The Society of Cost Estimating and Analysis

Cost Risk Allocation Theory and Practice

Christian Smart, Ph.D., CCEA

Director, Cost Estimating and Analysis

Missile Defense Agency/DOC

christian.smart@mda.mil

Presented at the 2011 ISPA/SCEA Conference

Introduction

"I can see that it works in practice, but does it work in theory?"
- Garrett Fitzgerald, Prime Minister of Ireland 1981-1987

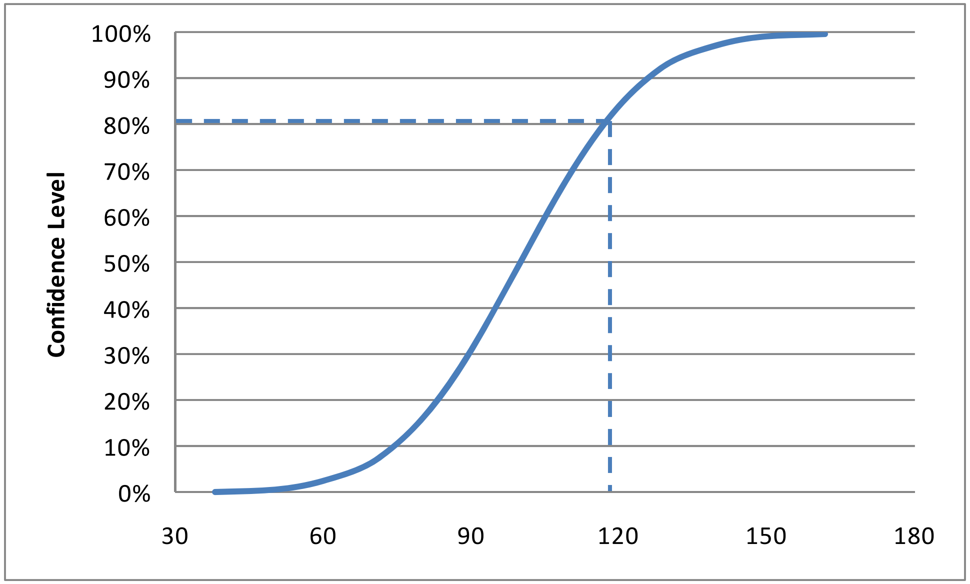
- **Cost risk allocation is a challenging problem**
  - **There are some existing heuristics, but little underlying theory**
  - **The purpose of this presentation is to fill that gap, as well as to present a method that is new to cost estimating**
    - **An optimal method for allocation is presented**
    - **Connect risk measurement with risk allocation**
    - **Show the connection with current heuristics as special cases of the optimal method**

The Society of Cost Estimating and Analysis

Confidence Levels

Cost Estimating

- **The output of a cost risk analysis is a probability distribution**
  - **Typically displayed as a cumulative distribution function, or "S-curve"**
  - **Confidence levels are simply percentiles of the distribution**



Risk Measurement

- Confidence levels are the most common way to set risk reserves
  - NASA policy dictates that programs budget to the 70th percentile, and that individual projects within a program are budgeted to at least the 50th percentile
  - The Weapons Systems Acquisition Reform Act states that project budget to the 80th percentile, or be able to explain why they do not budget to the 80th percentile
- Confidence levels are percentiles of the project risk analysis S-curve
- Other ways to measure risk include standard deviation, the mean, expected shortfall, and semi-variance

Percentiles Do Not Add

- **However, we are not solely interested in the risk of one project, but the risk of multiple projects combined**
  - **However we cannot simply add percentiles!**
- **As a notional example consider two independent normal distributions**
  - **One distribution has mean equal to *100* and standard deviation equal to *20***
  - **The second distribution has mean *300* and standard deviation of *80***
- **The sum of two independent normally distributed random variables is also normally distributed**
- **To combine the two distributions add the means and add the variances**
  - **Total mean is**

$$+ \quad =$$

  - **Total standard deviation is**

$$\sqrt{\quad + \quad} \approx$$

The Society of Cost Estimating and Analysis

Percentiles Do Not Add (2)

Cost Estimating

- The 80th percentiles for the individual distributions are 117 and 367, resp.
- The 80th percentile of the combined distribution is 469, but the sum of the two 80th percentiles is 484
- To see why this is the case note that the percentiles of a normal distribution are determined by the mean and the standard deviation
  - The standard deviations are not added when normal distributions are combined, rather the variances are combined
  - Sum of variances is $a^2 + b^2$
  - The standard deviation of this sum is $\sqrt{a^2 + b^2}$

Percentiles Do Not Add (3)

- **Since**

$$a^2 + b^2 < a^2 + 2ab + b^2$$

we can write

$$\sqrt{a^2 + b^2} < a + b$$

- **The left side of this inequality represents the risk of the combined distributions, while the right side represents the sums of the individual risks**
- **Combining two missions that are independent results in a diversification of risk**
  - **The total portfolio is not as risky on a relative basis as each individual project**

Example

- **As a notional example consider 10 independent normally distributed projects**

Project	Mean	Standard Deviation	
1	$100	$20	$117
2	$250	$50	$292
3	$300	$100	$384
4	$75	$10	$83
5	$490	$150	$616
6	$350	$90	$426
7	$280	$100	$364
8	$90	$10	$98
9	$100	$30	$125
10	$150	$40	$184
Sum	$2,185	$237	$2,690

80th Percentiles

Sum of 80th Percentiles

- **The sum of the 80th percentiles is equal to the 98th percentile on the aggregate distribution**
- **The 80th percentile for the aggregate is much lower at $2,385**

The Society of Cost Estimating and Analysis

Risk Allocation

Cost Estimating

- **Individual project estimates are at a confidence level, such as the 80th percentile**
- **These projects each contain numerous WBS elements, each of which must be funded in a manner consistent with the overall 80th percentile funding for the project**
- **Since the percentiles do not add we cannot simply budget each WBS element at the 80th percentile**
  - **Doing so will result in a confidence level much higher than the 80th at the project level**
  - **The goal is to allocate risk back to each individual WBS element so that each is funded in a manner so that the sum of the individual WBS allocations is the overall 80th percentile funding for the project**

Allocation Methods in Use

- **There are two widely-used, established methods for allocating risk from the aggregate project level to WBS elements**
  - **Proportional standard deviation method**
    - **Conceptually simple**
    - **Used in at least one cost estimating platform software**
  - **“Needs” method (see Book (2006))**
    - **More sophisticated**
    - **Overcomes shortcomings in the standard deviation method**
      - Risk ≠ Standard Deviation
      - Correlation should be accounted for in the method
    - **More complicated, but can be easily implemented in a spreadsheet**
    - **Institute for Defense Analysis (IDA) has endorsed**

Proportional Standard Deviation Method

- **Standard deviation is a measure of risk**
  - **A project's standard deviation represents its contribution to overall risk**
- **For independent random variables, the total variance is equal to the variance of the individual WBS elements**
- **Step 1: Calculate the overall standard deviation**

$$\sigma_{Total} = \sqrt{\sum_{i=1}^{N} \sigma_i^2}$$

- **Step 2: Calculate the 80th percentile at the total level**
  - **For a Normal distribution**

$$\mu_{Total} + z_{.80}\sigma_{Total}$$

Proportional Standard Deviation Method (2)

- **Step 3: Calculate the total risk dollars**
  - **If the basis of cost (the "point" estimate) is the mean, then the risk dollar amount is equal to**

$$\mu_{Total} + z_{.80}\sigma_{Total} - \mu_{Total} = z_{.80}\sigma_{Total}$$

  - **This is the amount that will be allocated among the individual WBS elements' point estimates**
- **Step 4: Calculate each WBS element's standard deviation percentage of the sum of the standard deviations**

$$p_i = \frac{\sigma_i}{\sum_{i=1}^{n} \sigma_i}$$

The Gap

- **The two methods commonly used in practice have limitations**
  - **Both are heuristics**
    - **While they are logical ways to allocate risk there is no underlying theory that leads one to believe either is optimal**
  - **There is nothing to connect the method of risk measurement with the allocation method**
    - **Should the needs method be used with percentile funding, or is the standard deviation method better?**
  - **The only constraint required by the needs and standard deviation allocation methods is that the allocation must be complete, that is, for a risk measure r_T and n WBS elements with respective allocation $r_1,\ldots r_n$, that**

$$\sum_{i=1}^{n} r_i = r_T$$

Gradient Allocation

- **A widely used method for allocating risk in finance and insurance**
  - **Has been found by many authors in many fields to be optimal**
- **Consistent with coherent risk measures (discussed in "Here There Be Dragons" by this author at last year's conference (Smart 2010))**
- **Ties together the notions of risk measure and allocation**
  - **Allocating along the gradient dictates the way that a risk measurement is allocated, which results in different allocation algorithms for different risk measures**

Defining Gradient Allocation

- **Gradient allocation involves allocating to each WBS element an amount equal to the gradient of the risk measure**
- **To define this, consider an *n*-element WBS with cost random variables denoted by $X_1,\ldots X_n$ and portfolio weights denoted by $\lambda_1,\ldots,\lambda_n$. The total cost for the project is found by summing the individual WBS elements, accounting for the weights, i.e.**

$$x(\lambda) = \sum_{i=1}^{n} \lambda_i x_i$$

The Society of Cost Estimating and Analysis

Defining Gradient Allocation (2)

Cost Estimating

- **If the total risk measure is denoted by *r* and the risk measure for each individual WBS is denoted by r_i then the gradient of *r* is defined as**

$$\frac{\partial r}{\partial \lambda_i}$$

which reflects the rate of change in the total risk relative to the rate of change in the portfolio weight for individual WBS elements

- **As long as the risk measure is positive homogeneous (a property shared by all the risk measures discussed in this presentation including confidence levels), the allocation is a complete allocation (see the paper or Smart (2010) for a definition of positive homogeneous risk measures)**

Defining Gradient Allocation (3)

- **The result of positive homogeneity is that**

$$r(\lambda) = \sum_{i=1}^{n} \frac{\partial r(\lambda)}{\partial \lambda} \lambda_i$$

so risk can be allocated to each constituent element by its gradient

  - **This allocation is complete: no additional constraint is needed to ensure this property holds**

- **Provides natural connection between risk measurement and risk allocation**
- **For a given risk measure, the allocation is derived directly from the risk measure and is specific to the risk measure utilized**

Optimality of Gradient Allocation

- **There are several arguments for the optimality of gradient allocation**
- **One due to economics was provided by Tasche (1999)**
  - **Risk should be viewed as relative to its performance**
- **In terms of cost analysis, this would be the expected cost relative to the risk, as measured by the ratio**

$$\frac{E(X)}{r}$$

The Society of Cost Estimating and Analysis

Economic Optimality

Cost Estimating

- **The economic performance criteria is then defined (Tasche 1999) as**

$$\frac{\partial}{\partial \lambda_i}\left(\frac{E(X(\lambda))}{r(\lambda)}\right)\begin{cases} >0 \;\; if \;\; \dfrac{E(X_i)}{r_i} > \dfrac{E(X(\lambda))}{r(\lambda)} \\ <0 \;\; if \;\; \dfrac{E(X_i)}{r_i} < \dfrac{E(X(\lambda))}{r(\lambda)} \end{cases}$$

- **The criterion states that those elements which have superior risk-adjusted performance should receive greater capital allocation**
- **Gradient allocation is the only allocation that meets this criterion**

Optimality: Criteria

- **Given the following criteria**
  - **The allocation must be**
    - **A linear function**
    - **Diversifying in the sense that** $r_i(X_i) \le r(X_i)$ **for all *i=1,…,n***
      - I.e., the risk allocated to the i^{th} element should be no larger than the risk measure for that particular element
    - **Continuous**
- **Gradient allocation is the only allocation method that meets all three criteria (Kalkbrenner 2005)**

Optimality: Game Theory

- The gradient allocation principle has also been derived from game-theoretic arguments
- Rather than the non-cooperative game theory that most people are familiar with, such as popularized in the prisoner's dilemma and in the film *A Beautiful Mind* risk allocation can be viewed as a cooperative game
  - Coalitions or elements work in accordance to allocate total risk
- It has been found with some simple criteria that the only allocation principle consistent with them is gradient allocation (Denault 2001). These criteria are
  - Diversifying allocation principle (same as Kalkbrenner)
  - Property of symmetry, which means that if by adding any set to the portfolio, any two subportfolios that contribute the same amount of risk will also receive the same allocation
  - Riskless item will receive only its cost in the allocation scheme, no more and no less

Optimality and Cost

- In terms of criteria for cost risk allocation, the notion of economic performance may be a good one for activities involving profit and loss, but is not a motivating factor for the cost of government projects where the activities are determined according to scientific pursuits, technological objectives, or the needs of national defense
- But diversification for an allocation makes sense regardless of the application
  - The amount allocated to a specific WBS element should be less than or equal to the contribution of that element to the overall risk
- Gradient allocation thus meets logical, sound criteria, is linked with and thus consistent with the risk measure used, and is naturally a complete allocation without requiring an explicit constraint

Application of Gradient Allocation

- **For confidence level or percentile funding, which is also referred to as "Value at Risk" or VaR (see Smart 2010), it has been shown that gradient allocation is given by**

$$E(X_i \mid X = VaR(X))$$

- **See the paper for a derivation**

The Society of Cost Estimating and Analysis

Percentile Funding Allocation

Cost Estimating

- **The result is simple, and even intuitive (even though the derivation is complicated)**
  - **However even though the formula appears simple, it is not easy to calculate in practice**
  - **This is not a simple, straightforward conditional expected value calculation, since for continuous distributions the probability that**

$$X(\lambda) = VaR_{\alpha}(\lambda)$$

will be zero

Percentile Funding Allocation (2)

- **In the case of continuous distributions a simple linear approximation can be found by noting that in the subject of linear regression**

$$E(X_i \mid X(\lambda))$$

represents the best estimate of X_i by X

- **Thus a simple linear approximation can be found by minimizing**

$$E((X_i - a - bX)^2)$$

Percentile Funding Allocation (3)

- This is well known as

$$b = \frac{Cov(X_i, X)}{Var(X)}$$

$$a = E(X_i) - bE(X)$$

where *Cov(X,Y)* is the covariance between *X* and *Y*

- Plugging in these values into the linear approximation ($a+bX=a+bVaR_\alpha$) yields

$$E(X_i \mid X(\lambda)) \approx E(X_i) + \frac{Cov(X_i, X)}{Var(X)}(VaR_\alpha - E(X))$$

Percentile Funding Allocation (4)

- **Note that "Var" in the above formula denotes variance while "VaR" denotes the "value at risk" or percentile**
- **Note that this approximation amounts to applying the covariance principle to the difference of the percentile at which the project is funded and the total expected value, or mean**
- **This linear approximation is similar to the needs method due to Book (2006)**
  - **The only difference is the use of one-sided moments in the needs method rather than the covariance and the variance**

The Society of Cost Estimating and Analysis

Hermann's Risk Measure

Cost Estimating

- In a recent technical note, risk allocation is posed as an explicit optimization problem (Hermann 2010)
- This begins with the risk measure

$$r = \int_{\mu}^{\infty} (x - \mu) f(x) dx$$

and proceeds to consider allocating this risk to individual WBS elements *1,…,n* by minimizing the sum of the individual expected shortfalls across WBS elements, i.e.,

$$Minimize \sum_{i=1}^{n} r_i^* = Minimize \sum_{i=1}^{n} \int_{\mu_i + r_i}^{\infty} (x_i - \mu_i - r_i) f(x) dx$$

such that $\sum_{i=1}^{n} r_i = r_T$ and $r_i \geq 0$ for *all i = 1,…n*

Restrictions on Hermann's Method

- **This novel method is notable for**
  - **Considering the issue of allocation as an optimization problem**
  - **For taking into consideration the entire right tail of the cost risk distribution in the allocation process**
- **The motivating factor for minimizing the sum of the expected shortfalls could be that risk dollars are not fungible across WBS elements**
  - **Money allocated is money spent**
  - **But this is not typically what is seen in practice since the allocation is below the contract value level, or across contract values**
    - **Risk is measured and allocated within a specific funding category and financial managers then have the ability to juggle and re-juggle allocations as needed**

Similarity to Proportional Standard Deviation Method

- As a result of looking at the sum of expected shortfalls, this method does not incorporate the impact of correlation, and thus is similar to the proportional standard deviation method
  - This similarity is not a superficial one

- In the case of normally distributed random variables

$$r = \int_{\mu}^{\infty} \frac{x-\mu}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)dx = \frac{\sigma}{\sqrt{2\pi}}$$

- In this case the funding level is

$$\mu + r = \mu + \frac{\sigma}{\sqrt{2\pi}} \approx 65.5th\ \ percentile$$

The Society of Cost Estimating and Analysis

Similarity to Proportional Standard Deviation Method (2)

Cost Estimating

- Given funding to the remaining expected risk exposure is, for a normally distributed random variable, equal to

$$r^{*} = \int_{\mu+r}^{\infty} \frac{x-\mu-r}{\sqrt{2\pi\sigma^{2}}} \exp\left(-\frac{(x-\mu)^{2}}{2\sigma^{2}}\right)dx$$

$$= \int_{\mu+r}^{\infty} \frac{x-\mu}{\sqrt{2\pi\sigma^{2}}} \exp\left(-\frac{(x-\mu)^{2}}{2\sigma^{2}}\right)dx - \int_{\mu+r}^{\infty} \frac{r}{\sqrt{2\pi\sigma^{2}}} \exp\left(-\frac{(x-\mu)^{2}}{2\sigma^{2}}\right)dx$$

$$= \frac{\sigma}{\sqrt{2\pi}} \exp\left(-\frac{r^{2}}{2\sigma^{2}}\right) - \int_{\mu+r}^{\infty} \frac{r}{\sqrt{2\pi\sigma^{2}}} \exp\left(-\frac{(x-\mu)^{2}}{2\sigma^{2}}\right)dx$$

Similarity to Proportional Standard Deviation Method (3)

- Employing a change of variable, letting $u = \frac{x-\mu}{\sigma}$ yields

$$\frac{\sigma}{\sqrt{2\pi}} exp(-\frac{r^2}{2\sigma^2}) - r\int_{\frac{r}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} exp(-\frac{u^2}{2})du$$

$$= \frac{\sigma}{\sqrt{2\pi}} exp(-\frac{r^2}{2\sigma^2}) - r(1-\Phi(\frac{r}{\sigma}))$$

Similarity to Proportional Standard Deviation Method (4)

- Using Lagrangian multipliers, the objective function with embedded constraint can be written as

$$\Lambda(r_1,\ldots,r_n,\lambda) = \sum_{i=1}^{n} \int_{\mu_i+r_i}^{\infty} (x_i - \mu_i - r_i) f(x) dx - \lambda\left(\sum_{i=1}^{n} r_i - r_T\right)$$

- In the case of normally distributed random variables

$$\frac{\partial \Lambda}{\partial r_i} = -\frac{r_i}{\sqrt{2\pi\sigma_i^2}} exp\left(-\frac{r_i^2}{2\sigma_i^2}\right) - \left(1 - \Phi\left(\frac{r_i}{\sigma_i}\right)\right) + r_i\left(\phi\left(\frac{r_i}{\sigma_i}\right)\frac{1}{\sigma_i}\right) - \lambda$$

which reduces to

$$\Phi\left(\frac{r_i}{\sigma_i}\right) - 1 - \lambda = 0$$

Similarity to Proportional Standard Deviation Method (5)

- This means that

$$\frac{r_i}{\sigma_i} = \Phi^{-1}(1+\lambda)$$

- The right side of the equation is constant and does not vary as i changes which implies that

$$\frac{r_i}{\sigma_i} = \frac{r_j}{\sigma_j} \quad \text{and}$$

$$r_i = \sigma_i \frac{r_j}{\sigma_j}$$

Similarity to Proportional Standard Deviation Method (6)

- **The constraint**

$$\sum_{i=1}^{n} r_i = r_T$$

can be written as

$$r_1 + \frac{\sigma_2}{\sigma_1} r_1 + \ldots + \frac{\sigma_n}{\sigma_1} r_1 = r_T$$

and thus

$$r_1 = \frac{\sigma_1}{\sum_{i=1}^{n} \sigma_i} r_T$$

The Society of Cost Estimating and Analysis

Similarity to Proportional Standard Deviation Method (7)

Cost Estimating

- **And thus for all j=1,…,n, it is true that**

$$r_j = \frac{\sigma_1}{\sum_{i=1}^{n} \sigma_i} r_T$$

- **Thus the percentage allocation is the proportional contribution of the *jth* random variable to the sum of the standard deviation values, which is the standard deviation principle mentioned at the beginning of this presentation.**

Cautionary Note

Cost Estimating

- The proportional standard deviation method is the one most often encountered in practice
  - Even included in a widely-used cost estimating platform
- However, this method is optimal only under restrictive conditions not frequently encountered
  - Non-fungibility among WBS elements (not typically seen in practice)
  - All risk distributions are normally distributed
    - Even for a large WBS, the central limit theorem will only apply to the total cost risk - the individual risks are typically skewed and not normally distributed

Expected Shortfall

- **Expected shortfall is a risk measure that is superior to percentile funding (Smart 2010)**
- **Expected shortfall (ES) is similar to *VaR*, but it looks at the expected overrun past a fixed percentile**
  - **Provides not only an indication that bad times have occurred (when the percentile is exceeded), but also a reserve set aside to deal with adverse conditions when they occur**
- **Defined as**

$$ES_\alpha = \frac{1}{1-F(Q_\alpha)}\int_{Q_\alpha}^{1} xf(x)dx = \frac{1}{1-\alpha}\int_{\alpha}^{1} VaR_u(X)du$$

The Society of Cost Estimating and Analysis

Expected Shortfall (2)

Cost Estimating

- In the case of continuous cost risk distributions, this risk measure is referred to as Conditional Tail Expectation (*CTE*)
- For example, $Q_{0.95}$ is the 95th percentile (McNeil et al., 2005)
- It is the "Tail Value at Risk" since in the case of continuous cost distributions it may be viewed as

$$CTE_{\alpha}=E[X|X>Q_{\alpha}]$$

Gradient Allocation and Expected Shortfall

- Suppose VaR is set at the α^{th} percentile
- Then the expected shortfall risk is defined as

$$r(\lambda) = \frac{1}{1-\alpha} \int_{\alpha}^{1} VaR_u(\lambda)\,du$$

- Calculating the gradient with respect to λ yields

$$\frac{\partial r}{\partial \lambda_i} = \frac{1}{1-\alpha} \int_{\alpha}^{1} \frac{\partial VaR_u(\lambda)}{\partial \lambda_i}\,du$$

Gradient Allocation and Expected Shortfall (2)

- Using the formula obtained for this partial derivative from the gradient allocation for *VaR* we find that

$$\frac{\partial r}{\partial \lambda_i} = \frac{1}{1-\alpha}\int_{\alpha}^{1} E(X_i \mid X(\lambda) = VaR_u(\lambda))du$$

- Let $v = VaR_u(X(\lambda)) = F^{-1}_{X(\lambda)}(u)$
- Then since $f_{X(\lambda)}(v)dv = du$

$$\frac{1}{1-\alpha}\int_{\alpha}^{1} E(X_i \mid X(\lambda) = VaR_u(\lambda))du =$$

$$\frac{1}{1-\alpha}\int_{VaR_\alpha}^{\infty} E(X_i \mid X(\lambda) = v) f_{X(\lambda)}(v)dv = \frac{1}{1-\alpha} E(X_i \mid X(\lambda) \geq VaR_\alpha)$$

Gradient Allocation and Expected Shortfall (3)

- **Thus for expected shortfall the gradient allocation formula is**

$$\frac{\partial r}{\partial \lambda_i} = \frac{1}{1-\alpha} E(X_i \mid X(\lambda) \geq VaR_\alpha)$$

- **Similar in form to the capital allocation for *VaR* (the only difference is that the equality in the conditioned expectation is now an inequality)**
  - **But more intuitive and easier to calculate than the *VaR* allocation.**
  - **For a Monte Carlo simulation, it is simply the contribution of the i^{th} element to the expected shortfall.**

Example

- **Consider the 10 individual projects shown in the table below**
  - **Each is assumed to be lognormally distributed with correlation equal to 20% among all projects**

Project	Mean	Standard Deviation
Project 1	1501	556
Project 2	804	219
Project 3	907	302
Project 4	875	400
Project 5	1450	420
Project 6	1271	419
Project 7	874	541
Project 8	1001	229
Project 9	1139	392
Project 10	981	485
Total	10803	

The Society of Cost Estimating and Analysis

Example – Risk Measurement

Cost Estimating

- **Note that the mean is also a risk measure**
- **For Value at Risk, Expected Shortfall, and the mean, the risk measures for the aggregate of the 10 projects in the example are displayed in the table below**

Risk Measure	Value
Mean	\$10,803
Value at Risk (70th Percentile)	\$11,695
Expected Shortfall (70th Percentile)	\$13,331

- **For a given percentile, the expected shortfall will always be greater than the accompanying value at risk measure**

Example – Risk Allocation

- **The results of applying the risk allocation methods to the 10 project example are displayed in the table below**

Allocation Method	Proj 1	Proj 2	Proj 3	Proj 4	Proj 5	Proj 6	Proj7	Proj 8	Proj 9	Proj 10
Proportional St. Dev.	14.0%	5.5%	7.6%	10.1%	10.6%	10.6%	13.7%	5.8%	9.9%	12.2%
Needs Method	16.2%	5.8%	7.8%	8.8%	12.8%	11.9%	9.1%	6.5%	10.7%	10.5%
Hermann's Method	14.8%	7.1%	8.7%	8.6%	13.2%	12.2%	7.0%	8.1%	11.0%	9.4%
Gradient – VaR (70%)*	15.3%	4.7%	6.9%	9.9%	10.5%	10.5%	14.7%	5.0%	9.6%	12.7%
Gradient – Expected Shortfall	9.3%	8.4%	10.4%	9.8%	14.2%	6.9%	8.1%	8.4%	12.9%	11.5%

- **Note that while there are some similarities between the methods there are also significant differences**

***Linear approximation used**

The Society of Cost Estimating and Analysis

Summary

Cost Estimating

- **Current risk allocation theory and practice and relatively new methods for risk allocation have been discussed**
- **The proportional standard deviation method and the needs method are heuristics that do not necessarily have optimal properties**
- **Risk allocation methods have not sought to distinguish between measurement and allocation, so risk measurement was also summarized**
  - **The twin problems of risk measurement and risk allocation are separate and distinct but related topics**
  - **A new risk allocation method that is becoming increasingly popular in finance and insurance was discussed, which is gradient allocation**
    - **Links together risk measurement and risk management, and in given certain criteria for allocating risk, proves to be the best method for an associated risk measurement method**

Summary (2)

- **The needs method falls within the gradient allocation framework**
  - **Similar to the best linear estimator for gradient allocation when value at risk is used for risk measurement**
- **Proportional standard deviation was found to be optimal under highly restrictive conditions not likely to be encountered in practice**
  - **Normally distributed cost elements at the WBS level**
  - **Lack of flexibility of risk dollars among WBS elements**

The Society of Cost Estimating and Analysis

References

Cost Estimating

1. **Anderson, T. P. "The Trouble With Budgeting to the 80th Percentile", presented at the 72nd Military Operations Research Society Symposium, 2004.**
2. **Artzner, P. et al., "Coherent Measures of Risk," Mathematical Finance 9:203-228, 1999.**
3. **Aubin, P., Mathematical Methods of Game and Economic Theory, Dover, Amsterdam, 2007.**
4. **Book, S.A. "Allocating 'Risk Dollars' Back to WBS Elements" LLC SSCAG/EACE/SCAF Meeting 19-21 September 2006.**
5. **Book, S.A., "Allocating 'Risk Dollars' Back to WBS Elements," presented to the SSCAG Risk Subgroup, 17-18 January 2007.**
6. **Denault, M., "Coherent Allocation of Risk Capital," Journal of Risk 4(1):7-21.**
7. **Hermann, W., "Optimal Risk Allocation: Theory, Heuristics, and Best Approximations," Ballistic Missile Defense System Analysis Team technical note, May, 2010.**
8. **Hermann, W., personal communication, April, 2010.**

The Society of Cost Estimating and Analysis

References (2)

Cost Estimating

9. Kalkbrenner, M., "An Axiomatic Approach to Capital Allocation," Mathematical Finance 15(3):425-437, 2005.

10. Markowitz, H., Portfolio Selection: Efficient Diversification of Investments, Wiley, Yale University Press, 1959.

11. McNeil,A.J., et al., *Quantitative Risk Management*, Princeton University Press, Princeton, 2005.

12 Sandberg, J., "Cost Risk Allocation: Objectives, Tendencies, and Limitations," presented at the Joint ISPA/SCEA Conference, June, 2007.

13. Smart, C., "Risk Analysis in the NASA/Air Force Cost Model," presented at the Joint ISPA/SCEA Conference, June 2005.

14. Smart, C., "Here There Be Dragons: Considering the Right Tail in Risk Management," presented at the Joint ISPA/SCEA Conference, June, 2010.

15. Smart, C., "The Portfolio Effect and the Free Lunch," presented at the Joint ISPA/SCEA Conference, June, 2009.

16. Tasche, D., "Risk contributions and performance measurement," Preprint, Department of Mathematics, TU-Munchen, 1999.

References (3)

17. Tasche, D., "Conditional Expectation as a Quantile Derivative," Preprint, Department of Mathematics, TU-Munchen, 2000.

18. Tasche, D. "Allokation des ökonomischen Kapitals auf Teilportfolios und Transaktionen," in *Handbuch Ökonomisches Kapital*, A. Becker, V. Gehrmann und H. Schulte-Mattler (Herausgeber), Fritz-Knapp Verlag, pp. 193-212, 2008.