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Cost Contingency Analysis using Polytopes

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Canada

Outline

- Background
- Contingency: Estimation vs. Analysis
- Contingency Analysis by Example
- Polytope Theory
- Contingency Analysis Procedure and Extensions
- Summary

Background

Uncertainty is inherent to cost estimation

- Uncertainties are risks to the project
- "Known-unknowns" are risks that the estimator is aware of

Cost contingency is the estimated cost of the known-unknowns

- an amount added to an estimate to allow for additional costs that experience shows will likely be required
- costs that will probably occur based on past experience, but with some uncertainty regarding the amount.

The objective is to prevent a project from experiencing cost overrun

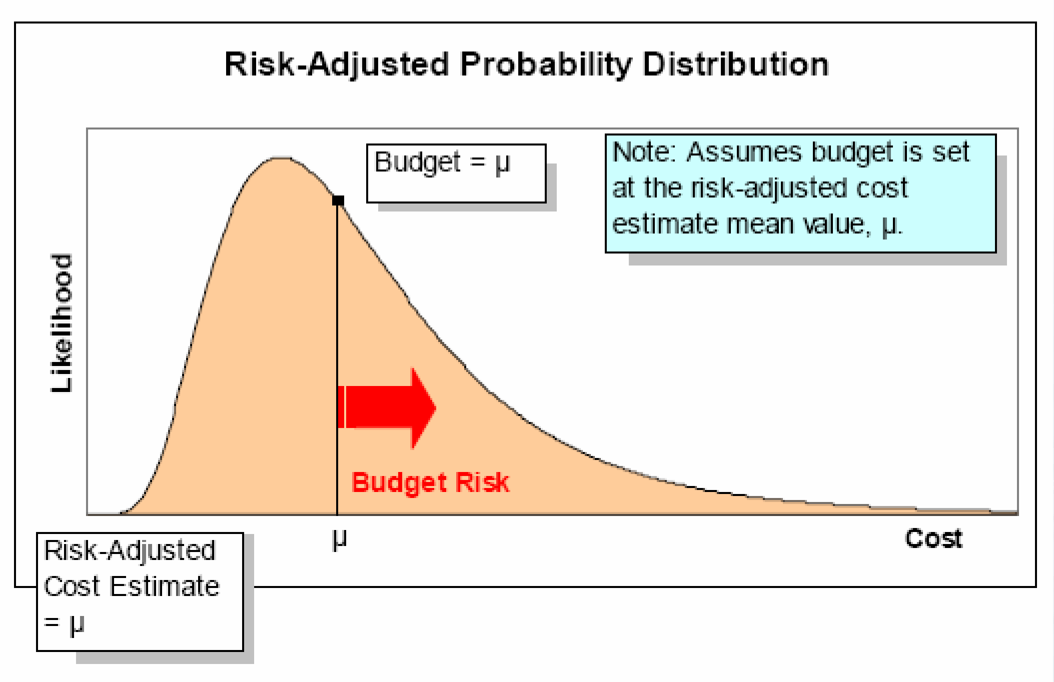
Background

Cost Contingency Estimation Approaches (i.e., cost risk analysis)

1. Expert judgment (based on previous experience)
2. Predetermined guidelines / rule-of-thumb (e.g., 30% of baseline estimate)
3. Simulation analysis (e.g., Monte Carlo) and selected confidence level
4. Parametric modelling

Background

Best practice cost estimation typically yields a cost probability distribution:



Risk adjusted estimate
- based on achieving a certain confidence level
- includes contingency

Image source: NATO report **RTO-TR-SAS-054**

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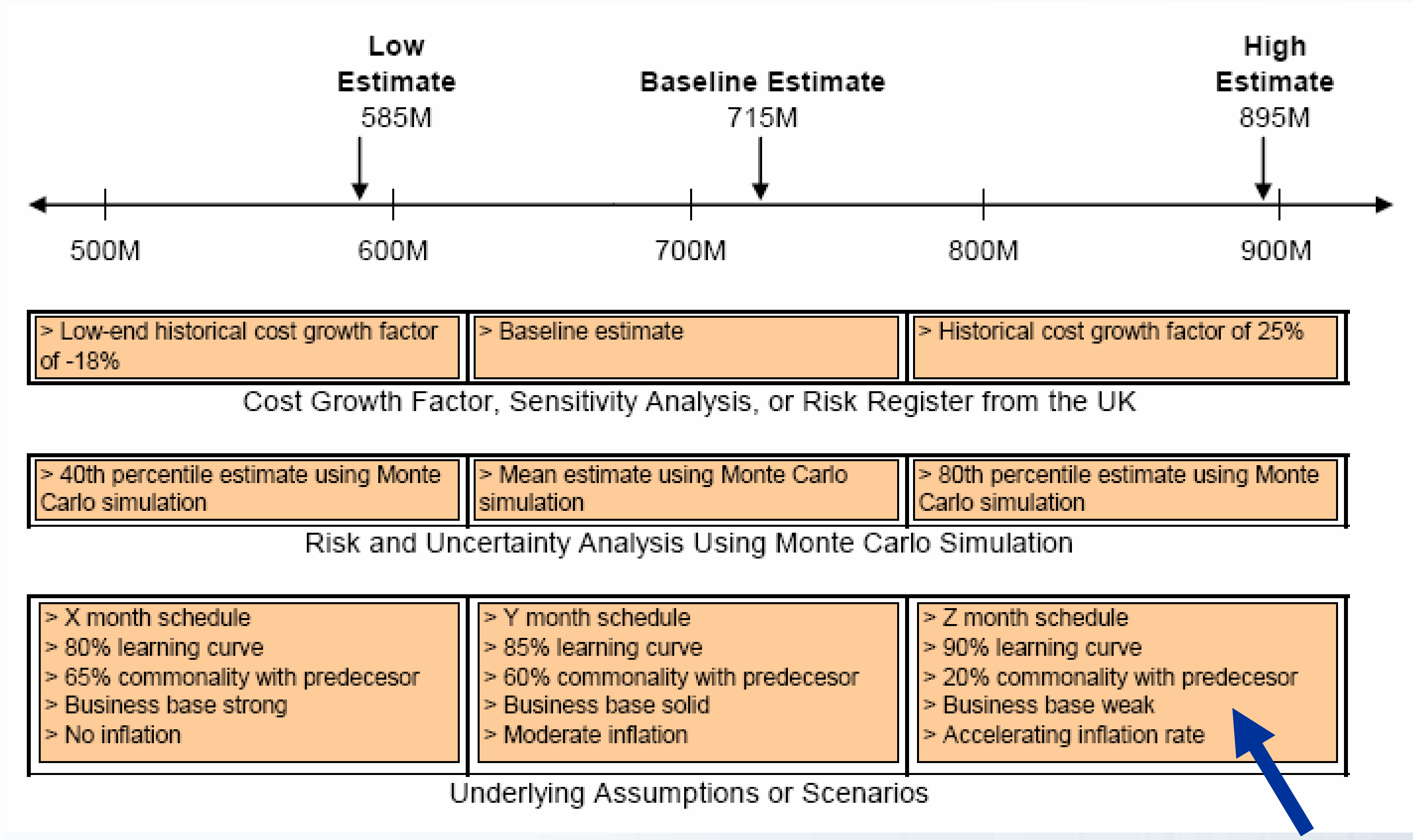
Background

Best practice presents decision maker with low, baseline, and high estimates:



Description of scenarios limited,
- typically focus on worst-case
- can mislead decision makers

Image source: NATO report **RTO-TR-SAS-054**

Estimation vs. Analysis

Cost Contingency Estimation Approaches

1. Expert judgment (based on previous experience)
2. Predetermined guidelines / rule-of-thumb
3. Simulation analysis
4. Parametric modelling

Contingency Analysis:

Simulation analysis that employs different qualitative assumptions (associated with probable events or phenomenon) to paint different scenarios, and tries to come up with the most optimal responses under the circumstances.

- BusinessDictionary.com

Estimation vs. Analysis

Cost Contingency Estimation Approaches

1. Expert judgment (based on previous experience)
2. Predetermined guidelines / rule-of-thumb
3. Simulation analysis
4. Parametric modelling

Cost Contingency Analysis:

The examination of a cost contingency estimate to fully understand the set of *scenarios* that would be covered, and which would not.

Scenarios are realizable combinations of known-unknowns

Contingency Analysis: Example

Canada is examining options for renewing its fighter capability to replace its aging CF-18 fleet

- Expected to retire between 2017 and 2020
- One option considered is the Lockheed Martin F-35A Lightning II
  - Also known as the Joint Strike Fighter (JSF)

- Canada is a partner in the multi-national JSF Program
- In 2010 Canadian Department of National Defence (DND) calculated that the expected cost of acquiring 65 F-35A was $9 billion dollars (CAD)

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Contingency Analysis: Example

In 2012, Government of Canada launched a Seven-Point Plan:

–Ensure that the Royal Canadian Air Force acquires the aircraft it needs through an open, transparent process

–Requires DND to provide annual updates to Parliament

–12/2012: released "Next Generation Fighter Capability Annual Update"

(http://www.forces.gc.ca/site/pri/2/pro/10/2012/annual_update-eng.pdf)

→Full life-cycle cost estimate of Joint Strike Fighter F-35A (for Canada)

Current baseline acquisition estimate at $8.4B

Contingency: $9B - $8.4B = $600M

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Contingency Analysis: Example

The report identified 4 main risks to acquisition cost:

1 - Foreign Exchange: CAD to USD

2 - Learning and Production rates: Unit Recurring Flyaway Costs

3 - Inflation

4 - Change in production numbers: reduction of int'l partner orders

Subject matter experts assessed associated likelihoods and impact

→ Cost risk analysis / contingency estimation

Note: all data and cost figures presented herein were extracted from Canada's Department of National Defence "Next Generation Fighter Capability Annual Update" publicly available at http://www.forces.gc.ca/site/pri/2/pro/10/2012/annual_update-eng.pdf

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Contingency Analysis: Example

Contingency on Acquisition: Acquisition contingency is primarily based on the statistical analysis technique of Expected Value. The Expected Value is the cost of a risk multiplied by the probability of the risk occurring.

Contingency Tables	Max Impact $Million	Mid-Point of Likelihood Ranges	Expected Value $Million
Foreign Exchange	1,800	30%	540
Inflation	500	30%	150
Learning/Production	1,700	10%	170
Number of Aircraft	500	50%	250
Other Acquisition Cost Risks	n/a	n/a	340
Total	**4,500**		**1,450**

The full amount of contingency suggested by the Life-Cycle Cost Framework is $1.45B

"If the full available acquisition contingency was required, the shortfall would be met by buying fewer aircraft" – *2012 Next Generation Fighter Capability Annual Update*

What scenarios does $600M of contingency cover? Which would not be?

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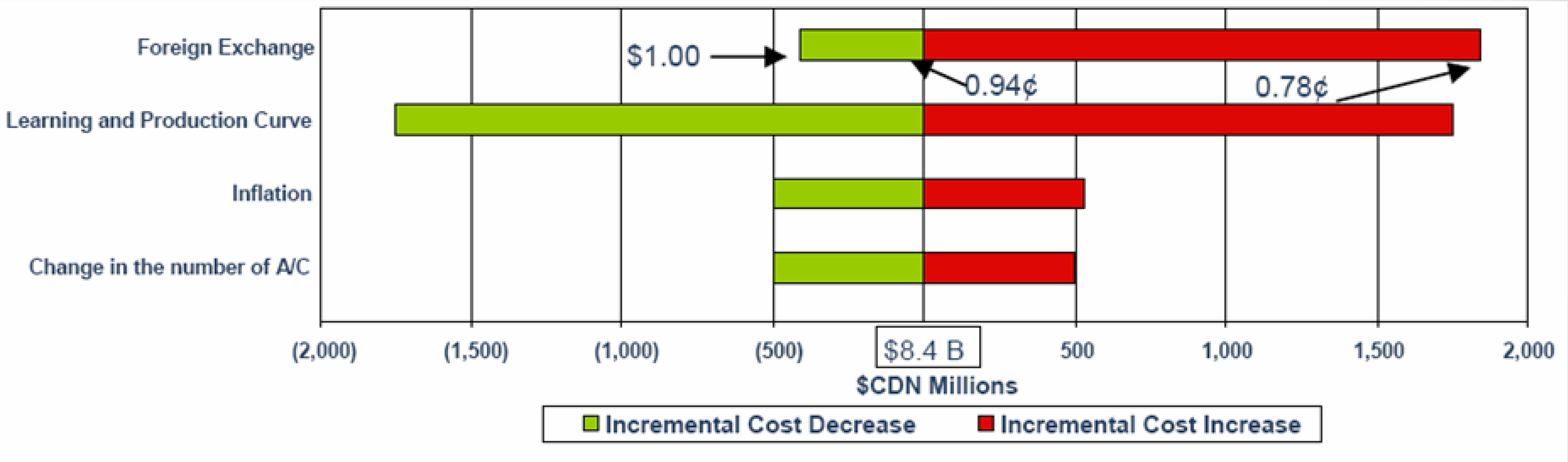
Contingency Analysis: Example

Tornado graph synthesized high-level results of subject matter experts estimates



Above results provide direction for risk mitigation efforts, but do not provide clarity into possible scenarios covered (or not) by contingency

What scenarios does $600M of contingency cover? Which would not be?

Contingency Analysis Procedure

Step 1: Identify main cost risk drivers (e.g., sensitivity analysis)

Step 2: Quantify impact of individual cost drivers

select suitable granularity

Step 3: Define lower and upper bounds for each cost driver

May be several such bounds, dependent on likelihood

Step 4: Express a system of variables and inequalities

Step 5: Convert to menu / list description of scenarios

Contingency Analysis: Example

Step 1: Identify main cost drivers

Foreign Exchange

Learning and Production rates

Inflation

Change in production numbers / buy profile

Note: all data and cost figures presented herein were extracted from Canada's Department of National Defence "Next Generation Fighter Capability Annual Update" publicly available at http://www.forces.gc.ca/site/pri/2/pro/10/2012/annual_update-eng.pdf

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Contingency Analysis: Example

Step 2: Quantify impact of individual cost drivers

Foreign Exchange:

1 change = $80M

Learning and Production rates:

1% point deviation change to learning or production rate = $5M

Inflation:

0.1% point change (over buy years) = $60M

Change in production numbers / buy profile:

1 unit change over Canada's buy profile = $1.2M

Note: all data and cost figures presented herein were extracted from Canada's Department of National Defence "Next Generation Fighter Capability Annual Update" publicly available at http://www.forces.gc.ca/site/pri/2/pro/10/2012/annual_update-eng.pdf

Contingency Analysis: Example

Step 2: Quantify impact of individual cost drivers

Foreign Exchange:

Define x_1 to be # of cents change in foreign exchange

Learning and Production rates:

Define x_2 to be % points deviation of learning or production rate

Inflation:

Define x_3 to be the # of 1/10th of % point change in inflation

Change in production numbers / buy profile:

Define x_4 to be the □Δ in production quantities over the buy profile

Contingency Analysis: Example

Step 3: Define lower and upper bounds for each cost driver

Foreign Exchange:

$-10 \leq x_1 \leq 10$ (# of cents change in foreign exchange)

Learning and Production rates:

$-2 \leq x_2 \leq 5$ (% points deviation of learning/production rate)

Inflation:

$0 \leq x_3 \leq 6$ (% point change in inflation / 10)

Change in production numbers / buy profile:

$-100 \leq x_4 \leq 500$ (?Δ production quantities over the buy profile)

Apply $600M constraint on contingency:

$$80\,x_1 + 5\,x_2 + 60\,x_3 + 1.2\,x_4 \leq 600$$

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Contingency Analysis: Example

Step 4: Express as a system of variables and inequalities

System of inequalities describes the entire set of feasible scenarios covered by the contingency amount ($600M)

Baseline *Upper limit of contingency*

$$0 \leq 80\, x_1 + 5\, x_2 + 60\, x_3 + 1.2\, x_4 \leq 600$$

$$-10 \leq x_1 \leq 10$$

$$-2 \leq x_2 \leq 5$$

$$0 \leq x_3 \leq 6$$

$$-100 \leq x_4 \leq 500$$

Contingency Analysis: Example

Step 4: Express as a system of variables and inequalities

System of inequalities describes the entire set of feasible scenarios covered by the contingency amount ($600M)

Baseline *Upper limit of contingency*

$$0 \leq 80\,x_1 + 5\,x_2 + 60\,x_3 + 1.2\,x_4 \leq 600$$

$$-10 \leq x_1 \leq 10$$

$$-2 \leq x_2 \leq 5$$

$$0 \leq x_3 \leq 6$$

$$-100 \leq x_4 \leq 500$$

Not very useful!

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Contingency Analysis: Example

Step 5: Convert to a menu / list description of scenarios

System of inequalities describes the entire set of feasible scenarios covered by the contingency amount ($600M)

$$0 \le 80\,x_1 + 5\,x_2 + 60\,x_3 + 1.2\,x_4 \le 600$$

$$-10 \le x_1 \le 10$$

$$-2 \le x_2 \le 5$$

$$0 \le x_3 \le 6$$

$$-100 \le x_4 \le 500$$

Not very useful!

Menu: list of explicit scenarios

	FOREX	RATE	INFLATION	UNITS	COST
a	1.6	-2	0	-100	0
b	-2.9	-2	6	-100	0
c	-10.0	-2	6	375	0
d	-10.0	-2	4	500	0
e	-10.0	-2	6	500	150
f	-4.4	-2	6	500	600
g	-10.0	5	6	500	185
h	-4.8	5	6	500	600
i	-10.0	5	3	500	0
j	4.6	-2	6	-100	600
k	-3.3	5	6	-100	0
l	-10.0	5	6	346	0
m	4.2	5	6	-100	600
n	-7.4	-2	0	500	0
o	0.1	-2	0	500	600
p	-7.8	5	0	500	0
q	-0.3	5	0	500	600
r	9.1	-2	0	-100	600
s	1.2	5	0	-100	0

Much more useful!

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Contingency Analysis: Example

Complete set of feasible scenarios covered by the contingency amount ($600M)

Menu: list of explicit scenarios

	FOREX	RATE	INFLATION	UNITS	COST
a	1.6	-2	0	-100	0
b	-2.9	-2	6	-100	0
c	-10.0	-2	6	375	0
d	-10.0	-2	4	500	0
e	-10.0	-2	6	500	150
f	**-4.4**	**-2**	**6**	**500**	**600**
g	-10.0	5	6	500	185
h	-4.8	5	6	500	600
i	-10.0	5	3	500	0
j	4.6	-2	6	-100	600
k	-3.3	5	6	-100	0
l	-10.0	5	6	346	0
m	4.2	5	6	-100	600
n	-7.4	-2	0	500	0
o	0.1	-2	0	500	600
p	-7.8	5	0	500	0
q	-0.3	5	0	500	600
r	9.1	-2	0	-100	600
s	1.2	5	0	-100	0

Scenario f:

Favourable foreign exchange by 4.4 cents

Improved learning/production rate by 2% points

Inflation worst-than-expected by 0.6% points

500 less F-35A units purchased by partners

Result: Depletes $600M contingency

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Contingency Analysis: Example

Complete set of feasible scenarios covered by the contingency amount ($600M)

Menu: list of explicit scenarios

	FOREX	RATE	INFLATION	UNITS	COST
a	1.6	-2	0	-100	0
b	-2.9	-2	6	-100	0
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d	-10.0	-2	4	500	0
e	-10.0	-2	6	500	150
f	-4.4	-2	6	500	600
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k	-3.3	5	6	-100	0
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o	0.1	-2	0	500	600
p	-7.8	5	0	500	0
q	-0.3	5	0	500	600
r	9.1	-2	0	-100	600
s	1.2	5	0	-100	0

Claim: this list completely characterizes all possible scenarios covered by contingency

Can take any *convex combination* of the scenarios to generate another scenario

e.g.,

		FOREX	RATE	INFLATION	UNITS	COST
0.3 x	r	9.1	-2	0	-100	600
+0.7 x	h	-4.8	5	6	500	600
=	t	**-0.6**	**2.9**	**4.2**	**320**	**600**

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Contingency Analysis:

In general, can define any corresponding system of inequalities to consider:

- relationships between cost drivers
- scenarios covered by or that exceed contingency level
- detailed constraints on budget overrun level

Drawback: requires linear inequality description

→ Not straightforward to model complex relationships between cost drivers

→ Suitable as a top-down analysis, not bottom-up

→ Suitable for high-level list of cost risk drivers (< 20)

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Contingency Analysis: Example

System of inequalities describes the set of scenarios exceeding the contingency amount by $100M.

Upper limit of contingency *Contingency + $100M*

$$600 \leq 80\,x_1 + 5\,x_2 + 60\,x_3 + 1.2\,x_4 \leq 700$$

$$-10 \leq x_1 \leq 10$$

$$-2 \leq x_2 \leq 5$$

$$0 \leq x_3 \leq 6$$

$$-100 \leq x_4 \leq 500$$

	FOREX	RATE	INFLATION	UNITS	COST
a	9.1	-2	0	-100	600
b	4.6	-2	6	-100	600
c	5.9	-2	6	-100	700
d	4.2	5	6	-100	600
e	5.4	5	6	-100	700
f	0.1	-2	0	500	600
g	-4.4	-2	6	500	600
h	-3.1	-2	6	500	700
i	-4.8	5	6	500	600
j	-3.6	5	6	500	700
k	1.4	-2	0	500	700
l	-0.3	5	0	500	600
m	0.9	5	0	500	700
n	10.0	-2	0	-100	670
o	10.0	-2	1	-100	700
p	10.0	-2	0	-75	700
q	10.0	4	0	-100	700
r	8.7	5	0	-100	600
s	9.9	5	0	-100	700

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Contingency Analysis: Example

System of inequalities describes the set of scenarios exceeding the contingency.

Upper limit of contingency *Unbounded!*

$$600 \leq 80\,x_1 + 5\,x_2 + 60\,x_3 + 1.2\,x_4$$

$$-10 \leq x_1 \leq 10$$

$$-2 \leq x_2 \leq 5$$

$$0 \leq x_3 \leq 6$$

$$-100 \leq x_4 \leq 500$$

	FOREX	RATE	INFLATION	UNITS	COST
a	9.1	-2	0	-100	600
b	4.6	-2	6	-100	600
c	10.0	-2	6	-100	1030
d	4.2	5	6	-100	600
e	10.0	5	6	-100	1065
f	0.1	-2	0	500	600
g	-4.4	-2	6	500	600
h	10.0	-2	6	500	1750
i	-4.8	5	6	500	600
j	10.0	5	6	500	1785
k	10.0	-2	0	500	1390
l	-0.3	5	0	500	600
m	10.0	5	0	500	1425
n	10.0	-2	0	-100	670
o	8.7	5	0	-100	600
p	10.0	5	0	-100	705

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Contingency Analysis

System of Inequalities

$$600 \leq 80\,x_1 + 5\,x_2 + 60\,x_3 + 1.2\,x_4$$

$$-10 \leq x_1 \leq 10$$

$$-2 \leq x_2 \leq 5$$

$$0 \leq x_3 \leq 6$$

$$-100 \leq x_4 \leq 500$$

How ?

Menu / List of Scenarios

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Answer: Convex Polytopes

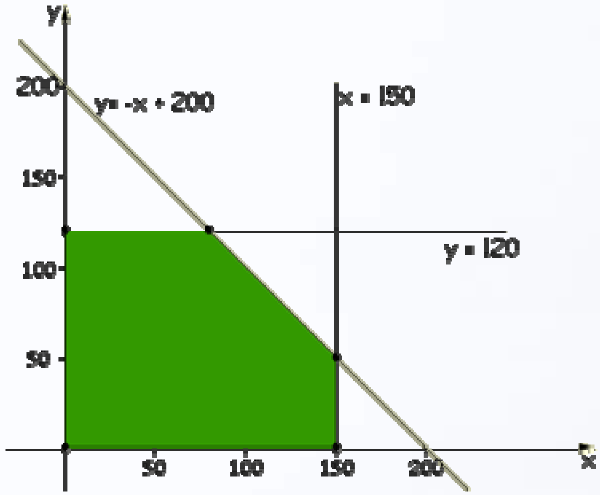
What is a Convex Polytope?

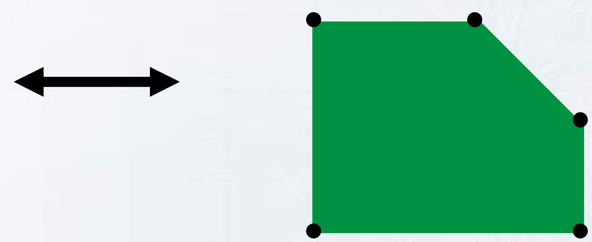
Main Theorem for Convex Polytopes

P is a bounded intersection of halfspaces/inequalities (H-polytope)

if and only if

P is a convex hull of a finite point set (V-polytope)





Representations are mathematically equivalent, but not algorithmically!

What is a Convex Polytope?

Main Theorem for Convex Polytopes

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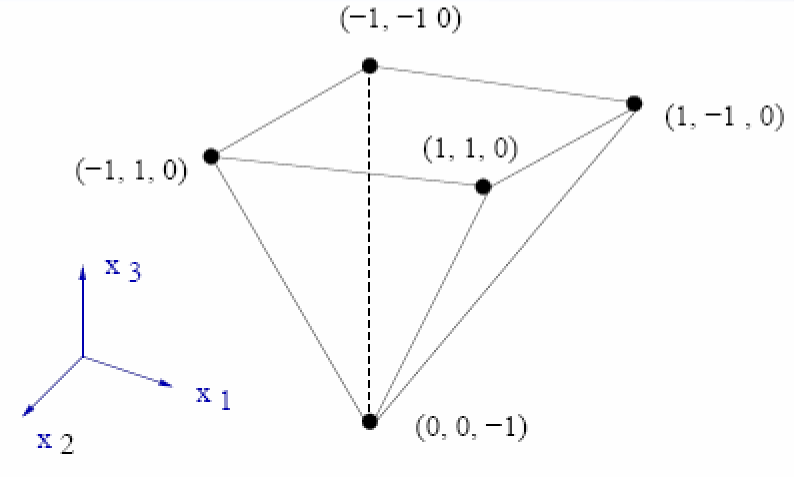
Example in three dimensions:



$$
\begin{aligned}
1 - x_1 \quad\quad + x_3 &\geq 0 \\
1 \quad\quad - x_2 + x_3 &\geq 0 \\
1 + x_1 \quad\quad + x_3 &\geq 0 \\
1 \quad\quad + x_2 + x_3 &\geq 0 \\
- x_3 &\geq 0
\end{aligned}
\quad \longleftrightarrow \quad
\begin{aligned}
v_1 &= (-1, 1, 0), \\
v_2 &= (-1, -1, 0), \\
v_3 &= (1, -1, 0), \\
v_4 &= (1, 1, 0), \\
v_5 &= (0, 0, -1)
\end{aligned}
$$

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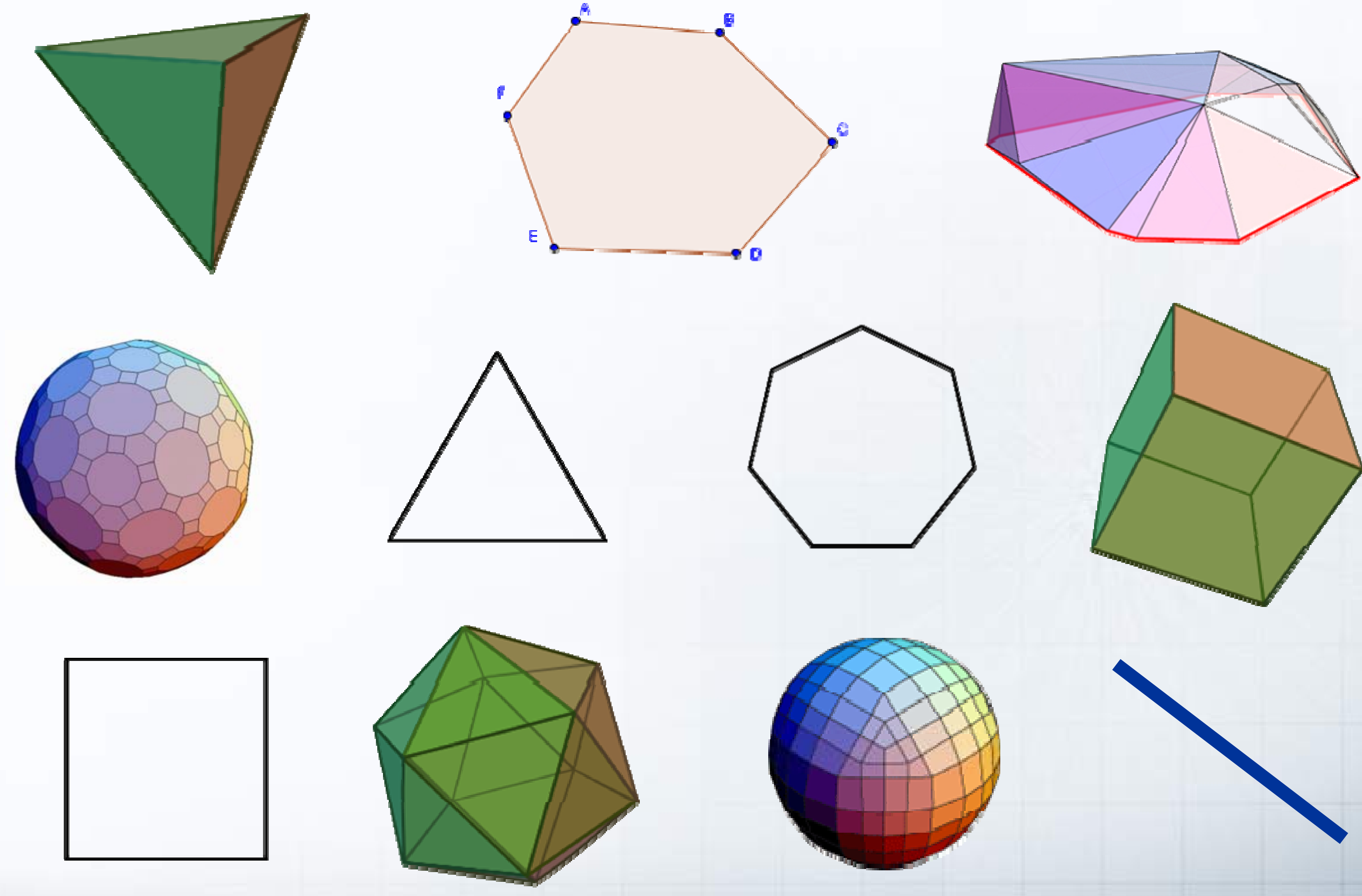
What is a Convex Polytope?

Here are some examples illustrated in 1D, 2D and 3D (projected to 2D)



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Main Theorem for Convex Polytopes

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if and only if

P is a convex hull of a finite point set (V-polytope)

In higher dimensions:

$$\begin{array}{ccccccc} a_{11}x_1 & + & a_{12}x_2 & + & a_{1n}x_n & \leq & b_1 \\ a_{21}x_1 & + & a_{22}x_2 & + & a_{2n}x_n & \leq & b_2 \\ & & & \cdots\cdots & & & \\ a_{m1}x_1 & + & a_{m2}x_2 & + & a_{mn}x_n & \leq & b_m \end{array}$$

$$x_1 \geq 0,\ x_2 \geq 0,\ \ldots,\ x_n \geq 0$$

⟷

Convex Hull of set of points $\{p_1,p_2,\ldots,p_k\}$, each p_i is a n-dimensional point

Representations are mathematically equivalent, but not algorithmically!

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Contingency Analysis

System of Inequalities

$$600 \le 80\,x_1 + 5\,x_2 + 60\,x_3 + 1.2\,x_4$$

$$-10 \le x_1 \le 10$$

$$-2 \le x_2 \le 5$$

$$0 \le x_3 \le 6$$

$$-100 \le x_4 \le 500$$

Menu / List of Scenarios

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o	8.7	5	0	-100	600
p	10.0	5	0	-100	705

Implementation: convex polytope software is available freely (open-source)

e.g., *lrs* available at *http://cgm.cs.mcgill.ca/~avis/C/lrs.html*

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Contingency Analysis: Example

Complete set of feasible scenarios covered by the contingency amount ($600M)

Menu: list of explicit scenarios

	FOREX	RATE	INFLATION	UNITS	COST
a	1.6	-2	0	-100	0
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e	-10.0	-2	6	500	150
f	-4.4	-2	6	500	600
g	-10.0	5	6	500	185
h	-4.8	5	6	500	600
i	-10.0	5	3	500	0
j	4.6	-2	6	-100	600
k	-3.3	5	6	-100	0
l	-10.0	5	6	346	0
m	4.2	5	6	-100	600
n	-7.4	-2	0	500	0
o	0.1	-2	0	500	600
p	-7.8	5	0	500	0
q	-0.3	5	0	500	600
r	9.1	-2	0	-100	600
s	1.2	5	0	-100	0

Claim: this list completely characterizes all possible scenarios covered by contingency

Can take any ***convex combination*** of the scenarios to generate another scenario

e.g.,

		FOREX	RATE	INFLATION	UNITS	COST
0.3 x	r	9.1	-2	0	-100	600
+0.7 x	h	-4.8	5	6	500	600
=	t	**-0.6**	**2.9**	**4.2**	**320**	**600**

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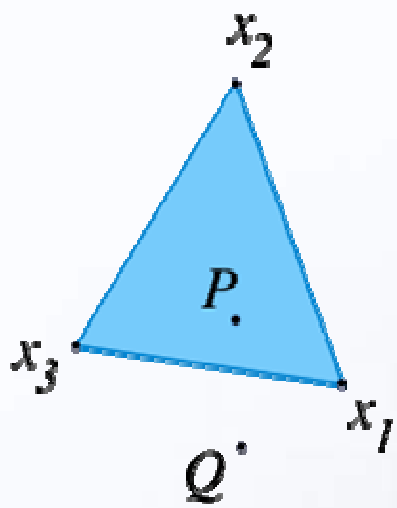
What is Convexity?

Convexity:



Point P *is* a convex combination of the three points x_1, x_2, and x_3. Q is *not.*

A convex combination is a linear combination of points where all coefficients are non-negative and sum up to 1

Given real points $x_1, x_2, \ldots, x_n$, a convex combination of these points is of the form $_1x_1 + \ _2x_2 + \ldots + \ _nx_n$, where $_i \geq 0$ and $_1 + \ _2 + \ldots + \ _n = 1$

Review of Contingency Analysis Procedure

Step 1: Identify main cost risk drivers (e.g., sensitivity analysis)

Step 2: Quantify impact of individual cost drivers

Step 3: Define lower and upper bounds for each cost driver

Step 4: Express a system of variables and inequalities

Step 5: Convert to menu / list description of scenarios

Extensions

Step 3: Define lower and upper bounds for each cost driver

→ can define several depending on confidence

e.g., on foreign exchange:

Recall that x_1 is the # of cents change in foreign exchange

Low Confidence: $-4 \leq x_1 \leq 4$

Medium Confidence: $-6 \leq x_1 \leq 6$

High Confidence: $-10 \leq x_1 \leq 10$

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Extensions

High Confidence:

$$0 \leq 80\,x_1 + 5\,x_2 + 60\,x_3 + 1.2\,x_4 \leq 600$$
$$-10 \leq x_1 \leq 10$$
$$-2 \leq x_2 \leq 5$$
$$0 \leq x_3 \leq 6$$
$$-100 \leq x_4 \leq 500$$

Medium Confidence:

$$0 \leq 80\,x_1 + 5\,x_2 + 60\,x_3 + 1.2\,x_4 \leq 600$$
$$-6 \leq x_1 \leq 6$$
$$-1 \leq x_2 \leq 3$$
$$0 \leq x_3 \leq 4$$
$$-75 \leq x_4 \leq 400$$

Low Confidence:

$$0 \leq 80\,x_1 + 5\,x_2 + 60\,x_3 + 1.2\,x_4 \leq 600$$
$$-4 \leq x_1 \leq 4$$
$$0 \leq x_2 \leq 2$$
$$0 \leq x_3 \leq 2$$
$$-50 \leq x_4 \leq 300$$

Compute scenarios for each

Each is a convex polytope

Extensions

High Confidence:

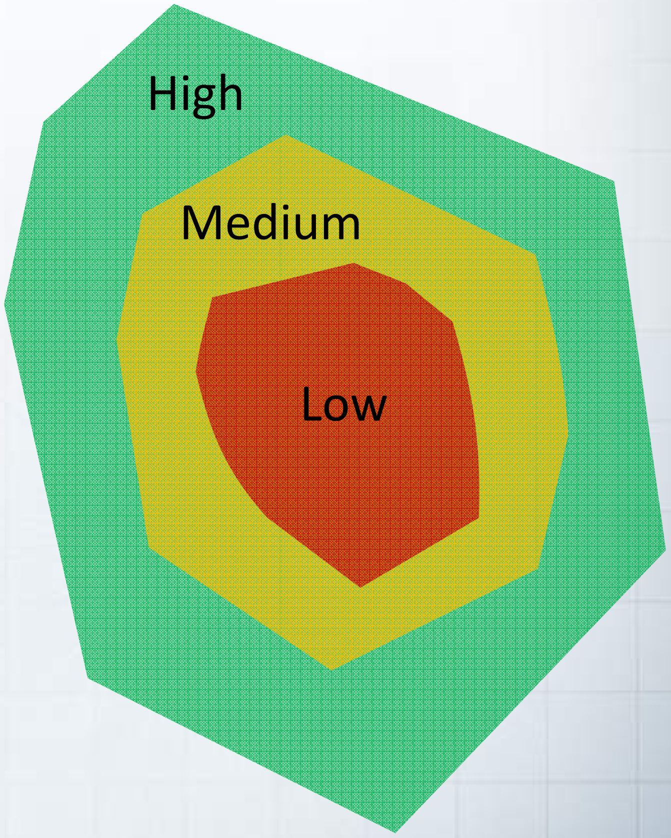
$$0 \le 80x_1 + 5x_2 + 60x_3 + 1.2x_4 \le 600$$
$$-10 \le x_1 \le 10$$
$$-2 \le x_2 \le 5$$
$$0 \le x_3 \le 6$$
$$-100 \le x_4 \le 500$$

Medium Confidence:

$$0 \le 80x_1 + 5x_2 + 60x_3 + 1.2x_4 \le 600$$
$$-6 \le x_1 \le 6$$
$$-1 \le x_2 \le 3$$
$$0 \le x_3 \le 4$$
$$-75 \le x_4 \le 400$$

Low Confidence:

$$0 \le 80x_1 + 5x_2 + 60x_3 + 1.2x_4 \le 600$$
$$-4 \le x_1 \le 4$$
$$0 \le x_2 \le 2$$
$$0 \le x_3 \le 2$$
$$-50 \le x_4 \le 300$$



Extensions

High Confidence:

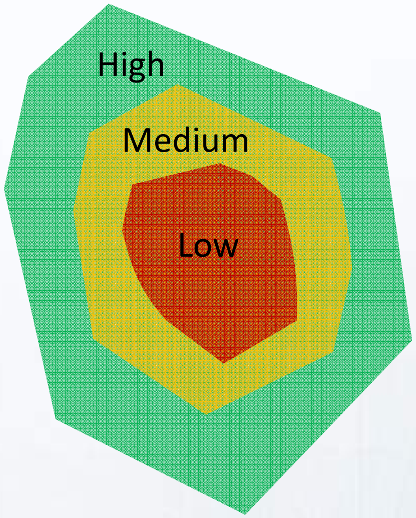
$0 \le 80x_1 + 5x_2 + 60x_3 + 1.2x_4 \le 600$
$-10 \le x_1 \le 10$
$-2 \le x_2 \le 5$
$0 \le x_3 \le 6$
$-100 \le x_4 \le 500$

Medium Confidence:

$0 \le 80x_1 + 5x_2 + 60x_3 + 1.2x_4 \le 600$
$-6 \le x_1 \le 6$
$-1 \le x_2 \le 3$
$0 \le x_3 \le 4$
$-75 \le x_4 \le 400$

Low Confidence:

$0 \le 80x_1 + 5x_2 + 60x_3 + 1.2x_4 \le 600$
$-4 \le x_1 \le 4$
$0 \le x_2 \le 2$
$0 \le x_3 \le 2$
$-50 \le x_4 \le 300$



A property of convex polytopes is that they have a computable volume

→Can examine the ratio of volumes:

High : Medium = 3.5 : 1

High : Low = 21.8 : 1

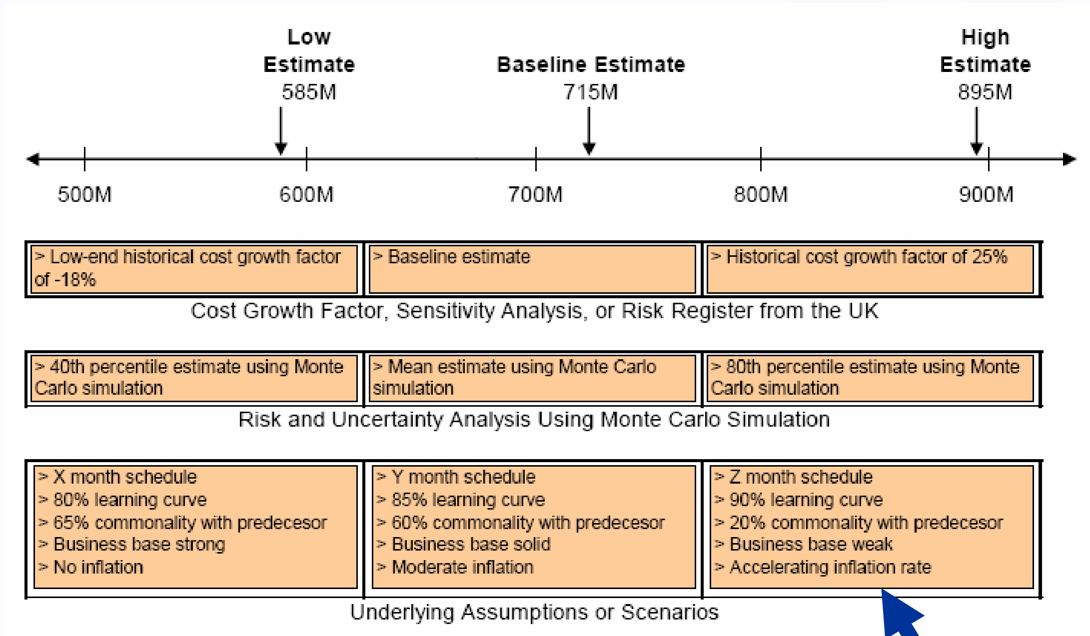
Medium : Low = 6.3 : 1

Interpretation:

scenarios covered at high confidence nearly 3.5 times more that # of scenarios covered at medium confidence

Bottom Line

Best practice presents decision maker with low, baseline, and high estimates:



Using contingency analysis with Polytopes facilitates generation of meaningful and comprehensive scenarios to decision makers

Image source: NATO report **RTO-TR-SAS-054**

DEFENCE DÉFENSE

Summary

- Contingency analysis is defined:

  The examination of a cost contingency estimate to fully understand the set of scenarios that would be covered, and which would not

- Proposed top-down methodology based on convex polytope theory

  Provides ability to comprehensively enumerate all possible scenarios concisely

  Provides ability to quantify comparison of contingency levels

  Assumes that impact of cost drivers and their relationships can be expressed as a system of linear inequalities

- Research continues into possible extensions / applications

Questions?

DEFENCE R&D DÉFENSE

DEFENCE DÉFENSE

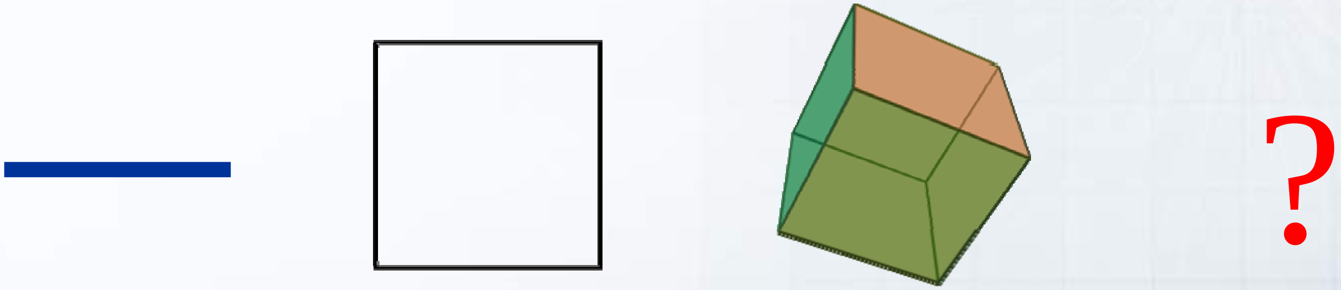
What is a Convex Polytope?

Hard to visualize high-dimensional convex polytopes, but can try!

Alicia Boole Stott (1860-1940)

- British Mathematician
- Coined the term “Polytope”
- Grasped 4D geometry at a very young age

The hypercube:



1D hypercube 2D hypercube 3D hypercube 4D hypercube

DEFENCE DÉFENSE

What is a Convex Polytope?

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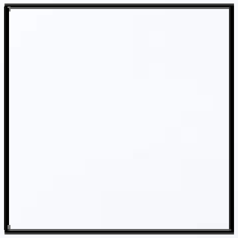
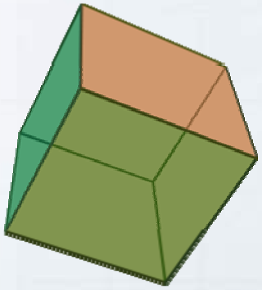
The hypercube:

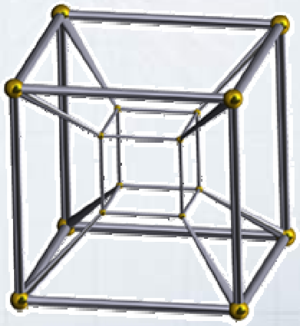
1D hypercube



2D hypercube



3D hypercube

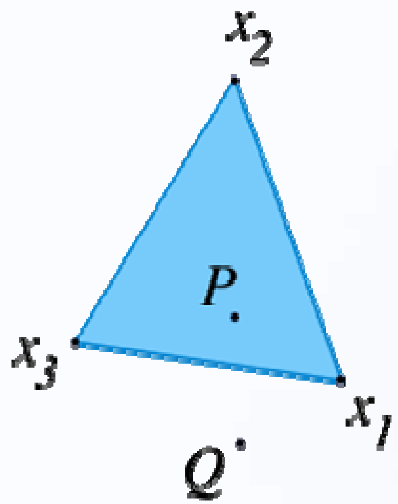


4D hypercube

Schlegel diagram (projection)

What is a Convex Hull?

The Convex Hull of a set of points (e.g., x_1, x_2, and x_3) is the set of all convex combinations of these points



A convex polytope is the convex hull of a finite set of points in real space