Here, There Be Dragons: Considering the Right Tail in Risk Management

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Abstract

The portfolio effect is the reduction of risk achieved by funding multiple projects that are not perfectly correlated with one another. It is relied upon in setting confidence level policy for programs that consist of multiple projects. The idea of a portfolio effect has its roots in modern finance as pioneered by Nobel Memorial Prize winner Harry Markowitz. However, in three recent ISPA-SCEA conference presentations, "The Portfolio Reconsidered" in 2007, "The Fractal Geometry of Cost Risk" in 2008, and "The Portfolio Effect and the Free Lunch" in 2009, the author has demonstrated that the portfolio effect is more myth than fact. However, current NASA and Department of Defense policy guidance relies heavily upon this chimerical effect. This is seen in policy that sets funding at a set percentile, typically at the 70th or 80th percentile. The inherent, optimistic belief is that the portfolio effect will allow total funding agency-wide to be much higher, above the 90th percentile. However, in the absence of such an effect, policy guidance that specifies funding at a relatively low percentile, like the 70th or 80th, will result in numerous overruns, insufficient reserves, and other financial difficulties at the agency level. Funding at such levels will result in overruns for 20-30% of missions, so cost growth will be a common occurrence. And by funding only at a percentile, there is no insight into how much will be needed in extra reserves. Depending upon the variance of the cost risk distribution and other characteristics, such as skewness and kurtosis, this amount can vary significantly. Thus the right tail must be taken into consideration when establishing reserves. A superior alternative has been proposed that measures this expected shortfall, called Conditional Tail Expectation. Also called Tail Value at Risk, its use is growing in popularity in a variety of industries, including insurance. Recently, the notion of coherence has been proposed and adopted for risk measures. The notion of coherence is discussed, and its relevance to Value at Risk and Tail Value at Risk is examined.

Introduction

"Western Europe conquered the word because of a technological revolution that started because of attempts to measure the world." Phillipe Jorion (Ref. 15) In the same way, attempts to measure risk will lead to better project management.

Nearly all risk management for DoD and NASA agencies seem to focus solely on finding a single percentile to budget against. NASA policy explicitly mentions 70% and 50%, for example, and some defense agencies budget to the 80% confidence level. In the financial industry this is known as "Value at Risk" and is the maximum loss not exceeded with a given probability (Ref. 10). Technically it is defined as

$$VaR_{\alpha} = inf\{l \in \mathbb{R}: P(L > l) \leq 1 - \alpha\} = inf\{l \in \mathbb{R}: F_L(l) \geq \alpha\}$$

That is, VaR is a percentile of the cost risk distribution. For example, the 95th percentile of a normal distribution with mean equal to \$600 and standard deviation equal to \$200 is approximately \$929, so $VaR_{0.95}$=\$929.

Percentile, or "Value at Risk," funding has some merits. Percentile funding, or "Value at Risk," is a common measure, and can be used to compare different programs and projects. It can be easily understood by senior managers. Risk reserves can be set in terms of percentiles. It is currently part of NASA policy and Department of Defense policy, and is commonly used in the banking industry. However risk management doesn't stop at that point. Even funding at a 70% confidence level means that there is roughly a one-in-three chance of experiencing an overrun. And not only that, but funding at the 70th percentile provides no information about what happens to the right of that point. If the 70th percentile is exceeded, how much additional funding is expected to be required? Funding at any set percentile is budgeting in the dark, ignoring the truly risky, bad-day events that projects should have the funds to pay when they occur. Funding against such low percentiles is whistling in the dark, hoping that tail events do not occur. And not all right tails are created equal. Some distributions have thin tails, but others, typically those which are more realistic for projects, have relatively fat tails. As on old maps that depicted dragons and other monsters in uncharted territory (see Figure 1), so who knows what risk lurk beyond the 70th percentile?

Figure 1. A section of the Carta Marina by Olaus Magnus, 16th Century.

Consider four different distribution types, each of which has the same 70th percentile, but different characteristics, as displayed in Figure 2. In Figure 2, four distributions, a triangular, normal, lognormal, and Pareto, are all displayed. Each has different characteristics. So although each distribution happens to have the same 70th percentile,

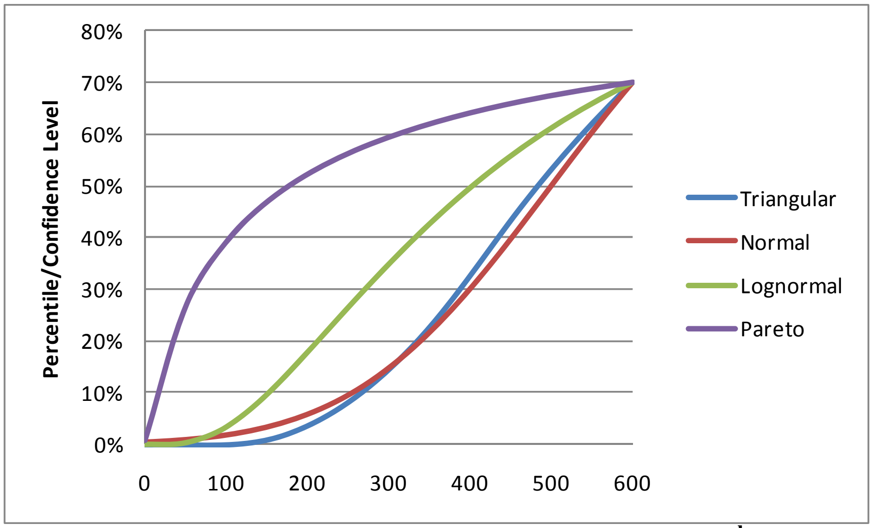

Figure 2. Four Different Distributions with the Same 70th Percentile.

should we expect them to have similar tail behavior? The obvious answer is no, and a cursory glimpse at the S-curves' tail characteristics, as shown in Figure 3, make this plain.

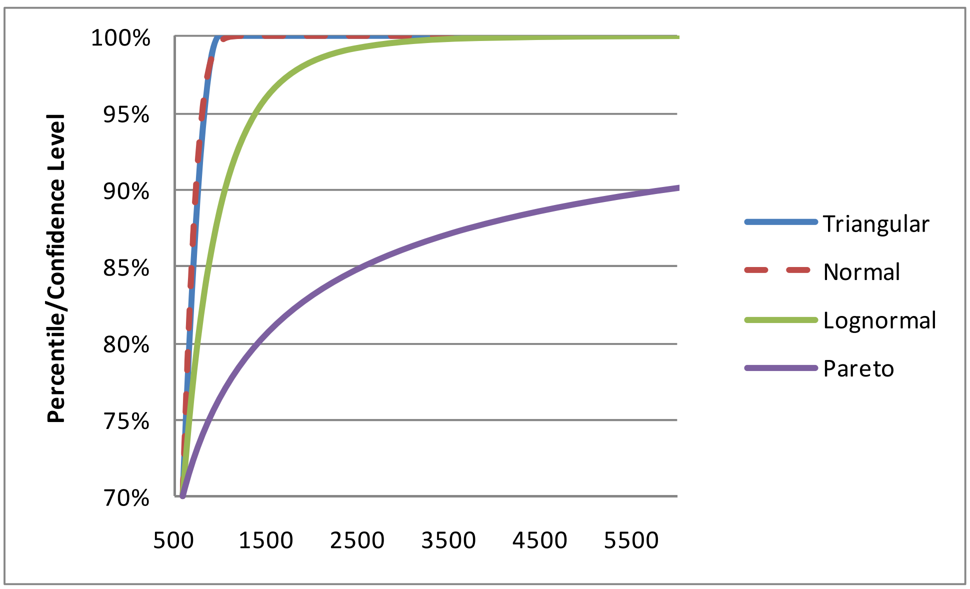

Figure 3. Tail Behavior of Four Distributions

The normal and triangular distributions both have thin tails. There is relatively little risk beyond the 70th percentile, while the lognormal has a fatter tail, and the Pareto has an extremely fat tail. The 70th percentile for all four distributions is $600. But to get to the 80th percentile, an additional $60 is needed for the normal, $80 for the triangular, but $170 is needed for the lognormal, and a whopping $830 more is needed for the Pareto. A comparison four the tail behavior of the four distributions is shown in Table 1.

Percentile	*Triangular*	*Normal*	*Lognormal*	*Pareto*
70th	$600	$600	$600	$600
80th	$680	$660	$770	$1,430
90th	$770	$750	$1,070	$5,880
95th	$840	$820	$1,410	$23,750
99th	$930	$950	$2,350	$600,000

Table 1. Comparison of Tail Percentiles for Four Distributions.

Note that there is significant tail risk for the lognormal and the Pareto. Indeed the 99th percentile for the Pareto is 1,000 times greater than the 70th percentile, indicating enormous risk, the type seen in financial markets and in catastrophic risks like natural disasters. Should four different projects, which follow the four different risk profiles seen in this example, be funded at the same level? That is what is indicated by 70th percentile confidence funding, but clearly funding a project that follows the triangular with $600, and the Pareto with $600 will have significantly different consequences in the 30% of those cases in which the 70th percentile is exceeded.

And does percentile budgeting even truly amount to effective risk management policy? Suppose that you are shopping for a new car. You mention that safety is your top concern. The salesman says he has a great, safe car available. You ask about the air bags. The salesman answers, “Of course the air bags work! Seventy percent of the time they work fine. Only 30% of the time, the air bags will fail to deploy.” Would you buy such a car? Hedge fund manager David Einhorn “Risk management is the air bag that must always work, but only in the multi-sigma event where you have an accident.” (Ref 7.) This is the complete opposite of percentile budgeting. Merrill Lynch, in its Sept. 28, 2002 10-Q filing, stated “VaR measures do not convey the magnitude of extreme events.” (Ref. 8). So percentile budgeting is misleading information. As Nassim Taleb, author of *The Black Swan*, has stated, “You’re worse off relying on misleading information than on not having any information at all. If you give a pilot an altimeter that is sometimes defective he will crash the plane. Give him nothing at all and he will look out the window.” (Ref. 9) The point is that average boring laid-back events are not what we should be safe-guarding against, but that’s what 70th percentile budgeting provides.

Coherent Risk Measures

A risk measure is a single number that is used to represent cost risk for a project or program. The variance of the distribution is a risk measure since it quantifies the spread in the cost distribution. Value at risk is another risk measure but there are many others.

What properties should a risk measure have? This issue has been studied in insurance specifically and in risk measurement in general. In a groundbreaking paper, Artzner et al. [Ref. 11] introduced the notion of coherent risk measures. One property important for a risk measure is that when two random variables are combined the risk measure of the portfolio should be no riskier than the sum of the individual random variables’ risk measures. That is, for any risk measure ρ, we should have that $\rho(X+Y) \leq \rho(X)+\rho(Y)$.

There should be some diversification benefit from combining risks, which is called *subadditivity*. Another desirable property for risk measures is that if one cost (X) is always higher than a second cost (Y), then the risk measure of X should be higher than the risk measure for Y. For example if the cost of system software is higher in every circumstance than the program management support, then the 70th percentile of the cost risk distribution should be higher for the software than for program management. This is the property of *monotonicity*, and can be stated in equation form as $X \leq Y \text{ for all possible outcomes} \Rightarrow \rho(X) \leq \rho(Y)$. A third desirable property is that the risk measure should be invariant of the currency in which the risk is measured, or whether cost is accounted for in thousands or millions of dollars. Also it means that an increase or decrease in exposure to the risk requires an equivalent change in the amount of capital needed to guard against this risk. This is the property of *positive homogeneity*, and can be expressed as $\rho(cX) = c\rho(X)$. And it is also important in measuring risk that if we add some certain fixed amount to a random variable, the risk does not change. This is the property of *translation invariance* and can be expressed as $\rho(X+c) = \rho(X)+c$. A coherent risk measure is defined as a risk measure $\rho(X)$ which has the four properties of *subadditivity*, *monoticity*, *positive homogeneity*, and *translation invariance*.

A simple and popular risk measure is the defined as the mean plus a fixed number of standard deviations, i.e., $\mu + k\sigma$, which is called the *standard deviation principle*. Note that this risk measure is subadditive, since

$$\mu_{X+Y} + k\sigma_{X+Y} = \mu_X + \mu_Y + k\sigma_{X+Y} \leq \mu_X + \mu_Y + k\sigma_X + k\sigma_Y$$

$$= \mu_X + k\sigma_X + \mu_Y + k\sigma_Y .$$

Also, the standard deviation principle is positive homogeneous, since

$$\mu_{c(X+Y)} + k\sigma_{c(X+Y)} = c\mu_{X+Y} + ck\sigma_{X+Y} = c(\mu_{X+Y} + k\sigma_{X+Y}).$$

And since standard deviation is not affected by a translation of the random variable, but the mean is shifted by exactly the translation, the standard deviation principle is translation invariant.

However, the standard deviation principle is not monotonic. To see this, consider a bivariate random variable defined as

$$p(X,Y) = \begin{cases} 0.25 \text{ for } X = 0, Y = 4 \\ 0.75 \text{ for } X = 4, Y = 4 \end{cases}$$

In this case, $\mu_X = 3, \mu_Y = 4, \sigma_X = \sqrt{3}, \sigma_Y = 0$. Note that even though $X \leq Y$ we have that

$$\mu_X + \sigma_X = 3 + \sqrt{3} > 4 = 4 + 0 = \mu_Y + \sigma_Y .$$

Note that the standard deviation principle is not the same as Value at Risk, unless we restrict our attention to normally distributed random variables. In this case *VaR* is a special case of the standard deviation principle with *k* set to satisfy whichever percentile is selected. In the case of the 70th percentile, $k \approx 0.5244$. In this case, *VaR* clearly satisfies the conditions of translation invariance, monotonicty, and positive homogeneity. It is also subadditive by the same rationale used for the standard deviation principle. Thus in the special case of normally distributed random variables, *VaR* is a coherent risk measure. However, in general, *VaR*, as a percentile of a cost distribution, is translation invariant, monotonic, and has positive homogeneity. However, it is not subadditive for non-normal random variables.

As an example, consider two random variables X, Y, each of which follows a Pareto distribution with $\alpha = \frac{1}{2}$, we have that $F(x) = 1 - x^{-\frac{1}{2}}$. Then by convolution

$$Pr(X + Y \leq z) = \int_1^{z-1}\int_1^{z-x} Pr(X = x)Pr(Y = y)\,dy\,dx$$

$$= \int_1^{z-1} \frac{1}{2} x^{-\frac{3}{2}} [-y]_1^{z-x}\,dx$$

$$= \int_1^{z-1} \frac{1}{2} x^{-\frac{3}{2}} \left(1 - (z-x)^{-\frac{1}{2}}\right) dx$$

$$= \int_1^{z-1} \frac{1}{2} x^{-\frac{3}{2}}\,dx - \frac{1}{2}\int_1^{z-1} x^{-\frac{3}{2}}(z-x)^{-\frac{1}{2}}\,dx$$

$$= \left[-x^{-\frac{1}{2}}\right]_1^{z-1} - \frac{1}{2}\int_1^{z-1} x^{-\frac{3}{2}}(z-x)^{-\frac{1}{2}}\,dx$$

$$= 1 - \frac{1}{\sqrt{z-1}} - \frac{1}{2}\int_1^{z-1} x^{-\frac{3}{2}}(z-x)^{-\frac{1}{2}}\,dx.$$

In order to integrate the remaining expression, set $u = \sqrt{x}$ and thus $u = \frac{1}{2\sqrt{x}}dx$. Therefore (ignoring integration limits for now),

$$1 - \frac{1}{\sqrt{z-1}} - \frac{1}{2}\int x^{-\frac{3}{2}}(z-x)^{-\frac{1}{2}}\,dx = 1 - \frac{1}{\sqrt{z-1}} - \int x^{-\frac{3}{2}}(z-x)^{-\frac{1}{2}}\,dx.$$

Now set $u = \sqrt{z}\sin(s)$, and thus $du = \sqrt{z}\cos(s)ds$. Then

$$\sqrt{z - u^2} = \sqrt{z - z\sin^2(s)} = \sqrt{z}\cos(s)$$

and

$s = sin^{-1}\left(\frac{u}{\sqrt{z}}\right)$.

Therefore,

$$1 - \frac{1}{\sqrt{z-1}} - \int \frac{1}{u^2\sqrt{z-u^2}}du = 1 - \frac{1}{\sqrt{z-1}} - \int \frac{csc^2(s)}{z}ds$$

$= 1 - \frac{1}{\sqrt{z-1}} - \left[\frac{cot(s)}{z}\right] + c$, where c is an arbitrary constant.

Note that

$$cot(s) = \frac{cos(s)}{sin(s)} = \frac{cos\left(sin^{-1}\left(\frac{u}{\sqrt{z}}\right)\right)}{\frac{u}{\sqrt{z}}} = \frac{\sqrt{cos^2\left(sin^{-1}\left(\frac{u}{\sqrt{z}}\right)\right)}}{\frac{u}{\sqrt{z}}}$$

$$= \frac{\sqrt{1 - sin^2\left(sin^{-1}\left(\frac{u}{\sqrt{z}}\right)\right)}}{\frac{u}{\sqrt{z}}} = \frac{\sqrt{1 - \frac{u^2}{z}}}{\frac{u}{\sqrt{z}}}$$

Thus,

$$1 - \frac{1}{\sqrt{z-1}} - \left[\frac{cot(s)}{z}\right] = 1 - \frac{1}{\sqrt{z-1}} - \frac{\sqrt{1 - \frac{u^2}{z}}}{z\frac{u}{\sqrt{z}}} = 1 - \frac{1}{\sqrt{z-1}} - \frac{\sqrt{1 - \frac{u^2}{z}}}{u\sqrt{z}}.$$

Since $u = \sqrt{x}$,

$$1 - \frac{1}{\sqrt{z-1}} - \frac{1}{2}\int_1^{z-1} x^{-\frac{3}{2}}(z-x)^{-\frac{1}{2}}dx = 1 - \frac{1}{\sqrt{z-1}} + \left[\frac{\sqrt{1 - \frac{x}{z}}}{\sqrt{xz}}\right]_1^{z-1}$$

$$= 1 - \frac{1}{\sqrt{z-1}} + \left[\frac{\sqrt{z-x}}{z\sqrt{x}}\right]_1^{z-1}$$

$$= 1 - \frac{1}{\sqrt{z-1}} + \frac{1}{z\sqrt{z-1}} - \frac{\sqrt{z-1}}{z}$$

$$= 1 - \frac{2\sqrt{z-1}}{z}$$

That is,

$$Pr(X + Y \leq z) = 1 - \frac{2\sqrt{z-1}}{z}$$

Note however that

$$Pr(2X \leq z) = Pr\left(X \leq \frac{z}{2}\right) = 1 - \left(\frac{z}{2}\right)^{-\frac{1}{2}} \geq 1 - \frac{2\sqrt{z-1}}{z} = Pr(X + Y \leq z),$$

when $z \geq 2$. Recalling that in this case $VaR_\alpha(X) = F^{-1}(\alpha)$ and noting that $VaR_\alpha(2X) = 2(1-\alpha)^{-2}$ and $VaR_\alpha(X) = (1-\alpha)^{-2}$ it follows that

$$VaR_\alpha(X + Y) > VaR_\alpha(2X) = VaR_\alpha(X) + VaR_\alpha(Y).$$

As another example, suppose there are two projects in a program, and that there is a 50% chance of a funding shortfall for the program. If the shortfall occurs, it will impact the project with the lesser amount of progress at that time. Assume both projects have the same schedule, that each schedule is independent, and that it is equally likely that each project's funding will be cut. There is then a 25% chance of a shortfall occurring to either project. A shortfall in funding will mean that funds will not be available when needed, leading to schedule delays, which will in turn lead to cost growth. Suppose that the impact of the shortfall will be to increase the cost by $20 million. Suppose that the budget for each project is $100 million and the funding shortfall is the only risk. Then the 70th percentile for each project is $100 ($120 million is the 75th percentile). So $VaR_{0.70}$ funding for the projects is $200 million. But $VaR_{0.70}$ funding for the program is $220 million, since the 50th percentile for the program is $220 million. Thus the overall portfolio is riskier than the sum of the individual projects. Thus not only does the portfolio not exist (Refs. 4 and 5), for percentile funding it is possible to have a reverse portfolio effect!

Conditional Tail Expectation

NASA and other agencies also need to set policy on a course of action once the 70% mark is exceeded. Percentile funding is not a risk management policy. As we have discussed it simply determines when a bad situation has occurred, but says nothing about what course of action to take once a bad event (cost has exceeded a specified percentile has occurred). Indeed if funding is set at the percentile, there is no guidance on how to proceed. A better policy is one that would have funds set aside in the case of a bad event. One useful risk measure that comes in handy in such cases is conditional tail expectation.

This is defined as the amount of cost growth to expect given that cost has exceeded a specified amount, that is

$$E\left[X \mid X > Q_\alpha\right]$$

where Q_α is a specified percentile. This risk measure is referred to as Condtional Tail Expectation (*CTE*). For example, $Q_{0.95}$ is the 95th percentile. This risk measure is also called the "Tail Value at Risk" and the expected shortfall (Ref. 10). It is the "Tail Value at Risk" since in the case of continuous cost distributions it may be viewed as

$$CTE_\alpha = \text{EMBED Equation.3} = \frac{1}{1-F(Q_\alpha)}\int_{Q_\alpha}^{1} x f(x)\,dx = \frac{1}{1-\alpha}\int_{\alpha}^{1} VaR_u(X)\,du$$.

The latter can easily be derived by setting $u = F(x)$ and making a simple substitution. Note also that CTE_α can be written as

$$CTE_\alpha = VaR_\alpha + \frac{\int_{VaR_\alpha}^{\infty}(x - VaR_\alpha)f(x)\,dx}{1-\alpha} = VaR_\alpha + \frac{E[X] - E[X_\alpha^{VaR}]}{1-\alpha}$$.

Note that for a normal distribution, $VaR_\alpha(X) = \mu + \sigma\Phi^{-1}(\alpha)$, and that

$$CTE_\alpha(X) = \mu + \sigma E\left[\frac{X-\mu}{\sigma}\middle|\frac{X-\mu}{\sigma} \geq Q_\alpha\left(\frac{X-\mu}{\sigma}\right)\right]$$

Note that

$$E\left[\frac{X-\mu}{\sigma}\middle|\frac{X-\mu}{\sigma} \geq Q_\alpha\left(\frac{X-\mu}{\sigma}\right)\right] = \frac{1}{1-\alpha}\int_{\Phi^{-1}(\alpha)}^{\infty} x\phi(x)\,dx$$

$$= \frac{1}{1-\alpha}\left[-\phi(x)\right]_{\Phi^{-1}(\alpha)}^{\infty}$$

$$= \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}$$

Where ϕ represent the standard normal density function and Φ^{-1} represents the inverse of the cumulative normal distribution. Therefore for a normal distribution,

$$CTE_\alpha(X) = \mu + \sigma\frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}$$

For a lognormal distribution,

$$CTE_\alpha = VaR_\alpha + \frac{E[X] - E[X \wedge VaR_\alpha]}{1-\alpha}$$

Note tha $E[X \wedge VaR_\alpha] = \int_0^{VaR_\alpha} \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{1}{2}\left(\frac{lny-\mu}{\sigma}\right)^2\right)dy$ t, and setting $z = \frac{lny-\mu-\sigma^2}{\sigma}$, the integral simplifies to

$$exp\left(\mu+\frac{\sigma^2}{2}\right)\int_{-\infty}^{\frac{lnVaR_\alpha-\mu-\sigma^2}{\sigma}} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{1}{2}z^2\right)dz = E[X]\left[\Phi\left(\frac{\ln VaR_\alpha-\mu-\sigma^2}{\sigma}\right)\right].$$

Thus the *CTE* for the lognormal distribution can be written as

$$VaR_\alpha + \frac{E[X] - E[X]\left[\Phi\left(\frac{\ln VaR_\alpha-\mu-\sigma^2}{\sigma}\right)\right] - VaR_\alpha(1-\alpha)}{1-\alpha}$$

$$= \frac{E[X]\left[1 - \Phi\left(\frac{\ln VaR_\alpha-\mu-\sigma^2}{\sigma}\right)\right]}{1-\alpha},$$

where Φ is the cumulative normal distribution function. For example, for a single project for which cost risk has been modeled as a lognormal distribution with mean equal to $100 million and standard deviation equal to $50 million, $\mu = 4.49$, $\sigma = 0.72$, and the 70th percentile is equal to

$$e^{4.49+z_{0.70}\cdot 0.72} \approx \$114.6\, million.$$

Thus, in this instance,

$$CTE_{0.70} = 100 \cdot \frac{1-\Phi\left(\frac{\ln 114.6 - 4.49 - 0.47^2}{0.47}\right)}{1-0.7} \approx \$159.7\ million.$$

Therefore, given that the 70th percentile has been reached, the expected amount needed to complete the project will be $160 million, roughly $46 million above the 70th percentile budget. This is 40% more than the budget.

In addition to providing a true risk management approach, in that the conditional tail expectation provides not only a trigger (the Value at Risk event), but also an additional amount of reserves set aside in case bad times occur, conditional tail expectation provides information about the right tail, additional detail relevant to a sensible risk management policy. And conditional tail expectation is a coherent risk measure. Most of the coherence properties follow naturally from properties of percentiles, in particular positive homogeneity and translation invariance. Monotonicity naturally follows, since if ***X*** is always less than or equal to ***Y***, the conditional expected value of ***X*** greater than some fixed value will always be less than the conditional expected value of $\boldsymbol{Y}$ for that same fixed value. To see that subadditivity holds note that

$$CTE_\alpha(X) + CTE_\alpha(Y) - CTE_\alpha(X+Y)$$

$$= E[X|X > Q_\alpha] + E[Y|Y > Q_\alpha] - E[X+Y|X+Y > Q_\alpha]$$

$$= E[X|X > Q_\alpha] - E[X|X+Y > Q_\alpha] + E[Y|Y > Q_\alpha] - E[Y|X+Y > Q_\alpha]$$

which can easily seen by definition to be greater than zero since both expected value differences are nonnegative, proving subadditivity.

Another problem with *VaR*, or percentile funding, that is solved by *CTE* is what the author has termed the "Lognormal Paradox." As discussed in Smart (Ref. 4), with funding levels at or below the 84^{th} percentile, for a common mean and standard deviation, a normal distribution will require more funding than a lognormal distribution even though the lognormal has a heavier right tail, and hence is riskier. This is contrary to common sense, which tell us that riskier events should require greater funding. See Figure 4 for a graphical comparison. As is evident from Figure 4, for percentiles between the 23^{rd} and 84^{th} percentiles, the normal distribution has higher percentile levels than the lognormal distribution. This is despite the fact that the means and standard deviations are the same and the lognormal is riskier than the normal distribution, all else being equal. The cause of the paradox is that percentile funding does not take into account the full right tail, which is where the lognormal's risk is located.

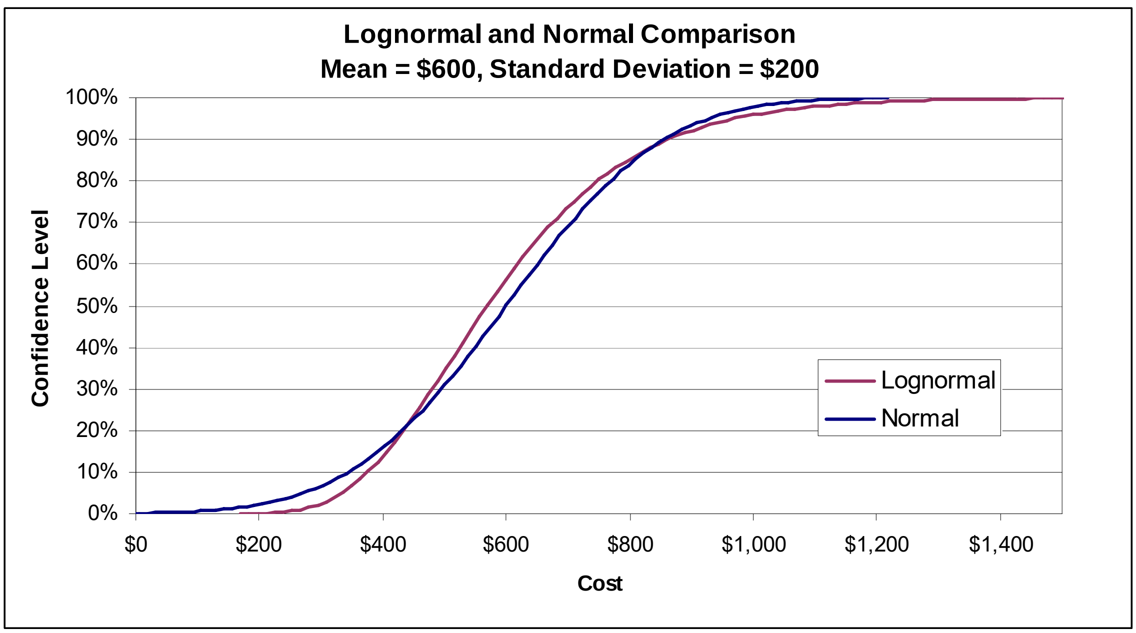

Figure 6. Comparison of a Lognormal and Normal Distribution Percentiles with Common Mean = $600 and Common Standard Deviation = $200.

However conditional tail expectation does not suffer this shortcoming, precisely because it takes the full right tail into account. See Table 2 for a comparison. For example in the

	Mean = \$600, Standard Deviation = \$200								
	α =	50.0%	60.0%	70.0%	80.0%	90.0%	95.0%	99.0%	99.9%
VaR	Normal	\$600	\$651	\$705	\$768	\$856	\$929	\$1,065	\$1,218
	Lognormal	\$569	\$618	\$675	\$748	\$863	\$971	\$1,211	\$1,552
CTE	Normal	\$760	\$793	\$832	\$880	\$951	\$1,013	\$1,133	\$1,273
	Lognormal	\$753	\$793	\$842	\$908	\$1,016	\$1,120	\$1,359	\$1,704

Table 2. Comparison of VaR and Conditional Tail Expectation.

table, the percentiles for the lognormal and normal do not cross until the 90th percentile for Value at Risk, while for *CTE* the lognormal is greater than the normal for all percentiles above the 60th percentile. Thus *CTE* has another advantage, in that it is a more sensible policy.

Note that *CTE* funding requires additional money above and beyond strict percentile funding. This decreases as a percentile of the percentile as the percentile increases. For lognormal funded at the 70th percentile the additional funding needed to make up the expected shortfall is approximately 25% greater, while at the 80th percentile it is 21% more.

Conditional tail expectation is simple to calculate. When a lognormal or normal distribution is used to represent total cost risk, as in the NASA/Air Force Cost Model, the formulas presented in this paper can be used to calculate the conditional tail expectation for any percentile. And conditional tail expectation can be easily calculated when Monte Carlo simulation is used to estimate cost risk. For example in a 10-trial Monte Carlo simulation of a normal distribution with mean equal to \$600 and standard deviation equal to \$200, whose trial values are shown in Table 3, the 70th percentile represents values above \$687.21, so to calculate $CTE_{0.70}$ we take the mean of the three values \$732.19, \$755.82, and \$779.58, which is equal to \$755.86. And since conditional tail expectation is a mean, its calculation should require fewer trials to accurately measure than percentiles, which require more trials.

1	379.69
2	450.73
3	451.91
4	504.46
5	548.09
6	661.94
7	687.21
8	732.19
9	755.82
10	779.58

Table 3. Monte Carlo Trial Values.

Conditional tail expectation was introduced in the late 1990s and quickly became the preferred standard for setting liabilities for insurance settings. In Canada, the "actuarial Standards of Practice promulgate the use of the CTE whenever stochastic methods are used to set balance sheet liabilities" (Ref. 12). It is also the basis for the Swiss Solvency Test (Ref. 13), which forms a major part of Swiss insurance policy. And the National Association of Insurance Commissioners recommends setting reserves using CTE (Ref. 14).

The Practical Impact of Conditional Tail Expectation

Percentile funding is still being implemented as a policy by government agencies. The question still outstanding is whether or not this policy will be effective in containing cost growth. Fewer missions overall should experience cost growth, but what about those which do? As we have shown percentile funding is not a true risk management policy as additional funding, perhaps a significant amount, will be required much of the time, as much as 30% or more. This will likely be more, since even with 70th percentile funding, cost risk analyses typically explicitly exclude extreme events that occur from time to time, such as strikes, "acts of God," and other external factors beyond a project's control. However, should the overall amount needed above and beyond the 70th percentile be a relatively small amount, perhaps a percentile funding policy will help to stem cost growth. But we will show that empirical data indicates this is yet another pipe dream.

To gain an understanding of how much additional funding will be required for percentile funding in practice, it is useful to examine historical cost growth data. As discussed by Smart (Ref. 5), for a data set of cost growth for 112 recent NASA missions, the minimum cost growth was -25.2% for Super Light Weight Tank (SLWT), an upgrade for the Shuttle Program from a more traditional aluminum structure to aluminum-lithium. The negative number means that costs under ran their initial budget by approximately 25%. (Contrary to popular belief, missions occasionally come in under budget.) For the current study, 14 such missions experienced under runs, which is12.5% of the missions studied. Only two of the missions hit their budget target spot on. Nine of the missions were within 5% of the initial budget, and 19 within 10% (either above or below).

The maximum cost growth among the missions studied was 385% for the Hubble Space Telescope and Space Telescope Assembly, which suffered from several sources of traditional cost growth, including funding constraints, launch vehicle delays, and under-estimation of the time and resources necessary to develop the requisite technology.

A range from -25% on the low side to over 350% on the high end is a wide range. The average cost growth for all missions was 53.0%, with median growth equal to 32.1%. The difference between the mean and median indicates a high degree of positive skew in the data, with most missions experiencing relatively small amounts of cost growth (half experienced growth less than 33%), with some missions experiencing extreme amounts of cost growth, such as Hubble and others. Overall, seventeen missions had cost growth in excess of 100%, which means cost more than doubled. While representing only about 15% of the cost growth data we will see that growth of this severity, while not common, occurs often enough to offset any hoped-for portfolio effect. See Figure 7 for a graphical summary of these data.

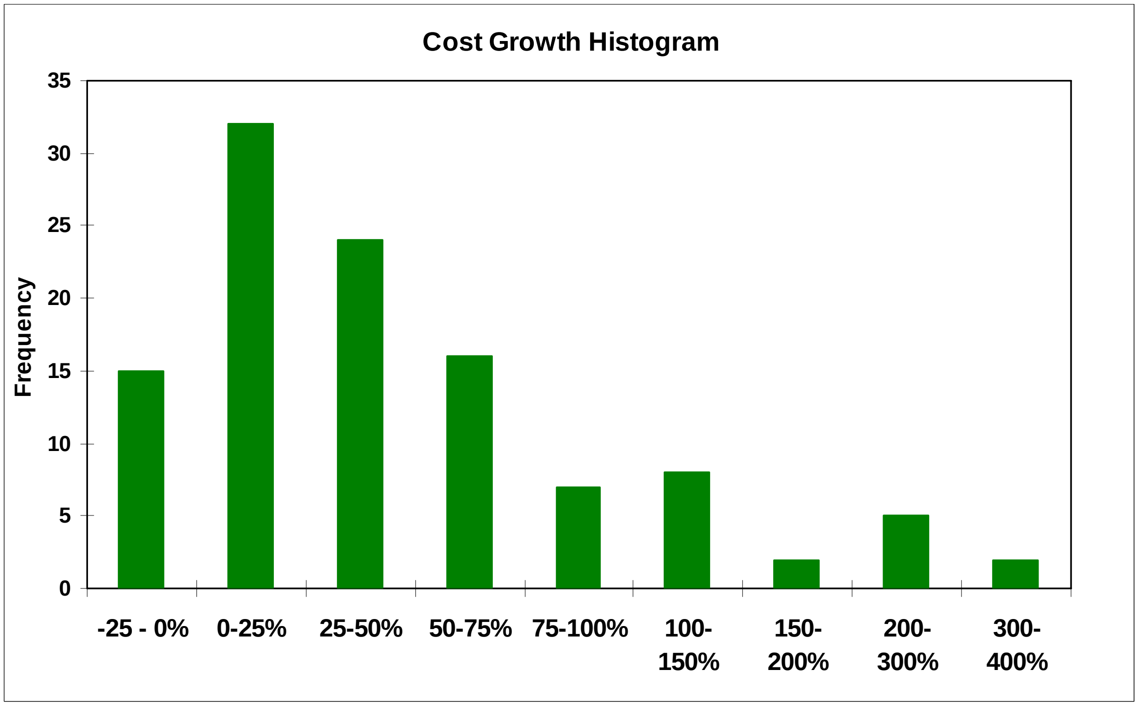

Figure 7. Graphical Summary of NASA Cost Growth.

Cost risk is the probability that an estimate will exceed a specified amount, such as $100 million or $150 million. Cost growth and cost risk are thus intrinsically related. Historical cost growth provides an excellent means for determining the overall level of risk for cost estimates. For example, if 95% of past programs have experienced less than 100% growth, we should expect that the ratio of actual cost to the initial estimate should be less than 100% with 95% confidence. Thus cost growth is the impact of cost risk in action. Because of uncertainty in historical data, cost models, program parameters, etc., the term "cost risk" is redundant. Thus characteristics of this cost growth data set determine characteristics seen in a cost risk distribution that is consistent with cost growth. In Smart (Ref. 5) it was shown that the cost growth data closely follow a lognormal distribution with a coefficient of variation equal to 100%. This implies a significant amount of cost risk, and is much higher than is typically modeled by cost analysts.

Table 4 displays the additional amount expected to be required if the budget is exceeded for lognormal cost risk distributions. Note that this amount ranges from around 10% if the budget is set at the 90^{th} percentile and the coefficient of variation of the lognormal cost risk distribution is 20%, to 185% if the budget is set at the 30^{th} percentile and the coefficient of variation is 100%. Note that for the case of NASA, with 70^{th} percentile funding and a coefficient of variation implied by the data of 100%, for missions that experience cost overruns beyond the 70^{th} percentile, on average an additional 89% funding will be needed to complete such projects. Here there be dragons indeed! This is a sobering amount. Thus 30% of the time, approximately 90% more money will be needed, and indicates that even if risk models are calibrated to empirical cost growth experience,

		Budget Set at						
		30th	40th	50th	60th	70th	80th	90th
Coefficient of Variation	20%	23.6%	20.5%	18.0%	15.9%	14.0%	12.2%	10.2%
	30%	38.0%	32.7%	28.5%	25.0%	21.9%	19.0%	15.8%
	40%	54.1%	46.1%	40.0%	34.9%	30.4%	26.2%	21.6%
	50%	72.0%	60.9%	52.4%	45.5%	39.4%	33.7%	27.7%
	60%	91.6%	76.8%	65.7%	56.7%	48.8%	41.5%	33.9%
	70%	112.7%	93.8%	79.7%	68.3%	58.5%	49.5%	40.1%
	80%	136.0%	112.0%	94.4%	80.5%	68.6%	57.6%	46.4%
	90%	160.0%	131.0%	110.0%	93.0%	78.8%	65.9%	52.7%
	100%	185.0%	151.0%	125.5%	105.8%	89.2%	74.2%	59.0%

Table 4. Additional Amount Expected if Budget Exceeded.

so, the average project should expect to experience 27% growth. And 30% of the time, missions will continue to experience an embarrassing amount of cost growth. While an improvement over the current 53% average growth, we can see that percentile funding will not be the hoped-for panacea, but only a band aid where major surgery is required.

Thus, reserve setting cannot stop with simply setting reserves at a relatively high confidence level. NASA and other agencies should expect to regularly spend much, much more.

Summary

Current risk management policy for NASA and other agencies consists largely of setting reserves at a fixed percentile, popularly known as "Value at Risk" or "VaR." This has much in common with the banking industry, which used this concept in setting risk reserves. However this policy ignores the tails of risk distributions, leading to a dangerous ignorance. And funding to a percentile does not even provide a cushion for when times are bad; they simply tell you times are bad. Percentile funding will not lead to an end to cost growth – empirical evidence suggest that 70th percentile funding will still result in a significant amount of cost growth on average. And running to Congress when a fixed percentile is exceeded and asking for more money is not a risk management policy, but reflects the lack of maturity and discipline required to fully implement sophisticated and meaningful risk management. And worst of all, percentile funding can result in a reverse portfolio effect, which means that NASA as a whole could be riskier than any single project! A better policy would be to use a risk measure that takes into account the tails of the distribution such as conditional tail expectation. Such a policy provides both a signal of a bad event (*VaR* is exceeded) as well as a cushion for the expected amount of money to guard against this event. Conditional tail expectation is a simple measure, represented by a single number just like percentile funding, and can be easily explained to senior management and project managers, since it is simply the additional amount of money required to fund a project in those cases when the percentile funding is breached. It need not be significantly more expensive than the current percentile funding policies. Since it takes into account the full right tail of the distribution a lower level threshold, such as the 60th or 70th percentile could be chosen for the trigger.

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