Ideas from Measurement Theory for Quantifying Intangibles

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Preliminaries

Abstract

Measurement theory is the mathematical study of measurement scales.

Psychologists, economists, and mathematicians developed measurement theory, in part, to provide a rigorous basis for quantifying subjective and intangible attributes. The theory has similar promise for intangibles in which the cost community has novel interests, such as ‘complexity’, ‘difficulty’, ‘newness’ of design, and ‘technology level’.

This presentation reports on progress in making these applications, while providing the audience a thumbnail tutorial to measurement theory concepts.

Acknowledgements

Thanks to Richard M. Soland and Fred S. Roberts, for inspiring an interest in measurement; and to Don Mackenzie for inspiring an interest in intangibles.

Apologia

We forego notational and conceptual rigor to facilitate this presentation. The references section identifies more careful treatments of measurement theory and its applications.

Agenda

- What is measurement theory?
- Why does it matter?
- Why focus on intangibles?
- What does measurement theory tell us about quantifying intangibles?

What is measurement?

[Measurement is] "*the assignment of numbers to represent properties of material systems … in virtue of the laws governing these properties.*" (Campbell, 1938, p. 126)

"Measurement of a property involves the assignment of numbers … to represent that property." (Torgerson, 1958, p. 14)

All of the above definitions suggest that measurement has something to do with assigning numbers that represent or preserve certain observed relations. (Roberts, 1979, p. 50)

What does it mean to preserve relations?

Let A be a set of objects. For all *a* in *A*, ***f*(*a*)** maps – or assigns – a number to each object. To measure, say, the weight of objects in *A*, *f* should satisfy the conditions, $\forall$ *a*, *b* in *A*:

for all

-$a\ H\ b \Rightarrow f(a) > f(b)$;

If *a* is 'heavier' than *b*, we want *f*(*a*) > f(*b*). We say that *f* with > preserves the *H* relation between *a* and *b*.

-$f(a) > f(b) \Rightarrow a\ H\ b$

Any numbers *f*(*a*) and *f*(*b*) preserving these relations satisfactorily quantify weight in this sense

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This thinking covers the case "*a* is just as heavy as *b*." When *a* is not heavier than *b* and ~ *b H a* it must be that *f*(*a*) = *f*(*b*) and we define *a* equivalent to *b* in this sense: $\sim a\ H\ b\ \&\ \sim b\ H\ a \Leftrightarrow a\ E_H\ b$

What does it mean to preserve relations?

To measure, say, the weight of each object, the mapping *f* should satisfy

$a \; H \; b \Leftrightarrow f(a) > f(b) \; \forall \; a, b \text{ in } A$

In the case of weight we normally expect more of *f* than that it merely preserves the rank ordering of all *a* in *A*. ‘Additivity’ fills this bill. For an object made up of the combination of *a* and *b* from *A*:

-($a \circ b$) should be in *A*

The ‘combined object’ $a \circ b$, should be in the set *A.*

-$f(a \circ b) = f(a) + f(b)$

The number we assign to the combined objects is the sum of the numbers we assign to *a* and *b* separately. We say that the arithmetic operation “+” preserves ‘additivity’ in $\circ$.

If $\forall \; a, b$ in *A*, *f* satisfies these conditions and $a \; H \; b \Leftrightarrow f(a) > f(b)$, then for any number $\alpha > 0$, $g = \alpha \times f$ is another mapping – more on this later.

We will call *f* a **measurement scale** or **scale** if it preserves the specified relations.

What is measurement theory?

Three main questions:
Representation, Uniqueness and Meaningfulness

- Representation. Does a measurement scale *f* exist – for the relations we want preserved and the *a* in *A*?

What properties of the objects *a* in *A* are necessary and sufficient for there to exist a suitable measurement scale – e.g., one that preserves additivity and not merely rank order?

- Uniqueness. What is the scale's uniqueness property?

We noted in the examples that *f* is not unique in preserving the specified relations. In the first, <u>any</u> *f* preserving $a\ H\ b \Leftrightarrow f(a) > f(b)\ \forall\ a, b$ also works; in the second, only $\alpha \times f$, $\alpha > 0$ works. A measurement scale with this last uniqueness property is a **ratio scale**.

In the ratio scale case, we can think of $\alpha \times f$ in terms alternative units for weight – e.g., kilograms, pounds. We convert between the units using appropriate $\alpha > 0$; in fact, <u>any</u> such α generates a new unit.

What is measurement theory?

Three main questions

- Meaningfulness. When measurement scales are not unique, do statements incorporating the scales fundamentally change when we replace them with other scales allowed by uniqueness properties?

  A statement incorporating measurement scales is 'meaningful' if its truth (or falsity) is invariant when we replace the scales with other allowable scales.

  Example: The predictions of a linear regression model incorporating a weight independent variable, expressed in pounds will not change if we rescale the weight data to kilograms and refit the model constants. The weight data are ratio scaled; the model predictions are meaningful in that they don't change under any admissible rescaling of the weight data.

Why does measurement theory matter?

Meaningfulness

It's all about meaningfulness

- Do the uses to which we've put measurement scales give meaningful results - in the invariance sense?

- How will we use the measurements?

For example, to develop

  - Regression models and other estimators
  - Significance tests, confidence intervals
  - Complexity factors, difficulty factors, metrics
  - Classifications providing a rank ordering – e.g., Technology Readiness Levels

- What uniqueness properties do we require for the uses to be meaningful – in the invariance sense?

Why does measurement theory matter?

Uniqueness

- What is the measurement scale *f* ’s uniqueness property? Some examples.
  - No other scale – *f* is an ‘absolute’ scale
  - If another scale *g* also works and:
    - $g = \alpha \times f + \beta$, $\alpha > 0$ – *f* is an ‘interval’ scale;
    - $g = \alpha \times f$, $\alpha > 0$ – *f* is a ‘ratio’ scale;
    - $g = f + b$ – *f* is a ‘difference’ scale;
    - $g = \Phi(f)$, where Φ is any monotone, strictly increasing function so that $g(a) > g(b)$ Φ if and only if $f(a) > f(b)$ – *f* is an ‘ordinal’ scale.

- What conditions are sufficient for a scale to have a specified uniqueness property? What conditions are necessary?

Why does measurement theory matter?

Representation

- Does a measurement scale *f* exist for the relations it must preserve and the *a* in *A*?
- What properties of the objects *a* in *A* are necessary and sufficient for there to exist a measurement scale *f* ?

Do scales with the desired uniqueness properties exist?

Why focus on intangibles?

- What is an intangible anyway?

1. Not tangible (*Dictionary.com*) **That helps … (sarcasm)**
2. An unquantifiable quality or asset (*Encarta Dictionary)* **That doesn't help …**
3. Incapable of being perceived by the senses (*Thefreedictionary.com*)

I haven't found any definitions without holes or that don't require indefensible assumptions. After trying many definitions for 'intangible' I've adopted the "know it when I see it" approximation and have moved on from there.

Push come to shove, I see intangible attributes as conceptual, man-made constructs while tangibles are physical attributes inherent in the object. As such, tangibles are thus 'directly perceivable' by the senses or indirectly by using devices; intangible attributes are not observable in this way. Tangible attributes – in my mind – include such attributes as weight, extent, duration, temperature.

Why focus on intangibles?

- Why focus on intangibles?
  - There are well-developed scales, procedures, and instruments for quantifying tangible attributes.
  - The intangible attributes that researchers have studied in depth – such as subjective probability and utility – provide a proof of principle that the required analysis is both possible and practical.
  - The intangibles that the cost community has occasion to quantify, or would like to quantify, are seemingly less mature in the needed analyses that provide a foundation for measurement. Filling these gaps promises improved understanding of the attributes and better utility in crafting cost models and cost estimates.

Some ideas from measurement theory

❑ Well-define the attributes

We don't need measurement theory to prescribe unambiguous definitions. However, measurement theory brings its own requirement.

Define "*a M b*" as "object *a* has more of an attribute *M* than does object *b*."

Every scale *f* preserves a relation of the form $\forall\, a, b$ in A, $a\ M\ b \Leftrightarrow f(a) > f(b)$. This is basic.

It is thus necessary to define the attribute well enough to make the discriminations *a M b*, *b M a*, or none of the above. Quoting KLST, "Little seems possible in the way of a careful analysis of an attribute until means are devised to say which of two objects or events exhibits more of the attribute" (1971, p. 32).

Note that the ordering pertains to the attribute itself and not to a proxy, index, correlate, or scale value for the attribute.

Some ideas from measurement theory

- How should the attribute behave after combining objects – i.e., $\circ$.

Recall that the ratio scale for weight preserved additivity, $f\,(a \circ b) = f\,(a) + f\,(b)$. By virtue of satisfying this condition, *f* is unique only up to multiplication by a positive scalar. Having this uniqueness property is generally good for the uses to which we put scales, from a meaningfulness standpoint.

However, intangible attributes, like complexity, difficulty, newness, or technology level may not add when combining objects. Focusing on complexity, that of $a \circ b$ may fall strictly between that of *a* and *b; or* it may be equivalent to the more complex of the pair. The scale value $f\,(a \circ \mathrm{b})$ would correspondingly fall strictly between $f\,(a)$ and $f\,(b)$; or equal their maximum. Which holds could have consequences for the uniqueness property of the scale *f* and the meaningfulness of uses to which we'd put it.

Consistent with prior measurement theory work, we may discover there are different kinds of complexity or different senses of combining *a* and *b*, each with its own properties for $a \circ b$ and the measurement scales that preserve it.

Some ideas from measurement theory

❑ Some non-physical attributes are 'multidimensional'

Asked why objects *a* have more of an attribute than objects *b* – *a M b* – we may identify several contributing factors – e.g., complexity may reflect both the number of interacting components and how they interact. As a result, we may now need:

- a measurement scale, f_i, $i = 1 \ldots n$ for each of the *n* factors – 'sub-attributes' – that distinguish objects in the 'primary attribute' – e.g., complexity; and
- a 'merging function' ***f*** for combining the sub-attribute scales into a scale for the primary attribute.

The three measurement questions – representation, uniqueness, meaningfulness – exist for sub-attributes as well as for primary-attributes. The theory addressing them has the name "conjoint measurement theory" (KLST, 1971; Roberts, 1979); and should sound familiar to those who've studied "multi-attribute utility" in decision theory (e.g., Keeney and Raiffa, 1976).

Recap

- What is measurement theory?
- Why does it matter?
- Why focus on intangibles?
- What does measurement theory tell us about quantifying intangibles?

References

Suggested readings

Roberts, F.S. *Measurement Theory*, *with Applications to Decisionmaking, Utility, and the Social Sciences.* Addison Wesley Publishing Company, 1979.

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