

# Exploring Methods of Conflating Data From Various Data Sources

Dr. Ashley Moses  
SCEA/ISPA Joint Annual Conference  
June 26 – 29, 2012

## Overview

- ▶ Introduction
- ▶ Brief history of problem
- ▶ Toy problem
- ▶ Conclusion

## Introduction

- ▶ When trying to create an estimate, one might find it useful to combine data from multiple sources to be as accurate as possible
- ▶ Different sources such as expert opinion, empirical data, or simulation data might need to be conflated to determine an accurate cost estimate
- ▶ This talk will discuss different methods that may be used to combine data from multiple sources and will explore these methods further through the use of a toy problem

## How the Problem Came About

- ▶ Working on a project that required the conflation of empirical data, simulation data from physics-based model, and SME opinion
- ▶ Researched problem of conflating data from multiple sources and came across paper by Coleman, Braxton, and Druker

## Brief History

- ▶ In their paper, “The Correct Use of Subject Matter Experts in Cost Risk Analysis,” Coleman, Braxton, and Druker, discussed which methods might be the best to combine opinions given by subject matter experts (SMEs)
- ▶ Additionally, they also discussed what adjustments are needed to make the estimates provided by SMEs more accurate

## Toy Problem

- ▶ Interested in estimating the cost of next trip to a new grocery store, Harris Teeter, to purchase
  - Loaf of bread
  - Gallon of milk
  - Dozen Eggs
  - 1 lb of grapes
  - 1 lb of bananas
  - Box of Cheerios
  
- ▶ Data sources
  - SME opinion (38 eager SMEs!) on prices for Harris Teeter
  - Empirical price data collected since Sept. 2009
  - Analogy model comparing prices of Safeway to Harris Teeter

# SME Opinions

How much would you expect to pay for a gallon of milk?			How much would you expect to pay for a dozen eggs?			How much would you expect to pay PER POUND for grapes?			How much would you expect to pay PER POUND for bananas?			How much would you expect to pay for a loaf of bread?			How much would you expect to pay for a standard size box of Cheerios?		
Min	Mode	Max	Min	Mode	Max	Min	Mode	Max	Min	Mode	Max	Min	Mode	Max	Min	Mode	Max
\$1.89	\$2.09	\$4.29	\$1.49	\$1.89	\$2.09	\$0.49	\$0.69	\$0.79	\$0.29	\$0.49	\$0.59	\$1.29	\$1.89	\$3.29	\$2.89	\$3.49	\$4.29
\$1.50	\$3.00	\$5.00	\$1.00	\$1.75	\$3.00	\$0.75	\$1.29	\$2.00	\$0.25	\$0.60	\$1.50	\$0.75	\$1.50	\$3.00	\$2.50	\$3.50	\$5.00
\$2.00	\$3.00	\$3.50	\$2.00	\$2.00	\$3.00	\$3.00	\$3.50	\$4.00	\$2.00	\$2.00	\$3.00	\$1.89	\$2.25	\$3.00	\$3.00	\$3.99	\$4.50
\$1.00	\$2.00	\$3.00	\$0.59	\$1.09	\$1.99	\$1.50	\$2.00	\$2.50	\$0.99	\$1.49	\$1.99	\$1.49	\$1.99	\$2.99	\$2.50	\$3.50	\$4.00
\$3.00	\$3.50	\$5.00	\$1.70	\$2.50	\$4.50	\$0.99	\$1.50	\$3.99	\$0.69	\$1.00	\$2.00	\$1.50	\$2.89	\$5.00	\$1.50	\$2.50	\$4.00
\$1.99	\$2.50	\$3.50	\$1.00	\$1.99	\$3.50	\$1.99	\$1.99	\$2.50	\$0.30	\$0.59	\$0.65	\$1.50	\$3.00	\$4.00	\$1.99	\$3.00	\$4.00
\$3.00	\$3.69	\$3.89	\$1.39	\$1.79	\$1.99	\$0.89	\$1.39	\$2.19	\$0.49	\$0.69	\$1.19	\$1.29	\$2.79	\$3.29	\$3.50	\$3.90	\$4.30
\$2.50	\$3.00	\$4.00	\$1.00	\$2.00	\$3.00	\$2.00	\$2.50	\$3.00	\$2.00	\$3.00	\$4.00	\$2.00	\$2.75	\$4.00	\$2.50	\$3.50	\$5.00
\$3.25	\$4.00	\$6.00	\$0.29	\$0.79	\$1.99	\$0.99	\$2.99	\$3.99	\$0.35	\$0.49	\$0.59	\$1.99	\$2.49	\$3.49	\$2.49	\$3.00	\$3.49
\$2.99	\$3.98	\$4.25	\$0.68	\$1.38	\$1.79	\$2.99	\$4.99	\$5.99	\$0.69	\$0.99	\$1.69	\$1.99	\$2.99	\$3.99	\$1.99	\$3.98	\$4.98
\$2.50	\$2.75	\$3.00	\$1.00	\$1.25	\$1.50	\$2.00	\$2.80	\$4.00	\$0.80	\$1.00	\$1.50	\$2.00	\$2.75	\$4.00	\$4.00	\$4.50	\$5.00
\$3.25	\$3.50	\$3.75	\$1.30	\$1.50	\$1.75	\$1.75	\$2.00	\$2.50	\$0.50	\$0.60	\$0.80	\$2.25	\$2.50	\$2.75	\$2.39	\$2.50	\$2.60
\$3.29	\$3.49	\$3.89	\$1.19	\$1.29	\$1.39	\$0.99	\$2.99	\$4.99	\$0.29	\$0.39	\$0.99	\$1.59	\$2.39	\$3.39	\$2.59	\$3.19	\$4.29
\$3.00	\$3.50	\$4.00	\$2.00	\$3.00	\$3.50	\$3.00	\$4.00	\$5.00	\$0.50	\$1.00	\$1.30	\$2.00	\$3.00	\$4.00	\$2.00	\$3.00	\$4.00
\$2.79	\$3.09	\$5.79	\$1.59	\$3.19	\$4.99	\$2.00	\$3.00	\$6.00	\$0.29	\$1.00	\$2.00	\$1.99	\$3.09	\$5.99	\$3.00	\$4.00	\$5.00
\$3.00	\$3.29	\$3.50	\$1.89	\$1.99	\$2.50	\$0.58	\$0.79	\$0.85	\$0.49	\$0.59	\$0.98	\$1.00	\$1.29	\$2.19	\$2.38	\$3.15	\$3.50
\$2.50	\$3.50	\$4.00	\$1.49	\$1.79	\$3.00	\$0.99	\$1.99	\$3.00	\$0.49	\$0.69	\$1.00	\$0.99	\$1.50	\$2.50	\$2.00	\$4.00	\$5.00
\$2.85	\$3.15	\$3.75	\$0.99	\$1.60	\$2.20	\$1.00	\$3.00	\$5.00	\$0.50	\$0.60	\$1.00	\$0.95	\$1.75	\$3.00	\$2.50	\$3.00	\$4.00
\$2.50	\$3.00	\$4.00	\$2.00	\$2.39	\$3.00	\$0.59	\$1.25	\$2.99	\$0.49	\$0.99	\$1.50	\$1.00	\$1.50	\$4.00	\$2.99	\$3.99	\$5.99
\$2.50	\$3.00	\$3.50	\$1.10	\$1.40	\$1.80	\$0.99	\$1.50	\$1.80	\$0.99	\$1.50	\$1.80	\$1.99	\$2.60	\$3.00	\$3.00	\$3.80	\$4.50
\$2.00	\$3.00	\$4.00	\$1.60	\$1.80	\$3.00	\$1.50	\$1.75	\$2.50	\$0.25	\$0.40	\$0.80	\$1.50	\$2.00	\$3.00	\$2.50	\$3.00	\$4.00
\$2.87	\$2.87	\$4.29	\$0.89	\$1.18	\$1.29	\$1.96	\$3.47	\$4.06	\$0.49	\$1.19	\$1.98	\$1.00	\$2.19	\$3.06	\$1.97	\$2.50	\$3.00
\$1.98	\$2.98	\$3.15	\$1.50	\$2.25	\$2.95	\$0.89	\$1.75	\$2.98	\$0.28	\$0.50	\$0.60	\$0.99	\$2.79	\$4.00	\$2.50	\$3.50	\$3.75
\$2.00	\$2.50	\$4.50	\$2.00	\$3.00	\$4.00	\$3.50	\$4.00	\$6.00	\$2.00	\$3.00	\$4.00	\$1.00	\$1.75	\$3.00	\$1.00	\$1.50	\$4.00
\$2.20	\$2.50	\$4.00	\$1.10	\$1.25	\$1.75	\$1.00	\$1.20	\$1.88	\$0.43	\$0.52	\$0.60	\$1.00	\$1.25	\$2.00	\$2.50	\$2.88	\$3.00
\$2.50	\$3.00	\$4.00	\$2.00	\$2.50	\$3.00	\$0.99	\$1.29	\$1.40	\$1.00	\$2.00	\$2.50	\$2.00	\$2.50	\$2.75	\$2.50	\$3.00	\$4.00
\$3.00	\$3.49	\$3.99	\$1.00	\$1.29	\$1.99	\$2.49	\$2.79	\$3.49	\$0.29	\$0.49	\$0.69	\$1.99	\$2.49	\$3.99	\$2.19	\$2.49	\$3.99
\$0.99	\$2.50	\$5.00	\$0.99	\$1.29	\$3.50	\$1.50	\$2.00	\$2.50	\$0.50	\$0.75	\$1.00	\$1.50	\$2.29	\$3.00	\$3.00	\$5.00	\$6.00
\$3.00	\$4.25	\$6.00	\$1.50	\$2.50	\$5.00	\$0.75	\$1.25	\$5.00	\$0.50	\$0.78	\$2.00	\$1.00	\$1.75	\$4.00	\$3.00	\$4.00	\$5.50
\$4.25	\$4.75	\$5.25	\$2.35	\$2.35	\$3.95	\$1.95	\$1.25	\$3.00	\$0.49	\$0.57	\$1.00	\$2.35	\$2.50	\$3.00	\$4.00	\$4.25	\$4.35
\$2.00	\$3.00	\$5.00	\$1.50	\$2.50	\$4.00	\$2.00	\$2.50	\$5.00	\$0.50	\$1.00	\$1.50	\$1.50	\$3.00	\$5.00	\$2.50	\$3.50	\$5.00
\$1.50	\$2.50	\$4.00	\$3.00	\$5.00	\$7.00	\$0.89	\$1.50	\$2.00	\$0.60	\$1.00	\$2.00	\$1.00	\$2.00	\$2.50	\$2.00	\$3.00	\$5.00
\$3.00	\$3.68	\$4.00	\$0.88	\$1.24	\$1.48	\$2.99	\$3.24	\$3.50	\$0.33	\$0.50	\$0.78	\$0.87	\$2.89	\$3.25	\$2.00	\$3.68	\$5.00
\$3.00	\$3.30	\$3.50	\$1.50	\$2.35	\$3.35	\$3.50	\$4.00	\$5.00	\$2.00	\$3.00	\$3.50	\$1.29	\$2.25	\$4.00	\$3.50	\$4.00	\$4.50
\$2.00	\$3.50	\$5.00	\$2.50	\$3.50	\$5.25	\$0.69	\$1.25	\$3.25	\$0.45	\$0.99	\$2.50	\$1.50	\$2.00	\$4.25	\$2.50	\$3.50	\$5.00
\$1.69	\$2.09	\$3.00	\$0.89	\$1.29	\$1.50	\$0.99	\$1.99	\$2.99	\$0.34	\$0.49	\$0.64	\$0.99	\$1.99	\$2.99	\$2.00	\$2.99	\$4.00
\$2.50	\$2.88	\$3.50	\$1.00	\$1.99	\$3.00	\$0.77	\$1.50	\$2.88	\$0.33	\$0.39	\$0.50	\$1.00	\$2.88	\$4.50	\$2.50	\$3.88	\$4.50
\$2.59	\$3.09	\$5.98	\$0.99	\$1.29	\$2.29	\$1.99	\$2.99	\$3.99	\$0.44	\$0.49	\$0.52	\$1.99	\$2.98	\$3.25	\$1.99	\$2.99	\$3.99

## Analogous Model

- ▶ Harris Teeter prices are similar to that of Safeway
- ▶ Based on results from a study done on the website *The Hill is home*, we will assume that

Harris Teeter Price  $\approx$  .93\*Safeway Price

	Gallon of Milk	Dozen Eggs	Grapes (\$/lb)	Bananas (\$/lb)	Bread	Cheerios
Safeway	\$4.39	\$2.89	\$2.98	\$0.45	\$2.69	\$2.50



## Point Estimates for Harris Teeter

- ▶ Milk  $(.93)*(\$4.39)=\$4.08$
- ▶ Eggs  $(.93)*(\$2.89)=\$2.69$
- ▶ Grapes  $(.93)*(\$2.98)=\$2.77$
- ▶ Bananas  $(.93)*(\$0.45)=\$0.42$
- ▶ Bread  $(.93)*(\$2.69)=\$2.50$
- ▶ Cheerios  $(.93)*(\$2.50)=\$2.33$

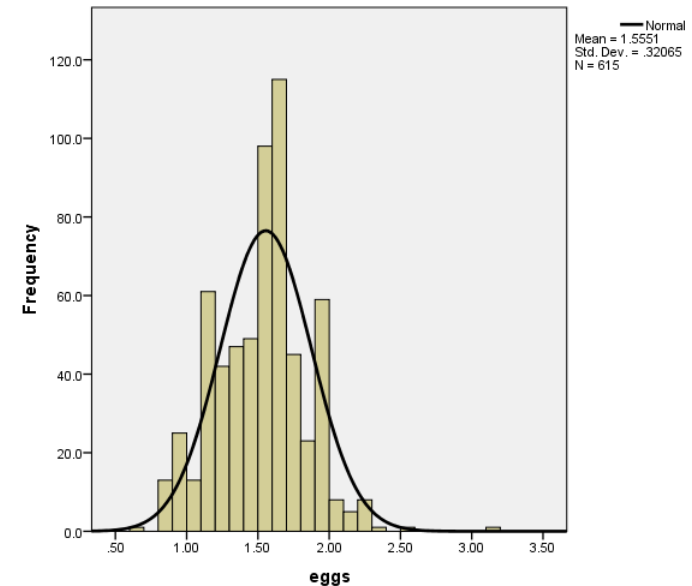
## Empirical Data

- ▶ Data from Sept 2009 to Mar 2012 from multiple grocery stores in Tampa Bay, FL

Date	milk	eggs	cereal	bread	grapes	bananas	Location
9/10/09	\$2.99	\$1.39	\$4.99	\$1.79	\$1.49	\$0.57	Publix
9/10/09	\$2.99	\$0.99	\$2.89	\$0.77	\$1.49	\$0.69	Sweetbay
9/10/09	\$2.49	\$1.42	\$3.50	\$1.09	\$0.99	\$0.69	Target
9/10/09	\$2.48	\$0.88	\$3.92	\$1.08	\$1.44	\$0.67	Wal-Mart
9/10/09	\$3.09	\$1.69	\$4.99	\$0.80	\$1.49	\$0.69	Winn-Dixie
9/17/09	\$2.99	\$1.19	\$4.99	\$1.79	\$1.49	\$0.57	Publix
9/17/09	\$2.99	\$1.39	\$2.89	\$1.15	\$1.99	\$0.69	Sweetbay
9/17/09	\$2.49	\$1.42	\$3.50	\$1.09	\$0.99	\$0.57	Target
9/17/09	\$2.68	\$0.88	\$3.92	\$1.08	\$1.44	\$0.67	Wal-Mart
9/17/09	\$3.09	\$1.69	\$4.99	\$1.25	\$0.99	\$0.69	Winn-Dixie
9/24/09	\$2.99	\$1.19	\$4.99	\$1.15	\$1.49	\$0.57	Publix
9/24/09	\$2.99	\$1.19	\$2.89	\$1.15	\$1.59	\$0.69	Sweetbay
9/24/09	\$2.49	\$1.09	\$2.09	\$1.09	\$0.99	\$0.57	Target
9/24/09	\$2.68	\$0.88	\$3.92	\$1.08	\$1.44	\$0.67	Wal-Mart
9/24/09	\$3.09	\$1.69	\$4.99	\$1.50	\$1.49	\$0.69	Winn-Dixie
10/1/09	\$3.09	\$1.19	\$2.50	\$1.59	\$1.49	\$0.69	Publix
10/1/09	\$3.09	\$1.19	\$2.89	\$1.15	\$1.29	\$0.69	Sweetbay
10/1/09	\$2.69	\$1.42	\$3.69	\$1.09	\$0.99	\$0.57	Target
10/1/09	\$2.68	\$0.88	\$3.92	\$1.08	\$1.44	\$0.67	Wal-Mart
10/1/09	\$3.09	\$1.69	\$4.99	\$1.50	\$1.49	\$0.69	Winn-Dixie
10/8/09	\$3.09	\$1.19	\$4.99	\$1.15	\$1.49	\$0.69	Publix

## Fitting Distributions to the Data

- ▶ Fitting the empirical data with distributions, we get the following:
  - Milk ~ Norm(\$3.38, \$0.32)
  - Eggs ~ Norm(\$1.55, \$0.32)
  - Grapes ~ Norm(\$1.46, \$0.22)
  - Bananas ~ Norm(\$0.64, \$0.09)
  - Bread ~ Norm(\$1.11, \$0.24)
  - Cheerios ~ Norm(\$4.06, \$0.76)



## Step 1 – Combine SME Opinions

- ▶ First step according to Coleman, Braxton, and Druker is to determine what we believe to be the underlying model
- ▶ Determine whether “Single Reality” or “Multiple Realities”

## Single Reality

- ▶ One (typically uni-model) distribution which we do not know
- ▶ Experts are presumed to know it to some degree of accuracy
- ▶ Example:
  - How many brown M&M's are there in a 1.69 oz bag of M&M's?

## M&M's Color Distribution

	Blue	Brown	Green	Orange	Red	Yellow	Total M&M's
<b>Percent expected</b>	24%	13%	16%	20%	13%	14%	----
<b>Percent observed</b>	18.36%	14.16%	18.44%	20.76%	14.20%	14.08%	----
<b>Qty. expected</b>	629	341	419	524	341	367	----
<b>Qty. observed</b>	481	371	483	544	372	369	2620
<b>Difference</b>	-148	+30	+64	+20	+31	+2	----
<b>Average per pack</b>	10.02	7.73	10.06	11.33	7.75	7.69	54.58
<b>Maximum in pack</b>	16	12	17	17	12	14	57
<b>Minimum in pack</b>	5	3	5	7	2	2	52
<b>Std. deviation</b>	2.82	2.19	2.59	2.54	2.62	2.65	1.32
<b>Variance</b>	1.74	7.98	4.80	6.70	6.44	6.87	7.03

## Multiple Realities

- ▶ There are  $k$  distributions (typically uni-modal) for which we generally know neither  $k$  nor the individual distributions
- ▶ Experts are presumed to know at least one each to some degree of accuracy
- ▶ Example:
  - What is the air-speed velocity of an unladen swallow? (African or European swallow?)

## Which is it?

- ▶ Multiple locations...
  - ▶ Multiple seasons (sales)...
  - ▶ Multiple brands...
- 
- ▶ All signs point to multiple realities



## Adjust SME Extrema

- ▶ Prior to combining, we must first adjust the SME extrema
- ▶ Use formulas from *Air Force Cost, Risk, & Uncertainty Handbook*

		Guidance and Control	Sys Test & Eval	Source or Formula
Low Bound	Low	595	225	Expert Opinion
Mode	Mode	700	250	Expert Opinion
High Bound	High	980	300	Expert Opinion
Uncertainty Captured	UncertCap	70%	70%	Expert Opinion
Skew	Skew	0.27	0.33	$(\text{Mode}-\text{Low})/(\text{High}-\text{Low})$
Uncertainty Not Captured	UncertNotCap	30%	30%	$1 - \text{UncertCap}$
Lower Interpretation	LowInterp	8%	10%	$\text{Skew} * \text{UncertNotCap}$
High Interpretation	HighInterp	78%	80%	$\text{UncertCap} + \text{LowInterp}$
High Tail Area	HighTail	22%	20%	$1 - \text{HighInterp}$
Absolute High	AbsHigh	1268.8	349.5	$\text{High} + (\text{High}-\text{Mode}) * \text{SQRT}(\text{UncertNotCap}) / (1 - \text{SQRT}(\text{HighTail}))$
Absolute Low		486.7	200.2	$(\text{Mode} - \text{Skew} * \text{AbsHigh}) / (1 - \text{Skew})$

# Adjusted SME Opinions

How much would you expect to pay for a gallon of milk?			How much would you expect to pay for a dozen eggs?			How much would you expect to pay PER POUND for grapes?			How much would you expect to pay PER POUND for bananas?			How much would you expect to pay for a loaf of bread?			How much would you expect to pay for a standard size box of Cheerios?		
Min	Mode	Max	Min	Mode	Max	Min	Mode	Max	Min	Mode	Max	Min	Mode	Max	Min	Mode	Max
\$1.66	\$2.09	\$6.82	\$1.17	\$1.89	\$2.25	\$0.33	\$0.69	\$0.87	\$0.13	\$0.49	\$0.67	\$0.68	\$1.89	\$4.71	\$2.33	\$3.49	\$5.04
\$0.10	\$3.00	\$6.87	\$0.28	\$1.75	\$4.21	\$0.25	\$1.29	\$2.66	-\$0.11	\$0.60	\$2.42	\$0.01	\$1.50	\$4.49	\$1.55	\$3.50	\$6.43
\$1.20	\$3.00	\$3.90	\$2.00	\$2.00	\$4.21	\$2.55	\$3.50	\$4.45	\$2.00	\$2.00	\$4.21	\$1.53	\$2.25	\$3.75	\$2.20	\$3.99	\$4.91
\$0.11	\$2.00	\$3.89	\$0.10	\$1.09	\$2.87	\$1.05	\$2.00	\$2.95	\$0.54	\$1.49	\$2.44	\$0.99	\$1.99	\$3.98	\$1.70	\$3.50	\$4.40
\$2.48	\$3.50	\$6.56	\$0.88	\$2.50	\$6.54	\$0.43	\$1.50	\$6.71	\$0.36	\$1.00	\$3.05	\$0.18	\$2.89	\$7.01	\$0.55	\$2.50	\$5.43
\$1.49	\$2.50	\$4.49	\$0.06	\$1.99	\$4.94	\$1.99	\$1.99	\$3.12	\$0.09	\$0.59	\$0.69	\$0.24	\$3.00	\$4.84	\$1.09	\$3.00	\$4.89
\$2.49	\$3.69	\$4.04	\$1.07	\$1.79	\$2.15	\$0.41	\$1.39	\$2.96	\$0.29	\$0.69	\$1.70	\$0.16	\$2.79	\$3.67	\$3.14	\$3.90	\$4.66
\$2.00	\$3.00	\$4.99	\$0.11	\$2.00	\$3.89	\$1.55	\$2.50	\$3.45	\$1.11	\$3.00	\$4.89	\$1.28	\$2.75	\$5.21	\$1.55	\$3.50	\$6.43
\$2.48	\$4.00	\$8.06	-\$0.22	\$0.79	\$3.21	-\$0.61	\$2.99	\$4.79	\$0.23	\$0.49	\$0.67	\$1.49	\$2.49	\$4.48	\$2.04	\$3.00	\$3.93
\$2.26	\$3.98	\$4.45	\$0.11	\$1.38	\$2.13	\$1.39	\$4.99	\$6.79	\$0.39	\$0.99	\$2.40	\$1.10	\$2.99	\$4.88	\$0.39	\$3.98	\$5.78
\$2.28	\$2.75	\$3.22	\$0.78	\$1.25	\$1.72	\$1.24	\$2.80	\$5.14	\$0.60	\$1.00	\$2.01	\$1.28	\$2.75	\$5.21	\$3.55	\$4.50	\$5.45
\$3.03	\$3.50	\$3.97	\$1.11	\$1.50	\$1.98	\$1.50	\$2.00	\$3.00	\$0.40	\$0.60	\$1.00	\$2.03	\$2.50	\$2.97	\$2.29	\$2.50	\$2.69
\$3.09	\$3.49	\$4.29	\$1.10	\$1.29	\$1.48	-\$0.80	\$2.99	\$6.78	\$0.18	\$0.39	\$1.66	\$0.85	\$2.39	\$4.32	\$2.00	\$3.19	\$5.37
\$2.55	\$3.50	\$4.45	\$1.20	\$3.00	\$3.90	\$2.11	\$4.00	\$5.89	\$0.09	\$1.00	\$1.55	\$1.11	\$3.00	\$4.89	\$1.11	\$3.00	\$4.89
\$2.45	\$3.09	\$8.87	\$0.13	\$3.19	\$6.63	\$0.96	\$3.00	\$9.13	-\$0.38	\$1.00	\$2.94	\$0.86	\$3.09	\$8.97	\$2.11	\$4.00	\$5.89
\$2.75	\$3.29	\$3.68	\$1.78	\$1.99	\$3.06	\$0.42	\$0.79	\$0.89	\$0.38	\$0.59	\$1.40	\$0.70	\$1.29	\$3.13	\$1.77	\$3.15	\$3.78
\$1.70	\$3.50	\$4.40	\$1.17	\$1.79	\$4.30	\$0.09	\$1.99	\$3.90	\$0.30	\$0.69	\$1.30	\$0.49	\$1.50	\$3.49	\$0.40	\$4.00	\$5.80
\$2.55	\$3.15	\$4.34	\$0.45	\$1.60	\$2.73	-\$0.79	\$3.00	\$6.79	\$0.39	\$0.60	\$1.43	\$0.18	\$1.75	\$4.20	\$2.00	\$3.00	\$4.99
\$2.00	\$3.00	\$4.99	\$1.63	\$2.39	\$3.58	-\$0.09	\$1.25	\$4.78	\$0.04	\$0.99	\$1.96	\$0.45	\$1.50	\$6.74	\$2.00	\$3.99	\$7.97
\$2.05	\$3.00	\$3.95	\$0.82	\$1.40	\$2.17	\$0.57	\$1.50	\$2.05	\$0.57	\$1.50	\$2.05	\$1.48	\$2.60	\$3.33	\$2.30	\$3.80	\$5.11
\$1.11	\$3.00	\$4.89	\$1.38	\$1.80	\$4.33	\$1.24	\$1.75	\$3.28	\$0.10	\$0.40	\$1.21	\$1.00	\$2.00	\$3.99	\$2.00	\$3.00	\$4.99
\$2.87	\$2.87	\$6.01	\$0.67	\$1.18	\$1.37	\$0.79	\$3.47	\$4.52	-\$0.15	\$1.19	\$2.70	-\$0.01	\$2.19	\$3.80	\$1.50	\$2.50	\$3.44
\$1.29	\$2.98	\$3.27	\$0.84	\$2.25	\$3.57	\$0.08	\$1.75	\$4.14	\$0.11	\$0.50	\$0.68	-\$0.52	\$2.79	\$5.02	\$1.77	\$3.50	\$3.93
\$1.46	\$2.50	\$6.65	\$1.11	\$3.00	\$4.89	\$2.96	\$4.00	\$8.15	\$1.11	\$3.00	\$4.89	\$0.28	\$1.75	\$4.21	\$0.45	\$1.50	\$6.74
\$1.87	\$2.50	\$5.64	\$0.94	\$1.25	\$2.28	\$0.79	\$1.20	\$2.60	\$0.35	\$0.52	\$0.67	\$0.74	\$1.25	\$2.78	\$2.22	\$2.88	\$3.09
\$2.00	\$3.00	\$4.99	\$1.55	\$2.50	\$3.45	\$0.76	\$1.29	\$1.48	\$0.20	\$2.00	\$2.90	\$1.60	\$2.50	\$2.95	\$2.00	\$3.00	\$4.99
\$2.56	\$3.49	\$4.44	\$0.71	\$1.29	\$2.70	\$2.19	\$2.79	\$4.20	\$0.11	\$0.49	\$0.87	\$1.47	\$2.49	\$5.55	\$1.86	\$2.49	\$5.63

## Method of Conflation

- ▶ Since multiple reality, Coleman, Braxton, and Druker suggest “sampling” method
- ▶ Sampling:
  - For each run of the Monte Carlo, randomly select an expert’s distribution and then randomly select a value from that distribution
  - Collection of all randomly sampled values forms the final conflated distribution

## Modification Opportunity

- ▶ One way to customize the sampling method would be to add different weightings
- ▶ SMEs are no longer picked completely at random – some will be picked more often than others
- ▶ Might do this when certain SMEs have more experience or more valuable opinions

## Another Method

- ▶ Averaging on each iteration
  - For each run of the Monte Carlo, randomly sample once from each of the distributions and then average those values
  - Collection of the averaged random sampled values form final distribution
  
- ▶ This method produces a good confidence interval for the mean of the experts but does not do a good job of capturing the full range of outcomes as it actually shrinks the standard deviation

## Step 2 – Combine All Three Sources

Potential different methods

### ▶ Sampling:

- For each run of the Monte Carlo, randomly select a source distribution and then randomly select a value from that distribution
- Collection of all randomly sampled values forms the final conflated distribution

### ▶ Weighted Sampling:

- Similar to sampling, but instead of randomly selecting source distributions, apply a weighting method so that some sources get picked more often than others

## Step 2 - Continued

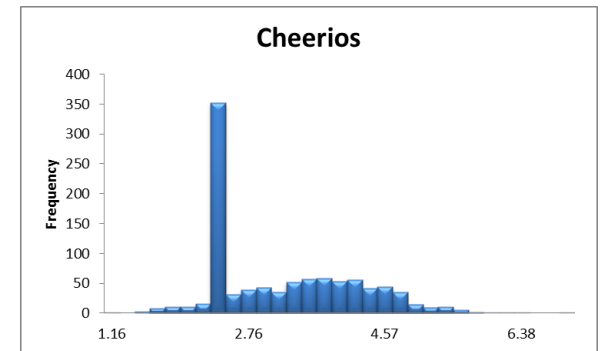
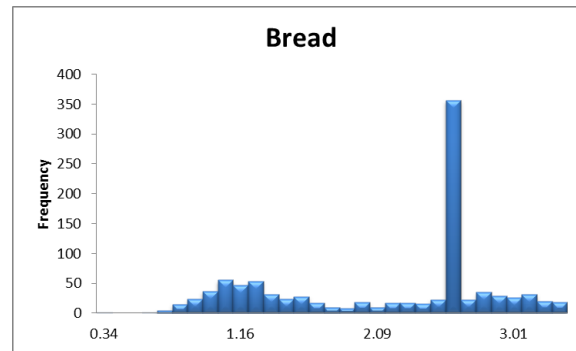
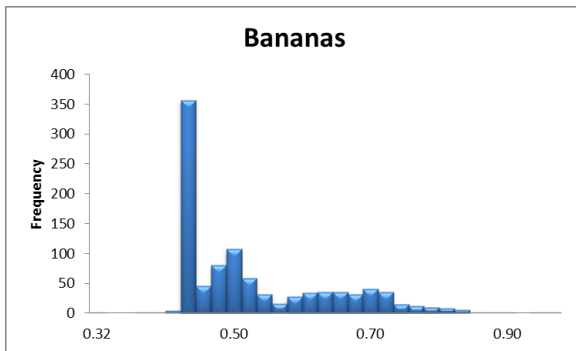
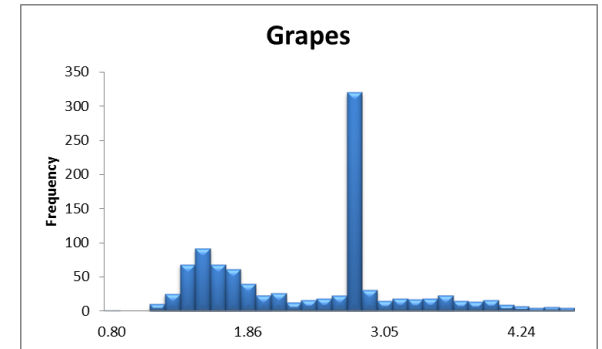
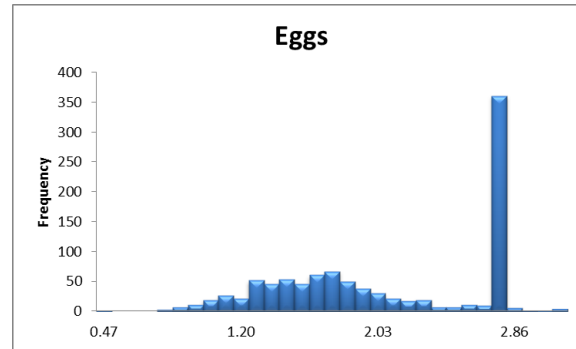
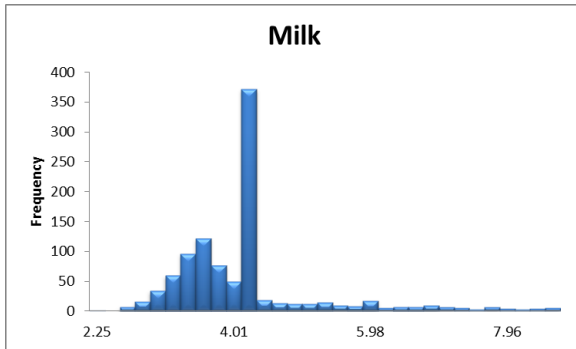
- ▶ Averaging on each iteration
  - For each run of the Monte Carlo, randomly sample once from each of the distributions and then average those values
  - Collection of the averaged random sampled values form final distribution
  - As with the SME case, this is probably not the best option for capturing the entire range of outcomes
  
- ▶ Supplement:
  - Rather than sampling from all three distributions, use certain aspects of one source to supplement weaker areas of other distributions
  - Ex: Use point estimate from analogy and standard deviation from empirical data to make a new distribution

## Monte Carlo Simulations

- ▶ Used RealTime Analytics: The Excel Tool (RTA), Booz Allen Hamilton proprietary technology
- ▶ 1000 Trials

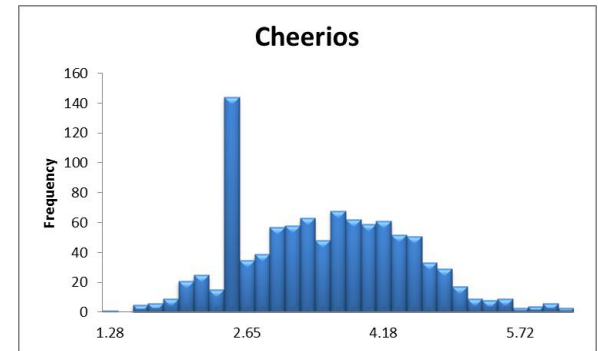
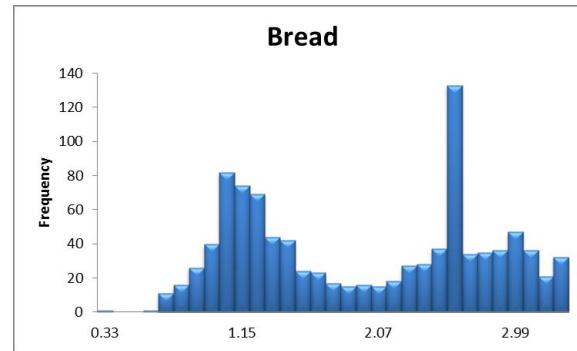
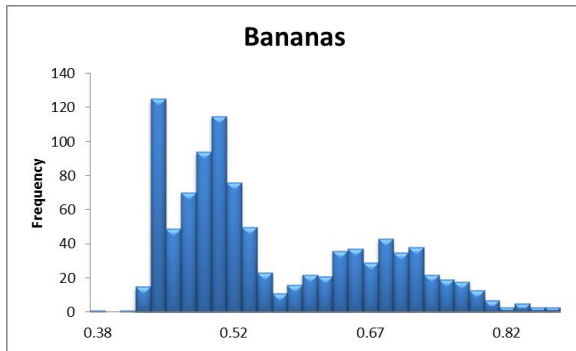
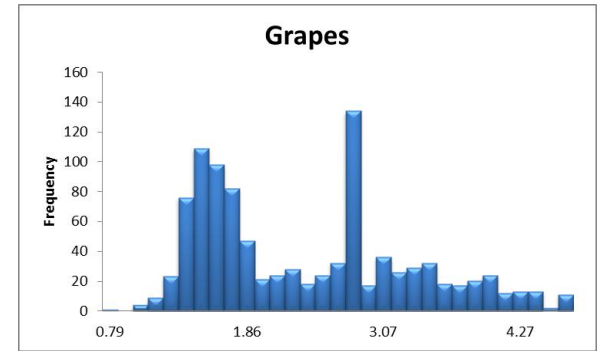
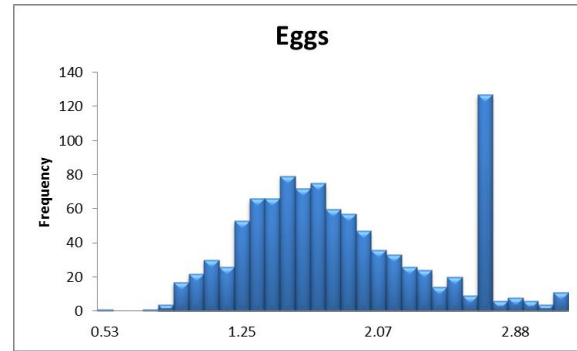
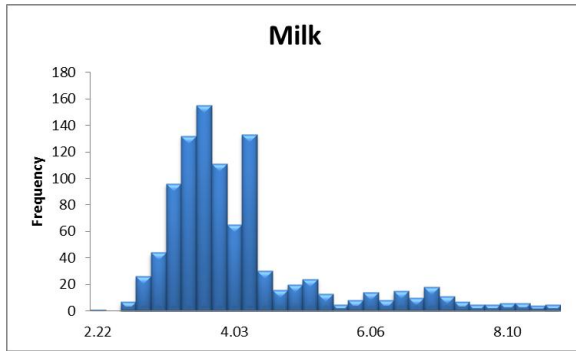


## Results: Sampling Method



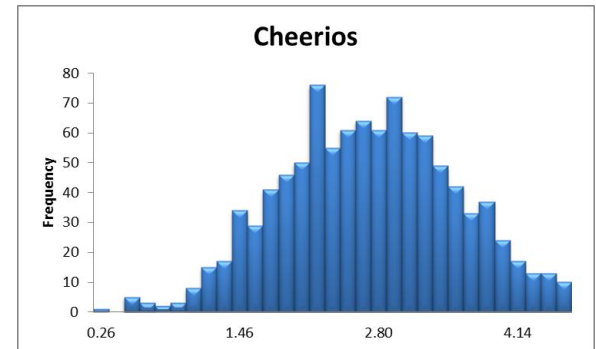
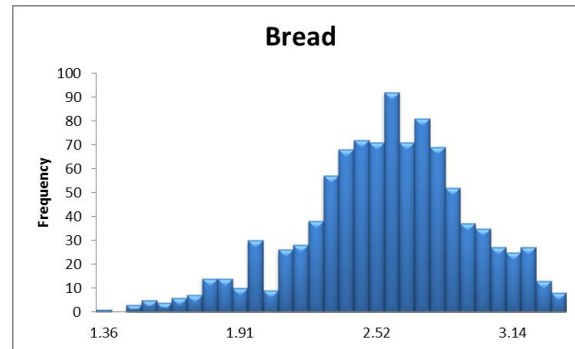
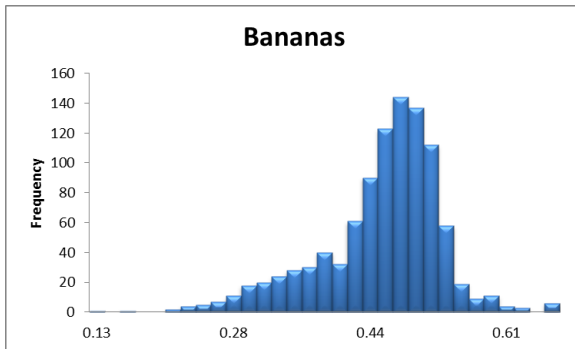
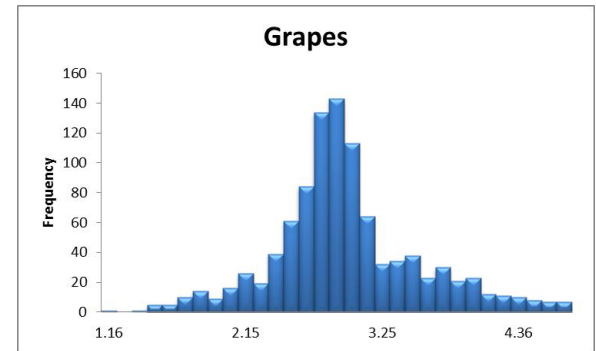
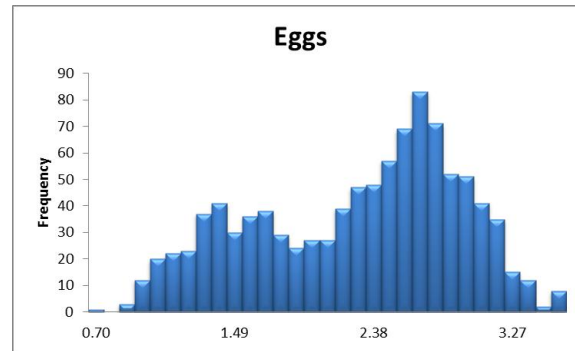
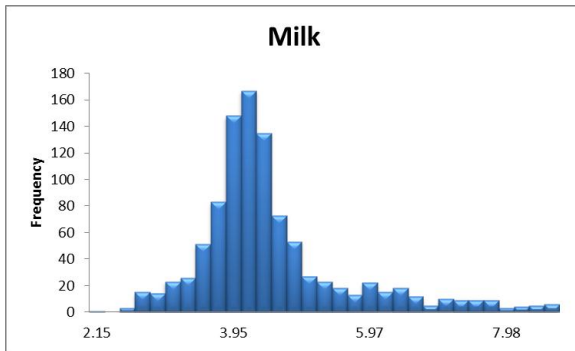
Each trial, randomly pick source, then randomly draw from said source.

## Results: Weighted Sampling Method









Each trial, pick source using categorical distribution (with probabilities of Empirical=.45, SME=.45, Analogy=.10), then randomly draw from said source.

## Results: Supplement



Used point estimate of analogy as mean for distributions from the empirical data; next, randomly pick distribution (analogy & empirical combo or SME), then randomly sample from said distribution.

# Actuals

Product Description	Delete ?	Item Price	Qty	Item Total
<b>Bakery</b>				
 Wonder Bread 20.00 oz	Classic <input type="checkbox"/>	\$2.99	1	\$2.99
<b>Beverages</b>				
 Harris Teeter Whole Milk 128.00 fl oz	1 Gallon <input type="checkbox"/>	\$2.99	1	\$2.99
<b>Breakfast</b>				
 Cheerios Cereal - Toasted Whole Grain Oat 8.90 oz	<input type="checkbox"/>	\$3.45	1	\$3.45
<b>Dairy</b>				
 Harris Teeter Eggs 1.00 doz	Grade A Large <input type="checkbox"/>	\$1.79	1	\$1.79
<b>Produce</b>				
 Fresh Bananas 8.00 oz	Please order the actual number of individual bananas you want. <input type="checkbox"/>	\$0.35	2	\$0.69
 Fresh White/Green Grapes 1.00 lb	<input type="checkbox"/>	\$3.49	1	\$3.49
SubTotal:				\$15.40



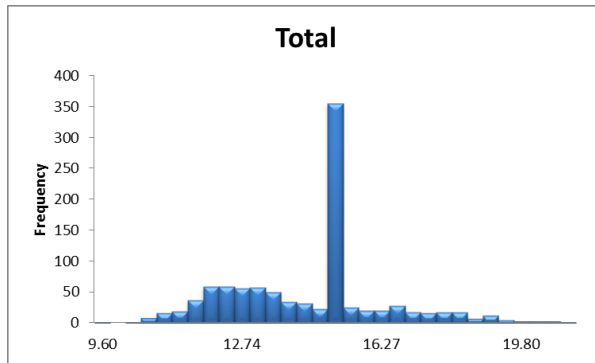
Fresh Bananas \$0.69 Price/lb \$0.35 Avg. Price/Each  
 Please order the actual number of individual bananas you want.  
 8.00 oz (avg.)

[Add to Cart](#) [Save to List](#)

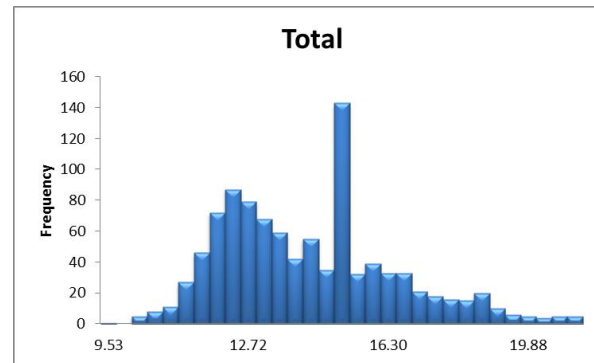


## Results: Totals

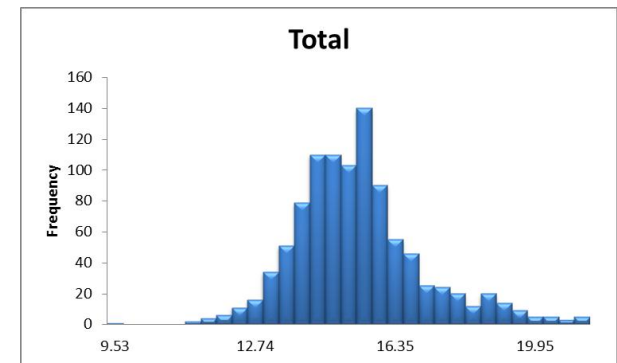
▶ Actual Total: \$15.40



Sampling



Weighted  
Sampling



Supplement

## References

- ▶ Coleman, R.L., Braxton, P.J., Druker, E.R. (2010). *The Correct Use of Subject Matter Experts in Cost Risk Analysis*, Naval Postgraduate School Acquisition Research Symposium, Monterey, CA.
- ▶ Krepp, Tim (2010). Grocery Wars – Harris Teeter vs. Safeway. *The Hill is home*. Retrieved March 2012, from <http://www.thehillishome.com/2010/03/harris-teeter-vs-safeway/>
- ▶ Madison, Josh (2007). M&M's Color Distribution Analysis. *JoshMadison*. Retrieved March 2012, from <http://joshmadison.com/2007/12/02/mms-color-distribution-analysis/>
- ▶ Market Basket: Finding cheap groceries in Tampa Bay (2012). Retrieved March 2012, from <http://www2.tbo.com/static/news-special-reports-data-bay/tbo-special-reports-database-holiday-food-prices/>
- ▶ U.S. Air Force (2007). Air Force Cost Risk and Uncertainty Analysis Handbook. Hanscom AFB, MA.

## Special thanks too...

▶ Dick Coleman for his time and suggestions

▶ My SMEs:

Rachel DiVincenzo

Barbara Jordan

Angie Brown

Lynn Laniewski

Robert Vallin

Alison Maliszewski

Julie Sutton

Amanda Croll

Cheri Hudgeons

Marina

Dombrovskaya

Darrin Speegle

Tony DiVincenzo

Cathy Tull

Deborah Wells

Dana Prewitt

and many more...

▶ Ray Britt for putting me on the project that helped me come across this problem