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Volatility and Cost Estimating

Abstract:

This paper will examine the properties and uses of implied volatility, stochastic volatility, and historic realized volatility. Further discussion will focus on what applications an assessment of market volatility brings to the field of cost estimating through the application of a volatility range derived for fuel prices at varying intervals over the course of a generic program's life cycle.

Stochastic Volatility and Cost Risk

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Introduction

Market shocks are events that cause a disruption in market equilibrium and cause fluctuations in the cost of commodities as well as inventory levels (Pindyck, 2004). These shocks are unpredictable, but there have been many attempts to predict future shocks through the study of market volatility. Through these studies, it has been observed that in times of economic uncertainty, changes in volatility tend to be more and more dramatic (Carr & Wu, 2005). The following figure compares the percent change of the VIX Index (a measurement of volatility for the S&P 500) to that of the S&P500 Index over a period of 20 years (Yahoo Finance Historic Data):

Figure 1: Percent Change in VIX and S&P500 Index[1]

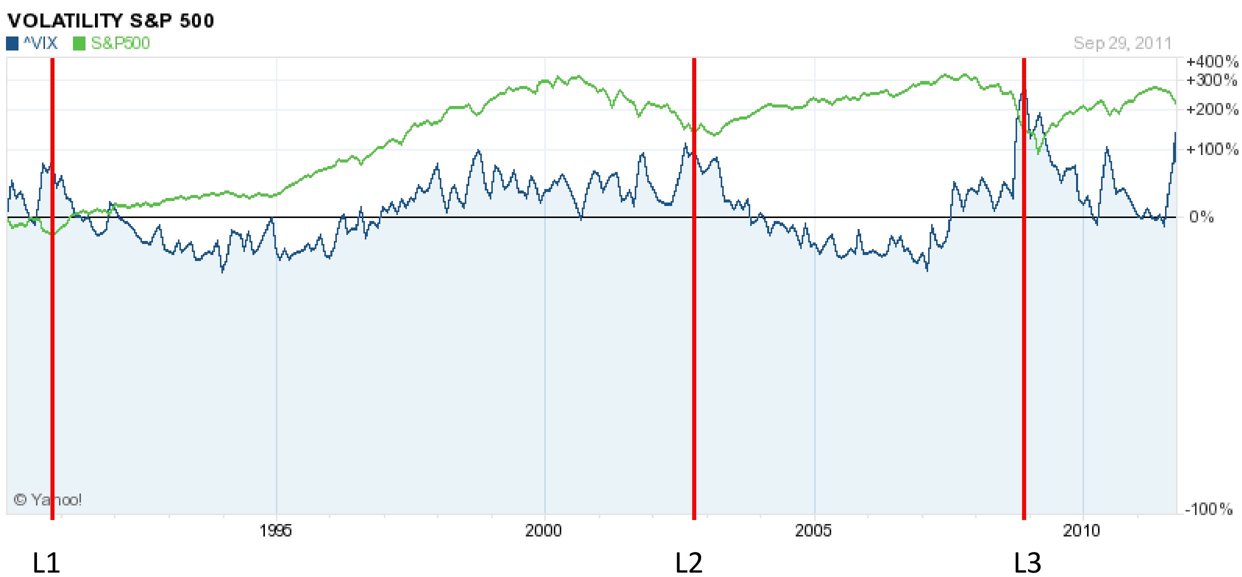

Figure 1 shows how changes in volatility increase during times of an economic slump. Each red line denotes a period of economic decline. L1 shows the recession that occurred in the early 1990s, L2 the end of the Tech bubble in the early 2000s, and L3 the recession that began in 2007/2008. It appears that the severity of the economic slumps impacts how much volatility changes. L1 and L3 represent recessions and the change in the VIX exceeds the change in the S&P500 at each of these two points. However, L2 represents one segment of the market performing poorly. It can be observed that the change in volatility does fluctuate during these years, but at the nadir of the slump, change in volatility does not exceed the change in the S&P 500 Index.

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[1] Figure 2, comparing the VIX to the Dow Jones Industrial Average (DJIA) is located in the Appendix. L1, L2, and L3 are also denoted on Figure 2; the results are similar to those shown in Figure 1. The primary difference is that the end of the Tech Bubble (L2) did not impact the DJIA as much as the S&P500.

The increase in volatility during times of economic decline indicate there is a greater risks of overrun for cost estimates on government systems due to uncertain commodity prices that are integrated into estimating models. The challenge is how to apply a market volatility measure to an estimate; especially since volatility is characterized by a "random walk" (i.e. past volatility does not indicate/predict what future volatility will be) (Harper, 2010). In the current fiscal environment of decreasing government spending and economic uncertainty, understanding and applying volatility to cost estimates becomes more important. To that end, this paper will examine different types of volatility and potential applications to the field of cost estimating.

Implied and Stochastic Volatility

Volatility can be defined as a statistical measure of the dispersion of returns for a given security or market index (Investopedia). It is measured using the standard deviation between returns from the same security or market index and is used to refer to the amount of uncertainty or risk around the changes in a security's value (Investopedia). The higher the volatility, the more the security's total value can be potentially spread out over a larger range of values (i.e. the more risk associated with that security) (Harper, 2010).

There are several different types of volatility. Implied volatility is used as a part of option pricing theory and can be defined as a forward-looking estimate based on market consensus, which is primarily calculated by equating a model-implied derivative price to the observed market price (Harper, 2010). This assumes that a derivative's underlying price follows a standard model for geometric Brownian motion as shown in the following equation (Tao, 2008):

$$dS_t = \mu S_t dt + \sigma S_t dW_t \qquad (1)$$

Where:
S_t is the Security Price at time t
μ is the constant drift (expected return) of S_t
σ is the constant volatility
dW_t is the standard Wiener process with zero mean and a unit rate of variance

Implied volatility captures volatility at a single moment in time and doesn't account for exogenous market shocks (Andersen & Benzoni, 2008). This was the primary flaw in the Black-Scholes model[2]; it did not implement stochastic volatility. Instead, the model looked at volatility as a moment in time, rather than as a continuously changing input (Andersen & Benzoni, 2008). Modeling continuous volatility mathematically is extremely complex; however,

[2] The Black-Scholes model is a mathematical model of a financial market that contains derivative investment instruments. From the resulting Black-Scholes formula, the price of options over time can be observed (Investopedia). The Appendix includes a brief description of the Black-Scholes formula for the price of a call option.

not taking volatility's stochastic nature into account contributed to the failure and subsequent bail out of Long Term Capital Management (Haubrich, 2007).

Another more complex approach to account for volatility is stochastic volatility. The underlying principle to go from implied volatility to stochastic volatility is to replace the measure of constant volatility, σ, with a function, v_t, that models the variance of the security price (S_t) over time (Pindyck, 2004). Stochastic volatility is primarily identified by two key principles (Andersen & Benzoni, 2008):

1. The idea of a second source of risk affecting the level of instantaneous volatility that should not be seen in isolation from the nature of the underlying asset or deliverable contract and
2. The application of continuous time[3]

Stochastic volatility can also model return variation dynamics that include an unobservable shock which cannot be predicted using available information (Andersen & Benzoni, 2008). Today, modern asset pricing theory uses continuous-time models (i.e. they work in-line with stochastic volatility rather than implied volatility) in order to obtain a more accurate estimate of the market (Tao, 2008). However, there are a variety of stochastic volatility models to choose from in order to quantify market volatility.

One commonly used and well known model of stochastic volatility is the Heston model. This model assumes that the randomness of the variance process varies as the square root of variance changes; that variance is a random process and exhibits a tendency to revert towards its long-term mean (Andersen & Benzoni, 2008). One key assumption of the model is that volatility appears proportional to the square root of its level and its source of randomness is correlated with the randomness inherent in the underlying asset's price (Andersen & Benzoni, 2008). This is expressed in the following equation:

$$dv_t = \theta(\omega - v_t)dt + \varepsilon\sqrt{v_t}dB_t \qquad (2)$$

Where:
v_t is a function of volatility at time t
ω is mean long term volatility
Θ is the rate at which volatility reverts to its long term mean
ε is the randomness associated with the underlying asset
dB_t and dW_t are Gaussian equations with zero mean and unit standard deviation (correlated to each other with correlation ρ)

Another well known model used to determine stochastic volatility is the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model. Although this model does not

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[3] The use of intraday transaction data has allowed for more genuine constructions of stochastic volatility models (Andersen & Benzoni, 2008).

render the current volatility known, there are many forms and variations of this model and it is generally well known (Andersen & Benzoni, 2008). GARCH assumes that the randomness of the variance process varies with the variance as opposed to the square root of the variance (as in the Heston model).

$$dv_t = \theta(\omega - v_t)dt + \varepsilon v_t dB_t \qquad (3)$$

As you can see, this equation is the same as that used for the Heston model, except the GARCH model does not use a radical.

Another well know stochastic volatility model, is the Stochastic Alpha, Beta, Rho (SABR) model (named after the key parameters in the model) (Andersen & Benzoni, 2008). This model attempts to capture and manage the volatility smile[4] phenomenon that is inherent in derivative markets. SABR does so by dynamically examining a single forward rate (e.g. London Inter-Bank Offer Rate (LIBOR) forward rate, forward swap rate, etc.) and the standard deviation as stochastic state variables over time. It is expressed in the following two equations:

$$dF_t = v_t F_t^{\beta} dW_t \qquad (4)$$
$$dv_t = \alpha v_t dZ_t \qquad (5)$$

Where:
v_t is stochastic state volatility at time t
F_t is the stochastic state forward swap rate at time t
W_t and Z_t are two correlated Wiener processes with correlation coefficient ρ (where $-1<\rho<1$)
β is constant parameter representing skewness (where $0\leq\beta\leq1$)
α is a constant parameter representing the volatility of volatility (where $\alpha\geq0$)

While these models apply continuous time to volatility, the complex modeling and math involved could make adding these equations to an already complex and diverse cost estimate very difficult; perhaps even unwieldy. Due to the quick turn-around time associated with most estimates and desire for a flexible model that can be easily used to run budget drills, an alternative attribute of volatility should be examined for application into point estimates. One such application is how volatility manifests in reality.

[4] Theoretically, for options with the same expiration date, the implied volatility should be unchanged, regardless of which strike practice is used (Options Guide, 2009). However, in reality, the implied volatility is different when different strikes are used. This disparity is known as volatility skew. A volatility smile is a long observed volatility skew pattern where at-the-money options tend to have lower implied volatilities than in-the-money or out-of-the money options, and another, more common skew pattern is called the volatility smirk (or reverse smirk) (Options Guide, 2009). The reverse skew pattern typically appears for longer term equity options and index options. See the Appendix for graphical representations of both the volatility smile and the volatility smirk.

Historic Realized Volatility

It is relatively easy to calculate historic realized volatility for an asset or market index. In order to calculate volatility, first an interval and a historic period are chosen (Harper, 2010). For example, the interval can be daily, weekly, or monthly, and the historic period can be any range of years over which data is available (e.g. 1 year, 5 years, 10 years etc.). Historic annualized volatility is captured using the following equations (Volatility Exchange):

$$v_t = 100 * \sqrt{\frac{252}{n}\sum_{t=1}^{n} R_t^2} \quad (6)$$

$$R_t = LN\frac{P_t}{P_{t-1}} \quad (7)$$

Where:
v_t is the historic realized volatility
n is the total number of trading days in the interval
252 represents the total number of trading days in a year
R_t is the continuously compounded daily return
P_t is the underlying asset price at time t
P_{t-1} is the underlying asset price for the interval immediately preceding time t

An important quality of historic realized volatility to note is that varying the duration of the interval examined can have a major impact on the level of volatility observed. For example, looking at the S&P500 and NASDAQ exchanges over a period of ten years (1994-2004), shows volatility ranging from 1.1% to 8.3%. The following table shows historic volatility for both exchanges using daily, weekly, and monthly intervals over the ten year period (Harper, 2010):

Table 1: Comparison of Volatility Intervals for the S&P500 and NASDAQ

Interval	S&P 500	NASDAQ
Daily	1.1%	1.8%
Weekly	2.4%	3.8%
Monthly	4.5%	8.3%

Table 1 shows that the realized volatility increases as the interval increases (but not in proportion to the change in the interval). This indicates that the standard deviation scale increases in proportion to the square root of time (Harper, 2010), which is true of random walk theory; giving further credence that volatility follows a “random walk”.

These calculations also indicate that the NASDAQ has more volatility than the S&P500. This disparity between the two indexes’ realized volatility could be related to differences between the underlying assets included in each index. The NASDAQ is traditionally home to many high tech stocks (approximately 5,000) while the S&P500 is an index of 500 stocks chosen primarily for their market size, liquidity, and industry (Investopedia). Stocks must be selected by S&P's Index Committee and the index itself is commonly used as a benchmark for the overall US stock market (Investopedia). This implies that a portfolio containing a variety of goods has less volatility than merely examining one single industry/commodity without reference to any other type of filtering or market research.

An additional look was taken at several of the Standard and Poor's Depository Receipt (SPDR) funds (for the period of December 1998 through October 2011). These are indexes traded on the New York Stock Exchange and are usually used by large institutions and traders to speculate on the overall direction of a particular commodity market (Investopedia). Specifically, the Energy, Materials, and Technology SPDRs were scrutinized over the time period using daily, weekly, and monthly intervals to see if there was any significant differences in volatility for one industry compared to another, or if volatility is relatively the same across industries. The following table shows the results (Yahoo Finance Historic Data):

Table 2: Comparison of Volatility Intervals for Specific Industries

Interval	Energy SPDR	Materials SPDR	Technology SPDR
Daily	9.6%	8.7%	9.5%
Weekly	51.2%	46.5%	51.3%
Monthly	218.0%	204.6%	225.4%

Table 2 confirms that the trading for a specific industry is more volatile than for the market as a whole. However, even though volatility is lower for the balanced portfolio, it still exists and can increase exponentially as the result of shocks to the market. How much impact volatility has on a specific index or program can be the result of how "balanced" that index/program is. A historical method can be used as an informative way to measure and analyze risk and uncertainty (Adkins, 2009).

Applications to Cost Estimating

Many programs that require cost estimates rely very heavily on commodities. Commodity prices are volatile and volatility itself varies over time (Pindyck, 2004). Changes in volatility can affect market variables by directly affecting the marginal value of storage and by affecting a component of the total marginal cost of production (i.e. the opportunity cost of producing the commodity now rather than waiting for more pricing information and producing it later) (Pindyck, 2004). Significant changes in these prices could have severely adverse affects on a program's Life Cycle Cost in the long run and during the current 5-year budget in the short run if Firm Fixed Price (FFP) contracts are not in place for the purchase of commodities like fuel and precious metals.

Since some of the commodities used in estimating are not predictable based on historic information due to market volatility and its random walk characteristics (e.g. fuel) (Andersen &

Benzoni, 2008), using realized volatility for commodity inputs as part of risk and uncertainty analysis could help to mitigate the risk inherent in a point estimate. However, because volatility itself exhibits random walk attributes, these calculations must be regularly updated based on commodity forecasts; otherwise this method would not account for future shocks to both the market and commodity prices (Andersen & Benzoni, 2008).

What platform/project is being estimated should determine the source of the volatility index or measure that is used when applying uncertainty to an estimate. For example, the Operations and Maintenance (O&M) of ground vehicles could depend heavily on the volatility associated with fuel, while an examination of volatility as related to electronics might be better suited when preparing an estimate for the development of a UAV. Furthermore, construction costs rely heavily on precious metals (i.e. copper) and the recent price increases in precious metals could easily cause the budget of a facility project to be breached. One source of intraday trading information that can be used to calculate historic volatility is the different SPDR Indexes. SPDRs trace certain commodities; for example there is an Energy Select Sector SPDR Fund that trades under the symbol XLE. In the same way, specific commodities can be tracked on the New York Mercantile Exchange (NYME) or Chicago Board of Trade (CBOT).

One way to apply this theory would be to use different risk ranges around inputs in a risk assessment model for the various commodity prices included in the estimate. The periods related to the risk ranges should also be varied in order to account for the different levels of volatility associated with the different time periods covered in the estimate. But first, an analyst must determine which commodities are crucial cost drivers for the estimate.

Look more closely at an O&M estimate for a generic ground vehicle as an example of this theory. Assume that the vehicles are expected to have a service life of 15 years. For the first year (or even two), the fuel price could be under a FFP contract; therefore there is little to no volatility around this input. However, there could be extreme volatility in the out-years for the fuel price. (Recall the increase in fuel prices in August of 2005 as a result of Hurricane Katrina. There was a sharp increase in fuel prices, before they dropped slightly and then remained relatively steady for several years.) One solution could be to examine the cost of different strike prices for varying fuel options (which relates to the volatility around barrels of oil) and create risk ranges for different year or different intervals based on the differences in the options' strike prices[5].

The following table provides an example of how to apply historic volatility around the cost of fuel associated with this O&M estimate:

[5] It is important to note that previous studies regarding fuel volatility (specifically gasoline) indicate that the past is not a good indicator of the future (Piesse & Van de Putte, 2004). Since this is the case, it might be better to model each year included in the estimate separately and give each year its own unique risk range to account for the high uncertainty associated with the fuel price input and the high level of demand inelasticity associated with fuel due to the ground vehicle's potential missions.

Table 3: Risk Inputs for Fuel Prices

	Year 1	Year 2	Years 3-5	Years 5-10	Years 10-15
Point Estimate Fuel Cost ($/gal)	$3.50	$3.50	$3.50	$3.50	$3.50
Historic Volatility (%)	2.38%	3.25%	7.34%	8.78%	9.56%
High Value ($/gal)	$3.58	$3.61	$3.76	$3.81	$3.83
Low Value ($/gal)	$3.42	$3.39	$3.24	$3.19	$3.17

First, the total life cycle was divided into intervals. The intervals are smaller for the earlier years of the program since it is expected that much more information (and therefore, less uncertainty) is available to estimators and planners. Next, historic volatility was calculated using the SPDR Energy Index and a daily interval (Yahoo Finance Historic Data) and the volatility equations (equations 6 and 7 described earlier in this paper). It can be observed from the table, that volatility increases as the time period examined increases. This makes the uncertainty range larger the further out from the current year (Year 1) that the estimate covers.

Once the historic volatility was calculated, it was used to build a range around the point estimate's fuel cost for each interval. From here, the analyst would simply choose the appropriate risk distribution for the input evaluated in order to create a confidence interval for each time period. This method of applying risk is both data driven (the ranges are derived based on historic costs related to a particular commodity) and can be used to account for the additional uncertainty inherent when estimating the far out-years for a program.

Conclusion

Not accounting for market shocks that cause fluctuations in commodity prices can cause programs to overrun their cost estimates. While applying stochastic volatility measurements may not be practical at this time, continual examination and forecasting of commodities can be done to account for the uncertainty that is inherent in markets. The use of an input based risk model to apply volatility adjustments to commodities used within the point estimate could be one way to employ volatility to account for potential market shocks. Additionally, breaking down the total life of the estimate into smaller intervals could help streamline the application of volatility since it is a measure that varies depending on the interval of time being observed (much as the uncertainty associated with a cost estimate changes as the program goes through different milestone decisions).

At the present time, this theory is unproven since this paper does not apply a commodity index to a specific program's point estimate. However, the relative ease of the formulas presented and their numerous potential applications to point estimates could be integrated into and applied as part of cost risk/uncertainty assessments for future Life Cycle Cost Estimates (LCCEs).

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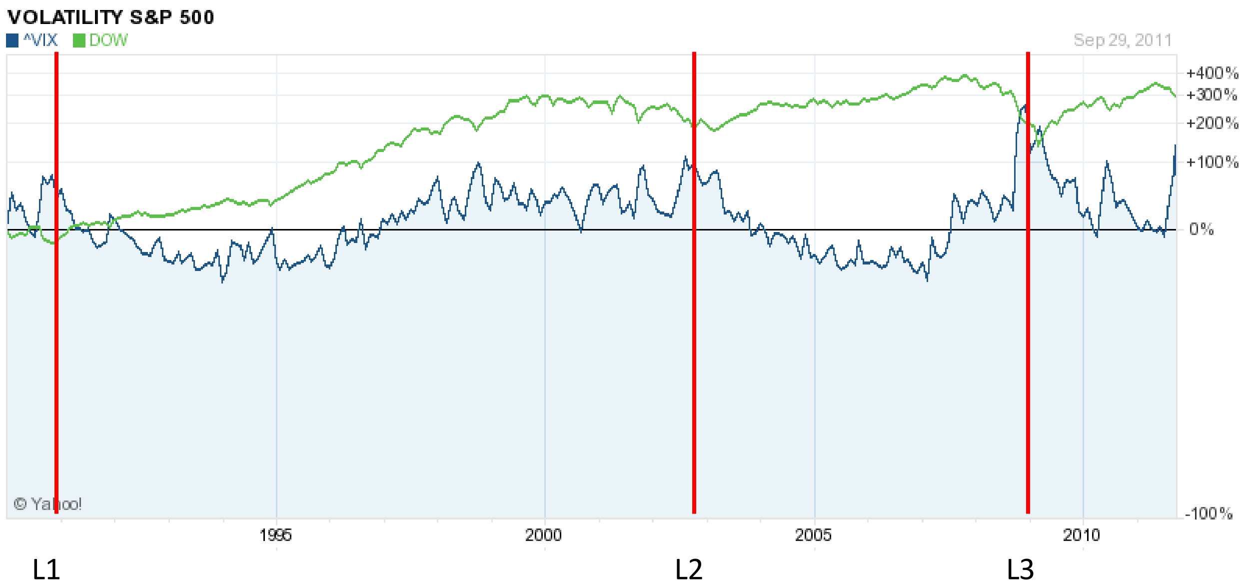
Appendix

Figure 2: Percent Change in VIX and S&P500 Index



Black-Scholes Formula (to price a call option)

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}$$ Where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}}$$ and $d_2 = d_1 - \sigma\sqrt{(T-t)}$

Where:
N (*) is the cumulative distribution function for the standard normal distribution
T-t is the time to maturity for the call option being priced
S is the spot price for the underlying asset
K is the strike price for the call option
r is the risk free interest rate (this is an annual rate and expressed in terms of continuous compounding)
σ is the volatility of returns on the underlying asset

Figure 3: Volatility Smile

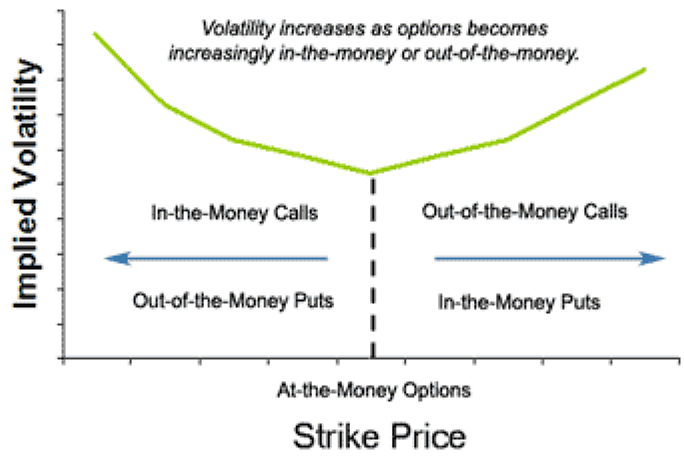


Figure 4: Volatility Smirk