

# MODELING THE RISK AND UNCERTAINTY OF INFLATION RATE PROJECTIONS

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*“Experience in controversies such as these brings out the impossibility of learning anything from facts till they are examined and interpreted by reason; and teaches that the most reckless and treacherous of all theorists is he who professes to let facts and figures speak for themselves, who keeps in the background the part he has played, perhaps unconsciously, in selecting and grouping them, and in suggesting the argument post hoc ergo propter hoc.”*

*Alfred Marshall*

**Brian J. Flynn and Peter J. Braxton**

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## **Abstract**

Accurately modeling the risk and uncertainty of inflation rate projections remains a holy grail in defense cost analysis. Seemingly imponderable questions abound, such as what statistical distribution for inflation to use in Monte Carlo simulations, with what parameters, and over what time frame. Simply employing a triangular distribution, with endpoints taken as plus and minus a percentage-point or two from the purported mode, seems wholly inadequate.

This paper focuses on the estimation of probability distributions of inflation rate forecasts. Inferences are drawn from analyses of *survey*- and *market-based* measures of inflation expectations. *Survey-based* measures examine projections of inflation made by government officials and professional forecasters. *Market-based* measures, on the other hand, focus on expectations inferred from the prices and yields of financial instruments actively traded on Wall Street, such as Treasuries and, more recently, inflation derivatives. As Economists note, focus sharpens and credibility rises when prices are set by market agents with “skin in the game,” such as pension, insurance, and hedge-fund managers, who put their “money where their mouths are.”

- **Survey-Based Measures**

- The Survey of Professional Forecasters. Administered by the Federal Reserve Bank of Philadelphia, this prestigious survey provides explicit observations on long-term *mean* inflation expectations, and, importantly, enables construction of near-term *probability distributions* of inflation as well.

- **Market-Based Measures**

- Treasury “Breakevens.” The delta between nominal and real yields, *after* adjusting for risk and liquidity premiums, provides observations on the market’s expectation of inflation up to 30 years out.
- Inflation Derivatives. Zero-coupon swaps in this nascent, *laissez faire* options market provide data on the expectations of market players hedging against the risk of inflation. “Cap and floor” data enable the construction of inflation probability density functions based on the daily interaction of agents who buy and sell protection against rising and falling price levels in the macro economy.

These distinct but related areas of investigation yield different but complementary measures of *mean* inflation expectations, and, in two cases, legitimate measures of *variance*. Together, they enable the development of a range of scientifically-sound, historically-based, and *market-consistent* values to employ in risk and uncertainty analyses. These values, in turn, support estimation of realistic S-curves, in *then-year dollars*, for our life-cycle cost estimates.

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# INTRODUCTION

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## Definitions of Inflation

In accordance with current economic theory and empirical reality, we define inflation from a macro-economic perspective as:

*“A sustained increase in the general level of prices in the economy.”<sup>1</sup>*

The prices of some goods and services may be falling during periods of inflation while the prices of other items are rising. In *general*, however, or on average, prices are increasing, and over a *sustained* period, with transitory fluctuations in values due to shocks or seasonality ignored.<sup>2</sup>

From a commodity or index construction point of view, inflation is an increase in the price of an *identical* good or service from one time period to another. Popular, well-constructed indices such as the Consumer Price Index (CPI) attempt to capture changes in pure “price relatives” in their market baskets. But success is never complete because of enormous, practical data-collection and measurement problems.

In defense cost-estimating, it’s important to measure inflation from both a temporal and spatial perspective; that is, over time and across various companies, facilities, and sectors of the economy such as shipbuilding, IT, and aerospace manufacturing.

## Measurement of Defense Inflation

The Office of the Under Secretary of Defense, Comptroller, (OUSD(C)), issues inflation guidance annually for use by Department of Defense Components in pricing the President’s Budget.<sup>3</sup> The inflation rates, for the Department’s investment accounts, are best estimates of

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<sup>1</sup>The definition contains no reference to the money supply, or to the growth rate in the money supply, or to the growth in the money supply relative to the growth rate in real GDP. These references resonate, perhaps unwittingly, in corners of the defense cost-analysis community today but not with Dr. Ben Bernanke and other subject matter experts of the Board of Governors of the Federal Reserve System. See, for example, “Monetary Aggregates and Monetary Policy at the Federal Reserve: A Historical Perspective,” Chairman Ben S. Bernanke, at the Fourth European Central Banking Conference, Frankfurt, Germany, 10 November, 2006.

<sup>2</sup> Dr. Alan Greenspan, former Chairman of the Board of Governors of the Federal Reserve System, popularized the Fed’s usage of the price index for core Personal Consumption Expenditures as the best measure of inflation in the economy. “Inflation Measure, Taylor Rules, and the Greenspan-Bernanke Years,” Yash Mehra and Bansi Sawhney, *Economic Quarterly*, Volume 96, Number 2, 2010, pages 123-151.

<sup>3</sup> “Inflation Guidance – Fiscal Year (FY) 2013 President’s Budget,” 10 Feb 2012, Office of the Under Secretary of Defense, Comptroller, page 1.

future *economy-wide* inflation, as measured by the Gross Domestic Product price index, in its current chain-linked Fisher formulation. The forecasts themselves are received by Comptroller from the Office of Management and Budget, which, in turn, generates them in conjunction with the President's Council of Economic Advisers and the U.S. Treasury. The three-member council, or Troika,<sup>4</sup> updates their forecasts of inflation and other economic aggregates every fall. Comptroller promulgates the projections along with appropriation outlay profiles, culled from U.S. Treasury data, thus enabling calculations of then-year dollar total obligational authority.

While generally regarded as *most-likely* forecasts in recent years, without the application of executive-branch wishful thinking, there's no *a priori* reason to believe that the GDP price index, which is heavily weighted by consumption expenditures,<sup>5</sup> is a good measure of defense inflation in the first place. Indeed, a generic market basket of goods and services, dominated by items such as hamburgers, cars, and haircuts, seems an ill-suited measure of changes in the prices of specialized labor, material, software, and tooling needed to design, develop, and build sophisticated weapon systems. Blind acceptance of indices prescribed by OUSD(C) seems a risky leap of faith.

The Department of Defense has allocated scant resources to the daunting task of developing concrete, repeatable, *sampling processes* and *index formulae* which summarize accurately and parsimoniously the changes in prices and quantities of tens if not hundreds of thousands of items bought and sold in the defense sector of the economy every year. Methodological problems abound. They include the need to distinguish changes in labor mix from changes in labor rates, to adjust for increases in productivity, and to account for the introduction of new weapon systems in the defense "market basket."

Despite these obstacles, several defense organizations nevertheless employ custom-built or unique inflation indices in their commodity areas of responsibility, such as ships, aircraft, and satellites. But the consistency of these indices with local labor-market conditions, their technical quality and accuracy, their statistical properties, and the degree to which self-fulfilling prophecies are at play remain open questions.<sup>6</sup> The magnitude of the *actually-incurred* versus *OSD-prescribed* inflation differential therefore remains somewhat conjectural.

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<sup>4</sup> The Department of Commerce, Bureau of Economic Analysis, may now be a fourth member of the forecasting group, as referenced in the [Economic Report of the President](#), 2012, page 74, Council of Economic Advisers, Washington, D.C., 17 February 2012.

<sup>5</sup> 70% of GDP in 2011 was consumption expenditures; [Ibid](#), with raw data on page 316.

<sup>6</sup> For additional details, see "Criteria for Evaluating the Quality and Application of Inflation Indices: Custom Built versus OSD Prescribed," Dr. Brian Flynn, 11 May 2012.

Several recent studies cite ship and aircraft indices from the Bureau of Economic Analysis (BEA) and the Bureau of Labor Statistics (BLS) as possibly legitimate measures of inflation for defense acquisition programs. A review of the methodology underpinning these indices, however, reveals their inapplicability.<sup>7,8</sup> BLS's Producer Price Index for "Military Self-Propelled Ships, New Construction," for example, measures output prices rather than input costs, does not account for changes in unit prices due to learning curves, uses expenditure weights of marginal validity, employs a limited formulaic construction, and is unable to confirm if the "big six" defense shipyards even participate in the voluntary sample surveys.<sup>9</sup> Construction of these indices is geared toward the commercial sector with large numbers of buyers and sellers in often highly competitive markets. The defense sector, on the other hand, is characterized by oligopoly and monopolistic competition on the supply side and monopsony on the buyer, foreign military sales aside.

### **The Estimation Imperative**

The requirement to generate cost estimates over the life cycle in then-year dollars confounds matters greatly, no matter which inflation indices are employed, OUSD(C) or custom-built. The stakes are huge. For example, a sample of commonly-denominated Major Defense Acquisition Programs (MDAPs) currently in development reveals that inflation accounts for 15% of the total cost of the programs, on average.<sup>10</sup> For Joint Strike Fighter alone, \$60B is required to fund inflation from the baseline through the end of production, or 17% of total acquisition cost.<sup>11</sup>

Further, deltas in annual inflation rates can generate a large dollar impact ten or twenty years hence, due to the effect of compounding. A multiplier for the delta is easily computed as

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<sup>7</sup> "Phone Con with Bureau of Economic Analysis, and Review of BEA's The Deflation of Military Aircraft," memo for the record, Dr. Brian Flynn, 6 July 2012. Also, see "Telecon with Bureau of Labor Statistics on the Producer Price Index," memo for the record, Dr. Brian Flynn, 21 June 2012.

<sup>8</sup> Even the subject matter experts at BLS and BEA for these indices counsel against their use for defense cost-estimating purposes.

<sup>9</sup> BLS employs a Laspeyres index, with the standard base-period weights. In reality, the numbers and mix of vessels under development and construction at the major shipyards changes yearly with deliveries and new starts. A Fisher index would capture this phenomenon. Further, the weights employed are not ship quantities. Instead, they're imperfect proxies equal to a ship's price as a percentage of a company's annual revenue. The weights are updated, on average, only once every 7.5 years. Monthly responses from an opaque "universe of domestic producers," which spans the range of major yards to repair shops, typically range from 70% to 80%, with some companies dropping out of the survey over time. It's completely unclear, then, *what* the index measures and for *which* companies.

<sup>10</sup> Selected Acquisition Report Summary Tables, 31 December 2011, OUSD(AT&L), ARA, 29 March 2012. Sample consists of all MDAPs with a base year for development of 2008.

<sup>11</sup> *Ibid.*

$$\text{Yearly Multiplier} = \frac{(1+x)^n + (1+y)^n}{2}, \text{ where } \quad (1)$$

$x$  and  $y$  are alternative yearly inflation rates and  $n = n^{\text{th}}$  year.

For example, a 100 basis-point delta in rates, from the current OSD projection of 1.8% per annum to a custom-built rate of 2.8% per annum, yields a multiplier of roughly 20% at the end of 20 years. A 200 basis-point delta yields a multiplier of 50%.

In any event, for *whichever* indices are employed in life-cycle cost estimates, the forecasts of their outyear values remains problematic and a major issue in generating realistic S-curves in then-year dollars. Seemingly imponderable questions abound, such as what statistical distribution for inflation to use in Monte Carlo simulations, and with what parameters. Simply employing a triangular distribution, with endpoints taken as plus and minus a percentage-point or two from the mode, seems wholly inadequate.

## Study Focus

This paper focuses on the estimation of probability distributions of inflation rate forecasts. Inferences are drawn from analyses of *survey*- and *market-based* measures of inflation expectations. *Survey-based* measures examine projections of inflation made by government officials and professional forecasters. *Market-based* measures, on the other hand, focus on expectations inferred from the prices and yields of financial instruments actively traded on Wall Street, such as Treasuries and, more recently, inflation derivatives. As Economists note, focus sharpens and credibility rises when prices are set by market agents with “skin in the game,” such as pension, insurance, and hedge-fund managers, who put their “money where their mouths are.”

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These distinct but related areas of investigation yield different but complementary measures of *mean* inflation expectations, and, in two cases, legitimate measures of *variance*. Together, they enable the development of a range of scientifically-sound, historically-based, and *market-consistent* values to employ in risk and uncertainty analyses. These values, in turn, support estimation of realistic S-curves, in *then-year dollars*, for our life-cycle cost estimates.

# SURVEY OF PROFESSIONAL FORECASTERS

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## Background

The Federal Reserve Bank of Philadelphia administers the Survey of Professional Forecasters. Begun by the American Statistical Association and the National Bureau of Economic Research in 1968, the quarterly survey captures forecasts of macroeconomic variables such as real growth, interest rates, and inflation. Respondents are professional forecasters from Wall Street financial firms, major banks, economic consulting firms, and academia.<sup>12,13</sup>

The survey produces a set of data “very rare” in economics, finance, and defense cost analysis; that is, *point* as well as *probabilistic* forecasts of inflation from the same sources, with parameters clearly defined and results professionally tabulated.<sup>14</sup>

Each of the survey’s respondents, with headcount varying between roughly 30 and 40, generates forecasts of the annualized rate of core inflation five- and ten-years out and *probabilistic* forecasts of the annualized core inflation rate one- and two-years ahead.<sup>15</sup>

## Probabilistic Projections

The short-term probabilistic forecasts are denoted

$P_t^1 = \text{Year one projections of the } i^{\text{th}} \text{ forecaster ;}$

$P_t^2 = \text{Year two projections of the } i^{\text{th}} \text{ forecaster.}$

Each of these *probabilistic projections*, it’s important to emphasize, is not *single point* estimate of the expected value of inflation but rather a *set* of probabilities reflecting how uncertain each respondent is about his or her own forecast. Each  $P_t^j$ ,  $j = 1, 2$  and  $i = 1$  to  $n$ , represents the chances in 100 that future inflation will fall into each of ten bins, with some of the actual responses shown in Table 1 for projections of inflation from 4<sup>th</sup> quarter 2012 to 4<sup>th</sup> quarter 2013.

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<sup>12</sup> “Introducing: The Survey of Professional Forecasters,” Dean Croushore, Philadelphia Fed, Business Review, November/December, 1993, page 7.

<sup>13</sup> The authors express their gratitude to Dr. Stefania D’Amico of the research staff of the Board of Governors of the Federal Reserve System for drawing our attention to the survey.

<sup>14</sup> “Consensus and Uncertainty in Economic Prediction,” Business Cycles: Theory, History, Indicators, and Forecasting, Victor Zarnowitz, University of Chicago Press, January 1992, page 492.

<sup>15</sup> The Fed survey produces forecasts of these measures of inflation: the GDP price index, the core Personal Consumptions Expenditures index, and the core Consumer Price Index. We focus on the latter.

Forecaster	< 0.0%	0.0-0.4%	0.5-0.9%	1.0-1.4%	1.5-1.9%	2.0-2.4%	2.5-2.9%	3.0-3.4%	3.5-3.9%	>= 4.0%
A	0	0	0	6	34	44	15	1	0	0
B	0	5	30	35	30	0	0	0	0	0
C	0	0	0	0.88	50.24	48.04	0.84	0	0	0
...										
n										
Mean	0.01	0.18	1.44	6.19	46.7	40.41	4.38	0.65	0.04	0.00

Table 1: Projections of Core CPI Inflation: 2012:Q4 to 2013:Q4

For example,  $P_A = \{0,0,0,6,34,44,15,1,0,0\}$  is the response of a forecaster with each element of the set corresponding to a specific interval of inflation one year out. Attaching “%” signs to the entries yields a set of probabilities.

### Conflation

The problem of combining or conflating these  $n$  subjective probability distributions into a single probability distribution,  $Q$ , where  $Q = f(P_1, \dots, P_n)$ , has been well studied.<sup>16,17</sup> Among the many methods proposed for computing the conflation is “the classical convex combination or weighted average,”<sup>18</sup> denoted for the one-year forecasts as

$$Q^1 = \sum_{i=1}^n w_i P_i^1, \quad \text{where } \sum_{i=1}^n w_i = 1. \quad (1)$$

Weights might be assigned to the projections of individual forecasters based on their track records. But past performance never guarantees future accuracy. And many studies highlight the virtues of a consensus point of view.<sup>19</sup> A commonplace alternative, then, is the application of equal weights,  $w_i = 1/n$ , for  $i = 1$  to  $n$ . Equation (1) in this case reduces to the average of the  $n$  entries in each bin or forecast interval. Results of the conflation are shown as the *mean* in the last row of Table 1.

<sup>16</sup> “Conflations of Probability Distributions,” Theodore P. Hill, Georgia Institute of Technology, Transactions of the American Mathematical Society, 363, 2011, pages 3351-3372.

<sup>17</sup> “Teaching Pigs to Sing: Improving the Fidelity of Assessments from Subject Matter Experts,” Peter J. Braxton, Technomics, Inc., and Richard L. Coleman, retired, SCEA Washington Area Chapter, 13 June 2012,

<sup>18</sup> Hill.

<sup>19</sup> “Introducing: The Survey of Professional Forecasters,” Dean Croushore, Federal Reserve Bank of Philadelphia, Business Review, November/December 1993, page 12.

Assuming a uniform distribution *withineach* bin, from lower to upper limits, the first two moments of the conflated distributions  $Q^1$  and  $Q^2$  are

$$\bar{\mu} = \sum_{i=1}^{10} p_i \cdot x_i \quad \text{and} \quad (2)$$

$$\sigma_i^2 = \sum_{i=1}^{10} p_i \cdot (x_i - \bar{\mu})^2 = \sum_{i=1}^{10} (p_i \cdot x_i^2) - \bar{\mu}^2, \quad (3)$$

where  $x_i$  is the midpoint of the  $i^{th}$  bin and  $p_i$  is the probability of inflation falling in that bin, or the raw numerical entry in the survey divided by 100.<sup>20</sup> The midpoints of the open-interval bookend bins of the survey ( $<0.0\%$  and  $\geq 4.0\%$ ) are set to -0.3 and 4.2 to enable the calculations.

Using results from equations (2) and (3), normal distributions are fit to conflated survey data, for selected quarters, since the Fed began eliciting probabilistic forecasts of inflation in 2007. As Figure 1 demonstrates, the probability distributions can and do change over time.

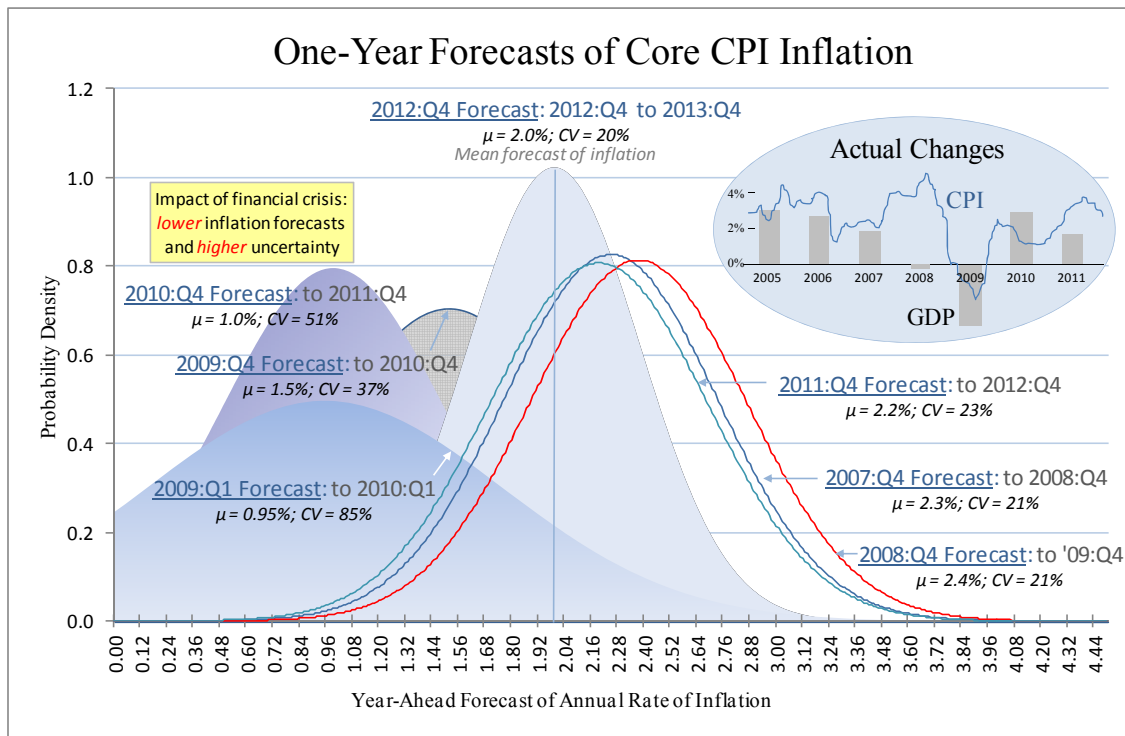


Figure 1: One-Year Forecasts of Core CPI Inflation

<sup>20</sup> Superscripts for forecast length are omitted from the equations for simplicity of exposition.

During the recession of December 2007 to June 2009, with these peak-to-trough dates in the economic cycle pegged by the National Bureau of Economic Research, the Consumer Price Index turned negative. It hit a low of -2% per annum in 2009. Mean forecasts of inflation fell by roughly 50 to 150 basis points as a consequence. And forecast uncertainty increased significantly, as measured by the coefficient of variation. From a figure of roughly 20% during periods of relative economic stability (2007, 2008), the CV reached a peak of 85% for the 2009:Q1 forecast. As Zarnowitz notes, "... increased volatility [of inflation] tends to be associated with decreased predictability. Thus the more variable inflation is, the less of it will be anticipated."<sup>21</sup>

### Year-Two Forecasts

The year-two probabilistic forecasts of the Fed survey,  $P^2$ , are 12-month projections of inflation measured for the year following the first-year forecasts, but made at the same point in time by the same group of forecasters. The forecast horizon, in other words, shifts an additional year into the future. As the insert in Figure 2 shows, these set of conflation of the second year,  $Q^2$ , reflect greater convergence but less certainty than their first-year  $Q^1$  counterparts.

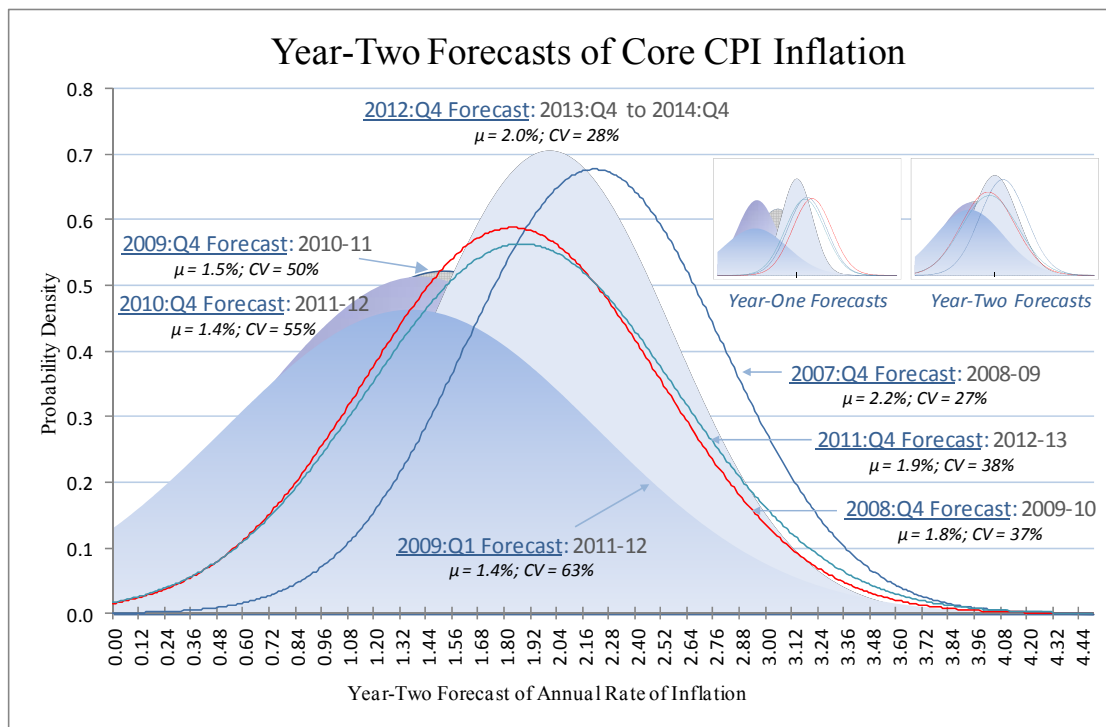


Figure 2: Year-Two Forecasts of Core CPI Inflation

<sup>21</sup> Zarnowitz, page 496.

The  $Q^2$ s are perhaps less susceptible to the immediate vagaries and vicissitudes of the marketplace. They seemingly discount the relatively rare occurrence of deflation by moving toward a supposed steady-state value of core inflation.

Reliance on the means and CVs computed from the *latest* Fed survey *only* may not necessarily be optimal for cost-estimating purposes. With ever-changing conflation, it's fruitful to evaluate their properties since the survey's inception, with partial entries shown in Table 2.

Survey			Year-One Forecasts			Year-Two Forecasts		
Year	Quarter	#	Mean	St Dev	CV	Mean	St Dev	CV
2007	1	1	2.25	0.55	0.24	2.20	0.57	0.26
2007	2	2	2.35	0.50	0.21	2.25	0.54	0.24
2007	3	3	2.25	0.49	0.22	2.17	0.53	0.24
2007	4	4	2.25	0.48	0.21	2.21	0.59	0.27
2008	1	5	2.18	0.62	0.28	2.15	0.72	0.33
2008	2	6	2.45	0.58	0.24	2.33	0.61	0.26
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:
2012	3	23	2.02	0.53	0.26	2.02	0.69	0.34
2012	4	24	1.99	0.39	0.20	2.00	0.57	0.28

Summary Statistics						
1st Quartile	1.43	0.50	0.23	1.62	0.60	0.28
Average (n = 24)	1.82	0.59	0.37	1.88	0.70	0.39
Average [without 2009/10]	2.10	0.56	0.27	2.06	0.66	0.33
3rd Quartile	2.25	0.67	0.47	2.15	0.78	0.47

Table 2: Moments of the Conflated Distributions

The average CV values at the bottom of the table of 37% and 39% are computed across all 24 density functions for their respective forecast horizons. The values drop by 100 and 60 basis points when the high-variance years of 2009 and 2010, as depicted in Figures 1 and 2, are excluded from the calculations. The repeatability of such outliers in future U.S. economic experience is a matter of conjecture and their exclusion from the sample therefore debatable.<sup>22</sup> The quartile values perhaps provide reasonable lower and upper bounds.

To complicate matters, on January 25<sup>th</sup> 2012, the Federal Reserve Open Market Committee set a target value for the rate of inflation of 2% per annum, “consistent over the longer run with ... statutory mandate” and announced publically to ... “keep longer-term inflation expectations

<sup>22</sup> The authors favor their inclusion since significant deviations from the mean, high and low, have been the norm in economic history.

firmly anchored.”<sup>23</sup> This was the first-ever public issuance of an inflation bogey in the Fed’s history, according to Mr. Tom Stark of the Philadelphia Federal Reserve Bank. He posits the possibility, but not the certainty, of structural change in the forecasts, as a result.

In any event, the average CVs are computed as the mean of the CVs from each of the survey’s conflation over time, or

$$\text{Mean } CV_j = \sum_{i=1}^n \frac{CV_{i,j}}{n}, \text{ where} \quad (4)$$

$j$  = the horizon of the forecast (year one or two),  $n$  = the number of surveys, and  $CV_i$  = the CV of the  $i^{\text{th}}$  conflated distribution.

### Dubious Assessments of Uncertainty

In many circumstances in finance, economics, and cost analysis, however, underlying probabilistic distributions are unobservable. The computational focus consequently changes from the mean of the CVs to the CV of the means:

$$CV_j = \frac{\sigma_j}{\mu_j}, \text{ where } \sigma_j = \frac{\sum_{i=1}^n [(P_{i,j} - \mu_j)]^2}{n-1}, \text{ and} \quad (5)$$

each  $P_{i,j}$  is a consensus *point prediction* of the value of inflation at a particular time for a certain forecast horizon. Use of the formula assumes that an *interpersonal* measure of dispersion is a good proxy for the dispersion of *intrapersonal* predictive probabilities. Only serendipitously will this hold. Indeed, application of equation (5) usually leads to an *underestimation* of uncertainty, which, in the case of the Fed survey data, can be demonstrated empirically. As Figure 3 shows, *CV* underestimates uncertainty by 10 percentage points for the year-one forecasts and by 23 percentage points for the year-two forecasts.

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<sup>23</sup> Press release, Board of Governors of the Federal Reserve System, 25 January 2012, [www.FederalReserve.gov](http://www.FederalReserve.gov), brought to our attention by Mr. Tom Stark, Philadelphia Federal Reserve Bank.

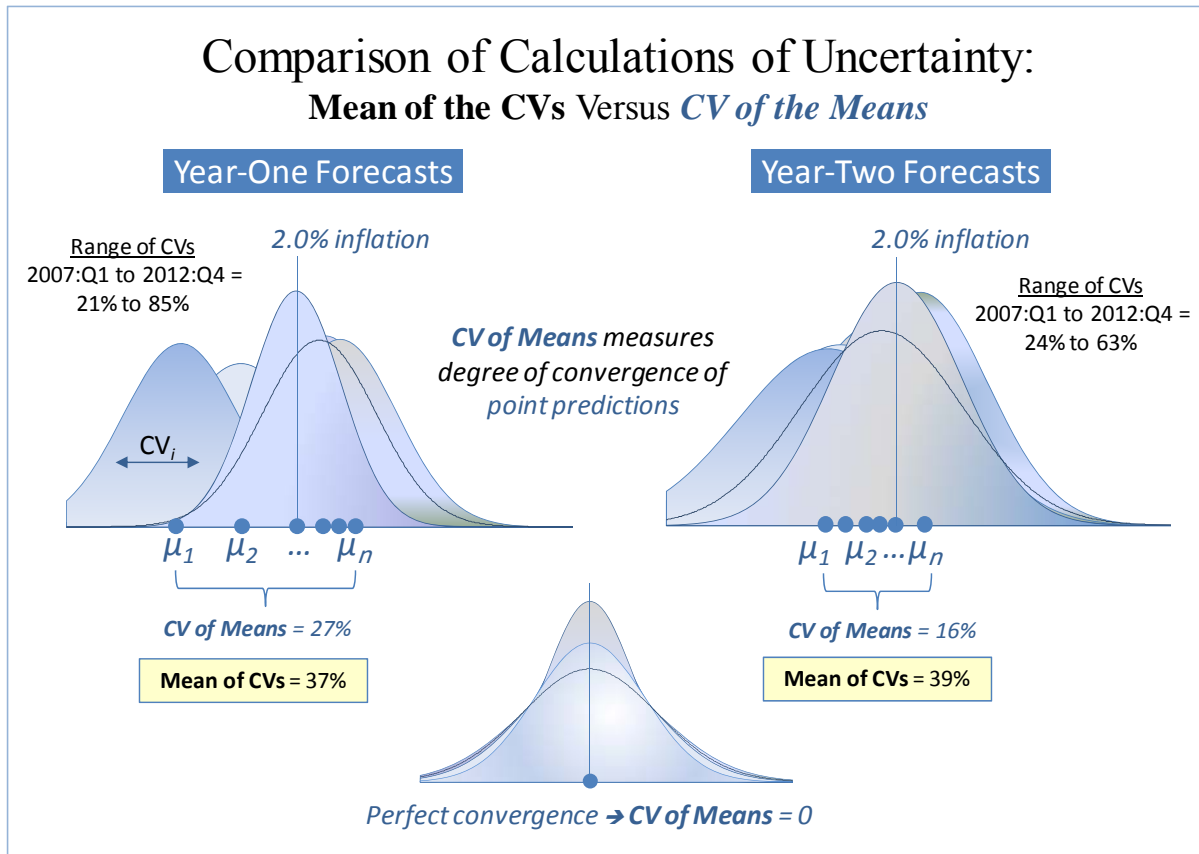


Figure 3: Comparison of Calculations of Uncertainty

Indeed, **CV** produces a value outside the range of observable data in year two [16% versus 24% to 63%]! The decrease in **CV** from year-one to year-two reflects a convergence of forecasters' opinion, over the six years under examination, of the expected value of inflation. In the hypothetical case of complete convergence of point predictions, **CV** yields the implausible value of zero while, of course, ignoring completely the demonstrable lack of confidence in the forecasts both individually and collectively. As Zarnowitz notes,

*"Inferences from point forecasts do not produce ... an informed assessment of uncertainty."*<sup>24</sup>

### Long-Term Forecasts

Shifting attention now to the longer-term forecasts of inflation, the Fed asks in these cases for point predictions *only* from the  $n$  survey participants rather than their probabilistic or binned estimates as in the case of the one- and two-year forecasts. The five-year and ten-year forecasts represent "average annual inflation" over the entire length of the forecast intervals,<sup>25</sup> with

<sup>24</sup> Zarnowitz, page 494.

<sup>25</sup> Survey of Professional Forecasters, pages 8 and 9.



responses since the survey start dates plotted in Figures 4 and 5. An increase in dispersion of the point predictions is evident for the period associated with the global financial crisis and its aftermath as well as a secular decline in mean predictions during the 1990s. For much of the past decade, both the five- and ten-year forecasts hover within a narrow band of roughly 2.0% and 2.5% inflation per annum. Unfortunately, as previously explained, it's simply impossible to make scientifically-sound inferences of forecast uncertainty from these point predictions and their dispersions. The first moments or means nevertheless remain valuable information.

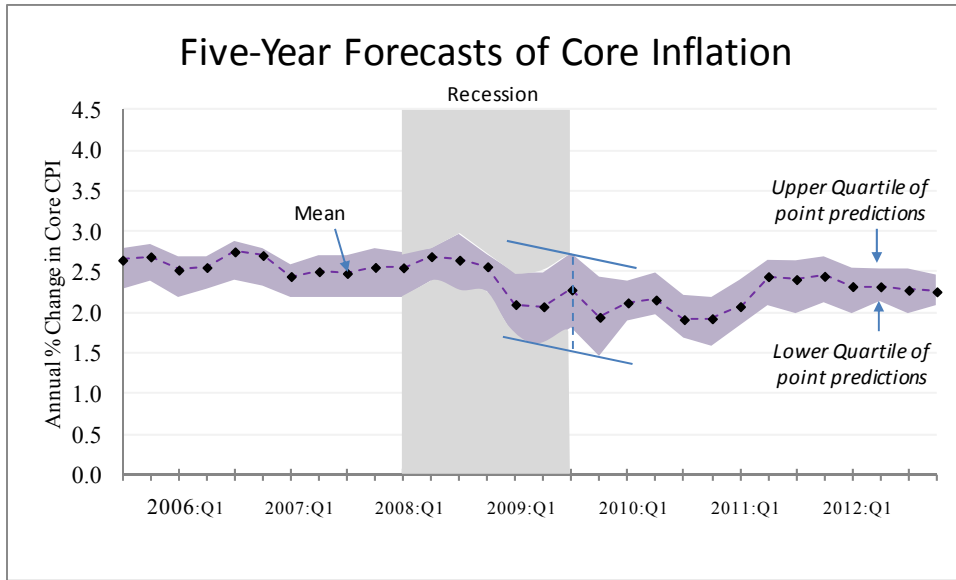


Figure 4: Five Year Forecasts of Core Inflation

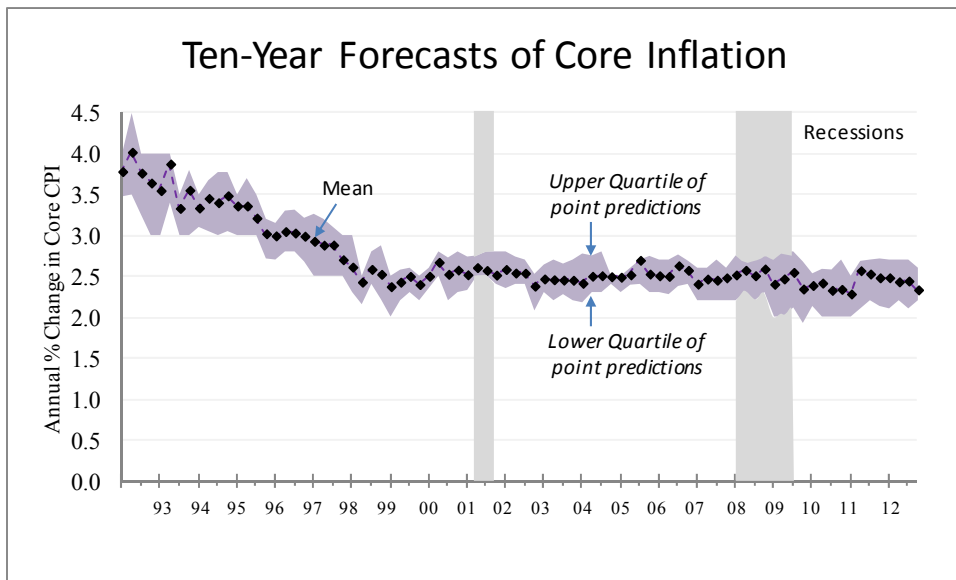


Figure 5: Ten-Year Forecasts of Core Inflation

# BREAKEVEN INFLATION ON TREASURY SECURITIES

## The TIPS Market

The U.S. Treasury issues bills, notes, and bonds to finance the nation's annual deficits and cumulative debt. Bills mature in a year or less, and, like zero-coupon bonds, do not pay interest prior to maturity. They're quoted for purchase and sale on an annualized discount basis. Notes mature in two to ten years, and pay interest every six months. Bonds are issued in 20- and 30-year maturities, with coupon payments also made every six months.

In 1997, Treasury Inflation Protected Securities (TIPS) were added to mix of financial instruments used for debt financing. They're sold today in maturities of 5, 7, 10, 20, and 30 years. Unlike nominal bills, notes, and bonds, the principal dollar value of a TIPS is automatically adjusted to the Consumer Price Index, providing a hedge against inflation so long as the security is held to maturity.<sup>26</sup> Further, while the coupon rate of the TIPS is constant, it generates a different amount of interest when multiplied by the inflation-adjusted principal.

Treasuries of all maturities are closely linked to the rate of overall inflation in the economy, as Figure 6 shows.

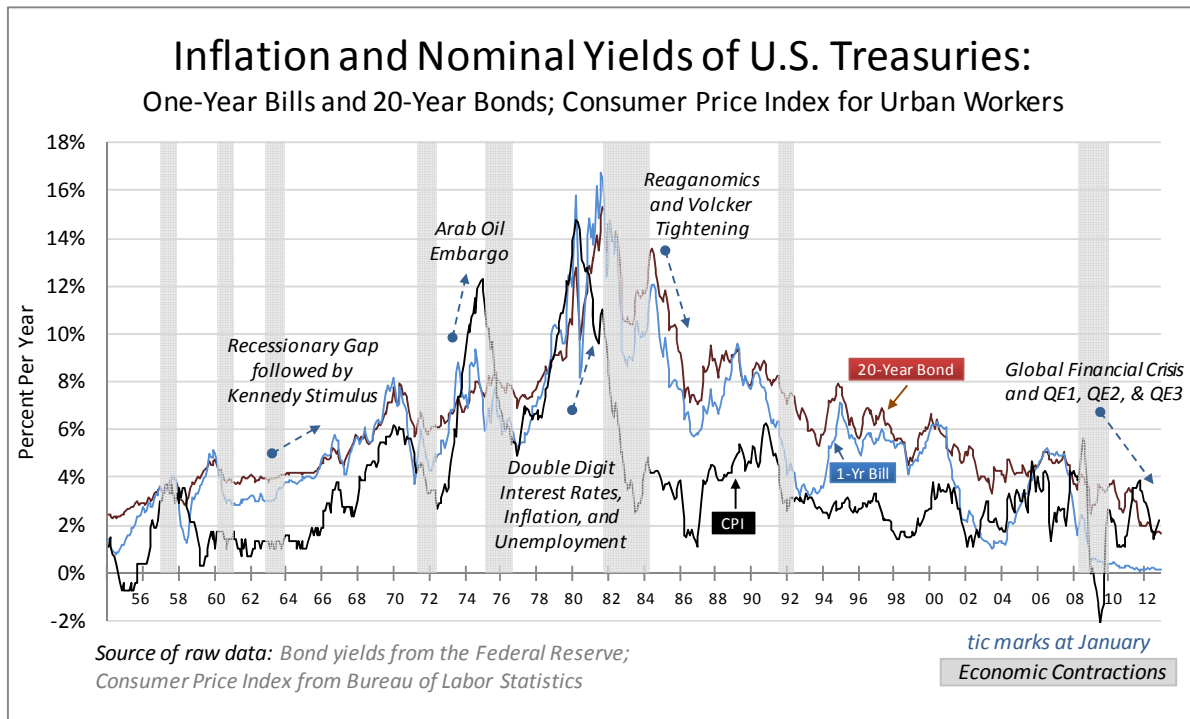


Figure 6: Inflation and Nominal Yields of U.S. Treasuries

<sup>26</sup> TIPS are indexed to the non-seasonally adjusted U.S. City Average All Items Consumer Price Index for all Urban Consumers (CPI-U), published by the Bureau of Labor Statistics (BLS). See [www.BLS.Gov](http://www.BLS.Gov) for more information.

The “Fisher equation” posits that the nominal interest rate,  $i$ , is composed of two parts, the rate of inflation,  $r$ , and a real return, or  $\pi$ .

$$i = r + \pi$$

(6)

TIPS, on the other hand, fully account for inflation. Supposedly, then, their yields reflect only  $\pi$ .

### Breakeven Inflation

The difference between yields on nominal and TIPS securities, of identical maturities, produces the so-called *breakeven* inflation rate. This is the degree of future inflation at which investments in both kinds of securities would be equally profitable. Investors will *breakeven* on purchases of either if *realized* inflation turns out to exactly match the breakeven value. If inflation runs higher, investors would have been better off with the inflation-indexed TIPS. Figure 7 shows the breakeven rates observable in the marketplace on a single day, 1 March 2013, for 5- to 30-year maturities.<sup>27</sup>

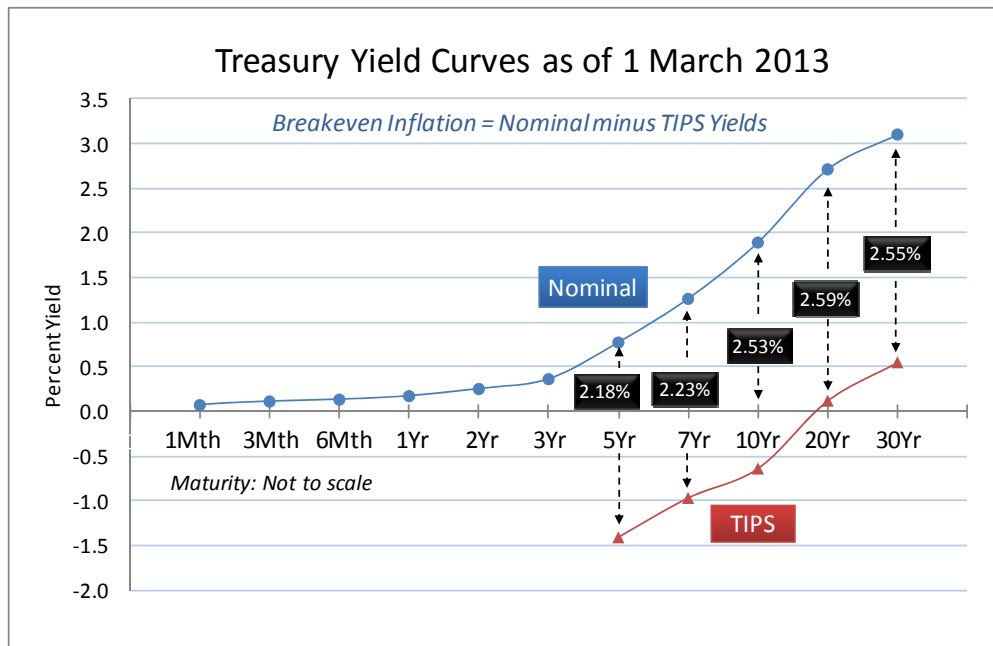


Figure 7: Treasury Yield Curves as of 1 March 2013

<sup>27</sup> Interesting, the 30-year breakeven rate is *lower* than the 20-year rate. This 20s/30s yield inversion, seen earlier in the year for 10s/30s yields, may be partly attributable to a “growing belief that the US could be the new Japan.” Risk Magazine, “Sponsored Forum: US Inflation Derivatives,” BNP Paribas, Abhishek Nadamani, Vice President, USD Inflation Trading, 14 Jan 2013.

As Federal Reserve points out, the use of breakeven inflation rates as a true measure of inflation expectations is confounded by the presence of *risk premi* in the market place.<sup>28</sup>

- Nominal Treasuries include a premium that inflation will overshoot its expected path.
- TIPS include a premium for holding an asset less liquid than a nominal security.

The risk premium on nominal Treasuries raises the nominal curve and *widens* the measure of breakeven inflation. The liquidity premium on TIPS, on the other hand, raises the real curve and *narrows* the breakeven rate. As the Fed indicates, “The yield premiums associated with both factors vary over time and often in offsetting ways, making it difficult to capture the residual expectations component of the breakeven inflation rate.”<sup>29</sup>

Consider, for example, the history of breakeven rates on 10-year securities, shown in Figure 8. During the global financial crisis, interest rates fell along with inflation. A flight to safety was apparent in financial markets. The TIPS yield spiked when Lehman Brothers failed, breaking a pattern of moving in tandem with nominal yields. Because they’re less liquid than nominal securities, TIPS are regarded as riskier. The TIPS liquidity premium might have risen to 100 basis points at this juncture, before returning again to normal levels.<sup>30</sup>

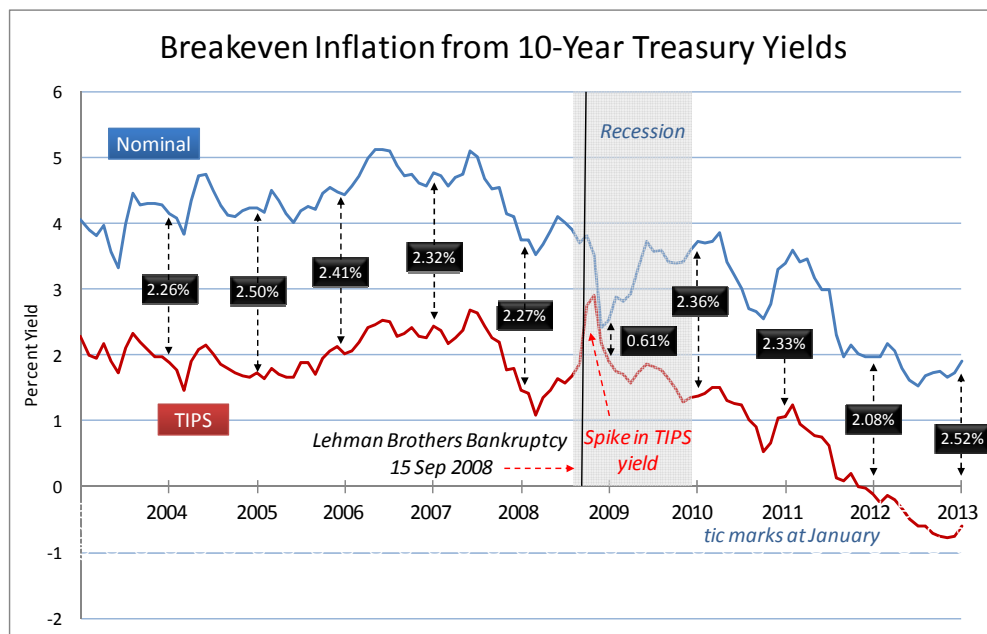


Figure 8: Breakeven Inflation from 10-Year Treasury Yields

<sup>28</sup> “Tips from TIPS: The Informational Content of Treasury Inflation-Protected Security Prices,” Stefania D’Amico, Don Kim, and Min Wei, 2010, Divisions of Research and Statistics and Monetary Affairs, Federal Reserve Board, Washington, D.C., page 1.

<sup>29</sup> “TIPS Liquidity, Breakeven Inflation, and Inflation Expectations,” Federal Reserve Bank of San Francisco, Economic Letter 2011-19, Jens Christensen and James Gillan, 20 June 2011.

<sup>30</sup> D’Amico.

In today's economic environment, and with the TIPS market growing over time, the total risk premium is estimated at slightly above 0 basis points to as much as 50.<sup>31</sup> Use of the midpoint as an expected value seems reasonable given the competing methodologies at play and lack of consensus within the Federal Reserve for either end of the range.

A procedure observed in the defense cost-analysis community not only erroneously uses breakeven rates as a *pure* measure of outyear inflation but egregiously adds to the mischief by using the variation in values over time to compute a coefficient of variation. This maneuver is identical in nature to computation of the CV of the means, shown in Figure 3.

### **Predictions versus Probabilities**

A breakeven inflation rate is the market's point expectation of future inflation, over a specified time horizon, duly adjusted for risk premia. The expectation is not a probability distribution. The point prediction sheds no light whatsoever on probabilistic values. Misguidedly computing CVs based on quarterly breakeven values over time for 5-, 7-, 10-, 20-, and 30-year maturities yields these corresponding values: 31%, 25%, 22%, 14%, and 11%. That is, the longer the maturity, the lower the CV. These CVs, of course, capture *only* the movement in *point predictions* over time. Lower CVs prevail for the longer-term maturities because of the market's reliance on secular inflation trends for these values. Extending this sequence's asymptotic trajectory a bit further would yield a CV of only 7% for forecasts 60 years out, and would imply virtually perfect knowledge of the annualized rate of inflation a millennium ahead!<sup>32</sup>

As is the case with the SPF's five- and ten-year forecasts of inflation, the TIPS-nominal security differential nevertheless provides valuable information to the cost analyst on long-term *mean* inflation expectations.

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<sup>31</sup> Christensen and Gillan.

<sup>32</sup> Fitting a power function to the data produces  $R^2 = 0.995$ ; F statistic = 636; and t-statistic on the coefficient = -25. The estimated elasticity of the CV, with respect to the time dimension, is -58%.

# INFLATION DERIVATIVES

## Zero-Coupon Swaps

Almost non-existent in the U.S. a couple of decades ago, inflation derivatives have grown along with the TIPS market.<sup>33</sup> As a measure of inflation expectations, they offer some distinct advantages over the bond-market “breakevens.” First, derivatives are traded over a wider range of maturities, often from one to 30 years out, thus providing insights on inflation near term and in the long run. Trading in a primary or dealers’ market, they avoid some of the liquidity issues faced by TIPS. Most importantly of all, a specialized form of these derivatives, caps and floors, can be used to estimate probability distributions for outyear inflation expectations.<sup>34</sup> That is, they provide information on both mean *and* variance. PDFs constructed from derivatives’ pricing in the open marketplace have the added credibility of reflecting the views of players “with skin in the game.”

In their simplest form, as zero-coupon vanilla swaps,<sup>35</sup> an inflation derivative is a bilateral contract between a *buyer* and *seller* of inflation protection, as shown in Figure 9.

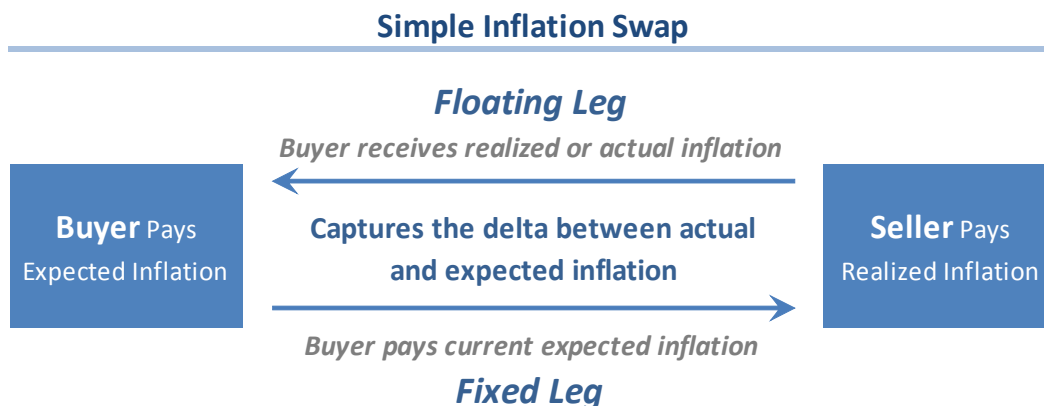


Figure 9: Simple Inflation Swap

<sup>33</sup> “Derivatives are financial instruments whose value is derived from the value of an underlying asset (such as gold, wheat or other commodities) or other financial instruments including bonds, or market benchmarks such as interest rates [and breakeven inflation rates]”; Financial Times.

<sup>34</sup> “The Economics of Options-Implied Inflation Probability Density Functions,” Yuriy Kitsul and Jonathan Wright, Division of Monetary Affairs, Federal Reserve Board, Washington, D.C., and, Department of Economics, Johns Hopkins University, Baltimore, MD, 6 June 2012.

<sup>35</sup> A swap is “an exchange of cash flows between two ... [parties] ... for a predetermined period or prescribed dates.”; *Global Derivatives; Products, Theory and Practice*, Eric Benhamou, page 105.

In this arrangement, the buyer agrees to pay a compounded, fixed rate of interest, or an inflation expectation,  $b$ , on a nominal dollar value over the tenor of the swap

$$\text{Expected Inflation} = \$\text{Notional} * ((1 + b)^{\text{tenor}} - 1) \quad (7)$$

The seller of protection, on the other hand, agrees to pay the cumulative percentage increase in the value of a pre-established price index over the identical tenor of the swap

$$\text{Realized Inflation} = \$\text{Notional} * \left( \frac{\text{CPI}_{t+\text{tenor}}}{\text{CPI}_t} - 1 \right) \quad (8)$$

Expected and realized inflation are typically measured in terms of the non-seasonally-adjusted Consumer Price Index for All Urban Workers, the same index used for TIPS breakeven inflation. There's no exchange of money until maturity. The breakeven swap rate,  $b$ , quoted daily in the marketplace for various maturities, is set at such a level that the market considers the value of the fixed leg equal to the value of the floating leg upon issuance, or

$$\text{Value}_{\text{swap}} = \text{Value}_{\text{fixed leg}} - \text{Value}_{\text{floating leg}} = 0 \quad (9)$$

As with Treasury breakevens, the inflation swap rate should not be interpreted as a pure measure of inflation expectations. Both buyers and sellers of inflation protection incur risks

$$\text{Buyer Side: Realized Inflation} < \text{fixed payment} \quad (10)$$

$$\text{Seller Side: Realized Inflation} > \text{fixed payment} \quad (11)$$

Seller-side risk is typically higher than buyer-side risk. In theory, inflation rates can rise unbounded above a mean expected value while, normally, the downside is limited to a small positive value. The swap rate, then, usually includes a net premium demanded by the seller as compensation for bearing asymmetric risk. The Federal Reserve Bank of Cleveland estimates this net risk premium at roughly 100 basis points today on a 30-year swap rate.<sup>36</sup>

Figure 10 plots historical yield curves. Interestingly, deflation was regarded as a distinct possibility during the financial crisis of 2008 and 2009. The swap rate of -3.99% on the first of December 2008 meant that sellers of swaps were willing to receive 96 cents on the dollar to provide inflation protection a year out!

In a steady-state economic climate, swap rates seem to fall in a 2% to 3% band. Shorter-term tenors exhibit more volatility than longer-term. The 30-year maturities converge to a value of roughly 3% per annum, unadjusted for risk. This convergence, of course, does not imply less

<sup>36</sup> 30-year swap rate as of 1 March 2013.

uncertainty regarding the level of inflation three decades out, but, rather, lack of variation in the market's expectation of a *mean*, long-term value.

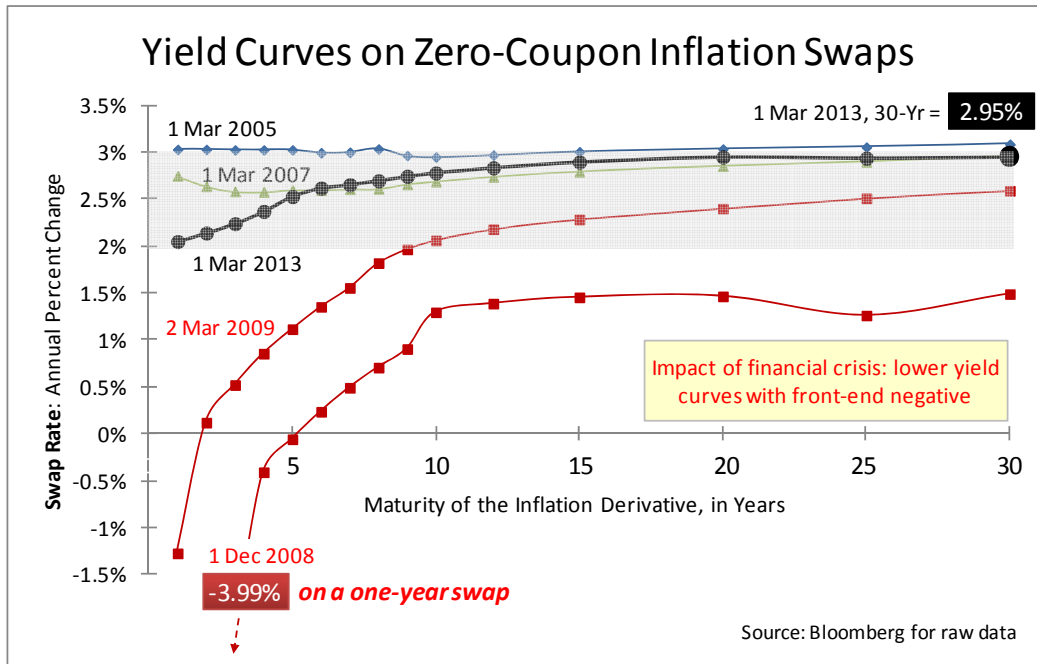


Figure 10: Yield Curves on Zero-Coupon Inflation Swaps



## Cap and Floor Derivatives

Within the last three or four years, a new and growing market has emerged for inflation derivatives, “caps and floors,” which are option-like securities whose payoff is linked to realized changes in a pre-established index, such as the headline CPI.<sup>37,38</sup> The seller of the cap promises to pay, upon contract expiration, an unlimited amount of inflation on a notional principal,  $\$N$ , above a certain threshold inflation rate,  $k$ , over the life of the contract,  $T$ , or

$$\$N \cdot \max \left[ \left( \left( \frac{CPI_T}{CPI_0} \right)^{\frac{1}{T}} - 1 \right) - k, 0 \right], \text{ where} \quad (12)$$

$k$  is also referred to as the strike of the cap, such as 2% inflation per annum. The buyer of the cap, then, such as a pension fund that's required to earn a minimum *real* rate of return on client money, is protected against any rise in inflation above  $k$  percent per year. Their risk exposure to inflation, from a portfolio perspective, is *capped* at this value. In return for the protection, the borrower makes an up-front payment  $P_0(k, T)$ . As Kitsul and Wright note, “the contract is effectively a call option on inflation.”<sup>39</sup> If the actual annualized rate of inflation ends up higher than  $k$ , the buyer is in-the-money. Otherwise, the option expires out-of-the-money, and worthless.

In the case of a floor, the payment is

$$\$N \cdot \max \left[ k - \left( \left( \frac{CPI_T}{CPI_0} \right)^{\frac{1}{T}} - 1 \right), 0 \right] \quad (13)$$

Using the parlance of Wall Street,<sup>40</sup>

### ***Inflation Index Options***

*Index options are options on the average annual inflation rate over the lifetime of the option. An index **CAP** pays if average (annually compounded) inflation exceeds the **strike rate**, while an index **FLOOR** pays out when average inflation is below the **strike**. Alternatively, it is intuitive to think of an index options as an option on the **price level**. That is, the quoted strike rate of inflation implies a strike for the price level. A **CAP** makes a payout on expiry that is linear in the excess of the actual final price level over that strike.”*

<sup>37</sup> Options are contracts giving the right to buy or sell an asset at a point in the future at a price set now, called the strike price. An option gives the *right* to make the transaction but is not an *obligation* to do so.

<sup>38</sup> The cap and floor market in the U.S. seems to have reached critical mass beginning in March 2009, based on statistics provided in “Inflation, Hedging It and Trading It,” Deutsche Bank, 2011, section 6.1.

<sup>39</sup> Kitsul and Wright, page 3.

<sup>40</sup> Screenshot from Bloomberg’s trading window kindly provided by the Federal Reserve Bank of Cleveland.

The theoretical underpinnings required to derive implied PDFs from option prices on the caps and floors flow from the expression for the market price  $P_o$  of the call option at time  $t_0$ , which equals the discounted value of its expected payoff:

$$P_o(k, T) = e^{-rT} E[\max(S_T - k, 0)], \text{ where } \square \quad (14)$$

$S_T$  is the spot market price of the asset at the delivery date, a clearly stochastic variable with a probability distribution, and where  $r$  is the interest rate.<sup>41</sup> Equivalently,

$$P_o(k, T) = e^{-rT} \int_k^{\infty} (S_T - k) f(S_T) dS_T, \text{ where } \quad (15)$$

$f(S_T)$  is the PDF of the value of the option at maturity. The first derivative of  $P_o$  with respect to the strike price is a function of the cumulative distribution

$$\frac{\partial P}{\partial k} = e^{-rT} (1 - F(S_T)), \quad (16)$$

with the second derivative, as is well known in the literature,<sup>42</sup> yielding the PDF for inflation,  $f(S_T)$

$$\frac{\partial^2 P}{\partial k^2} = e^{-rT} f(S_T) \quad (17)$$

### Estimation of the PDF

Inflation call options, for a given maturity date in the future, are traded at different strike prices. The *call-price function*, with an actual example shown in Figure 11, relates the prices of call options of the same maturity to their strike price. The curve slopes downward because the higher the cap, or the minimum inflation per annum covered the next twenty five years, the greater will be the exposure to inflation. In simple words, it costs more to protect against inflation at 1% per annum than it does at 6% per annum because the latter figure is far less likely than the former.

<sup>41</sup> 'Option-Implied Probability Distributions and Currency Excess Returns,' Allan M. Malz, Federal Reserve Bank of New York Staff Reports, Number 32, November 1997.

<sup>42</sup> "Prices of State-Contingent Claims Implicit in Option Prices," Douglas T. Breeden and Robert H. Litzenberger," *The Journal of Business*, Vol. 51, No. 4 (Oct., 1978), pages 621-651.

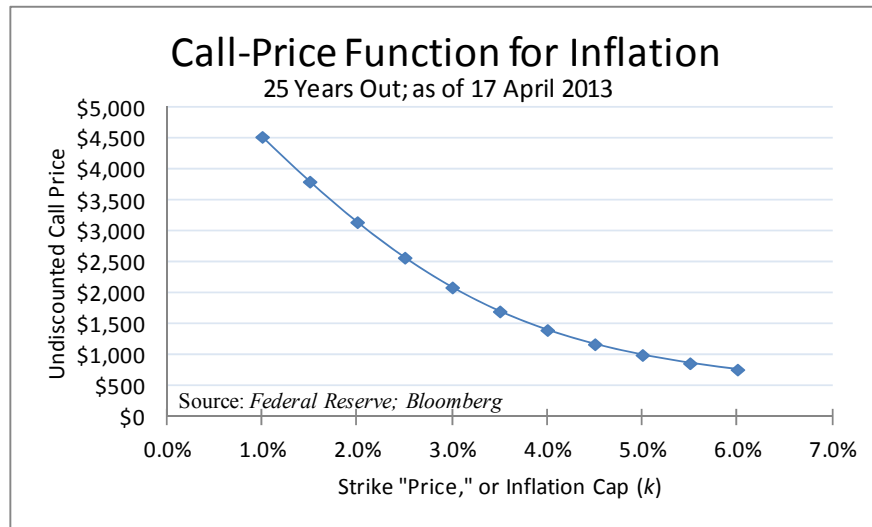


Figure 11: Call-Price Function for Inflation

This relationship enables estimation of the probability density function for the asset; in this case, the future rate of inflation. Intuitively, as the Bank of England notes,

*“A call option with a lower strike price will always be worth more than a higher strike option. This is because the option with the lower strike price will have a higher pay-off if exercised and has a higher probability of delivering a positive pay-off. This additional probability reflects the chances that the underlying asset price will lie between these strikes. If we have option prices for a range of strikes, it is possible to infer what the probabilities are of the underlying asset price at maturity lying between each of them, by examining the relative prices of options with adjacent strikes.”<sup>43</sup>*

A number of analytical techniques are employed to derive PDFs from option prices, for a wide range of derivatives, not just inflation caps and floors.<sup>44</sup> Among the most prominent are:

- Nonparametric
  - Constructs densities, in the form of histograms, directly from market data.
  - Fits a continuous distribution to discrete values.

<sup>43</sup> “Recent developments in extracting information from options markets,” *Bank of England Quarterly Bulletin*, February 2000, Clews, R., N. Panigirtzoglou and J. Proudman, 2000, page 51.

<sup>44</sup> “Estimating Implied Probabilities from Option Prices and the Underlying,” *Chapter 35, Handbook of Quantitative Finance and Risk Management*, Bruce Mizrahi, pages 515-529.

- Parametric
  - Assumes a functional form for the PDF, such as a combination of two lognormal distributions.
  - Estimates the PDF in such a manner that the implied call-price function matches an observed set of data as closely as possible, as in maximum-likelihood estimation.
  
- “Smile” Interpolation
  - Filters observed option prices to remove non-arbitrage restrictions of monotonicity and convexity;
  - Transforms (strike, price) data into (delta, implied volatility) data using a Black-Scholes formulation;
  - Fits a smooth function or spline through the transformed data;
  - Transforms the data from the spline back into (strike, price) space; and
  - Computes the second derivative using a numerical algorithm.

Employing a numerical technique suggested by a member of Dr. Ben Bernanke’s research staff, Figure 12 plots a PDF for inflation 25-years ahead, based on prices in the market place prevailing on 17 April 2013.<sup>45</sup> The estimated mean is 2.8% per annum inflation and the estimated CV 60%. The area between the dotted lines depicts the probability that inflation will fall 100 basis points to either side of the mean, or between 1.8% and 3.8% per annum, over the next quarter century. The area is 44% of the total area under the curve; therefore, the market’s assessment of this probability equals that same percentage.

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<sup>45</sup> Technique suggested by Dr. Yuriy Kitsul in a visit by the authors to the Federal Reserve Board. Using the call-price function illustrated in Figure 11, the first derivative,  $\partial C/\partial k$ , is approximated by  $(c_2 - c_1)/(k_2 - k_1)$ , where the  $c$ ’s and  $k$ ’s are call and strike prices respectively. Symmetric numerical derivative was applied at  $x$ -values from 1.5% to 5.5%, inclusive. The second derivative,  $\partial^2 C/\partial k^2$ , is then approximated similarly. In using this procedure, inflation floors should, ideally, be converted to inflation caps using the call-put parity equation. This gives direct observations on zero and negative values of inflation.

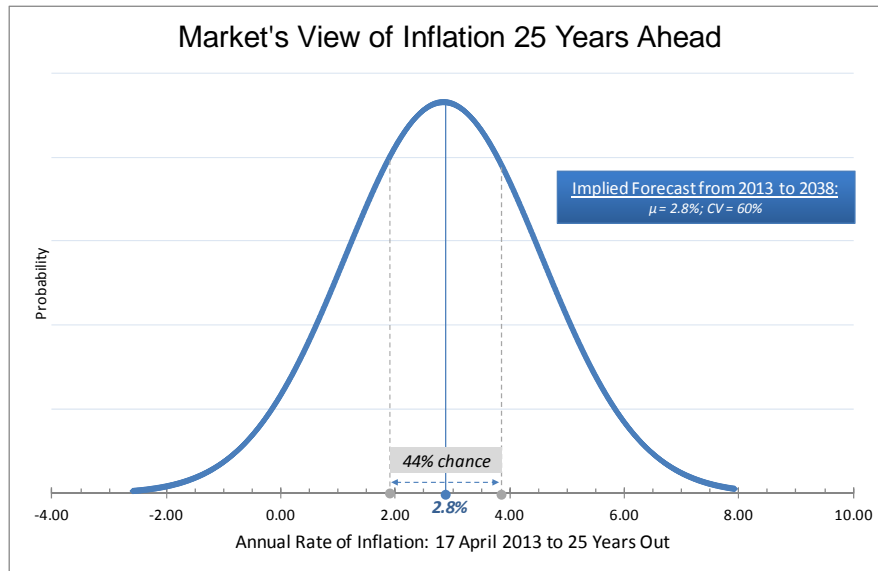


Figure 12: Market's View of Inflation 25 Years Ahead

It's important to emphasize that Figure 12 represents the view of an open, active, vigorous, and growing market in inflation protection, with a volume of \$1.8 billion USD in 2011.<sup>46</sup> Agents in the market include pension funds, insurance companies, utilities, speculators, and hedge-fund managers, with large sums of client or individual monies at stake.

That said, the PDF is generated under an assumption of risk neutrality, where investors are neither risk takers nor risk avoiders. If investors are risk averse, an inflation premia would distort the expectations derived from the cap and floor data. However, the financial literature reveals that any such premia is likely insignificant for these types of instruments.

Using data from Bloomberg, PDFs will be estimated for 1-, 10-, 20-, and 30-year maturities, and will be published on the website of the Naval Center for Cost Analysis. Figure 12, of course, is sufficient to demonstrate the results and utility of estimates from market data.

<sup>46</sup> Deutsche Bank.

## HISTORICAL FORECAST ACCURACY

### GDP Deflator

While the focus of this paper is the CV (uncertainty) of inflation forecasts, not their accuracy (risk), as an excursion we followed the methodology outlined in DeCarlo<sup>47</sup> to approximate the standard deviation of inflation projections using the standard deviation of forecast errors from past projections. This follows the so-called “pseudo-iid” thought process. While this may still be a reasonable approach, the forecast errors are anything but independent and identically distributed (iid). In particular, forecast errors are dependent on the prevailing inflation rate at the time of projection, as illustrated in the graph below. The relationship is strongest for the one-year look-ahead, which is shown in the graph. Rates are overestimated during times of low inflation (left-hand side) and underestimated during times of higher inflation (right-hand side). The interpretation is that that “correction” of the pendulum swing is overestimated in either direction.

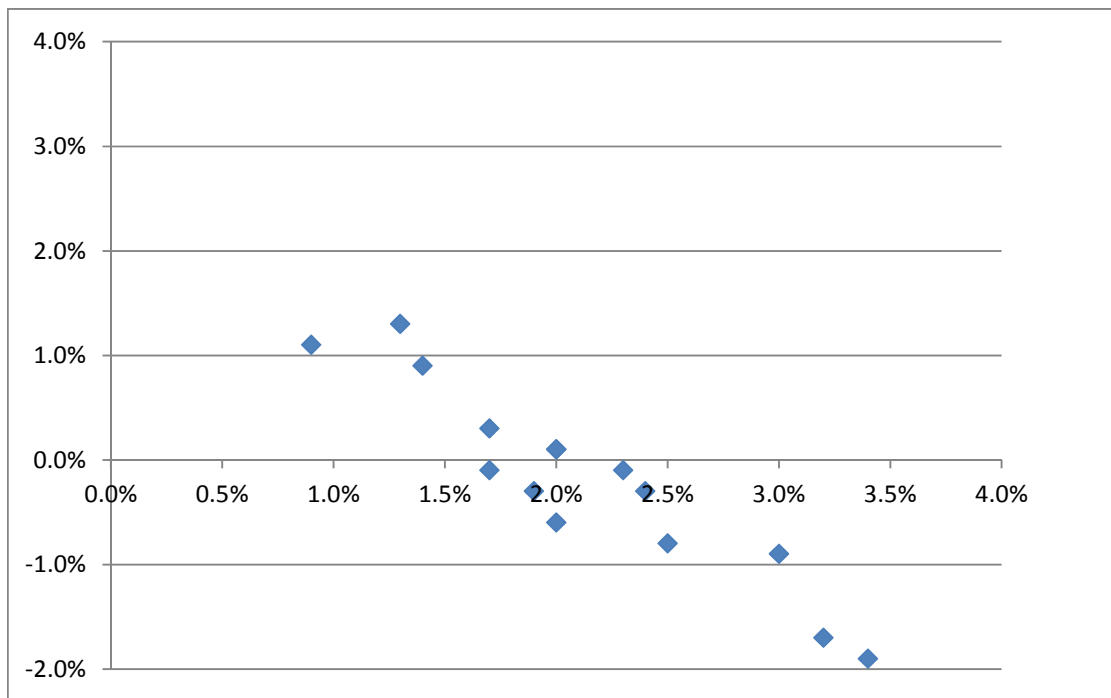


Figure 13: Forecast Error as a Function of Prevailing Inflation Rate

We took the GDP Deflator data for FY1998-2012 from the USD(Comptroller) “Green Book.” The table shows the forecast errors for look-ahead from one to 10 years. The

<sup>47</sup> “Inflation Cost Risk Analysis to Reduce Risks in Budgeting,” Michael DeCarlo, Stephanie Jabaley, Eric Druker, SCEA/ISPA 2012.

amount of available data naturally dwindles as the gap gets larger. The correlations shown are between the annual rates themselves.

**Table 3: Forecast Accuracy Summary Statistics**

<b>look-ahead (years)</b>	<b>1</b>	<b>2</b>	<b>5</b>	<b>7</b>	<b>10</b>
<b>n</b>	15	14	11	9	6
<b>mean</b>	-0.2%	-0.2%	-0.1%	-0.1%	0.5%
<b>std dev</b>	0.9%	0.9%	0.8%	0.7%	0.5%
<b>correl (rate)</b>	61.9%	10.6%	-56.9%	-34.3%	27.3%

The decreasing standard deviation should be viewed with a degree of caution, since it is based on a smaller and smaller dataset over a specific historical period.



## SUMMARY OF ESTIMATES

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### **Compilation of Values**

Table 3 reveals several important findings. First, mean point predictions of inflation in the marketplace trend higher than the flat OSD-prescribed rates. Witness the five- and ten-year forecasts from the Survey of Professional Forecasters as well as breakeven rates from Treasury securities and values implied from cap and floor derivatives.

Summary of Estimates of Means and CVs of Inflation Expectations

Source or Technique	Time Frame of Inflation Projections in Years							
	1	2	5	7	10	20	25	30
<b>Mean Values of Annualized Inflation (unless noted otherwise)</b>								
<b>Survey of Professional Forecasters<sup>1</sup></b>								
Average for 2007:Q1 to 2012:Q4	1.82%	1.88%						
Average for 2007:Q1 to 2012:Q4 without 2009/10	2.10%	2.06%						
2012:Q4	1.99%	2.00%	2.26%		2.34%			
2013:Q1	x	x	2.25%		2.33%			
<b>Breakeven Inflation on Treasury Securities</b>								
Trade data of 1 March 2013								
Raw delta between nominal and TIPS securities			2.18%	2.23%	2.53%	2.59%		2.55%
Adjusted for risk and liquidity premia					2.28%	2.34%		2.30%
<b>Yield on Zero-Coupon Inflation Swaps</b>								
Trade data of 1 March 2013								
Raw rate	2.05%	2.14%	2.53%	2.66%	2.78%	2.95%	2.94%	2.95%
Adjusted for risk (by Federal Reserve Bank of Cleveland)	1.05%	1.17%	1.29%	1.36%	1.47%	1.75%	1.86%	1.95%
<b>Cap and Floor Derivatives</b>								
Trade data of 17 April 2013								
	x				x	x	2.80%	x
<b>OSD Comptroller Guidance</b>								
Issued 27 Feb 2013 (GDP Price Index; from OMB)								
	1.90%	1.90%	1.90%	1.90%	1.90%	1.90%	1.90%	1.90%
<b>Coefficients of Variation for Inflation Expectations</b>								
<b>Survey of Professional Forecasters<sup>2</sup></b>								
Average for 2007:Q1 to 2012:Q4	37%	39%						
Average for 2007:Q1 to 2012:Q4 without 2009/10	27%	33%						
2012:Q4	20%	28%						
2013:Q1	x	x						
<i>Conflation of probabilistic forecasts</i>								
<b>Cap and Floor Inflation Derivatives</b>								
Trade data of 17 April 2013								
	x				x	x	60%	x
<i>Preliminary estimate = 60% CV 25years out</i>								
<b>Historical Forecast Accuracy</b>								
GDP Deflator data FY1998-FY2012								
	48%	45%	41%	39%	24%			
<i>Historical standard deviation divided by 1.90%</i>								

<sup>1</sup>Year 2 represents the projected rate of inflation from the end of the first year to the end of the second year

<sup>2</sup>Year 2 represents the CV for the distribution of year-two inflation projections

Table 4: Summary of Estimates of Means and CVs of Inflation Expectations

Second, estimates of coefficients of variation are substantial. There're likely significantly higher than values typically employed within the defense cost-analysis community.

Third, the CVs appear to increase over time, jumping from roughly 20% near-term to much higher several years out.

## Escalation Percentages

In modeling the risk and uncertainty of inflation rate projections, the means and CVs presented here apply only to the escalation dollars in the cost estimate, or the difference between then-year dollars and base-year dollars. Escalation, as a percent of total then-year dollar cost, varies from program to program, and depends upon length and spending profile of the acquisition, as well as the magnitude of the inflation rates themselves. Table 4 presents useful benchmark values for the escalation percentages at Milestones B and C, based on data presented in Selected Acquisition Reports for DoD MDAPs.<sup>48</sup>

Escalation as a Percent of Total Then-Year Dollar Current Estimate										
n = sample size										
Milestone and Base Year	All DoD		DoD Level		Air Force		Army		Navy	
	Value	n	Value	n	Value	n	Value	n	Value	n
<b>Milestone B</b>										
1980 to 1984	42%	22	21%	1	35%	10	84%	4	30%	7
1985 to 1989	20%	24	21%	3	21%	5	22%	10	15%	6
1990 to 1994	17%	16	13%	3	34%	2	9%	7	26%	4
1995 to 1999	19%	18		0	19%	7	11%	6	28%	5
2000 to 2004	17%	35	10%	3	17%	8	17%	14	21%	10
2005 to 2009	13%	28	14%	1	4%	5	15%	8	15%	14
2010 to 2012	14%	11	17%	3	13%	3	11%	3	15%	2
Average: 2000-12	15%		14%		11%		14%		17%	
<b>Milestone C</b>										
1980 to 1984	26%	16		0	41%	4	24%	2	20%	10
1985 to 1989	20%	13	36%	1	26%	2	16%	2	18%	8
1990 to 1994	17%	17	17%	3	18%	4	14%	3	19%	7
1995 to 1999	13%	19	8%	1	11%	4	8%	7	19%	7
2000 to 2004	14%	25	8%	1	13%	6	13%	7	16%	11
2005 to 2009	10%	19		0	8%	6	10%	6	10%	7
2010 to 2012	10%	10	6%	1	11%	2	9%	5	11%	2
Average: 2000-12	11%		7%		11%		11%		12%	

Table 5: Escalation as a Percent of Total Then-Year Dollar Current Estimate

Several points bear emphasis. First, the benchmark percentages fall significantly after the 1980-to-1984 period, corresponding to a decline in economy-wide inflation [Figure 6] and, more specifically, a decrease in OSD-prescribed raw rates from xx% per annum in 1980 to xx% per annum in 1985. OSD rates today are far lower than three decades ago, and the percentages follow suit. Second, Navy percentages at Milestone B are currently higher than values for Air Force and Army, perhaps due to the relatively long time periods required for ship design and construction. Finally, values over the past few years may be artificially constrained. An increasingly common practice in the acquisition and cost communities is to use relatively high,

<sup>48</sup> Raw data and pivot table kindly provided by Mr. Richard Lee, Technomics.

unique inflation rates to generate then-year dollars but to *deflate* these estimates, in SAR reporting, using *lower* OSD values. This produces an artificially high base-year dollar and a correspondingly lower escalation percentage. Use of average values since 2000 therefore seems prudent.

Combining the values of Table 4 with estimated measures of variance from the Survey of Professional Forecasters and from inflation derivatives provides estimates of CVs to employ for escalation dollars in cost estimates. At Milestone B, for example, with two or three decades of development and production ahead, combining the CV of 60% with 15 percentage points of escalation dollars yields a CV of 9% [ $60\% \times 0.15$ ], for application to the *inflation part* of the acquisition cost estimate.

## CONCLUSIONS

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Survey- and market-based measures of inflation expectations form the basis for reliable, scientifically-sound estimation of a range of means and CVs to employ in risk and uncertainty analyses. Based on the foregoing analysis, the following conclusions are offered:

- Measurement of actual defense inflation remains conjectural, at best. An analysis of the methodology underpinning measures of defense inflation from the Bureau of Labor Statistics and the Bureau of Economic Analysis reveal the inapplicability of the use of their indices. Even subject matter experts at the bureaus counsel against employment of their ship and aircraft indices for defense cost-analysis purposes! Methodology of these indices is geared toward large competitive markets rather than oligopoly and monopsony which characterize defense.
- Inflation is a significant portion of then-year dollar cost estimates. 15% of then-year dollar acquisition cost seems a reasonable estimate at Milestone B and 11% at Milestone C. Milestone B values for the Department of the Navy are larger than of those for other defense components, perhaps because of the relatively long periods required for ship design and construction.
- Means of CVs do not equal the CV of the means. Confusion between the two calculations can and will result in serious estimation errors for inflation uncertainty. More specifically, measuring the variation in *point predictions* over time is not the same as measuring the *probability* underlying the point predictions themselves.
- Probabilistic estimates of inflation can and do change, as demonstrated by the variability in PDFs for first-year and second-year forecasts from the Survey of Professional Forecasters *before, during, and after* the financial crisis of 2008 and 2009. CVs clearly respond to changes in the economic environment, growing larger in times of financial stress and uncertainty. At the height of the crisis, the CV almost quadrupled from a steady-state value.

- Likewise, point predictions of the *mean* inflation rate change according to macro-economic currents. Front ends of the yield curves on zero-coupon inflation swaps, for example, were *negative* during the financial crisis, as the threat of deflation loomed large. They reached a nadir of -3.99% on a one-year vanilla swap at the end of 2008!
- Cap and floor derivatives, a market that's reached critical mass in the U.S. only since 2009, are an *invaluable* measure of inflation uncertainty. *Very rarely if ever* in defense cost analysis are entire *probability distributions* for stochastic variables *at hand* and *measurable*. The market-based values, implied from real-time quotes from those with *skin in the game*, enable solid estimates of CVs for inflation up to 30-years out. The values are likely substantially higher than those previously employed in defense cost analysis.

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