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Parametric O&S Cost Estimation Using Markov Chains and the Influence Function Method

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Introduction

- **Parametric Methods in O&S Estimation**
  - Influence Function Method (IFM) first developed by Galorath Incorporated
    - Reduced user inputs
    - Evolutionary development as more data becomes available
    - Uniform mathematical structure for IFM estimates
  - Using Markov chains
    - Cost per mission approach feasible vs. period costs
    - Prediction of average system life
    - When each movement is assigned an estimated cost, cumulative movements of the system from state to state generate cumulative costs

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What is Presented in the Paper

- The Influence Function Method
- Markov Chains
- Operations Cost of an Unmanned Aerial Vehicle (UAV)
- Estimating Three Level Maintenance Cost of a Reported Failure
- A Truly Parametric Support Model
- The TORPID Missile and Markov Chains Example
- The Markovland Navy Example

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Traditional Approach in Constructing CERs

- Requires suitable historical data
- Normalization process
- Assumption that cost of interest y can be calculated using the equation y = f(x) fitted to the normalized data, where x could be one or more independent variables that "drive" the cost and f(x) is chosen based on the perception of the nature of the relationship
- Use methods of regression that are data hungry by nature
- Requires the model builder to use judgment and expertise at several steps in the process

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The Influence Function Method

- IFM recognizes that for many new technologies there is not enough data to build a model based on regression methods
- When properly structured, IFM model tends to mature along with the technology it estimates
- The IFM process begins with the identification of a cost of interest within a fairly narrowly defined technology family.
- Typically, the technology family is sufficiently narrowly defined that not more than about seven key technical / performance parameters (KTPPs) need be used to create a reasonably accurate estimate

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IFM Estimating Process

- **The uniform mathematical structure for IFM estimates**

$$\text{Cost of interest} = \text{E}\left[\prod_{\text{all k}} \text{m}_\text{k}()\right]\left[\sum_{\text{all n}} w_n f_n()\right]$$

- **E = is the normalized value of the desired cost of interest from a particular project called the prime exemplar**
- **$m_k()$ = simple mathematical functions or constants that have been found useful for universally adjusting the estimated result when that can be done without significant error**
- **n = counter for KTPPs that have been selected as costs drivers**
- **w_n = numerical weights assigned to the KTPPs. These weights must be assigned such that $\Sigma w_n = 1$ across all n. The weights represent the average relative cost effect of the KTPPs across all feasible design variations within the technology family—they are initially set using the Analytic Hierarchy Process**

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IFM Estimating Process (cont.)

- **Two KTPP Types**
  - Discrete
    - Example – material of manufacture, number of axes of rotation
    - Number of discrete KTPPs should be limited to three
    - $f_n()$ is simple for the discrete KTPPs, comprising only a numerical value assigned to each design option
  - Continuous
    - Characterized by units of measure and by a design range expressed using those units
    - Typical and valid units are count (quantity), watts, centimeters, hertz, psig, years, etc.
    - The design range is expressed as a minimum value and a maximum value for which the model is expected to be valid
    - $f_n()$ are mathematical functions (called influence functions) defined on the design range

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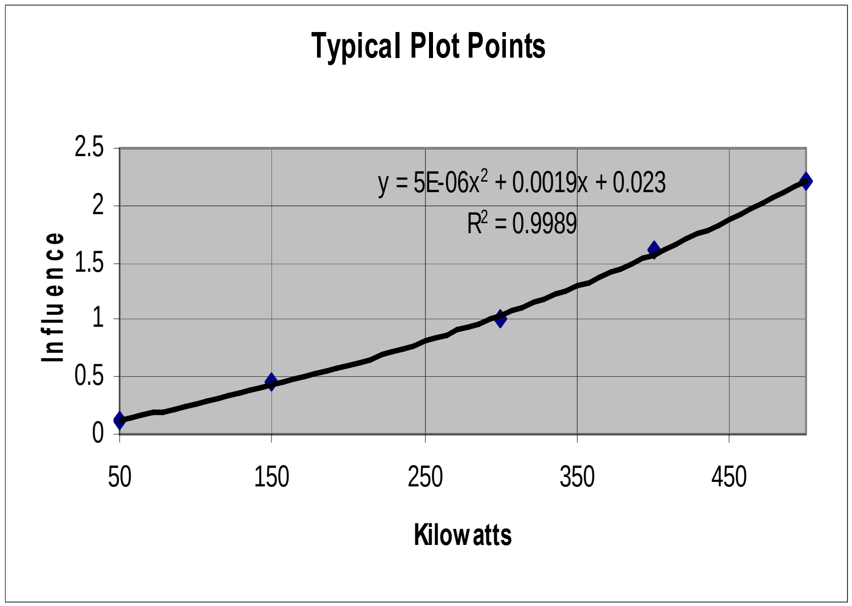
Setting up the Plot Points

- Group of experts completes the entries in the white cells. Note that the exemplar is always assigned a value of unity
- Once E and all of the $m_k()$, $f_n()$, and wn have been assigned, a working cost model results
- Due to its unique mathematical properties, it is precisely accurate at the prime exemplar. This is, of course, entirely intentional

Radiated Power	Kw	Influence
Maximum	500	2.2
Intermediate #1	400	1.6
Exemplar	300	1
Intermediate #2	150	0.45
Minimum	50	0.12



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IFM Estimating Process (cont.)

- To determine how accurate is it away from the prime exemplar
  - Use expert judgment
  - "Cumulative Calibration" Process
    - As each new exemplar is added, a calibration process is activated
    - Based on Monte Carlo Simulation -- all of the functions $m_k()$ and $f_n()$, and the weights w_n are randomly varied, and at each iteration the cost estimated for the new exemplar is compared to its known value. This is done several thousand times.
- The mathematical nature of the model preserves its ability to estimate the prime exemplar without error.
- What happens if a new exemplar cannot be modeled to acceptable accuracy?

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Markov Chains

- The main idea of value in the theory of Markov chains is that a system can and will, from time to time, randomly move from one state of being to another state of being, and that these movements are susceptible to expected value calculations.
- In Markov chain theory, movement from one state to another is probabilistically controlled.
- Cumulative movements from state to state generate cumulative costs.

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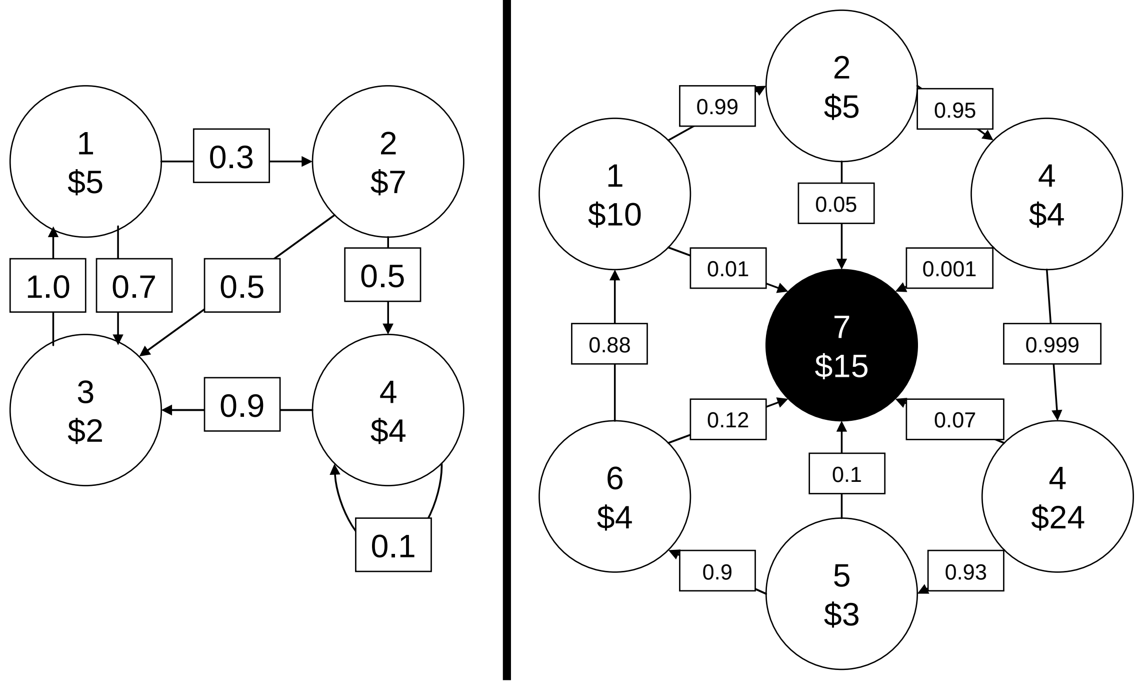
Markov Chains Examples

Figure 1 - A Four State Markov Chain with Costs at Each State

Figure 2 -- A Seven State Markov Chain with Costs at Each State & One Trapping State



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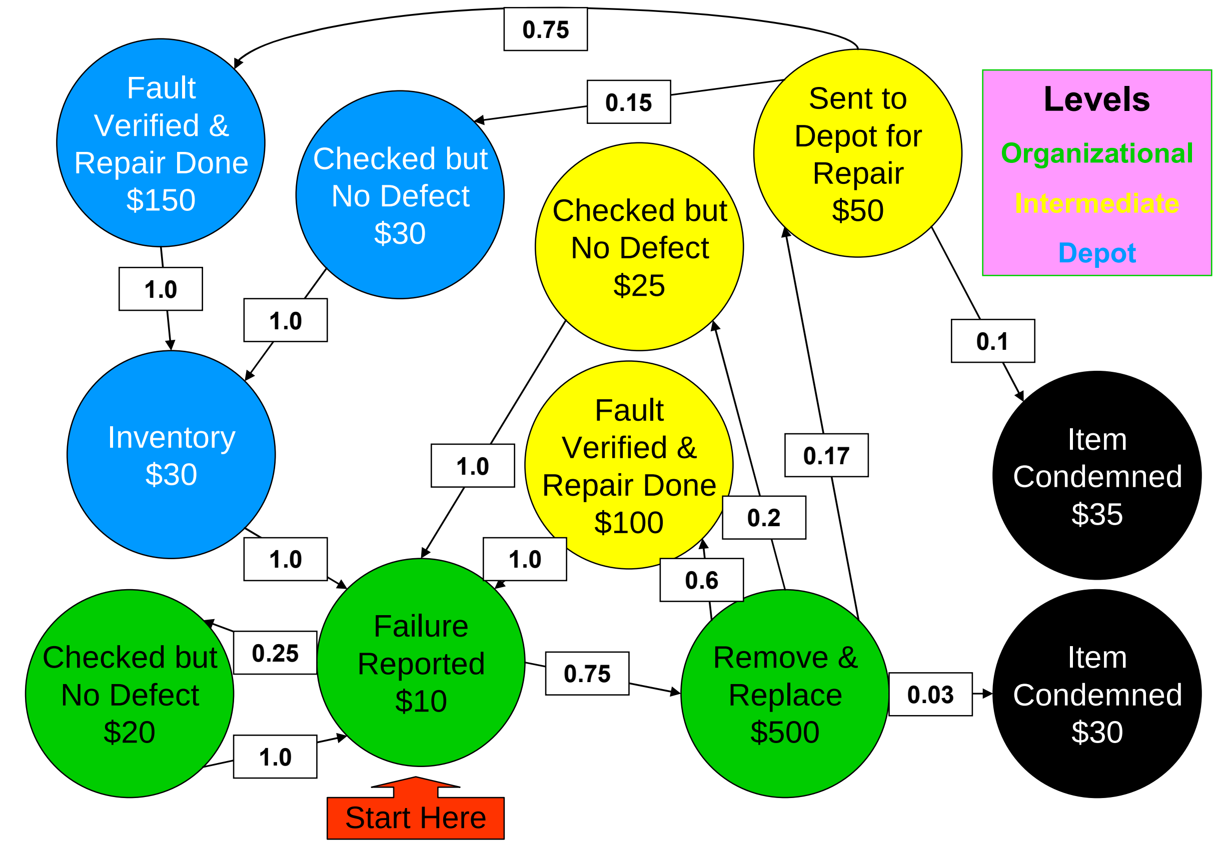
Markov Chains -- Background

- **The practical upper bound to the number of states the system can be in is around fifty**
- **The probability of movement from the current state to another state is governed only by the current state, and therefore, it's "memoryless"**
  - Markov Chains can not be used in the systems whose history significantly influences what happens next
- **Markov chains with trapping states are useful in modeling systems subject to attrition and for other purposes**
- **Markov chains can be approximately "solved" using Monte Carlo simulation, but they can also be exactly solved by solving systems of linear equations**
- **When one or more trapping states are present, finding the expected number of visits to each state before trapping occurs**
- **"Movement" from one state to another does not imply either physical movement or passage of a particular amount of time, unless those features are specifically built into the model**

Estimating Three Level Maintenance Cost of a Reported Failure with a Markov Chain

Figure 3 – Markov Chain Model for Three Level Maintenance



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Estimating Three Level Maintenance Cost of a Reported Failure with a Markov Chain (cont)

Code	FR	R&R	CNDO	FVRDI	CNDI	STDFR	FVRDD	CNDD	INV	ICI	ICD
FR	0	0.75	0.25	0	0	0	0	0	0	0	0
R&R	0	0	0	0.6	0.2	0.17	0	0	0	0.03	0
CNDO	1	0	0	0	0	0	0	0	0	0	0
FVRDI	1	0	0	0	0	0	0	0	0	0	0
CNDI	1	0	0	0	0	0	0	0	0	0	0
STDFR	0	0	0	0	0	0	0.75	0.15	0	0	0.1
FVRDD	0	0	0	0	0	0	0	0	1	0	0
CNDD	0	0	0	0	0	0	0	0	1	0	0
INV	1	0	0	0	0	0	0	0	0	0	0
ICI	0	0	0	0	0	0	0	0	0	1	0
ICD	0	0	0	0	0	0	0	0	0	0	1

The numbers in the matrix are the transition probabilities. Note that the matrix is arranged so that the trapping states are at the bottom. Four distinct areas can be identified in the matrix; they are color coded

Q	R
0	I

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Estimating Three Level Maintenance Cost of a Reported Failure with a Markov Chain (cont)

- **The transition matrix row and column labels are defined as follows:**
  - FR = Failure Reported
  - R&R = Remove & Replace
  - CNDO = Checked Out but No Defect (Organization)
  - FVRDI = Fault Verified & Repair Done (Intermediate)
  - CNDI = Checked Out but No Defect (Intermediate)
  - STDFR = Sent to Depot for Repair
  - FVRDD = Fault Verified & Repair Done (Depot)
  - CNDD = Checked Out but No Defect (Depot)
  - INV = Inventory
  - ICI = Item Condemned (Intermediate)
  - ICD = Item Condemned (Depot)
- **The numbers in the matrix are the transition probabilities**

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Estimating Three Level Maintenance Cost of a Reported Failure with a Markov Chain (cont)

(I-Q)^-1 matrix								
28.3688	21.2766	7.0922	12.766	4.25532	3.61702	2.71277	0.54255	3.25532
27.0355	21.2766	6.75887	12.766	4.25532	3.61702	2.71277	0.54255	3.25532
28.3688	21.2766	8.0922	12.766	4.25532	3.61702	2.71277	0.54255	3.25532
28.3688	21.2766	7.0922	13.766	4.25532	3.61702	2.71277	0.54255	3.25532
28.3688	21.2766	7.0922	12.766	5.25532	3.61702	2.71277	0.54255	3.25532
25.5319	19.1489	6.38298	11.4894	3.82979	4.25532	3.19149	0.6383	3.82979
28.3688	21.2766	7.0922	12.766	4.25532	3.61702	3.71277	0.54255	4.25532
28.3688	21.2766	7.0922	12.766	4.25532	3.61702	2.71277	1.54255	4.25532
28.3688	21.2766	7.0922	12.766	4.25532	3.61702	2.71277	0.54255	4.25532

The calculated matrix $(I-Q)^{-1}$, provides the expected number of passages through each state before trapping occurs, given some particular starting state, assuming a start at Failure Reported

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Estimating Three Level Maintenance Cost of a Reported Failure with a Markov Chain (cont)

Expected number of passages through the states before condemnation are:								
FR	R&R	CNDO	FVRDI	CNDI	STDFR	FVRDD	CNDD	INV
28.3688	21.2766	7.0922	12.766	4.25532	3.61702	2.71277	0.54255	3.25532
Estimated costs of each passage are:								
10	500	20	100	25	50	150	30	30
Expected costs by state until condemnation:								
283.688	10638.3	141.844	1276.6	106.383	180.851	406.915	16.2766	97.6596
							Total =	13,149

When the $32 expected condemnation cost is added to the cost of transiting through the various states prior to condemnation, the result is $13,180. We also learn, from the (I-Q)-1 matrix, that prior to condemnation the subject component, or its replacement, has the following expected life events.

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A Truly Parametric Support Model

- **The traditional three level support model using Markov Chains is not a true parametric model unless perchance the state costs came from historical data that has been fitted to CERs.**
- **Reasonable KTPPs for many maintenance situations:**
  - MTBF = mean time between failures (hours)
  - MTTR = mean time to repair (hours)
  - ERC = equipment replacement cost (e.g., $), as estimated by SEER-H or otherwise
  - IPRR = in-place replacement rate (%)
  - SMI = scheduled non-failure maintenance interval (hours)
  - CNR = condemnation rate (%)
- **The IFM equation can be written as follows for each element of the system, where SCPY is the support cost of interest**

$$SCPY = (OHPYR)(ESCPY)[w_1 f_1(MTBF) + w_2 f_2(MTTR) + w_3 f_3(ERC) + w_4 f_4(IPRR) + w_5 f_5(SMI) + w_6 f_6(CNR)]$$

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A Truly Parametric Support Model

- **Choose ESCPY = $50,000, appropriately normalized support cost per year per item, from a prime exemplar project that has similar hardware. For the prime exemplar, 45% is the operating hours per year ratio, also known as the duty cycle.**

KTPP	Units	Min	Max	Prime Exemplar	Influence Weights
MTBF	Hours	10,000	50,000	22,000	28.60%
MTTR	Hours	1	12	5.5	22.20%
ERC	$	100	3,000	1,650	21.30%
IPRR	%	10	50	20	11.20%
SMI	Hours	500	1500	800	7.30%
CNR	%	1	3.5	2.5	9.40%

- **Suppose that we want to estimate the annual maintenance cost of a similar item with the following characteristics using the initial (uncalibrated) model.**
  - The same duty cycle as the prime exemplar, MTBF = 35,450 hours
  - MTTR = 3 hours, ERC = $2,750, IPRR = 13%, SMI = 750 hours, CNR = 1.5%

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A Truly Parametric Support Model

			Calculated	Prime Exemplar =		$52,550
KTPP	Units	Value	Influence	Weight	Wtd Infl.	Cost
MTBF	hours	35450	1.611253674	28.60%	0.460818551	24,216
MTTR	hours	3	5.45E-01	22.20%	0.121094443	6,364
ERC	$	2750	1.83E+00	21.30%	0.3903647	20,514
IPRR	%	13	6.25E-01	11.20%	0.070014173	3,679
SMI	hours	750	9.38E-01	7.30%	0.06843865	3,596
CNR	%	1.5	5.36E-01	9.40%	0.050425765	2,650
				100.00%	= Sums =	$ 61,019