Capturing Risk Interdependencies: The CONVOI Method

Blake Boswell
Mike Manchisi
Eric Druker

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Introduction

Introduction: Shortcomings of Most Risk Analysis Methodologies

- Programs identify risks with the intention of them being independent, and not affected by other risks, however it is rare that this is the case
  - Systems are often dynamic and interconnected with resulting costs that are heavily influenced by risk interdependencies
  - It is common for risks that occur at the outset of a system to be determining factors in the likelihood of occurrence of later risks

- Analyzing interdependencies among risks is problematic due to the independent nature in which risk data is created
  - Risks are often generated by groups of technical experts focused on *their part* of the system
    - Program's risk registers are populated with marginal probabilities of risk occurrence that have not been examined aggregately
    - Even if generated by the whole program team it is rare for the relationships between risks to be examined

- Efforts for incorporating risk interdependence through correlation exist, but with no rationale, only with the goal of increasing the CV

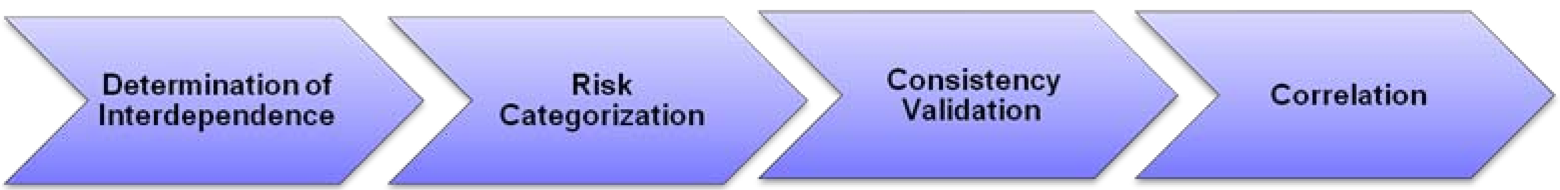
Introduction: Benefits of Analyzing Risk Interdependence

- With CONVOI, accounting for risk interdependence is **simple**, requiring almost no additional work outside the scope of traditional risk analysis/management efforts, and provides the following benefits:
  - More accurate cost and schedule risk assessments to allow for more accurate program budgeting
  - The ability to validate likelihood of occurrence for some risks
  - A glimpse into the root causes of risks, leading to more optimal risk mitigation plans
- Addresses two of the most common complaints about risk register based risk analysis
  - Underestimation of mean costs
  - Underestimation of variability
- To accomplish this, risks from a program's risk register are correlated in the risk analysis using a cutting edge algorithm that can be incorporated into all COTS risk analysis tools
  - Crystal Ball, @Risk, @Risk for Project, ACEIT, Risk+
  - The process can also be incorporated into custom tools as needed

The CONVOI Process

The CONVOI Process: Overview

- The CONVOI method involves four tasks which are divided into two main phases: Consistency Verification and Conditional Odds Integration
  - The **Consistency Verification Phase** makes up the first three tasks and involves comparative analysis of risks in a programs risk register
    - Determination of Risk Interdependencies
    - Risk Categorization
    - Consistency Validation
  - The **Conditional Odds Integration Phase** involves the correlation of risks based on conditional and marginal probabilities of occurrence
    - The result is a correlation matrix compatible with COTS risk analysis tools
    - Allows for the correlation of random variable draws so that they maintain the desired marginal and conditional probabilities



The CONVOI Process: Consistency Verification

- **Determination of Risk Interdependencies**
  - The first task in the Consistency Verification phase is the collection of marginal and conditional probabilities of risk occurrence
    - These probabilities are estimated by technical experts
    - Data can be gathered from the program's risk register or through consults with SMEs
- **Risk Categorization**
  - In this task risks are categorized according to their dependence relations with other risks. Risks are classified as *Drivers* or *Passengers*
    - *Driver Risk* - A Driver risk is a risk that is not affected by the occurrence of other risks. The probability of occurrence of a Driver risk is treated as a constant. (Denoted by *D*)
    - *Passenger risk* – A Passenger risk is a risk that is affected by a Driver risk. Probability of occurrence of a passenger risk varies with the occurrence or non-occurrence of its related Driver risk. (Denoted by *y*)
    - In order for there to be an established relationship between a Passenger and Driver there must be known estimates for the marginal probability of the Driver, $P(D)$, and the conditional probability of the Passenger given the Driver, $P(y|D)$

The CONVOI Process: Consistency Verification

- The following are important characteristics of the Driver – Passenger relationship:
  - *There is no implied causal relationship between Drivers and Passengers*
    - The phrase “affected by” is only meant to entail a measurable relationship between a Passenger risk and a previously occurring Driver risk
  - *Driver risks are not statistically independent*
    - Since a Driver risk impacts the likelihood of occurrence of other risks in the system, it has a conditional dependence relationship with those risks
    - Examples of possible Driver risks are events exogenous to the system or risks that occur at the outset of the system
  - *Drivers and Passengers can have multiple relationships*
    - A Driver risk can impact the likelihood of occurrence of multiple Passengers. A passenger risk can be affected by the occurrence of multiple Driver risks

- **Risk Validation**
  - The final task is determining a range of feasible values for passenger risks
    - The feasible range is created through application of the Law of Alternatives

The CONVOI Process: Conditional Odds Integration

- In the Conditional Odds Integration risks are correlated based upon marginal and conditional probabilities of risk occurrence
  - The correlation algorithm was developed through the analysis of the dependence structures resulting from correlated Monte Carlo simulations
    - The algorithm utilizes a mapping of Rho based upon input marginal and conditional probabilities
    - The result is the ability to instantly retrieve a correlation coefficient that when applied to correlated Monte Carlos simulations will create random number draws that preserve the desired marginal and conditional probabilities
    - The process for correlating normal random variables followed most closely in the development of CONVOI is the Lurie-Goldberg Method [1]

- Find matrix L such that series of transformations

$$\underset{indep.normal}{X} \xrightarrow{L} \underset{mult.normal}{Y} \xrightarrow{\Phi} \underset{uniform}{U} \xrightarrow{F^{-1}} \underset{desired}{V}$$

  lead to random variables with desired correlations and marginal distributions
  - L: Cholesky factor transforms independent normals to correlated normals
  - Φ: normal c.d.f. transforms correlated normals to correlated uniforms
  - F^{-1}: transforms correlated uniforms to correlated random variables with desired marginal distributions F

[1] Goldberg, Matthew S, Lurie, Phillip M. *Correlating Random Variables*, 32nd DoDCAS, Williamsburg, VA. February 1999

CONVOI: A Case Study

CONVOI: A Case Study

- Consider two risks from a satellite program's risk register:
  - Late delivery of accelerometer (probability 75%, impact $400K)
  - Availability of Environmental Testing Facilities (probability 20%, impact $100K)

- After interviews with SMEs are conducted it is revealed that if the accelerometer is delivered late then environmental testing facilities are almost certainly not going to be available

CONVOi

Risk Consistency Verification and Conditional Odds Integration Tool

Generate List | Check | Clear | Size of List: 100

#	Risk Description	P(#)	Cost Impact	Schedule Impact	Driver	P(#|D)	Error	min	max	Rho
1	Late delivery of accelerometer	0.75	$ 400,000.00	Tri(4 weeks, 8 weeks, 10 weeks)	0	0				
2	Contract delay	0.6	$ 20,000.00	Tri(1 week, 2 weeks, 4 weeks)	0	0				
3	Availability of Enviormental Testing Facilities	0.2	$ 100,000.00	Tri(2 weeks, 4 weeks, 6 weeks)	1	0.99				
4	Delay of development process	0.23	$ 10,000.00	Tri(1 week, 2 weeks, 3 weeks)	2	0.26				
5	Excess staff	0.48	$ 5,000.00	Tri(1 week, 2 weeks, 3 weeks)	2	0.75				
6										
7										

Instructions | CONVOi

Initial probabilities of risk occurrences are entered into the P(#) column.

Risk 3 is affected by the occurrence of risk 1, therefore a 1 is put in the driver column, and the conditional probability is entered in the P(#|D) column.

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CONVOI: A Case Study

CONVOi

Risk Consistency Verification and Conditional Odds Integration Tool

Size of List: 100 | Generate List | Check | Clear

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2	Contract delay	0.6	$ 20,000.00	Tri(1 week, 2 weeks, 4 weeks)	0	0	Driver				
3	Availability of Enviormental Testing Facilities	0.2	$ 100,000.00	Tri(2 weeks, 4 weeks, 6 weeks)	1	0.99	Passenger	P(#) must be in the following range:	0.7425	0.9925	
4	Delay of development process	0.23	$ 10,000.00	Tri(1 week, 2 weeks, 3 weeks)	2	0.26	Passenger	Risk is Consistent			0.22
5	Excess staff	0.48	$ 5,000.00	Tri(1 week, 2 weeks, 3 weeks)	2	0.75	Passenger	Risk is Consistent			0.89
6											
7											

Instructions | CONVOi

- Checking for consistency reveals that the initial estimate of the likelihood of occurrence for the *Availability of Environmental Testing Facilities* risk is not feasible. The range of consistent probabilities is given in the min and max columns.

- Once risks are verified as consistent a correlation matrix is generated.
  - This correlation matrix can be input into any COTS tool or homegrown risk engine.

CONVOi **Correlation Matrix**

1	0	1	0	0
0	1	0	0.22	0.89
1	0	1	0	0
0	0.22	0	1	0
0	0.89	0	0	1

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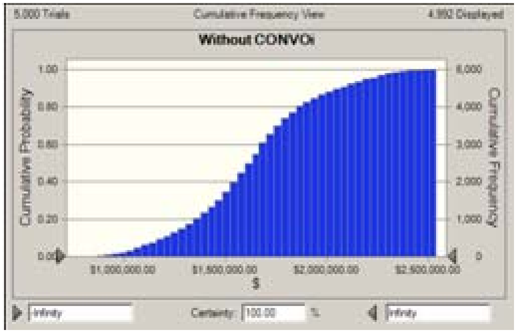
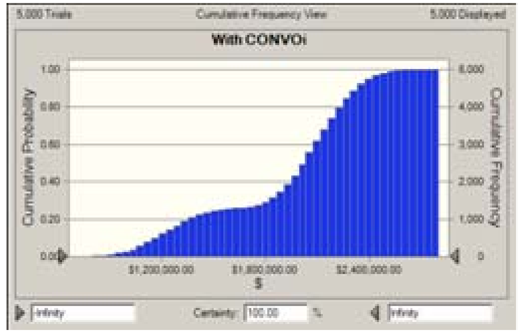
CONVOI: A Case Study

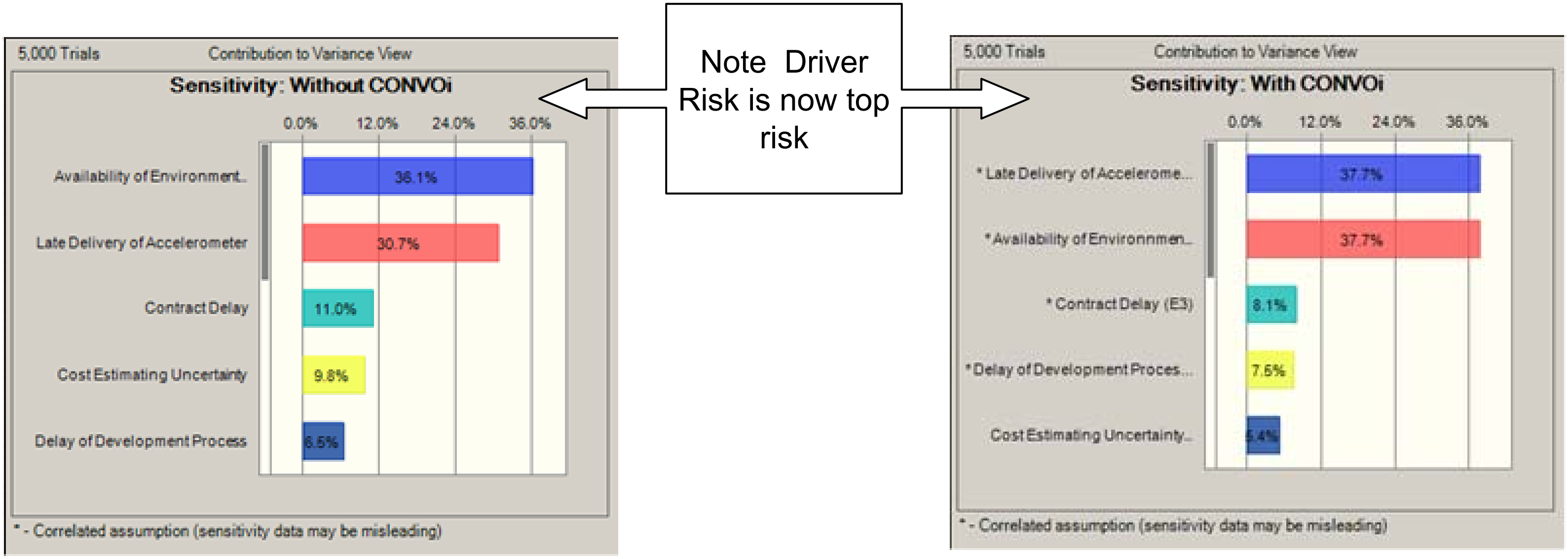




	Without CONVOi	With CONVOi
Mean	$1,638,589.49	$1,903,272.02
Median	$1,633,114.10	$2,042,525.58
Standard Deviation	$320,997.63	$432,086.69
Variance	$103,039,476,879.81	$186,698,903,546.96
Skewness	0.1797	-0.7924
Kurtosis	2.96	2.46
Coeff. of Variability	0.1959	0.227
Minimum	$760,568.85	$713,005.89
Maximum	$2,651,932.23	$2,802,199.60
Mean Std. Error	$4,539.59	$6,110.63

- The results of risk analysis following the CONVOI process are more accurate
  - Increasing the likelihood of occurrence of the *Availability of Environmental Testing Facilities* risk increases the most likely cost of the satellite by 16%
  - The introduction of correlation increases the coefficient of variability by 16%
- Two of the most common complaints about risk register based risk analysis are that its resulting most likely cost and variability are too small
  - This method addresses both of those issues without using arbitrary correlation coefficients

CONVOI: A Case Study



- Sensitivity analysis on the standard risk program shows the *Availability of Environmental Testing Facilities* risk *as* having the greatest cost impact on the program

- CONVOI, which focuses on the interdependencies between risks, reveals that cost risk is actually being driven by the *Late delivery of the Accelerometer* risk because of the influence it has on the *Availability of Environmental Testing Facilities* risk

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Consistency Verification

The CONVOI Process: Consistency Verification

- Consistency Validation

  - *The Law Of Alternatives*
    - **Definition:** Let $A_1, A_2, A_3, \ldots, A_n$ be a set of nonempty subsets of a sample space S. If the events $A_1, A_2, A_3, \ldots, A_n$ are mutually exclusive and $\cup A_i = S$, then the sets $A_1, A_2, A_3, \ldots, A_n$ are called a *partition* of S
    - **Theorem:** If $A_1, A_2, A_3, \ldots, A_n$ is a partition of the sample space of an experiment and $P(A_i) > 0$ for i = 1, 2, 3, ... , n, then

$$P(S) = \sum_n P(S|A_n)P(A_n)$$

Image from: http://en.wikipedia.org/wiki/File:Set_partition.svg

The CONVOI Process: Consistency Verification

- CONVOI utilizes the Law of Alternatives to determine consistency ranges for passenger risks under the following scenarios
  - One or more Passengers affected by a single Driver
  - One or more Passengers affected by multiple Drivers
  - One or more Passengers affected by a single Passenger
  - One or more Passengers affected by multiple Passengers
- All scenarios require that Driver risks meet the requirements set forth in the definition of a *partition*
  - Two or more Drivers/Passengers impacting a single risk must be mutually exclusive with respect to each other
  - The union of the occurrences of Driver/Passenger risks that impact a Passenger must account for all possible outcomes of that Passenger

The CONVOI Process: Consistency Verification

- Passengers affected by single Drivers
  - The conditions set forth in the definition of *partition* are met and therefore the total probability of the Passenger risk y can be expressed as follows

$$P(y) = P(D)P(y|D) + P(D')P(y|D')$$

- If $P(y|D')$ is known, then the total probability of the occurrence of the Passenger can be determined explicitly
- If $P(y|D')$ is unknown, a range of possible values for $P(y)$ can be determined due to the fact that $P(y|D') \in [0,1]$
- The total probability for $P(y)$ is maximized when $P(y|D')=1$ and minimized when $P(y|D')=0$ therefore the range of possible values for $P(y)$ is defined as follows

$$P(D)P(y|D) \leq P(y) \leq P(D)P(y|D) + P(D')$$

The CONVOI Process: Consistency Verification

- Passengers affected by multiple Drivers
  - Applying the Law of Alternatives $\mathrm{P(y)}$ can be defined as follows

$$P(y) = \sum_n P(y|D_n)P(D_n)$$

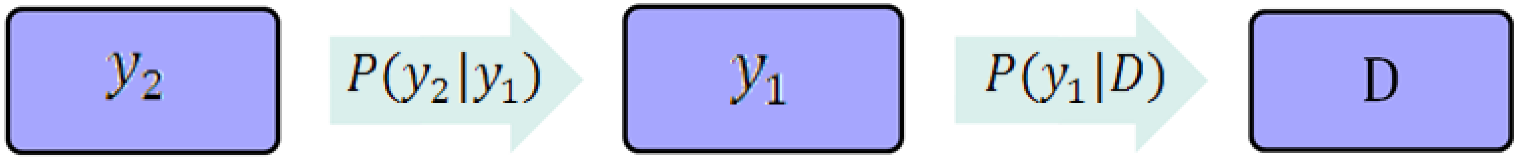
  - If the Drivers form a partition of y then $\mathrm{P(y)}$ can be defined explicitly.
  - If the occurrences of the driver risk do not account for the entire sample space then a new Driver must be defined

$$D_{n+1} = \bigcap_n D'_n \quad \text{such that} \quad P(D_{n+1}) = 1 - \sum_n P(D_n)$$

  - Analogous to the one driver case the unknown conditional probability $P(y|D_{n+1}) \in [0,1]$
  - Hence a range of possible values for $\mathrm{P(y)}$ can be defined as follows

$$\sum_n P(y|D_n)P(D_n) \le P(y) \le \sum_n P(y|D_n)P(D_n) + 1 - \sum_n P(D_n)$$

The CONVOI Process: Consistency Verification



- Passengers affected by a single Passenger
  - Similar to a Passenger affected by a single Driver the consistency range is as follows

$$P(y_1)P(y_2|y_1) \leq P(y_2) \leq P(y_1)P(y_2|y_1) + P(y_1')$$

- $P(y_1) \in [\min_{y_1}, \max_{y_1}]$ where min and max are the bounds of the range for $P(y_1)$
- Algebraic manipulation of the RHS of the range for $P(y_2)$ reveals

$$P(y_1)P(y_2|y_1) \leq P(y_2) \leq 1 - P(y_1)\left(1 - P(y_2|y_1)\right)$$

- It follows that the consistency range for $P(y_2)$ is established by setting $P(y_1)$ equal to its minimum possible value.
- This is the case no matter how many degrees of separation there are between the Passenger and Driver.

The CONVOI Process: Consistency Verification

- Passenger affected by multiple Passengers
  - The consistency range of a Passenger affected by multiple Passengers is similar to the range defined for Passengers affected by multiple Drivers and can be rearranged as follows

$$\sum_n P(y|y_n)P(y_n) \le P(y) \le 1 - \sum_n P(y_n)(1 - P(y|y_n))$$

- $P(y_n) \in [\min_{y_n}, \max_{y_n}]$, for each n
- The range for $P(y)$ can be established by setting each $P(y_n)$ to its minimum value
- This result applies no matter how man degrees of separation exist between y and each D_n
- This result also applies to a Passenger affected by a mix of Passengers and Drivers as long as the set of Passengers and Drivers form a partition of y

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The CONVOI Process: Consistency Verification

- The calculations involved in the Consistency Verification Phase can be implemented quickly and with little effort
  - In the CONVOI tool, the calculations are automated via the use of VBA for Excel
- Upon completion of the Consistency Verification calculations, a range of possible values has been established for all marginal passenger probabilities
  - Initial estimates of marginal Passenger probabilities that do not fall within the defined consistency range must be revisited
    - One possible method for correcting inconsistent marginal probability estimates is to redefine the probability as the closest value within the feasible range
    - Discussing the inconsistency with a technical expert can produce a new more accurate estimate for the marginal probability of the Passenger, or may reveal errors in the estimated Driver probability and conditional probability
    - Once probability values are updated the calculation process is reiterated
- As a result of the analysis in the Consistency Verification Process risks in a programs risk register are more accurate

Conditional Odds Integration

The CONVOI Process: Conditional Odds Integration

- CONVOI measures correlation with Pearson's Product Moment Correlation Coefficient (Rho)

$$\rho_{x,y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_y} = \frac{\text{E}[XY] - \text{E}[X]\,\text{E}[Y]}{\sqrt{\text{E}[X^2] + \text{E}[X]^2}\sqrt{\text{E}[Y^2] + \text{E}[Y]^2}}$$

  - Pearson's correlation coefficient ranges from −1 to 1. A value of 1 implies that there exists a perfect increasing linear relationship between two variables. A value of −1 implies a perfect decreasing linear relationship between two variables. A value of 0 implies that there is no linear correlation between the variables

- Most commercial risk programs (e.g. Crystal Ball & @Risk) use Spearman's rank correlation rather than Pearson's correlation because it is easier to simulate
  - Rank correlation is concerned with whether or not the function is monotonic

- Past research has shown that in cost risk simulations using the most common families of distributions (Normal, Log-Normal, Triangular, Bernoulli), the two yield fairly similar results[1]

[1] Robinson, M and Salls, W. *More on Correlation Accuracy in Crystal Ball Simulations (or What We've Now Learned about Spearman's R in Cost Risk Analyses)*. Presented at the 2004 SCEA Conference, Manhattan Beach, CA, June 2004

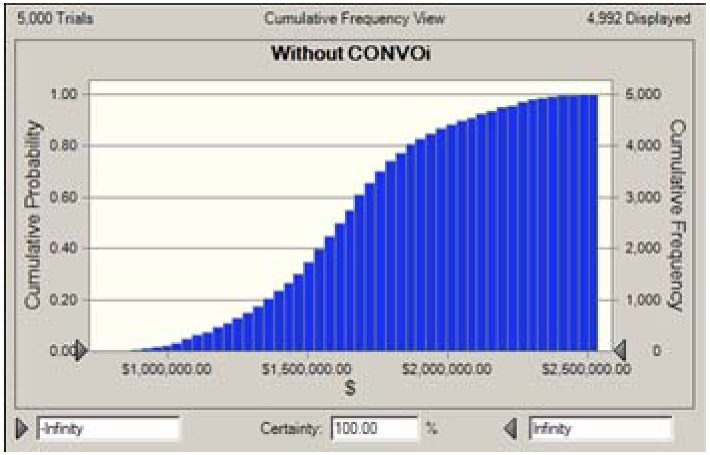
Conclusion

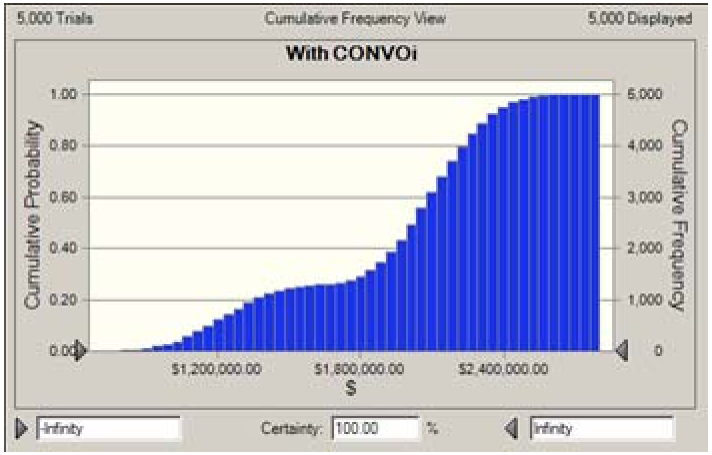
Conclusion

- CONVOI improves accuracy of cost distribution simulations by incorporating risk interdependencies
  - The first main phase of the CONVOI process involves comparative analysis of the risks in a programs risk register, providing for a more complete and accurate view of the data
  - The second main phase of the CONVOI process provides a method for associating Pearson’s Product moment Correlation Coefficient to risks in a program’s risk register based on risk interdependencies
    - The result is more accurate Coefficient of Variation which allows for more realistic representations of the dispersion of cost distributions
- The CONVOI process is simple and requires little effort beyond the scope of traditional risk analysis.
  - With the exception of data collection, the entire process is performed within Excel; and is automated with VBA
- CONVOI is effortlessly incorporated into COTS risk analysis tools

BACKUP SLIDES

CONVOI: A Case Study





- %16 increase in mean cost due to consistent cost values.

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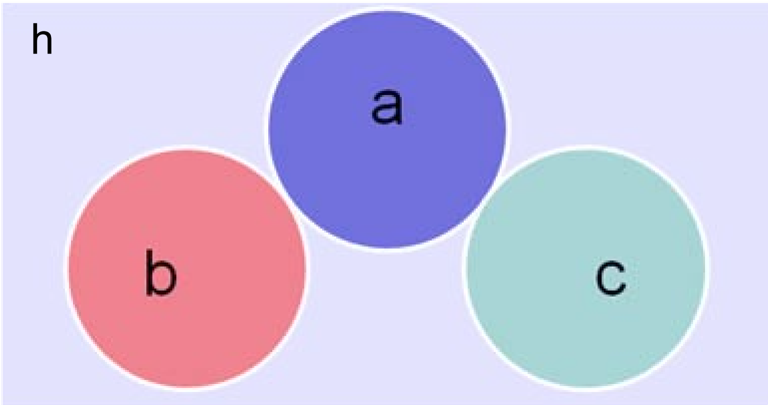
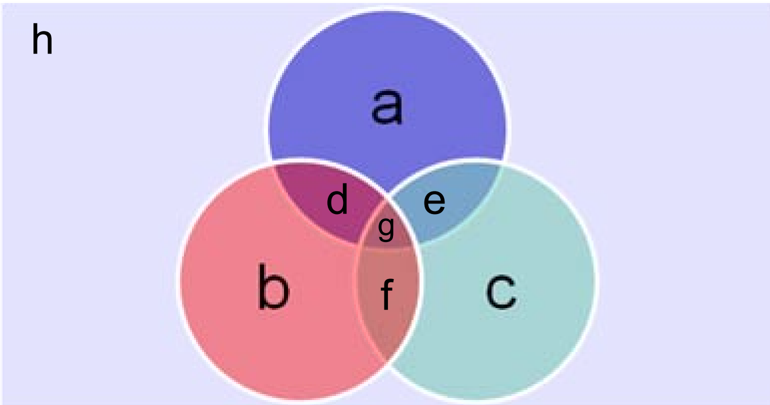
The CONVOI Process: Consistency Verification





- Passengers affected by potentially inclusive Drivers
  - Without the assumption of mutually exclusive Drivers the partition of a Passenger becomes much more complex and uncertainty increases
    - Considering the figure on the left, under the assumption of mutually exclusive Drivers the set of subsets {a,b,c,h} form a partition of the space
    - Potential inclusion is displayed in the figure on the left. The set of subsets {a,b,c,d,e,f,g,h} is required to form a partition of the space
    - The increase in the cardinality of the partition results in an increase in the terms that make up the Law of Alternatives equation. The number of known values derived from the Driver/Passenger relationship remain the same

The CONVOI Process: Consistency Verification

- Because of the unknown variables that result from allowing potential inclusion, a general range for Passenger probability cannot be determined
- However, there are two cases under potential inclusion in which a consistency range can be defined
  - If all Drivers increase the likelihood of occurrence of a Passenger, and it is safe to assume that any intersection of two or more Drivers will not decrease the likelihood of a Passenger's occurrence then a bound for the minimum of $P(y)$ can be determined
  - If all Drivers decrease the likelihood of occurrence of a Passenger

$$\min[\{P(y|D_1), P(y|D_2), \ldots, P(y|D_n)\}]\, P(D_n) \leq P(y)$$

  - or all Drivers decrease the likelihood of occurrence of a Passenger such that
  - or $P(y|D_n) \geq P(y)$ for all n. Then $P(y)$ will be bounded below by the product of the smallest conditional probability and the related Drivers marginal probability
  - If no Drivers increase the likelihood of occurrence of the Passenger, $P(y|D_n) \leq P(y)$ for all n. Then $P(y)$ will be bounded above by the product of the largest conditional probability

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