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Use of Weibull Failure Rates

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Introduction to Failure Rates

- Mean Time Between Failures = 1/ Failure Rate
  - Failure Rate often expressed per million operating hours
- Failure Rates easy to deal with because they are additive for components of a system

$$\Lambda_s = \lambda_a + \lambda_b + \lambda_c + \lambda_d + \lambda_e$$

$$1/\lambda = MTBF$$

MIL HDBK 217

$$\lambda_p = \lambda_b \pi_T \pi_A \pi_R \pi_S \pi_C \pi_Q \pi_E$$

where:

λ_p is the part failure rate,

λ_b is the base failure rate usually expressed by a model relating the influence of electrical and temperature stresses on the part,

π_E and the other π factors modify the base failure rate for the category of environmental application and other parameters that affect the part reliability.

Environmental Factor - π_E

Environment	π_E
G_B	.5
G_F	2.0
G_M	4.0
N_S	4.0
N_U	6.0
A_{IC}	4.0
A_{IF}	5.0
A_{UC}	5.0
A_{UF}	8.0
A_{RW}	8.0

The Challenge

- Vendor estimated reliability using commercial physics-of-failure model techniques
- Vendor System MTBF estimated at near threshold of 1000 hours
- Dem-Val MTBF = 250

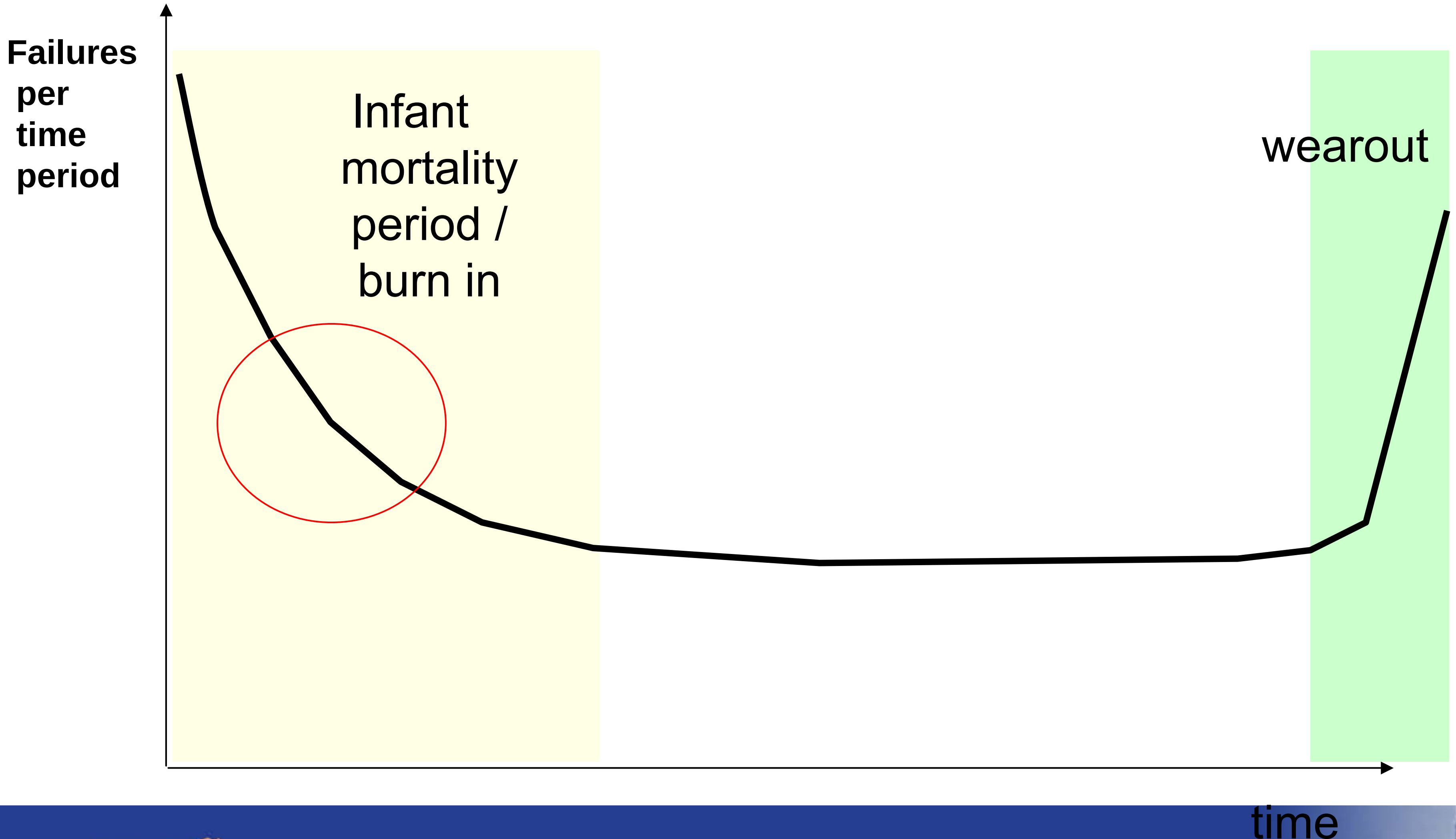
$$MTBF^* = \frac{\Sigma n(\text{elapsed time intervals})}{\text{failures during demonstration}}$$

* MIL HDBK 189

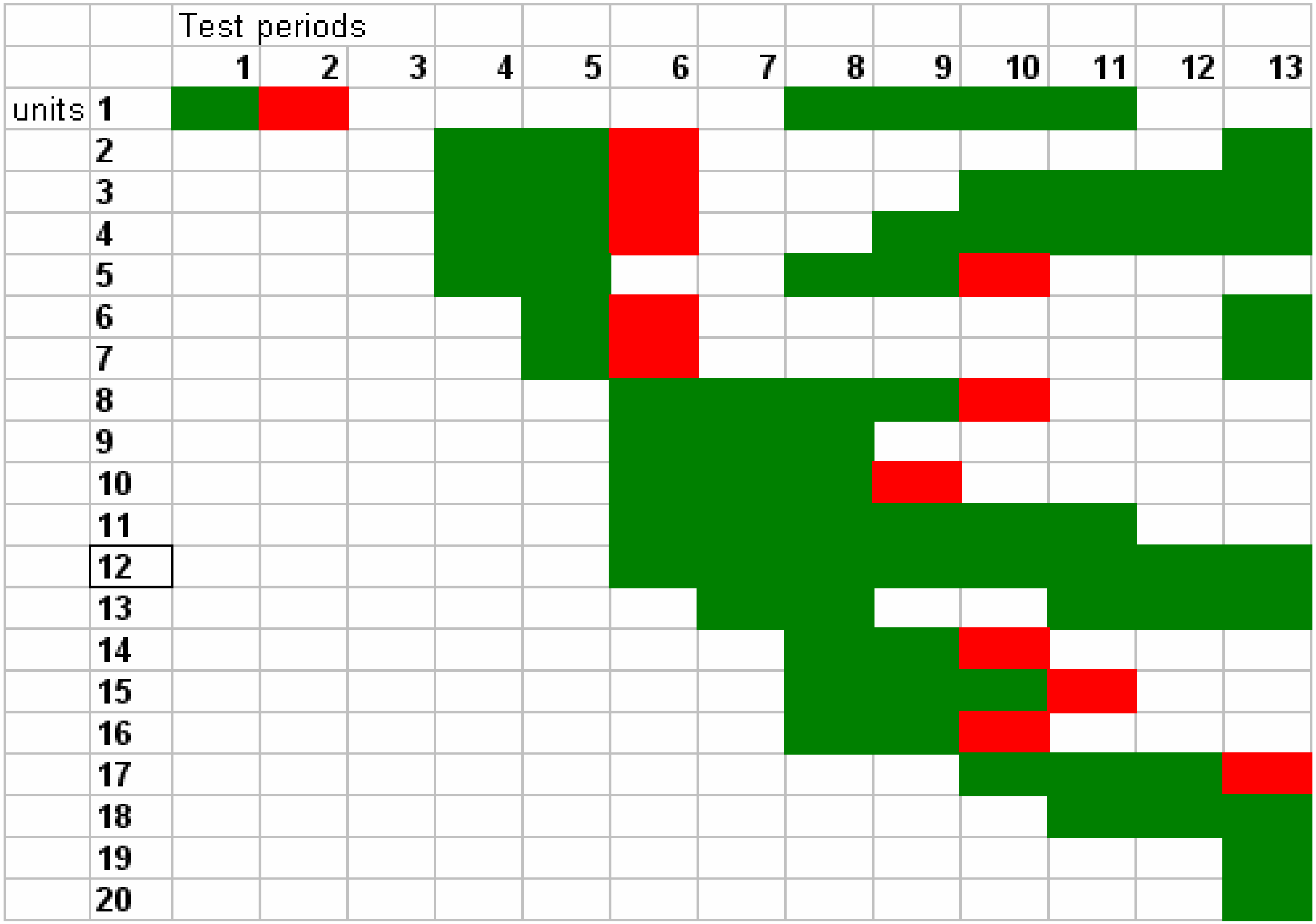
n = all units in demonstration evaluation

- Concern that we are measuring the infant mortality failures

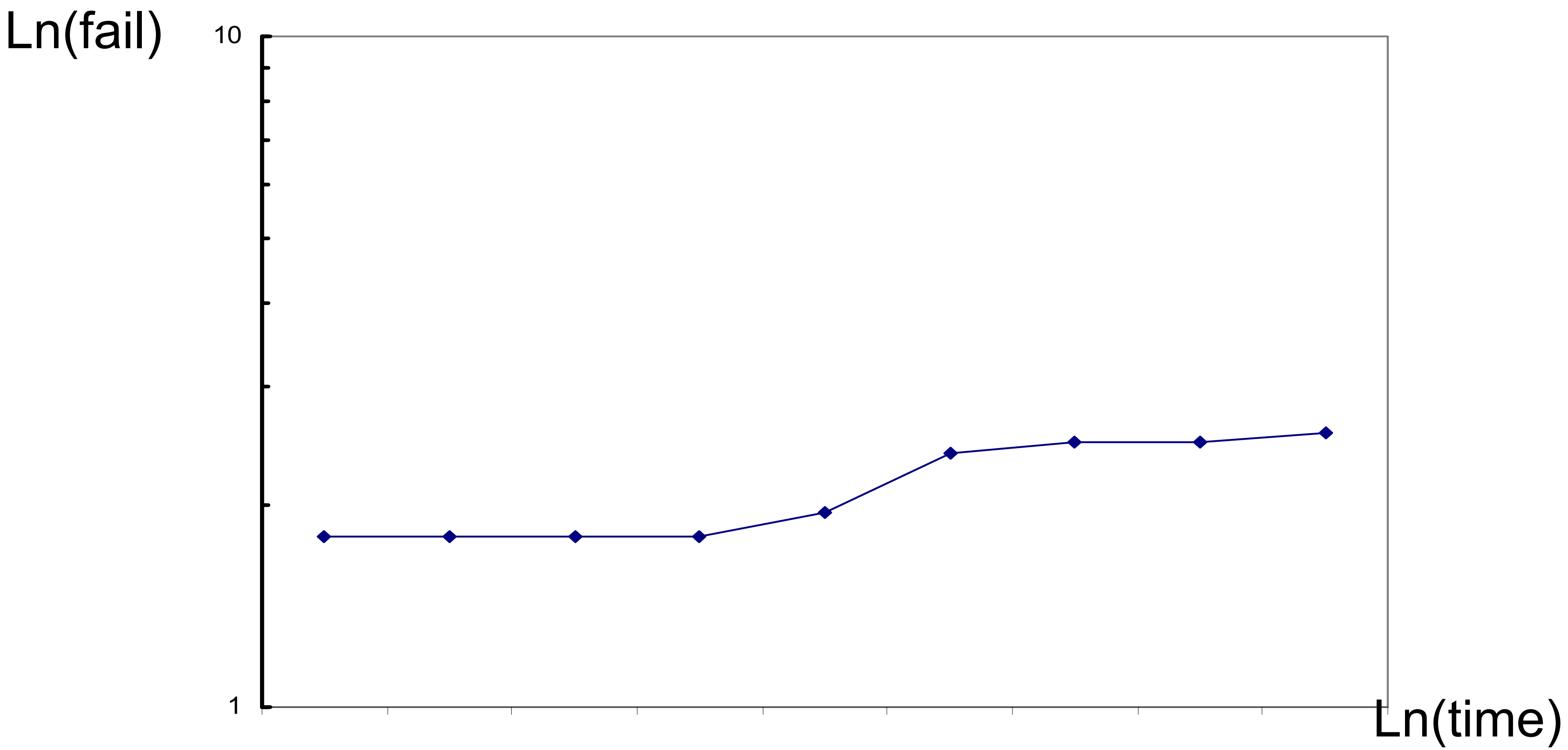
Vendor's Hypothesis



Test Results (Run til Hard Failure)

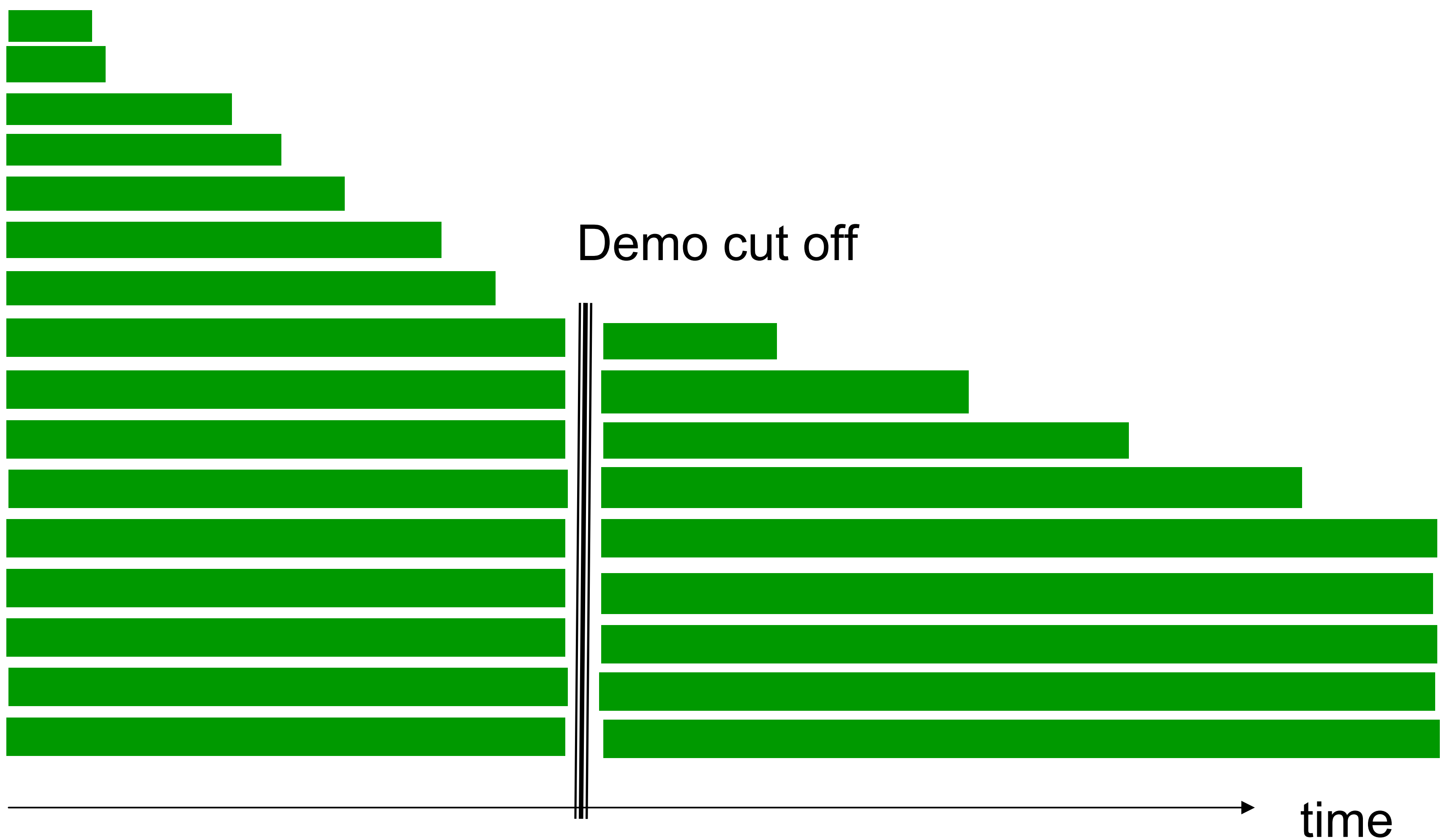


Gompertz Curve, Plotted in ln-ln Space



MIL HDBK 189 Section 5.2.6.1.2

Curtailed Sample Due to Time-Limited Test



What to do with Limited Sample?

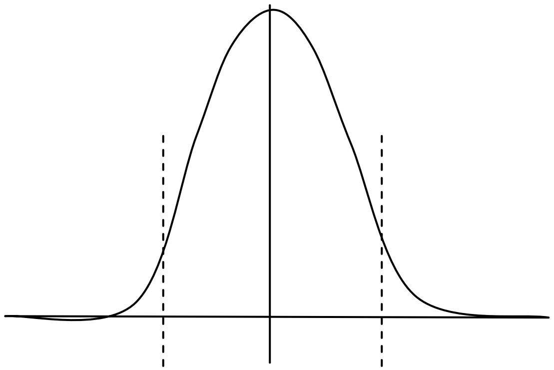
Choices

- Chi-Square Adjustment (confidence estimating)
- Fit to Weibull
- Ignore it and go back to simple mean

Chi-Square Adjustment

- Used for small sample sizes; the distribution depends on degrees of freedom
- Can be used with zero failures

$M_{confidence} = 2T / \chi^2_{2r+2}(\alpha)$



[time-terminated test]

T = time period (hours)

(1- α) = confidence level; for two-tailed, divide alpha by 2
e.g., for two-tailed 80% confidence, alpha = .10

$\chi^2_{2r+2}(\alpha)$ = Chi-Square distribution with 2r+2 degrees of freedom
where r is number of failures under test
Use Lookup table for (.90, 2r+2), (.10, 2r+2)

For failure terminated test d.f. = 2r

Ref. Neubeck , 2004

Chi-Square Conclusions

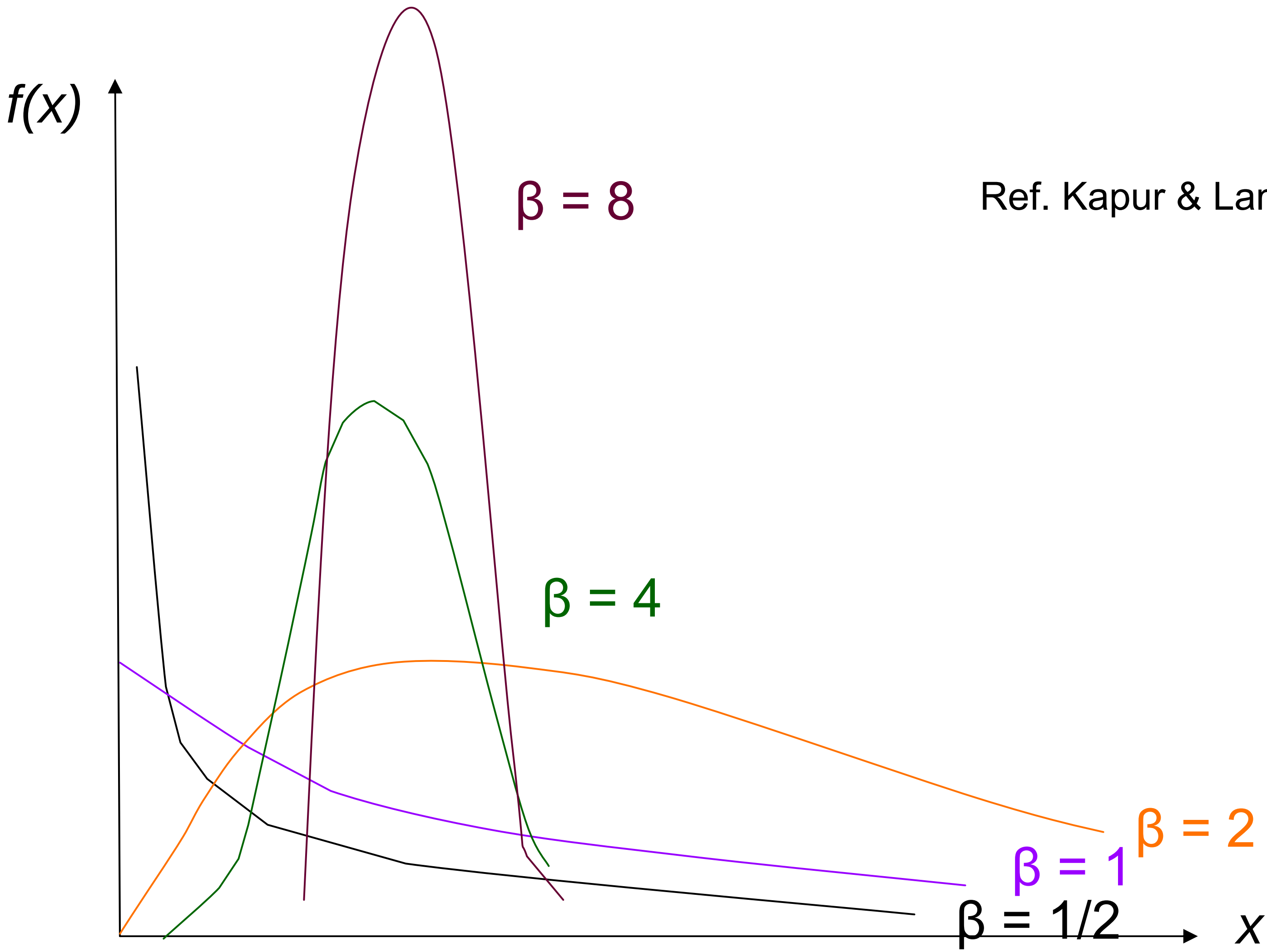
- Adequately deals with the curtailed sample problem
- Assuming away the impact of non-uniform start dates
- Gets us an 80% confidence interval (upper bound) of 630
- Still does not agree with vendor
- No explicit credit for infant mortality stage

Why Weibull

- A distribution to represent real world data allows us to make predictions and make confidence estimates on those predictions
- The Weibull can be adjusted to mimic real world data, using its scale (aka characteristic life) and shape (or slope) parameters
- Weibull shape parameter of <1 mimics a system with failures that decrease over time, indicative of infant mortality
  - defective items fail early and the failure rate decreases over time as they fall out of the population
- A constant failure rate (shape = 1) suggests that items are failing from random events
  - it is also an exponential distribution
- Increasing failure rate (shape >1) suggests "wear out:" parts are more likely to fail as time goes on
- Shape parameter ≈ 3.5, the Weibull is identical to the Gaussian distribution
  - A mature system under full rate production would likely have a Gaussian failure rate

Weibull PDF

Ref. Kapur & Lamberson p 295.



Weibull Estimation and Fit Steps

1. Order failures (achieve adjusted rank)
2. Estimate F(t) matching ordered failures

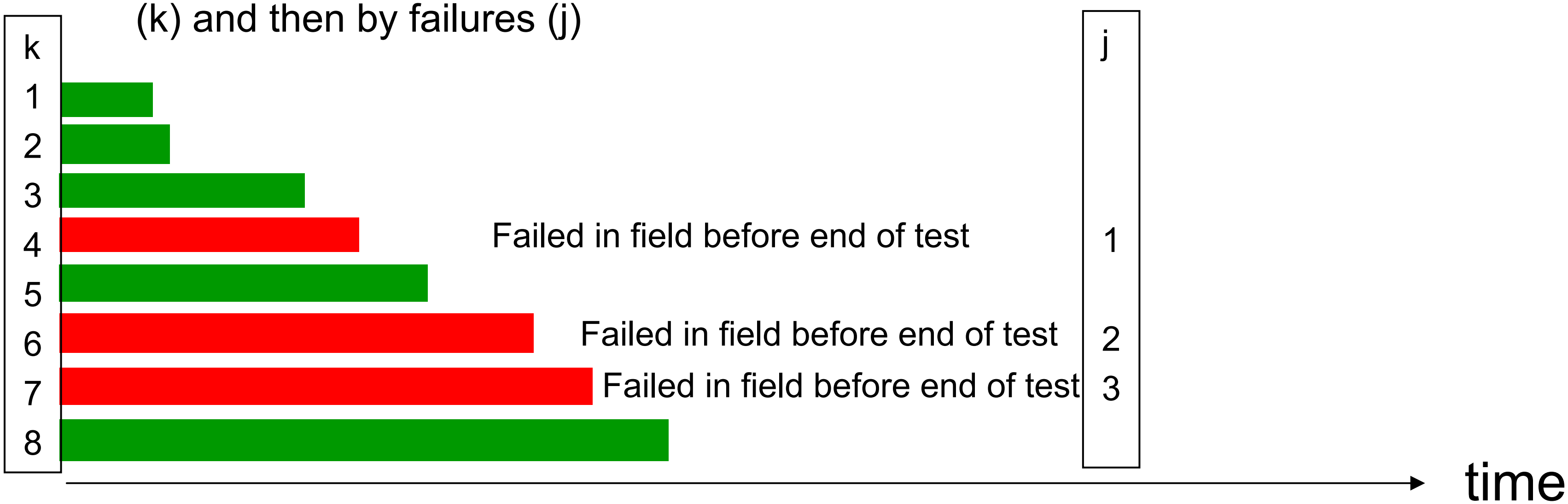
   Use adjusted rank to determine adjusted probability of failure F(t), also known as median rank of F(t)
3. Regress log of time elapsed on log of median rank of adjusted probability of failure to get rates of change of time and probability (CDF)
4. From regression, the slope is the shape parameter and the inverse log of the intercept is the scale parameter of Weibull distribution (CDF)
5. Find mean of Weibull to get MTBF

Weibull Fit Steps

1. Order failures to create x values for CDF

(Analogous to constructing a pdf from data; create the abscissa buckets)

A. Rank all units (failed and non failed during demo) by hours accumulated (k) and then by failures (j)



B. Rank failed units by a proportionate amount of accumulated hours. N = number of units in demonstration (here N=8)

$$r(j) = \text{rank of failed unit} = r(j-1) + \frac{N+1 - r(j-1)}{N+1 - (k(j)-1)}$$

And $r(0) = 0$

Weibull Fit Steps

2. Find probability of failure F(t) associated with adjusted failure ranks

$$F(t) = \frac{r(j) - 0.3}{N + 0.4}$$

Here the median rank is used; median more robust in a skewed distribution

This formula is an approximation for the probability of any unit having failed at time t

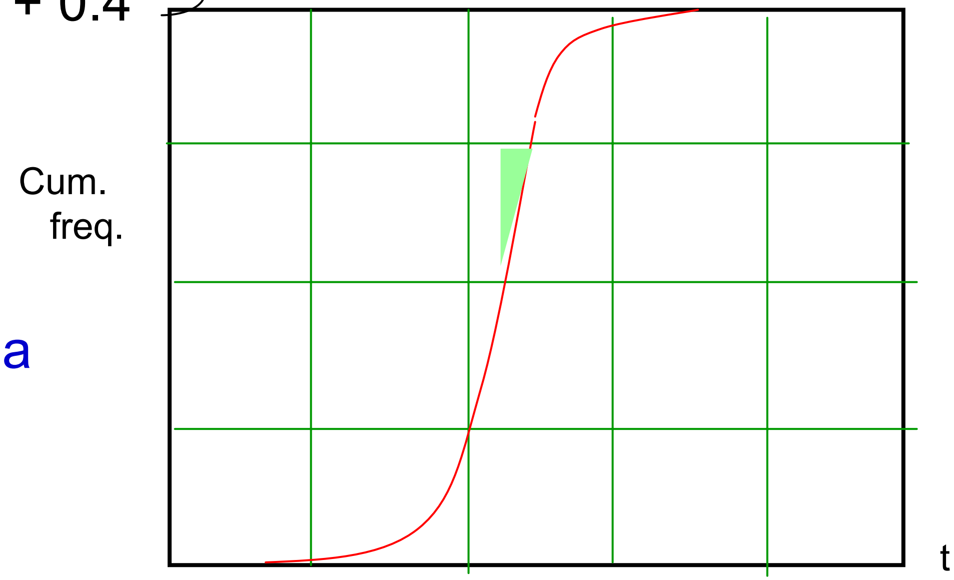
- For details, see Kapur & Lamberson
- For tables see Pearson, *Tables of the Incomplete Beta Function*, 1932.

Weibull Fit Steps

3. Regress log of time elapsed on log of rate of change of adjusted probability of failure to get rates of change of time and probability

Regress $\ln(t(j))$ on $\ln\left[\ln\left(\dfrac{1}{1 - \dfrac{j - 0.3}{N + 0.4}}\right)\right]$

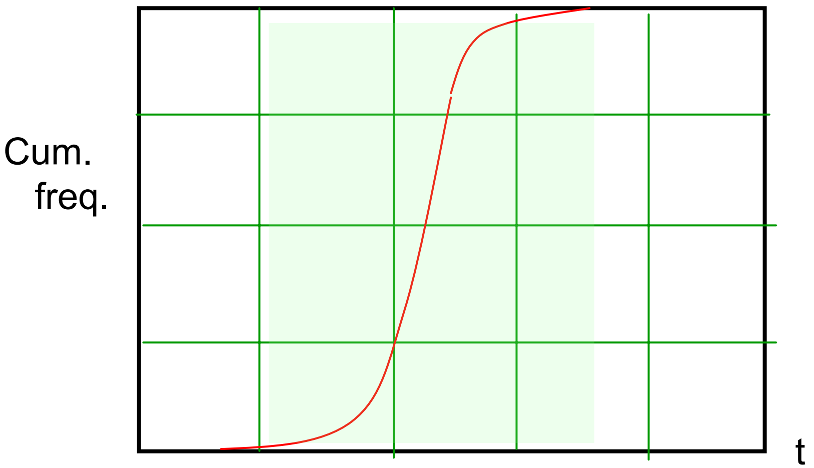
CDF



Double ln linearizes the CDF, so a linear regression can be fit to it [Dodson, 2008]

Weibull Fit Steps

4. From regression, the slope is the shape parameter and the inverse log of βln*y* is the scale (width) parameter of Weibull distribution



5. Find mean of Weibull to get mean expected life (MTBF)

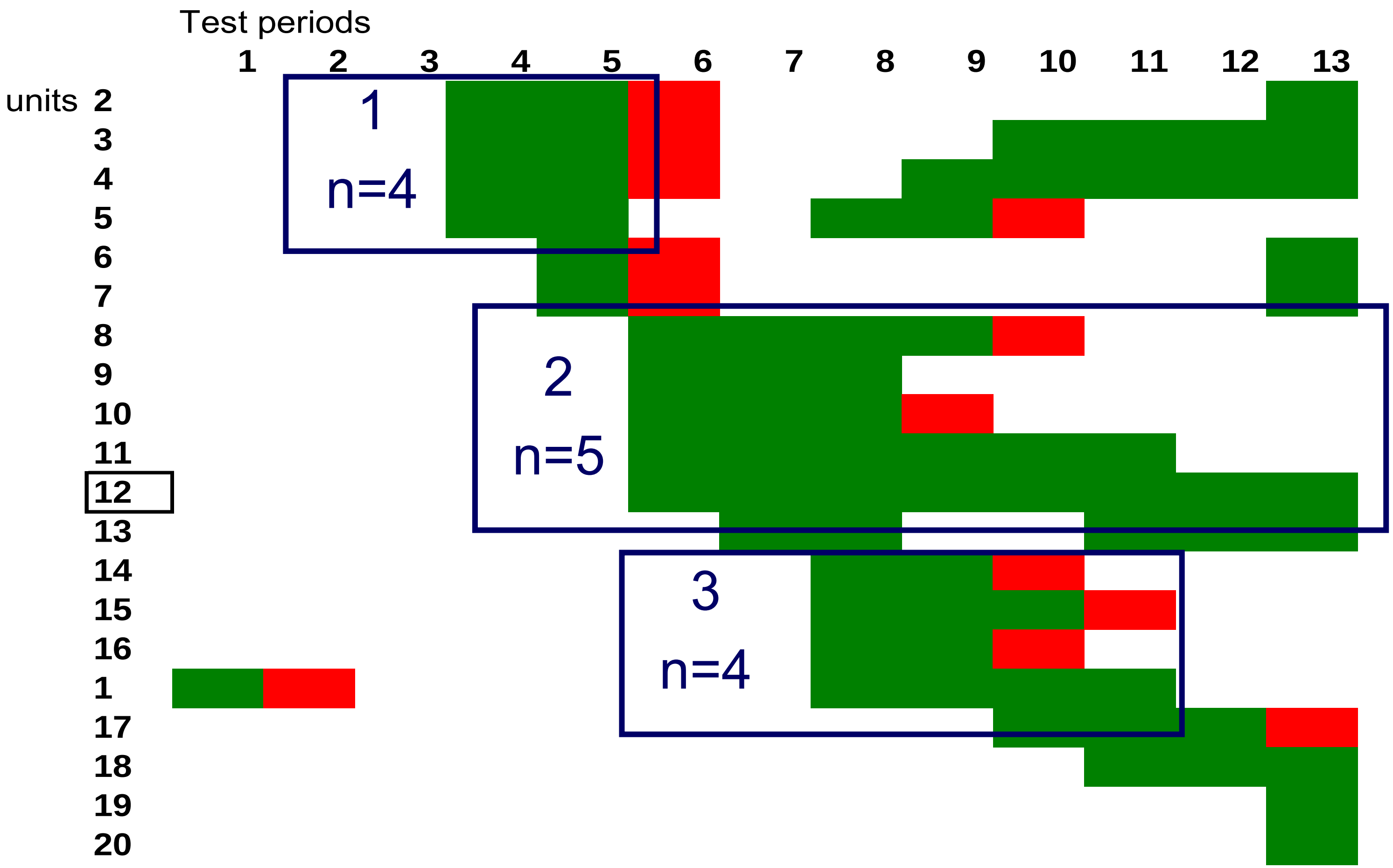
   $u = a\Gamma(1+1/b)$

   where b = shape parameter; a = scale parameter

Weibull Fit Steps

- Problem that we have non-uniform start dates

Subsamples with Same Start Dates



Subsample Result

subsamples				adj rank	f(t)	ln (t)	ln ln					
tot hrs	rank by hours	hrs at fail	rank by failure	0		x	v	x^2	beta	alpha	Weibull mean	arith mean
	1	194.2	4	1.25	0.22	5.27	-1.41	27.76	-0.07	0.61	#NUM!	235
	2	226.4	1	2.50	0.50	5.42	-0.37	29.40				
	3	239	2	3.75	0.78	5.48	0.43	29.99				
	4	279	3	5.00	1.07	5.63						
tot hrs	rank by	hrs at fa	rank by	0					beta	alpha	Weibull mean	arith mean
254	2	254	1	1.5	0.22	5.54	-1.38	30.66	-0.14	0.00	#NUM!	340
425	3	425	2	3	0.50	6.05	-0.37	36.63				
52	1			3.6	0.61							
472	4			4.8	0.83							
852	5			6								
tot hrs	rank by	hrs at fa	rank by	0					beta	alpha	Weibull mean	arith mean
114	1	114	1	1.25	0.18	4.74	-1.64	22.43	2.30	232.43	232	210
178	2	178	2	2.5	0.41	5.18	-0.65	26.85				
227.8	3	227.8	3	3.75	0.64	5.43	0.02	29.47				
321	4	321	4	5	0.87	5.77	0.71	33.31				

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Subsample Result: Lessons Learned

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1. Not enough samples: log of adjusted rank remains negative

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1. Not enough samples: log of adjusted rank remains negative
2. Total hours to hours at failure inversion

Subsample Result

subsamples				adj rank	f(t)	ln (t)	ln ln					
tot hrs	rank by hours	hrs at fail	rank by failure	0		x	y	x^2	beta	alpha	Weibull mean	arith mean
	1	194.2	4	1.25	0.22	5.27	-1.41	27.76	-0.07	0.61	#NUM!	235
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This looks pretty good!

Lessons Learned

- Weibull is very flexible
  - Too flexible if you don't have a sanity check in place
- To fit the Weibull ideally you need
  - A well-behaved test sample
    - Same start times
    - Hours at failure increasing with hours of operation
    - At least 3 failures
    - About half the units in failure
- This project was not a good candidate for the Weibull

All Samples

beta	alpha	Weibull mean	Arithmatic mean
0.308	966.4	5798	230
1 (infant mort)	966.4	966	230

15 failures out of 22 tests

References

- AMSAA Reliability Growth Guide, Technical Report TR-652, September 2000
- Dodson, Brian, “The Weibull Analysis Handbook,” American Society for Quality, 2006.
- Handbook for Reliability Test Methods, Plans and Environments for Engineering, Development, Qualification, and Production (MIL HDBK 781)
- Kapur, K.C. and L.R. Lamberson, “Reliability in Engineering Design,” New York: Wiley, 1977.
- Military Handbook of Reliability Prediction of Electronic Equipment (MIL HDBK 217 vF)
- Reliability Growth Management (MIL HDBK 189)
- Samaniego, F.J. and Y. S. Chong, “On The Performance Of Weibull Life Tests Based On Exponential Life Testing Designs,” National Academy of Science: Statistics, Testing, and Defense Acquisition: Background Papers, National Academies Press, 1996-1997.
- Neubeck, Ken, Practical Reliability Analysis, Pearson Education, NJ 2004.
- www.mathpages.com – Click on Probability and Statistics, Weibull Analysis