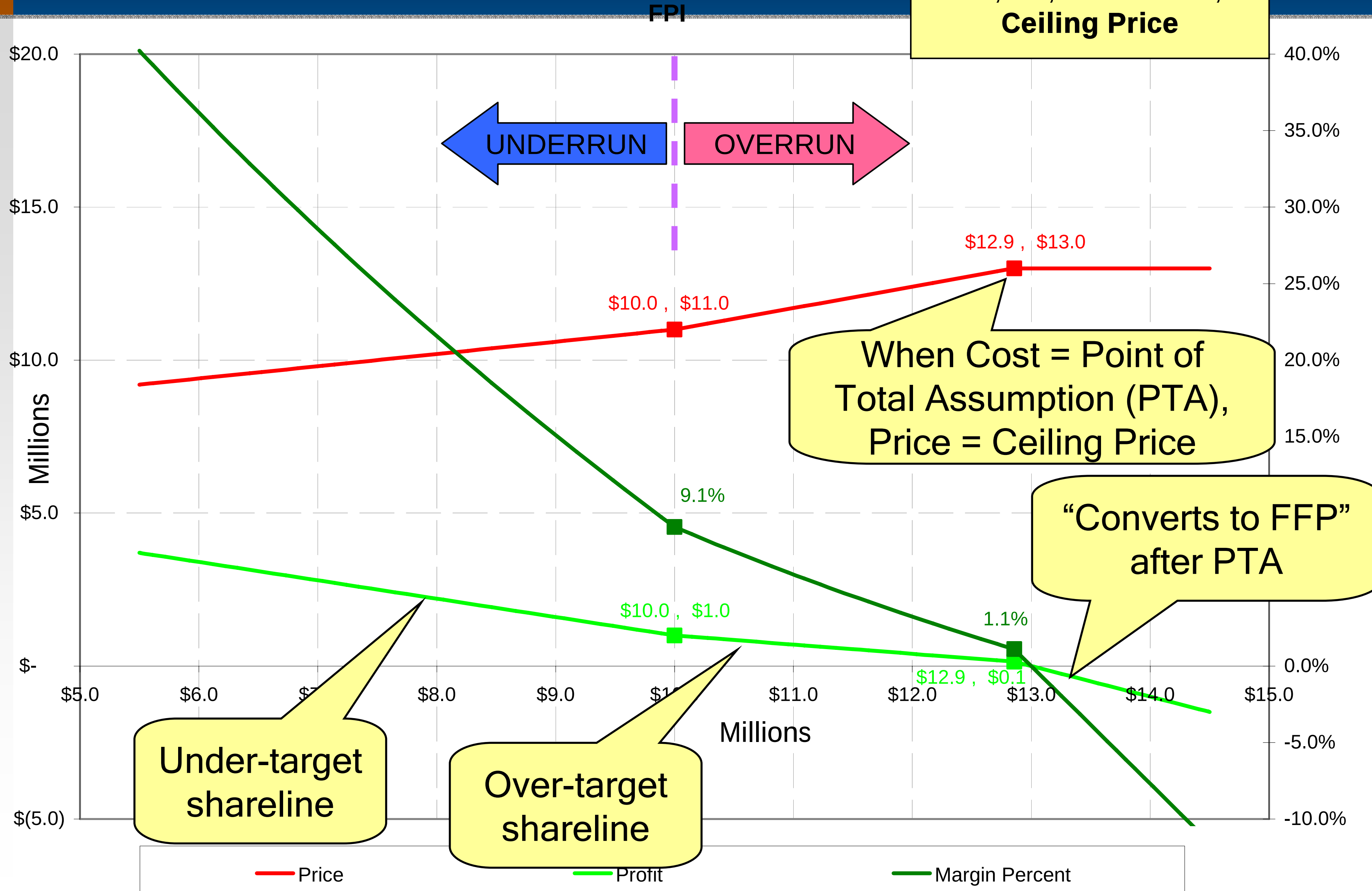
Fixed-Price Incentive (FPI)

Contract Data Elements:
TC, **TP**, **Sharelines**, **Ceiling Price**



Cost Estimating Body of Knowledge (CEBoK), Module 14 Contract Pricing, SCEA, 2009.

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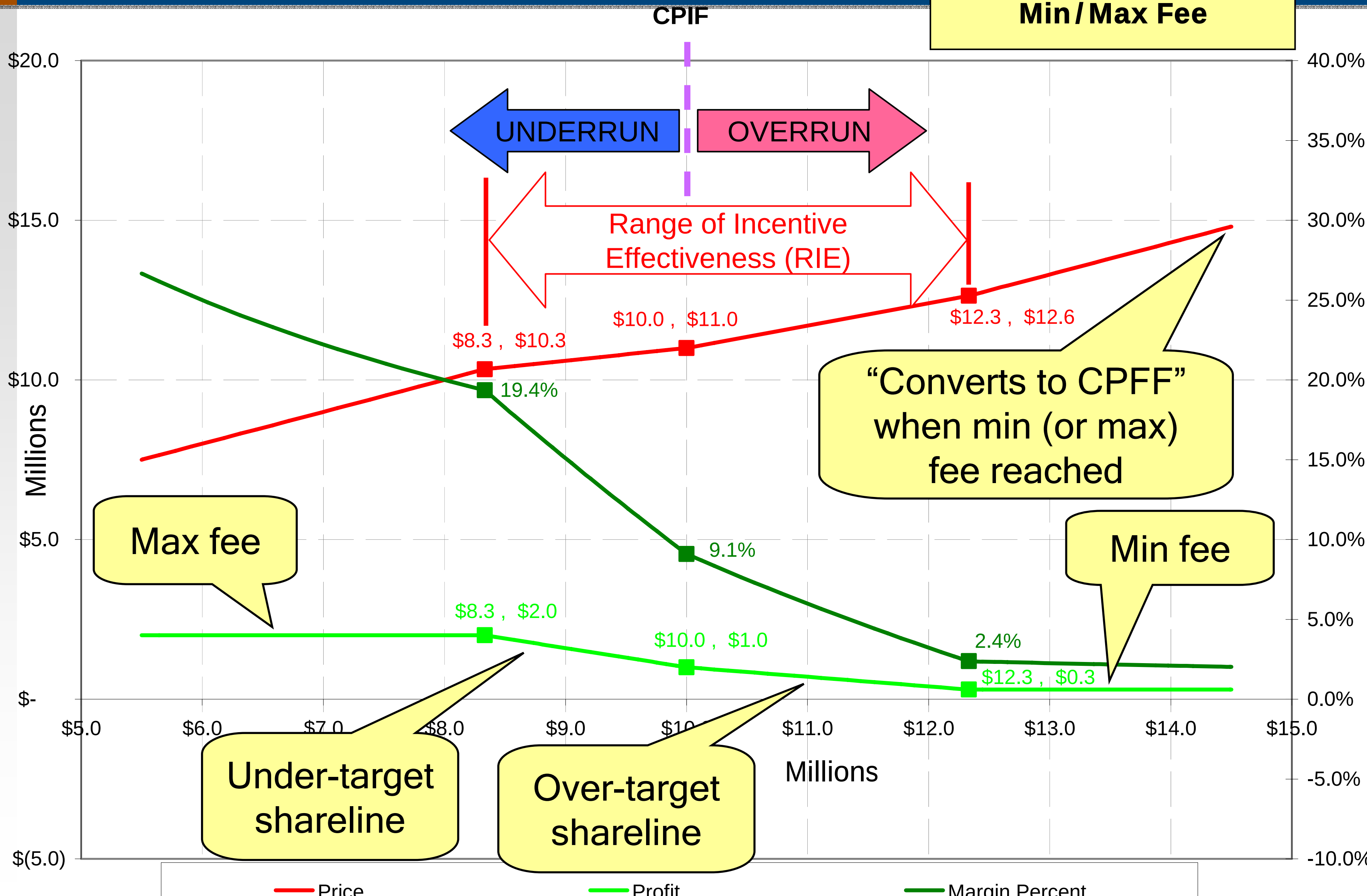
Cost-Plus-Incentive Fee (CPIF)

Contract Data Elements:
TC, **TF**, **Sharelines**, **Min / Max Fee**



Cost Estimating Body of Knowledge (CEBoK), Module 14 Contract Pricing, SCEA, 2009.

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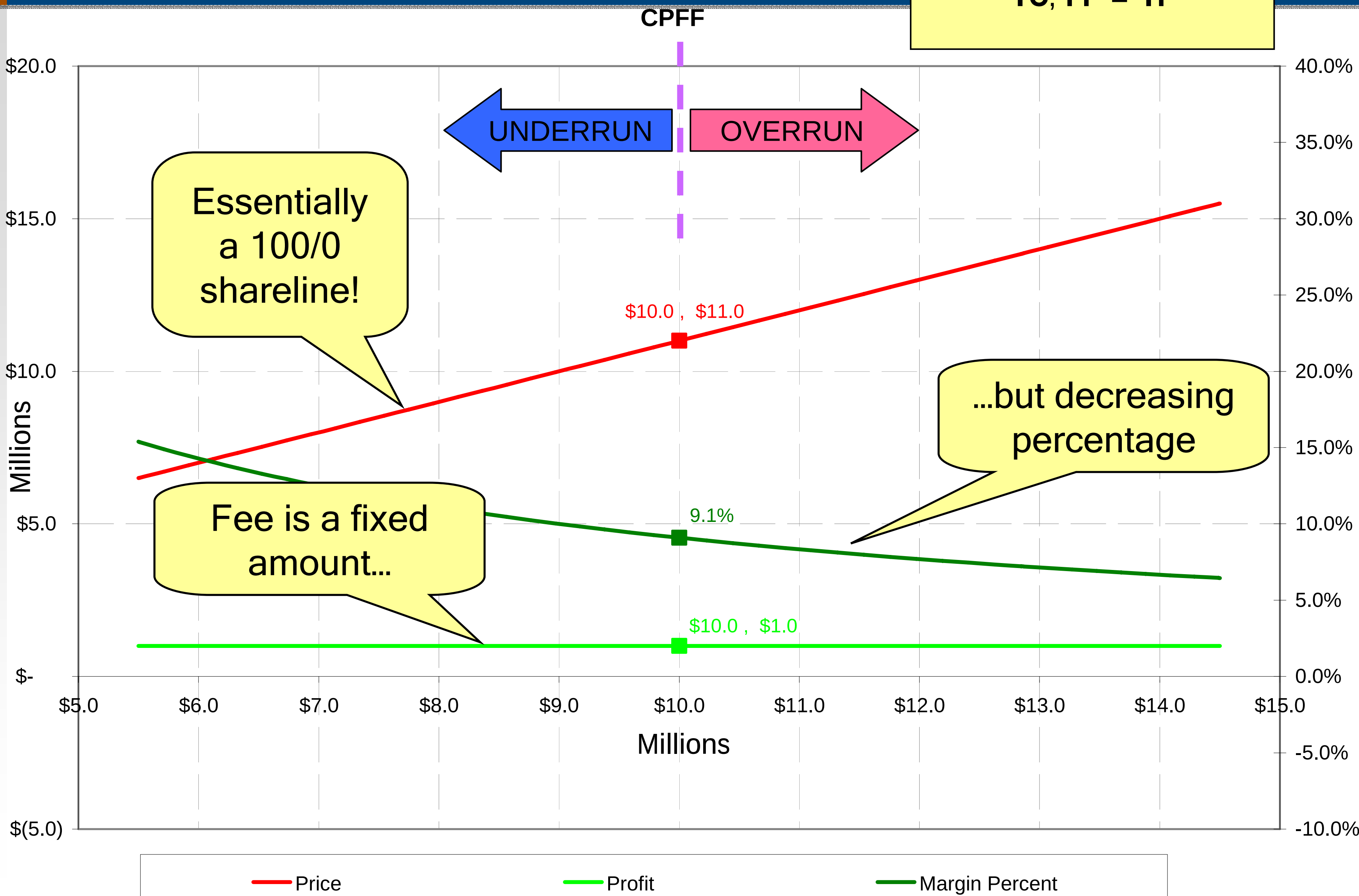
Cost-Plus-Fixed-Fee (CPFF)

Contract Data Elements:
TC, FF = TF



Cost Estimating Body of Knowledge (CEBoK), Module 14 Contract Pricing, SCEA, 2009.

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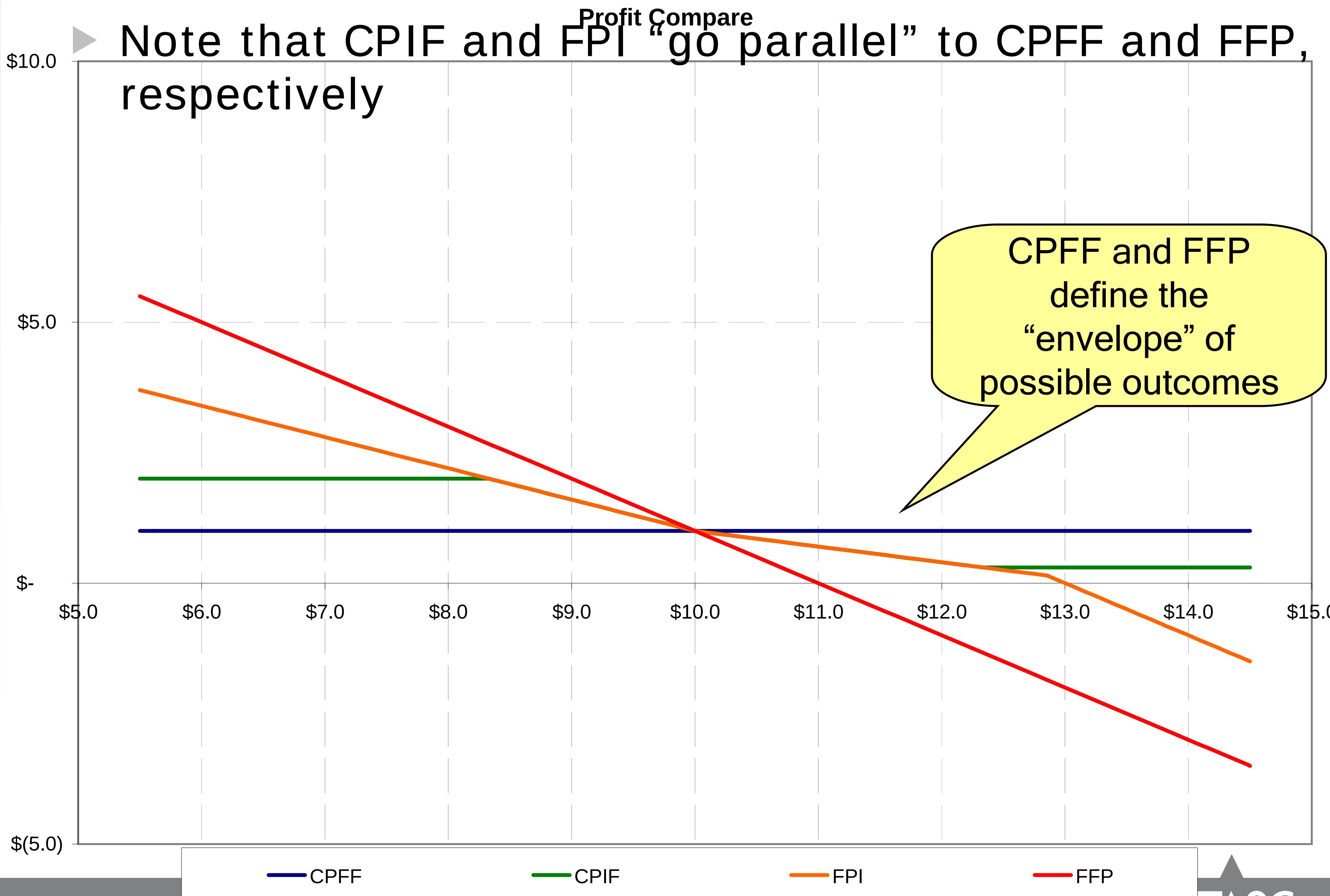
Contract Types Comparison – Profit (Y)

- Note that CPIF and FPI "go parallel" to CPFF and FFP, respectively



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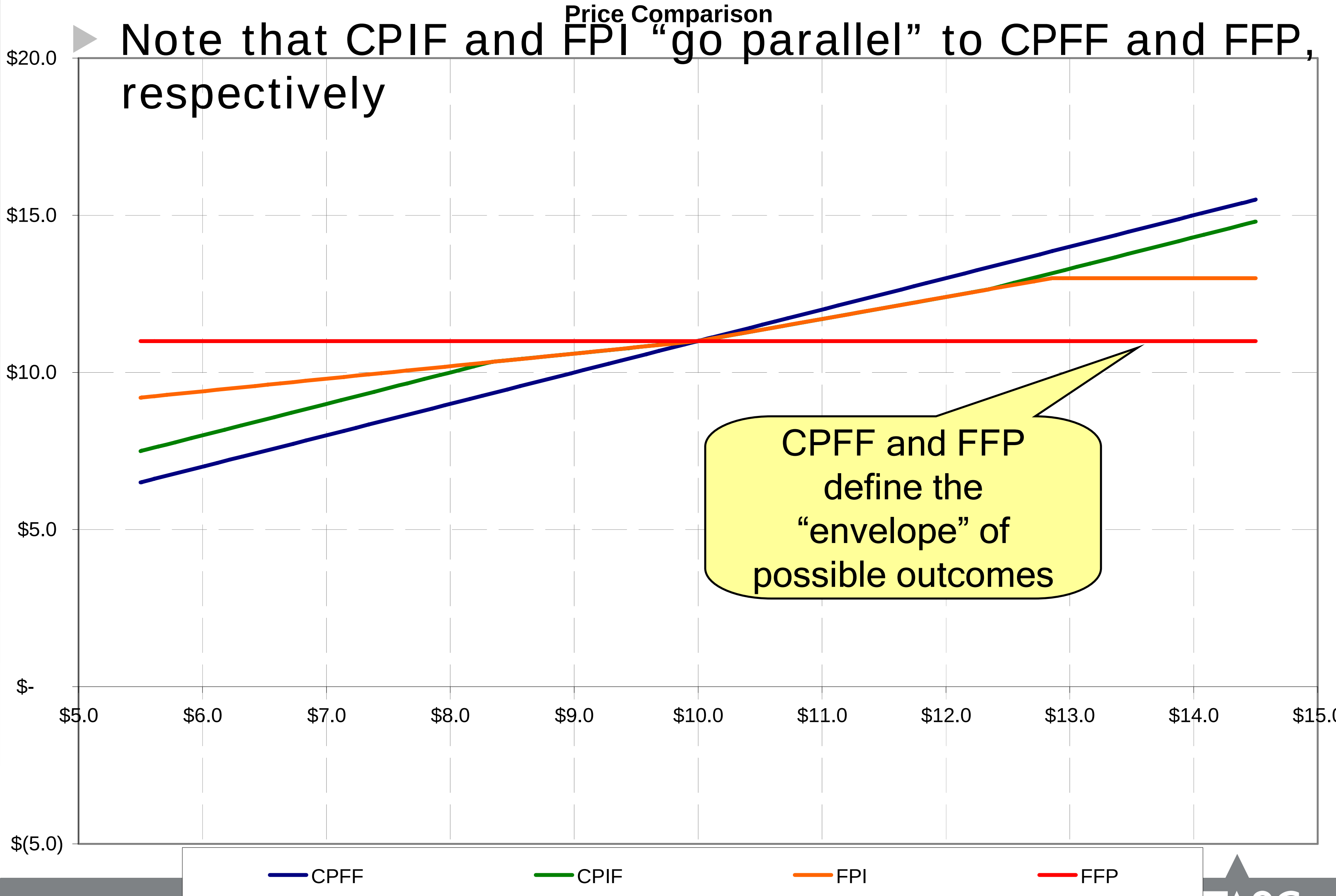
Contract Types Comparison – Price (X+Y)

- Note that CPIF and FPI "go parallel" to CPFF and FFP, respectively



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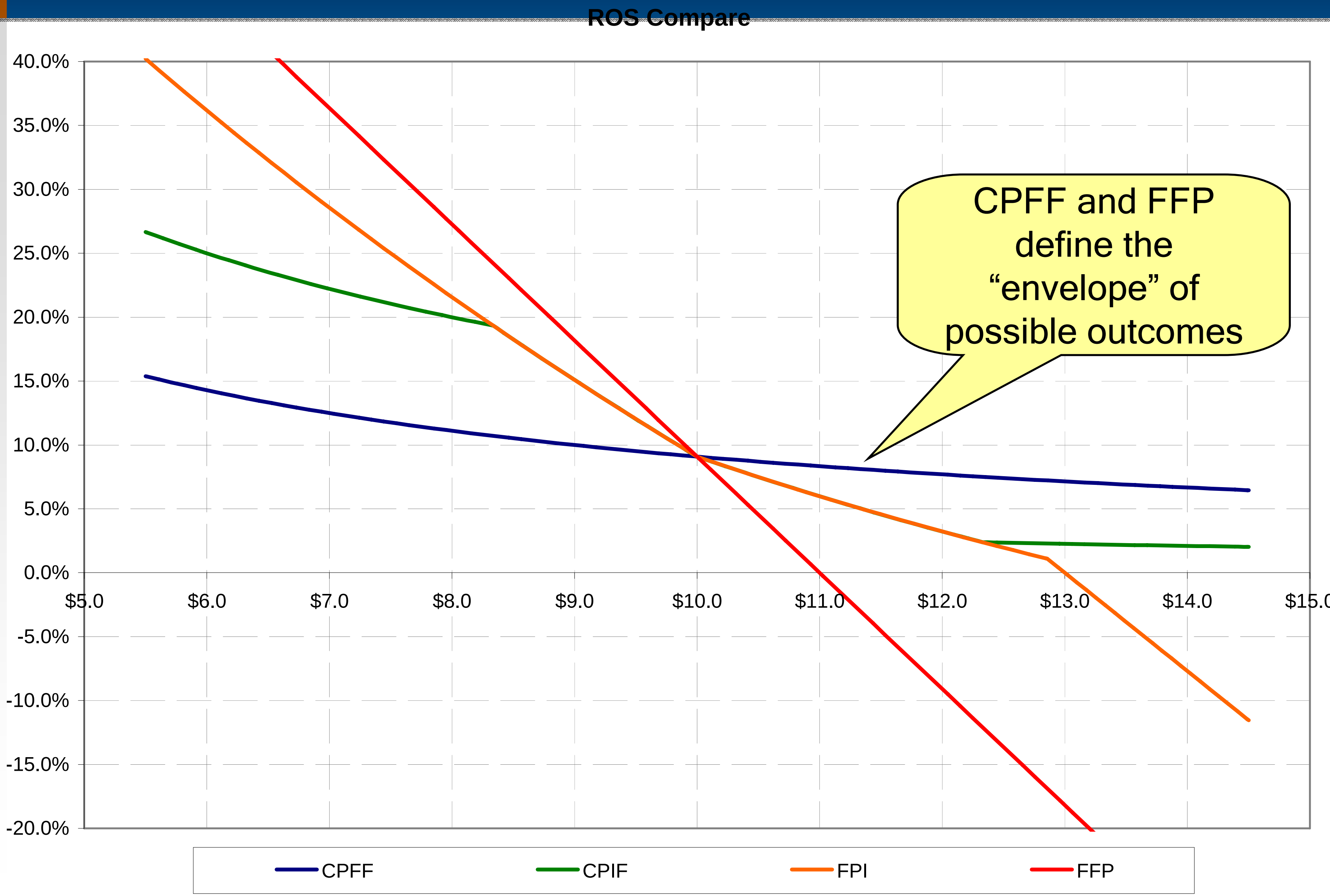
Contract Types Comparison – ROS (Y / (X+Y))



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Distribution of ROS (Analytical)

- Without Ts and Cs
- Transformation of random variables!
  - We math nerds always get excited about real-world applications of something we learned in school and thought we'd never use again!
- Define random variables:
  - X = Cost
  - Y = Profit (Fee) = f(X), where f is determined by contract type
    - Bright green line from earlier contract type graphs
    - Piecewise linear function for all major contract types (FFP/FPI/CPIF/CPFF)
    - Monotonically non-increasing function of Cost
      - In fact, monotonically decreasing except for CPFF
  - X+Y = Price
    - Monotonically non-decreasing function of Cost
      - In fact, monotonically increasing except for FFP
  - Z = ROS = Y/(X+Y) = 1 - X/(X+Y)
    - Monotonically decreasing function of Cost (for *all* contract types)

Distributions of Profit, Price, and ROS are continuous but not smooth at "break points"

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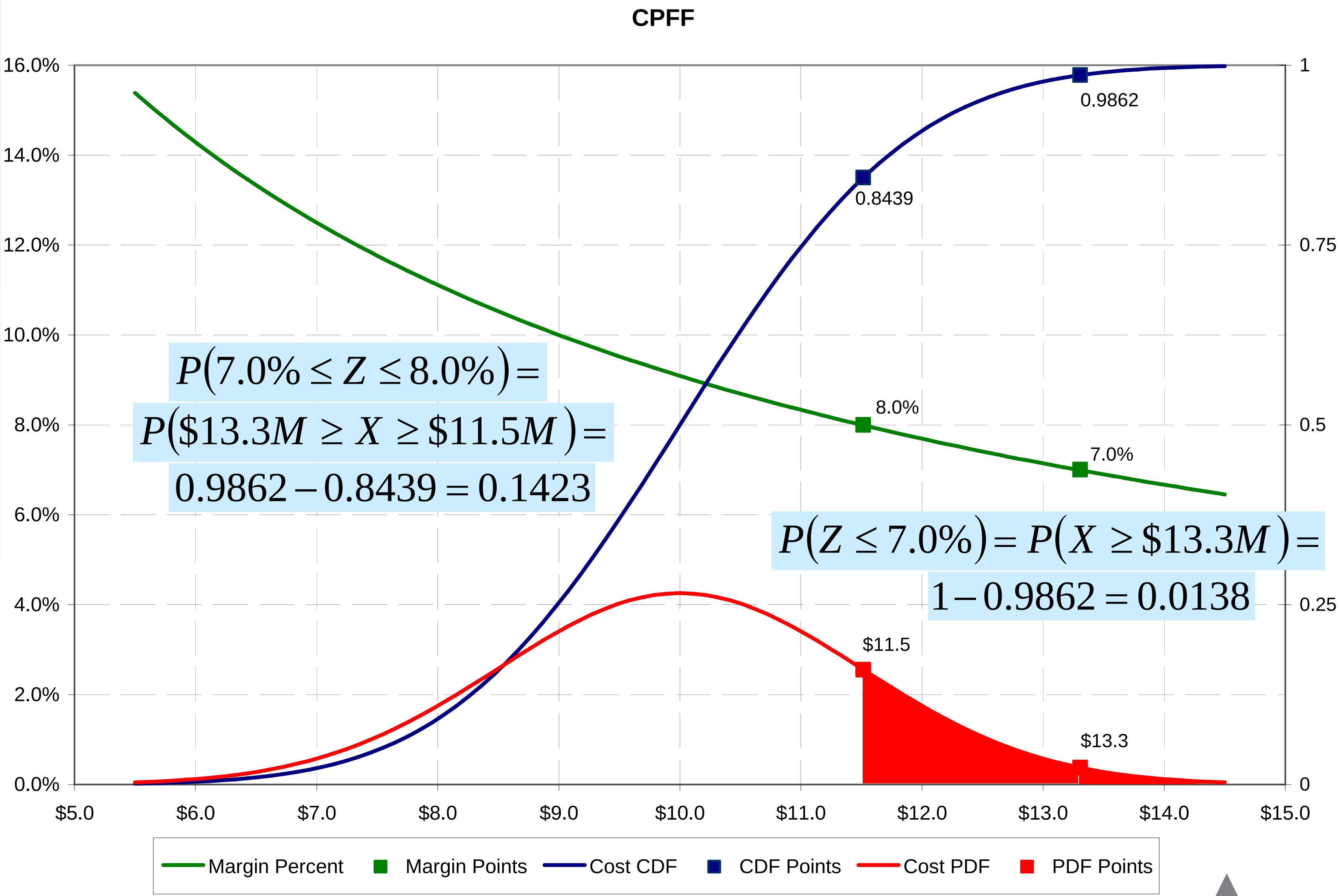
Distribution of ROS – Geometric Interpretation



$$P(7.0\% \le Z \le 8.0\%) = P(\$13.3M \ge X \ge \$11.5M) = 0.9862 - 0.8439 = 0.1423$$

$$P(Z \le 7.0\%) = P(X \ge \$13.3M) = 1 - 0.9862 = 0.0138$$

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Distribution of ROS – The "Easy Way"

- Using the Cumulative Distribution Function (CDF) and logic (cf. Cadenza)

$$F_Z(z) = P(Z \le z) = P\left(\frac{Y}{X+Y} \le z\right) = P\left(1 - \frac{X}{X+f(x)} \le z\right) =$$

$$P\left(1 - z \le \frac{X}{X+f(X)}\right) = P\left(X + f(X) \le \frac{X}{1-z}\right) = \boxed{P\left(f(X) \le X\frac{z}{1-z}\right)} =$$

$$P(X \ge g(z)) = 1 - P(X \le g(z)) = 1 - F_X(g(z))$$

- The formula for g(z) depends on f(X) and hence contract type
  - Since f(X) is piecewise linear, there's always a simple solution
  - We'll enumerate the solutions for the four basic contract types
- The outlined step has interesting conceptual and geometric interpretations
  - Probability that Profit is less than profit percentage times cost! [slap forehead]
  - As z goes from 0 to 1, the line y = (z/(1-z))x traces out 90 degrees, starting from the x-axis and rotating counterclockwise to the y-axis
  - Intersects the decreasing Profit function further and further to the left
  - Hence captures a bigger and bigger chunk of the right part of the PDF of cost!

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Distribution of ROS – The "Hard Way"

- Using the Probability Density Function (PDF) and Jacobeans (!)
- Agrees with PDF derived from CDF from the "Easy Way"

$$p_Z(z) = \frac{d}{dz} F_Z(z) = -F_X'(g(z)) \cdot g'(z) = -p_X(g(z)) \cdot g'(z)$$

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Distribution of ROS – FFP

- FFP = Target Price = Target Cost + Target Profit
- Profit = FFP – Cost $Y = f(x) = TP - X$
- Linear function (slope of -1)

$$Z = \frac{TP - X}{TP}$$

$$P\left(TP - X \le X\frac{z}{1-z}\right) = P\left(X \ge TP(1-z)\right) = 1 - P\left(X \le TP(1-z)\right)$$

Linear Combinations property: X is Normal implies Z is Normal

$$F_Z(z) = 1 - F_X\left(TP(1-z)\right) \Rightarrow p_Z(z) = TP \cdot p_X\left(TP(1-z)\right)$$

Take derivative, apply chain rule

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Incentive Formula – FPI

- Over-Target Shareline Adjustment until Point of Total Assumption (PTA)
  - Converts to FFP
- Under-Target Shareline Adjustment
- Piecewise linear function (three regimes)

$$Y = f(X) = \begin{cases} TF + CS_{under}(TC - X) & X \le TC \\ TF - CS_{over}(X - TC) & TC < X \le PTA \\ CP - X & X > PTA \end{cases}$$

$$X = TC \Leftrightarrow Z = \frac{TF}{TP}$$

$$X = PTA \Leftrightarrow Z = \frac{CP - PTA}{CP}$$

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Distribution of ROS – FPI

$$P\left(\left(TF + CS_{under}\left(TC - X\right)\right) \le \left(\frac{z}{1-z}\right)X\right) = 1 - P\left(X \le \frac{\left(TF + CS_{under}TC\right)\left(1-z\right)}{CS_{under} + GS_{under}z}\right)$$

$$P\left(\left(TF - CS_{over}\left(X - TC\right)\right) \le \left(\frac{z}{1-z}\right)X\right) = 1 - P\left(X \le \frac{\left(TF + CS_{over}TC\right)\left(1-z\right)}{CS_{over} + GS_{over}z}\right)$$

$$P\left(CP - X \le \left(\frac{z}{1-z}\right)X\right) = 1 - P\left(X \le \left(1-z\right)CP\right)$$

$$F_Z\left(z\right) = \begin{cases} 1 - F_X\left(\dfrac{\left(TF + CS_{under}TC\right)\left(1-z\right)}{CS_{under} + GS_{under}z}\right) & z \ge \dfrac{TF}{TP} \\ 1 - F_X\left(\dfrac{\left(TF + CS_{over}TC\right)\left(1-z\right)}{CS_{over} + GS_{over}z}\right) & \dfrac{CP - PTA}{CP} \le z < \dfrac{TF}{TP} \\ 1 - F_X\left(\left(1-z\right)CP\right) & z < \dfrac{CP - PTA}{CP} \end{cases}$$

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Distribution of ROS – FPI

Take derivative, apply chain rule

$$p_Z(z)=\begin{cases}\left(\dfrac{TF+CS_{under}TC}{\left(CS_{under}+GS_{under}z\right)^2}\right)p_X\left(\dfrac{\left(TF+CS_{under}TC\right)\left(1-z\right)}{CS_{under}+GS_{under}z}\right) & z\geq\dfrac{TF}{TP}\\ \left(\dfrac{TF+CS_{over}TC}{\left(CS_{over}+GS_{over}z\right)^2}\right)p_X\left(\dfrac{\left(TF+CS_{over}TC\right)\left(1-z\right)}{CS_{over}+GS_{over}z}\right) & \dfrac{CP-PTA}{CP}\leq z<\dfrac{TF}{TP}\\ CP\cdot p_X\left(\left(1-z\right)CP\right) & z<\dfrac{CP-PTA}{CP}\end{cases}$$

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Incentive Formula – CPIF

- Over-Target Shareline Adjustment down to Min Fee
  - Converts to CPFF
- Under-Target Shareline Adjustment up to Max Fee
  - Converts to CPFF
- Piecewise linear function (four regimes)

$$Y = f(X) = \begin{cases} MF & X \le RIE_{low} \\ TF + CS_{under}(TC - X) & RIE_{low} < X \le TC \\ TF - CS_{over}(X - TC) & TC < X \le RIE_{high} \\ mF & X > RIE_{high} \end{cases}$$

$$X = RIE_{low} \Leftrightarrow Z = \frac{MF}{RIE_{low} + MF}$$

$$X = TC \Leftrightarrow Z = \frac{TF}{TP}$$

$$X = RIE_{high} \Leftrightarrow Z = \frac{mF}{RIE_{high} + mF}$$

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Distribution of ROS – CPIF

$$P\left(MF \le \left(\frac{z}{1-z}\right)X\right) = 1 - P\left(X \le \left(\frac{1-z}{z}\right)MF\right)$$

$$P\left(mF \le \left(\frac{z}{1-z}\right)X\right) = 1 - P\left(X \le \left(\frac{1-z}{z}\right)mF\right)$$

$$P\left(\left(TF + CS_{under}\left(TC - X\right)\right) \le \left(\frac{z}{1-z}\right)X\right) = 1 - P\left(X \le \frac{\left(TF + CS_{under}TC\right)\left(1-z\right)}{CS_{under} + GS_{under}z}\right)$$

$$P\left(\left(TF - CS_{over}\left(X - TC\right)\right) \le \left(\frac{z}{1-z}\right)X\right) = 1 - P\left(X \le \frac{\left(TF + CS_{over}TC\right)\left(1-z\right)}{CS_{over} + GS_{over}z}\right)$$

$$F_Z(z) = \begin{cases} 1 - F_X\left(\left(\frac{1-z}{z}\right)MF\right) & z \ge \dfrac{MF}{RIE_{low} + MF} \\ 1 - F_X\left(\dfrac{\left(TF + CS_{under}TC\right)\left(1-z\right)}{CS_{under} + GS_{under}z}\right) & \dfrac{TF}{TP} \le z < \dfrac{MF}{RIE_{low} + MF} \\ 1 - F_X\left(\dfrac{\left(TF + CS_{over}TC\right)\left(1-z\right)}{CS_{over} + GS_{over}z}\right) & \dfrac{mF}{RIE_{high} + mF} \le z < \dfrac{TF}{TP} \\ 1 - F_X\left(\left(\frac{1-z}{z}\right)mF\right) & z < \dfrac{mF}{RIE_{high} + mF} \end{cases}$$

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Distribution of ROS – CPIF

Take derivative, apply chain rule

$$p_Z(z)=\begin{cases} \dfrac{MF}{z^2}\,p_X\left(\left(\dfrac{1-z}{z}\right)MF\right) & z \geq \dfrac{MF}{RIE_{low}+MF} \\ \left(\dfrac{TF+CS_{under}TC}{\left(CS_{under}+GS_{under}z\right)^2}\right)p_X\left(\dfrac{\left(TF+CS_{under}TC\right)(1-z)}{CS_{under}+GS_{under}z}\right) & \dfrac{TF}{TP} \leq z < \dfrac{MF}{RIE_{low}+MF} \\ \left(\dfrac{TF+CS_{over}TC}{\left(CS_{over}+GS_{over}z\right)^2}\right)p_X\left(\dfrac{\left(TF+CS_{over}TC\right)(1-z)}{CS_{over}+GS_{over}z}\right) & \dfrac{mF}{RIE_{high}+mF} \leq z < \dfrac{TF}{TP} \\ \dfrac{mF}{z^2}\,p_X\left(\left(\dfrac{1-z}{z}\right)mF\right) & z < \dfrac{mF}{RIE_{high}+mF} \end{cases}$$

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Distribution of ROS – CPFF

- Fixed Fee amount = TF
  - Linear (constant) function

$$Y = f(x) = TF$$

$$Z = \frac{TF}{TF + X}$$

$$P\left(TF \le \left(\frac{z}{1-z}\right) X \right) = P\left(X \ge \left(\frac{1-z}{z}\right) TF \right) = 1 - P\left(X \le \left(\frac{1-z}{z}\right) TF \right)$$

$$F_Z(z) = 1 - F_X\left(\left(\frac{1-z}{z}\right) TF\right) \Rightarrow p_Z(z) = \frac{TF}{z^2} p_X\left(\left(\frac{1-z}{z}\right) TF\right)$$

Take derivative, apply chain rule

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Incentive Formula – FPI

- Over-Target Shareline Adjustment until Point of Total Assumption (PTA)
  - Converts to FFP
- Under-Target Shareline Adjustment
- Piecewise linear function (three regimes)

$$R = \begin{cases} TP - GS_{under}(TC - X) & X \le TC \\ TP + GS_{over}(X - TC) & TC < X \le PTA \\ CP & X > PTA \end{cases}$$

$$X = 0 \Leftrightarrow R = X + Y = TP - GS_{under} TC$$

$$X = TC \Leftrightarrow R = X + Y = TP = TC + TF$$

$$X = PTA \Leftrightarrow R = X + Y = CP$$

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Distribution of Price (R = X+Y) – FPI

$$P\left(TP - GS_{under}\left(TC - X\right) \le r\right) = P\left(X \le TC - \frac{TP - r}{GS_{under}}\right)$$

$$P\left(TP + GS_{over}\left(X - TC\right) \le r\right) = P\left(X \le TC + \frac{r - TP}{GS_{over}}\right)$$

$$P\left(R = CP\right) = 1 - P\left(X \le PTA\right)$$

discrete "chunk" of probability

$$F_R(r) = \begin{cases} F_X\left(TC - \dfrac{TP - r}{GS_{under}}\right) & TP - GS_{under}TC \le R < TP \\ F_X\left(TC + \dfrac{r - TP}{GS_{over}}\right) & TP \le R < CP \\ 1 & R \ge CP \end{cases}$$

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Distribution of ROS – FPI

Take derivative, apply chain rule

$$p_R(r) = \begin{cases} \dfrac{1}{GS_{under}} p_X\left(TC - \dfrac{TP - r}{GS_{under}}\right) & TP - GS_{under}TC \le R < TP \\ \dfrac{1}{GS_{over}} p_X\left(TC + \dfrac{r - TP}{GS_{over}}\right) & TP \le R < CP \\ 1 - F_X(PTA) & R = CP \end{cases}$$

discrete "chunk" of probability

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Distribution of ROS – Pathological Cases

- Comparison graphs for cases:
  - Base case: Base cost = Target Cost ($10.0M), standard deviation = $1.5M (15% CV)
  - Aggressive cost: True base cost is $11.5M instead of $10.0M
  - Padded cost: True base cost is $8.5M instead of $10.0M
  - Understated variability: True standard deviation is $3.0M instead of $1.5M
- Summary table across all contract types:

MONTE CARLO	Base case ($10M)				Padded cost ($8.5M)				Aggressive cost ($11.5M)				Understated variability			
	FFP	FPI	CPIF	CPFF	FFP	FPI	CPIF	CPFF	FFP	FPI	CPIF	CPFF	FFP	FPI	CPIF	CPFF
20th percentile	-2.2%	5.1%	5.2%	8.1%	11.4%	10.5%	10.3%	9.3%	-16.0%	1.3%	2.3%	7.3%	-13.6%	2.0%	2.4%	7.4%
median (50th percentile)	8.9%	9.1%	9.0%	9.1%	22.9%	18.2%	18.2%	10.5%	-4.6%	4.6%	4.6%	8.0%	9.4%	9.1%	9.6%	9.1%
mean	9.0%	11.0%	10.6%	9.3%	22.8%	19.1%	16.6%	10.8%	-4.5%	4.4%	5.8%	8.1%	9.3%	12.5%	12.5%	7.5%
80th percentile	20.3%	16.8%	16.7%	10.2%	34.2%	26.9%	21.6%	12.1%	6.9%	8.4%	8.3%	8.9%	32.0%	25.4%	21.2%	11.9%

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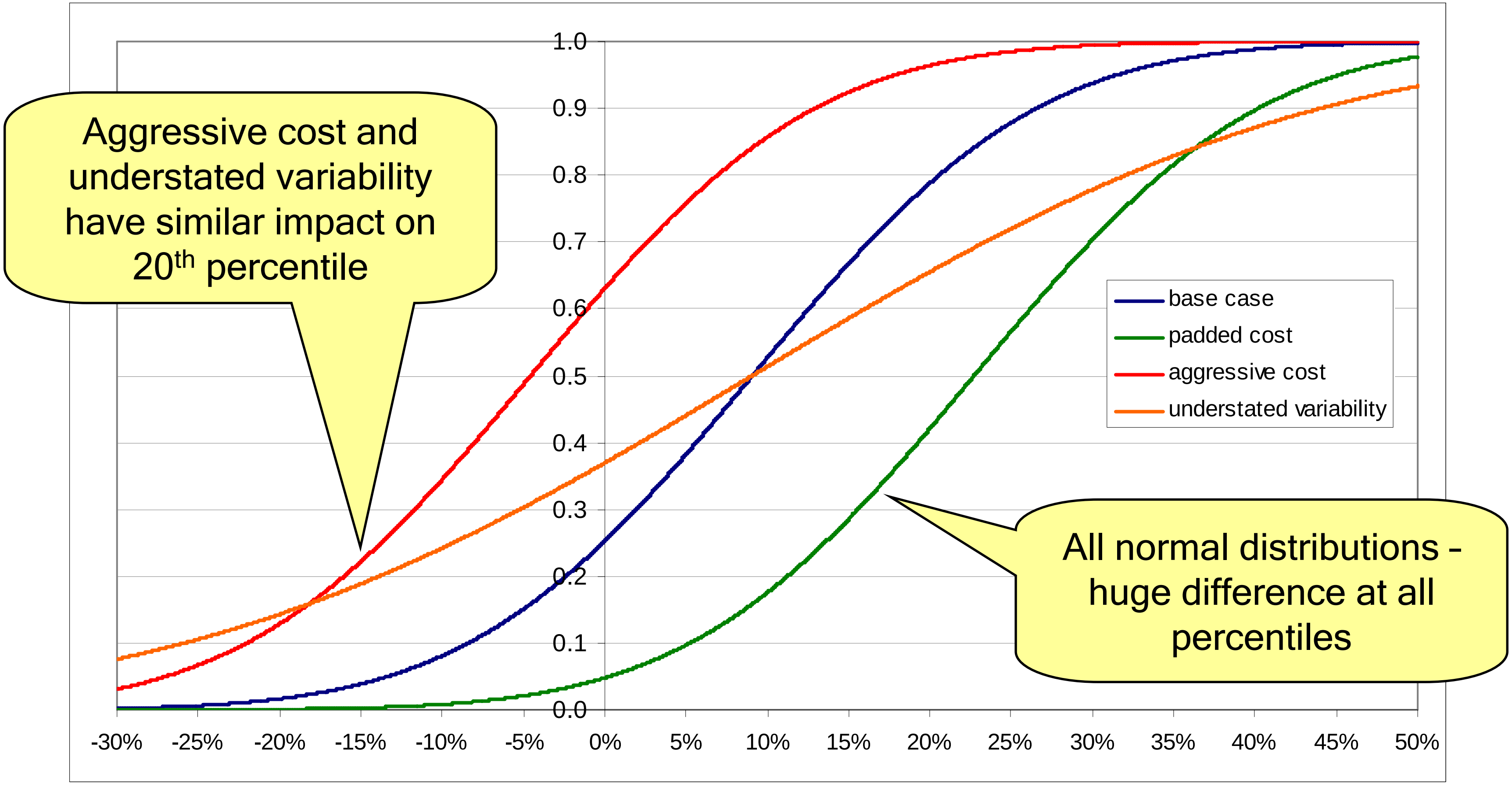
FFP – Pathological Cases



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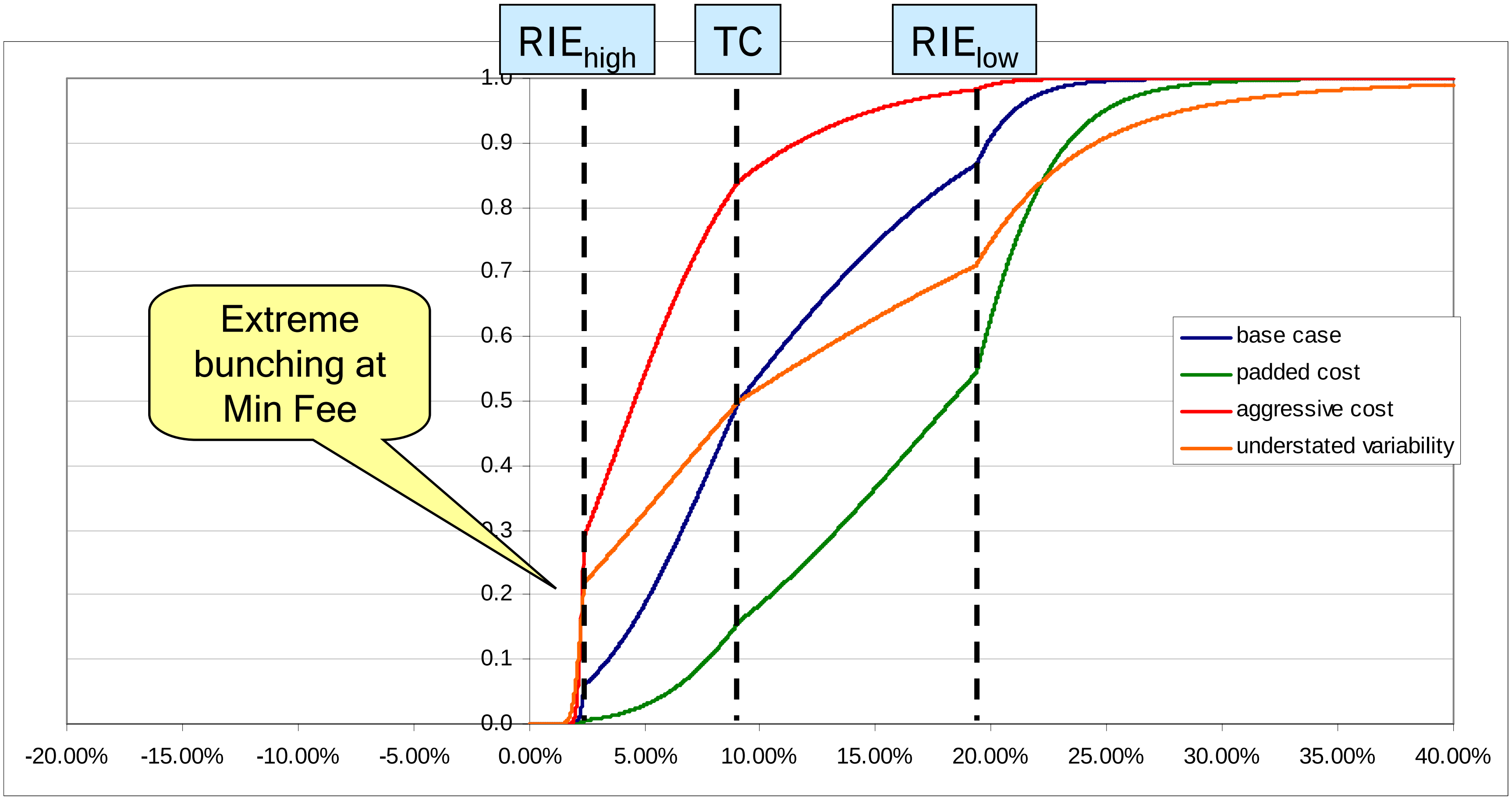
CPIF – Pathological Cases



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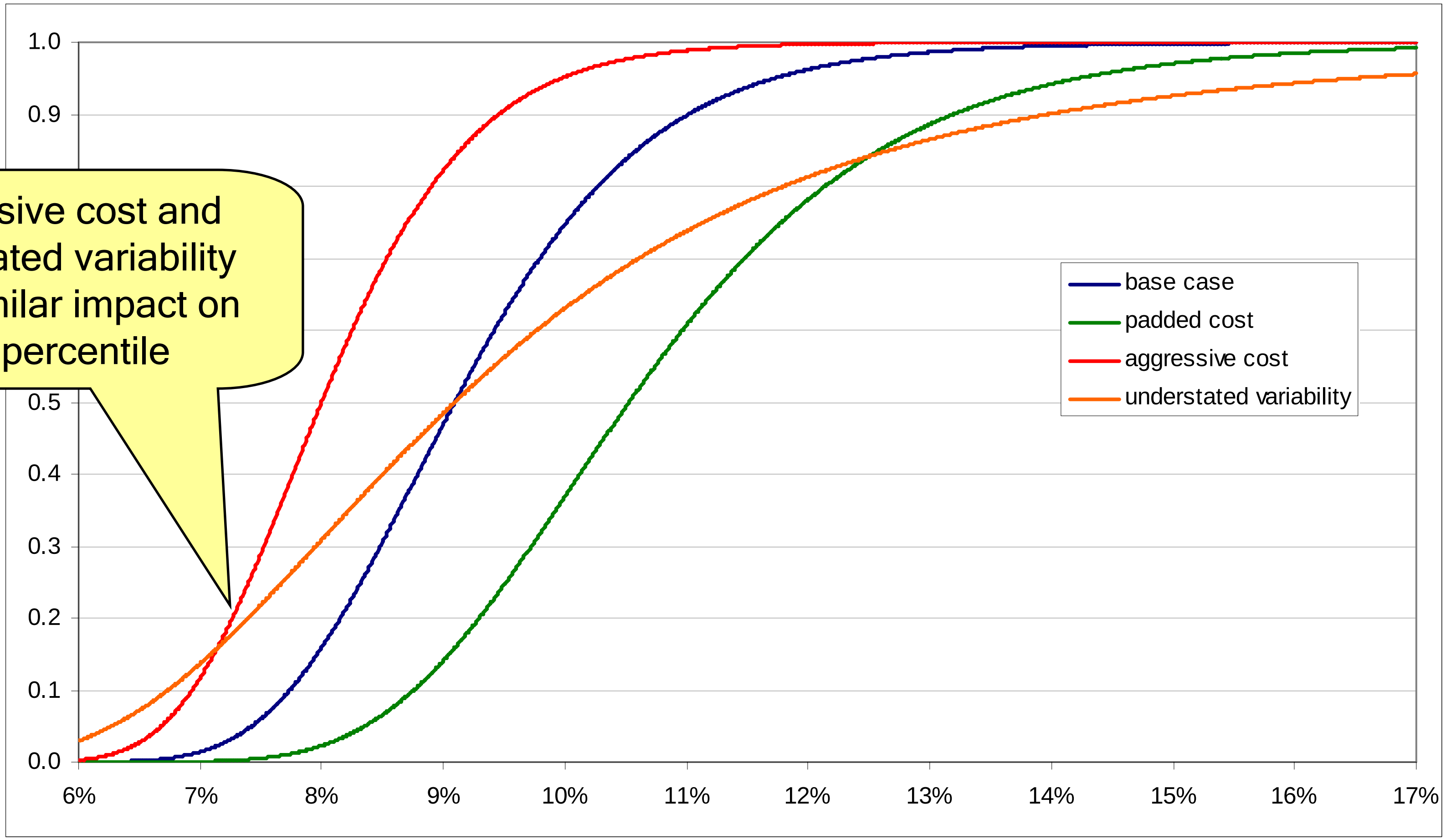
CPFF – Pathological Cases



Aggressive cost and understated variability have similar impact on 20th percentile

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