A Canadian F-35A Joint Strike Fighter Cost Estimation Model

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Abstract

The United States Joint Strike Fighter Program Office (JPO) has routinely provided the Canadian Department of National Defence (DND) with the latest projection of the unit recurring flyaway (URF) cost of the F-35A Joint Strike Fighter (JSF) that Canada should expect to pay. Until recently, DND lacked the ability to generate an independent estimate or perform sensitivity analysis.

Defence Research & Development Canada Centre for Operational Research & Analysis developed an F-35A URF cost estimation model. The cost estimation methodology is a *quantity effect* model employed by the RAND Corporation that combines cost improvement and production rate effects. The model is used to:

- reverse engineer JPO cost projections to determine best-fit learning and production rate slopes; and,
- provide a secondary, independent URF cost projection of future F-35A production lots based on the latest (June 2011) estimate-at-completion (EAC) actual cost data (for partially-completed F-35A low-rate-initial-production (LRIP) lots).

The study provided the Canadian Project Management Office Next Generation Fighter Capability the means to further scrutinize JPO cost estimates. The model facilitates sensitivity analysis such as determining the fiscal impact on Canada should international partners cancel or downsize their F-35 orders. Furthermore, the estimated risk and uncertainty in the prediction is provided to facilitate the selection of an appropriate level of confidence (in the prediction) for contingency planning.

*Data requests are subject to approval of the Canadian Department of National Defence and the U.S. Joint Strike Fighter Program Office.

1 INTRODUCTION

1.1 Background

The F-35 Joint Strike Fighter (JSF) is a single-engine, stealthy (radar-evading), supersonic multi-role fighter. It is manufactured in three versions: a conventional-takeoff-and-landing (CTOL) variant (F-35A) as seen in Figure 1, an aircraft-carrier version (F-35C), and a short-takeoff/vertical landing (STOVL) version (F-35B). Over three thousand F-35 JSFs are currently planned to be built for the United States (U.S.) and international partners including Canada. Pratt & Whitney is manufacturing the F135 propulsion systems and the Lockheed Martin Corporation is manufacturing the air vehicle and is responsible for final assembly (air vehicle and engine) of each F-35. In 2001, the F-35 program started its 10-year System Development and Demonstration (SDD) phase [21] building 22 test aircraft. Low-rate initial production[1] (LRIP) started in 2007 and by the end of 2011 about thirty new aircraft are scheduled to have rolled off the production line.

Figure 1: Lockheed Martin F-35A CTOL (photo credit: Lockheed Martin Corporation).

Canada has announced that it intends to purchase 65 F-35A CTOL jets for delivery between 2016 and 2022. In order to secure production, Canada will likely have to commit to procurement as early as 2012. To date, Canada has relied on the U.S. JSF Program Office (JPO) for projected costs. As a signatory to the JSF Production, Sustainment and Follow-On Development Memorandum of Understanding [12], signed in 2006, the JPO has routinely provided Canada with the latest projection of the unit recurring flyaway (URF) cost of the F-35A Joint Strike Fighter [5] (the price that Canada should expect to pay). Canadian project office representatives go over cost estimate details specific to Canada with the JPO at least once annually. Figure 2 depicts the URF costs and production rates of the F-35A CTOL projected by the JPO (based on June 2011 cost estimates and production profile plan provided by the JPO to the Canadian project office). The URF cost of an aircraft is the cost of purchasing the aircraft (including related management, hardware, airframe, vehicle systems, mission systems, propulsion, and engineering change order costs) as it rolls out of the production line (spare parts, support equipment, etc. is not included). The currency and year(s) of the cost amounts are masked to protect the sensitivity of the data.

In March 2011 DND's F-35A cost estimate came under public scrutiny as a result of a Parliamentary Budget Officer report [18]. As part of a response, DND presented the JPO projected cost figures and pro-

[1]Low-rate initial production is used in U.S. military procurement programs to indicate the phase of initial, small-quantity production of a weapon system.

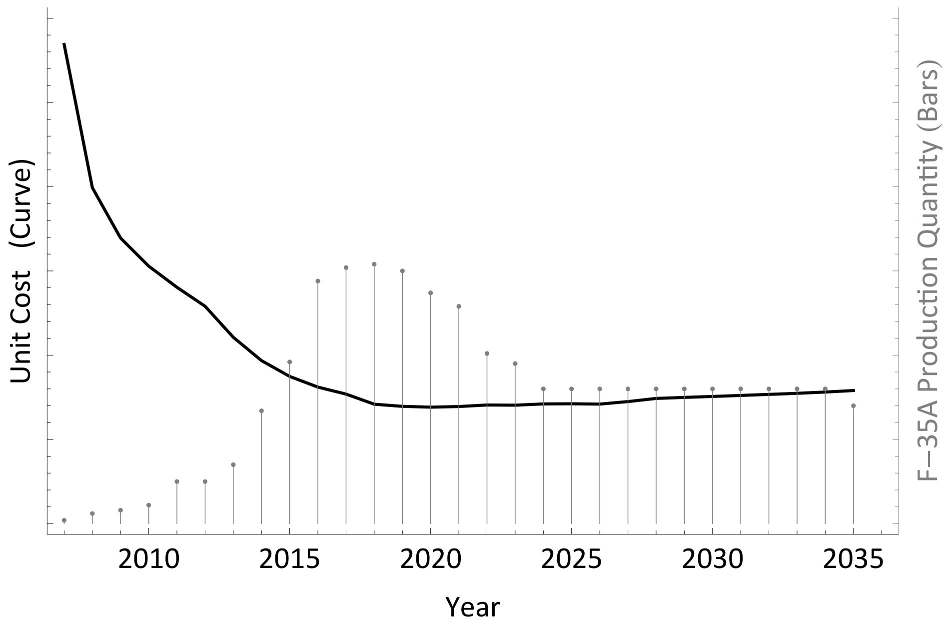

Figure 2: Yearly unit recurring flyaway costs and production rates of F-35A CTOL projected by the JPO (as of June 2011).

duction plans. DND argued that although current URF costs are high, well over 100M USD per aircraft, as the production line becomes more efficient and production capacity increases, the URF costs will go down. Furthermore, DND asserted that:

> *"Canada's average URF for 65 CTOL aircraft acquired between 2016 and 2022 is at the least expensive time of production. Canada's URF [cost] estimate is due to the delivery times that will be at around the peak of production efficiency."* [5]

Affordability has been heralded as the cornerstone of the F-35 program. The claim is that it would be achieved through a very high level of common parts and systems across the three versions of the aircraft. Streamlined assembly methods were to cut production time significantly [21]. The theory is that a large quantity ordered over time would lead to accumulated experience in producing the same system year after year, reducing the unit cost. This is referred to as the cost improvement effect. A second factor, termed the production rate effect results from changing the quantity of aircraft produced in a given year (or time period), with high production rates likely reducing the unit cost through greater operating efficiency and the spreading of fixed costs over more units.

In May 2011 Defence Research & Development Canada offered to provide the Canadian Project Management Office (PMO) Next Generation Fighter Capability (NGFC) a model to independently estimate the average unit recurring flyaway (URF) cost that Canada will likely pay for their F-35A CTOL aircraft. The cost estimation methodology used herein is a *quantity effect* model employed by the RAND Corporation that combines cost improvement and production rate effects [3, 23]. The model is used to:

- reverse engineer JPO cost projections to determine best-fit learning and production rate slopes; and,
- provide a secondary, independent URF cost projection of future F-35A production lots based on the latest (June 2011) estimate-at-completion (EAC) actual cost data (for partially-completed F-35A low-rate-initial-production (LRIP) lots).

The EAC cost data for completed LRIP lots represent the actual production costs incurred for each lot. EAC costs are not settlement costs—all cost overruns are included.

1.2 Objective

The goal of the study was to provide the Canadian DND with an F-35A CTOL URF cost estimation model that could be used to generate secondary, independent estimates, reverse engineer JPO cost projections, and perform sensitivity analysis. The study provides the Canadian F-35A project office the means to further scrutinize JPO cost projections and defend Canadian DND's estimates. For rigour and defensibility the methodologies and data were made explicit. The models presented herein facilitate sensitivity analysis such as determining the fiscal impact on Canada should international partners cancel or downsize their F-35 orders. Furthermore, risk and uncertainty analysis of the prediction is provided to facilitate the selection of an appropriate level of confidence (in the baseline estimate) for contingency planning.

1.3 Scope

The intent of the study was to provide Canadian DND decision makers with an ability to scrutinize JPO cost projections in an independent manner. The JPO employs several cost analysts dedicated to providing detailed, 'bottom-up', F-35 cost estimates. Furthermore the JPO is privy to confidential Lockheed Martin Corporation and Pratt & Whitney data inaccessible to the author. As a result, simplifying assumptions had to be made:

- Only URF F-35A CTOL costs are considered and predicted. These costs do not include potential modifications that might be incorporated in a 'partially-common' subset of the aircraft, such as a drag chute.
- Only F-35A CTOL data (production numbers and costs) as of June 2011 were considered. The JPO production profile indicates that 475 F-35C carrier variants and 405 F-35B STOVL aircraft will be produced. These two variants represent nearly 30% of the total projected F-35 production. While it is claimed that all variants share a high degree of commonality, the author did not attempt to model this. Including F-35B and F-35C quantities may have some favourable effect on lowering the URF costs of F-35A aircraft, however it can also be argued that potential production line changes to switch between variants can have a detrimental effect.[2]
- Due to 100% propulsion system commonality with the F-35A, F-35C production numbers were considered when projecting the costs of the engine.
- A major assumption of the quantity effects model used to generate a baseline cost estimate is that accumulated experience in producing the **same system** year after year can reduce the unit cost. Planned or unanticipated design or engineering changes can negate some of the achieved learning. Ideally the system's development and testing phase should be complete. This is clearly not the case as evident by a July 2011 F-35 Program Executive Officer statement on the cost growth of the first three years of production: "concurrent testing and production will continue to generate modification changes until testing completes" [6]. The JPO confirmed that any development and testing costs incurred are not included in EAC costs, however any retrofit costs (e.g., component remove and replace costs) are included.

[2]Results generated prior to publication of this report show that including the quantities of all three variants increases cost projections for the engine.

- Another major assumption of the quantity effects model is that trends in costs experienced during LRIP lots 1-3 will continue through future lots. This assumes the continuation (extrapolation) of the typical cost improvement (learning) curve observed for the first three lots.

- Due to the limited data set and to ensure validity of the mathematical models, in some cases it was necessary to rely on historical U.S. military aircraft production and learning rate percentages.

- An in-depth analysis and understanding of the JSF program's risks and uncertainties are outside the scope of the research presented in this paper. While there is no true substitute for this analysis, as a proxy, a top-down risk/uncertainty model developed by the U.S. Naval Center for Cost Analysis (NCCA) was applied. NCCA analyzed the cost outcomes of one hundred U.S. Department of Defense major acquisition programs in the past thirty years and subsequently proposed their top-down risk/uncertainty model.

- All cost figures presented herein are masked to protect the sensitivity of the original data. The cost axis of all graphs presented is also hidden. Results are presented as a percent or percent change.

1.4 Outline

In Section 2 the EAC cost data for completed or partially-completed F-35A LRIP lots is presented and validated. Current JPO production profile plans and cost projections are also presented. Section 3 exposes the quantity effects model suitable for projecting F-35A costs (Annex A details the mathematical theory of learning curves leading to the development of the quantity effects model). In Section 4 the model is then applied to the F-35A data and planned production profile to yield an independent baseline estimate of Canada's average F-35A URF cost. The baseline estimate is then adjusted using risk and uncertainty analysis in Section 5. The paper concludes in Section 6 by highlighting results.

2 DATA

2.1 Reported Cost Data

As of June 2011, the Lockheed Martin Corporation has completed, partially completed, or has started four LRIP lots. The JPO provided the Canadian PMO NGFC with completion rates and estimated-at-completion (EAC) average URF costs for lots 1-3 broken down into the air vehicle (includes airframe, mission systems, and vehicle systems) and engine components. The EAC cost for LRIP lot 4 was not available. EAC costs are developed by the JPO and are based on actual production costs collected to date. EAC costs most closely approximate the actual production costs incurred for each lot. They are not settlement costs; all cost overruns are included. All EAC costs presented also include any contractor incentive fees.

Table 1 lists the planned production quantities, completion rates (as of June 2011), and the average URF EAC costs reported by the JPO for LRIP 1-3 for both the F-35A air vehicle and F-135 engine. The completion rates and EAC cost data are presented separately for the air vehicle and engine. The last column presents the combined URF (air vehicle and engine) EAC cost. The costs are blacked-out to protect the sensitivity of the data.

2.2 JPO Plans and Projections

Table 2 presents the JPO production planning profile as of June 2011. The table list the number of F-35A units that international participants plan to buy in each year from 2007 to 2035 (the country names are replaced by "Partner #" or "Foreign Military Sale (FMS) #"). The year of delivery is assumed to be two years after the buy year indicated. For each buy year, the table indicates the total number of units to be

Table 1: LRIP 1-3 completion rates and estimate-at-completion costs.

			F-35A Air Vehicle		F135 Engine		
Lot	Buy Year	Planned Production	% Completion	EAC Cost	% Completion	EAC Cost	Total EAC Cost
LRIP 1	2007	2	88%	[redacted]	99%	[redacted]	[redacted]
LRIP 2	2008	6	82%	[redacted]	88%	[redacted]	[redacted]
LRIP 3	2009	8	56%	[redacted]	45%	[redacted]	[redacted]

purchased (lot size) and the JPO's predicted average (avg) unit cost (blacked out). Although much of the table is masked, it provides the reader with an appreciation of the pattern and overlap of international orders. Each cost amount is an incremental average lot cost—the average cost for a particular lot, not the average cumulative cost of the entire program. The incremental lot cost is defined as the total cost of a lot divided by the number of units produced in the lot. The last column of the table lists the projected number of F-35C to be produced per year (used later for propulsion system cost projection). Taking a product-sum of the quantities and JPO's predicted costs, one can compute the average URF (per aircraft engine included) by dividing by the total quantity to be procured.

Table 2: JPO F-35A CTOL production planning profile as of June 2011.

Buy Year (TY)	Partner 1	Partner 2	Partner 3	Canada	Partner 5	Partner 6	Partner 7	Partner 8	FMS 1	Lot Size	Cumulative	Avg Cost Per Unit	F-35C
2007	X									2	2	[redacted]	
2008	X									6	8		
2009	X					X				8	16		
2010	XX					X				11	27		4
2011	XX									25	52		7
2012	XX	X	X							25	77		7
2013	XX	X						X		35	112		19
2014	XX	X	X			X	X	X	X	67	179		14
2015	XX	X	X	X		XX		XX	X	96	275		28
2016	XX	X	XX	X	X	XX	XX	XX	X	144	419		31
2017	XX	X	XX	X	XX	XX	XX	XX		152	571		36
2018	XX	X	XX	XX	XX	XX	XX	XX		154	725		36
2019	XX	X	XX	XX	X	XX	XX	XX		150	875		44
2020	XX	X	XX	XX		XX		XX		137	1012		40
2021	XX	X	XX	X		XX		XX		129	1141		27
2022	XX	X				X		X		101	1242		24
2023	XX	X						X		95	1337		34
2024	XX									80	1417		34
2025	XX									80	1497		34
2026	XX									80	1577		34
2027	XX									80	1657		22
2028	XX									80	1737		
2029	XX									80	1817		
2030	XX									80	1897		
2031	XX									80	1977		
2032	XX									80	2057		
2033	XX									80	2137		
2034	XX									80	2217		
2035	XX									70	2287		
TOTALS:	1763	69	100	65	30	85	56	100	19	2287	2287		475

3 METHODOLOGY

A pillar of the JSF project is affordability, intended to be partly achieved through the large quantity of aircraft produced. The theory is that a large quantity ordered over time will lead to accumulated experience in producing the same system year after year, reducing the unit cost. This is referred to as the *cost improvement effect*.[3] A second factor, termed the *production rate effect* results from changing the quantity of aircraft produced in a given year (or time period), with high production rates likely reducing the unit cost through greater operating efficiency and the spreading of fixed costs over more units.

Annex A builds up the theory of cost improvement and production rate effect models. Without losing continuity, the reader can continue to read on (to Section 3.1) to proceed to the final mathematical model used for cost estimation.

3.1 The Quantity Effects Model

The *quantity effects model*, as termed by the RAND Corporation [3, 23], is a combination of cost improvement and production rate curves:

$$LAC_i = T_1 \times [\bar{Q}_i(b)]^b \times r_i^c. \tag{1}$$

where

- LAC_i is the average incremental cost of the aircraft, or equivalently the cost of the aircraft at the midpoint, of lot i,
- T_1 is often associated with the cost of the first aircraft produced. However, this parameter is typically estimated, not inputted, and represents the cost intercept (at zero production).
- $\bar{Q}_i(b)$ is defined as the midpoint of lot i, but it is not simply the middle point of the number of aircraft produced, but rather the likely fractional point where half of the lot's total cost is expended,
- b is the cost improvement slope, and 2^b is the cost improvement slope percentage,
- r_i is the production rate, proxied by the number of aircraft produced in lot i, and
- c is the production rate slope, and 2^c is the production rate percentage.

The quantity effects model was used by RAND in 2008 for a study for the U.S. Air Force and the U.S. Navy on the topic of explaining the rising cost of fixed-wing aircraft [3]. RAND examined the cost improvement (CI) effect and production rate (PR) effect for 24 military aircraft programs for which there were at least five annual buys and for which the midpoint and lot size correlations were less than the absolute value of 0.6 (avoiding systems with higher correlation potentially leading to statistically misleading results). The restricted data set is claimed to include a "diverse array of Air Force and Navy programs over the past 50 years, including attack, cargo, electronic, patrol, and training aircraft". Table 3 summarizes RAND's findings (in some table entries the statistic was not available (n/a)). The observed learning slope is rather conservative in comparison to National Aeronautics and Space Administration (NASA) guidelines[4] [17], however it should be noted that NASA states that their guidelines are to be used only when a statistical analysis of actual cost data for similar products is unavailable. Furthermore, NASA's guidelines do not consider production rate slopes.

[3] The terms cost improvement and learning are used interchangeably.
[4] NASA suggests using a cost improvement slope percentage of 85%.

Table 3: Historical cost improvement and production rate slope percentages.

	Military Aircraft	Fighter Jets
Mean CI slope %	0.97	0.93
Standard deviation of CI slope %	0.13	n/a
Mean PR slope %	0.89	0.78
Standard deviation PR slope %	0.23	n/a
Number of aircraft	24	6

In addition, RAND also analyzed the historical production rate effect for the F100 engine variants (-100/-200 and -229) of F-15 and F-16 fighter jets and reported an average PR slope of 97% [23, p.25].

The RAND Corporation also used the same quantity effects model in 2007 for a study for the U.S. Office of the Secretary of Defense assessing the cost savings of proceeding with multi-year contracts for the F-22A Raptor [23]. A goal of their research was to estimate the cost improvement and production rate slopes for the F-22A using Equation (1). However the actual F-22A production history showed that the production rate and unit midpoint values were so highly correlated that determining the production rate and cost improvement slopes simultaneously through multivariate analysis was deemed invalid. To overcome this problem RAND used the average historical production rate slope percentages of 89% and 97%, respectively, for the air vehicle and propulsion system [3].

3.2 Nonlinear Multivariate Regression Model

To determine the quantity effects of the F-35A production, a nonlinear multivariate regression model based on Equation (1) is applied. The regression model has the following form:

$$\text{Ln}(LAC_i) = \text{Ln}(T_1) + b \times \text{Ln}(\bar{Q}_i(b)) + c \times \text{Ln}(r_i) + \varepsilon_i \tag{2}$$

where

$$\bar{Q}_i(b) = \left(\frac{\left[(Q_i + 0.5)^{1+b} - (Q_{i-1} + 0.5)^{1+b} \right]^{\frac{1}{b}}}{(1+b) \times (Q_i - Q_{i-1})} \right) \tag{3}$$

and T_1, b, and c are parameters to be estimated and ε_i is the error term. The derivation of the lot midpoint, $\bar{Q}_i(b)$, is provided in Annex A. Combining Equations (2) and (3) reduces the regression to:

$$\begin{aligned} \text{Ln}(LAC_i) = {} & \text{Ln}(T_1) - \text{Ln}(1+b) - \text{Ln}(Q_i - Q_{i-1}) \\ & + \text{Ln}\left((Q_i + 0.5)^{1+b} - (Q_{i-1} + 0.5)^{1+b} \right) + c \times \text{Ln}(r_i) + \varepsilon_i. \end{aligned} \tag{4}$$

Equation (2) results from taking the natural logarithm of Equation (1) and adding an error term. As a result of the log-transformation of the lot average cost model, the uncertainty in the prediction outputted by the model can be presented by a log-normal distribution. The log-normal distribution is a probability distribution of a random variable whose logarithm is normally distributed. The logarithmic transformation is commonly used for positive data; the log-normal distribution domain of zero to infinity is more suitable for modelling costs than a normal distribution which includes the negative domain. The majority of total cost estimates for weapon-system acquisition programs modelled by the U.S. Deputy Assistant Secretary of the Navy (Cost & Economics) are log-normally distributed and often skewed right.

Wolfram *Mathematica*'s nonlinear least-squares model solver was used to solve instances of Equation (4).

4 MODEL APPLICATION AND COST ESTIMATE

Using the quantity effects model developed in Section 3, regression analysis was performed to obtain the following results:

- In Section 4.1 the quantity effects model is fitted to the entire June 2011 JPO planning profile and cost projections to determine the (statistically) most likely cost improvement and production rate effects as per the JPO predictions.
- In Section 4.2 the quantity effects models are fitted to the estimated-at-completion costs for completed or partially completed LRIP lots 1-3. The air vehicle and engine costs are treated separately. The fitted models are then used to project future F-35A lot costs, and in particular to estimate the average URF price Canada can anticipate to pay.

4.1 Quantity Effects of JPO F-35A Cost Projections

To determine the quantity effects of the F-35A production, the nonlinear multivariate regression model of Equation (4) is applied to the complete set of JPO data. The quantity effects curve was fitted to the June 2011 JPO projected costs (Table 2) for all planned lots from 2007 to 2035. For each lot $i = 1 \ldots 29$, LAC_i was set to the JPO projected average aircraft cost, Q_i was set as the JPO projected aircraft production sequence number of the first aircraft of lot i, and r_i was set to the lot size.

Figure 3 graphs both the JPO projections (solid line) and the fitted quantity effects curve (dashed line). The correlation of lot midpoint and production rate is 0.31, less than the RAND limit of 0.6, indicating that fitting both the production rate and cost improvement slopes should yield statistically reliable results. The best fit parameters are $T_1 = 319.8$, cost improvement slope percentage of 94% and production rate slope percentage of 89%. The cost improvement slope percentage indicates that 6% savings/efficiencies are realized everytime the cumulative number of aircraft produced doubles. The producation rate slope percentage indicates that 11% savings/efficiencies are realized everytime the total aircraft produced in a lot doubles. Table 4 presents the parameter confidence intervals, standard errors, and statistical tests ($R^2 = 0.9999$). Applied to Canada's buying profile, the average URF cost predicted by the quantity effects model is within 1% of JPO's estimate.

Table 4: Parameter estimates of the quantity effects curve fitted to the F-35A CTOL costs projected by JPO.

Parameter	Estimate	Standard Error	95% Confidence Interval	t-Statistic	P-Value
T_1	319.8	10.7	(297.8, 341.8)	29.8	≈ 0
CI Slope %	0.94	0.01	(0.93, 0.95)	162.9	≈ 0
PR Slope %	0.89	0.01	(0.86, 0.91)	85.1	≈ 0

Statistically, the best fit CI and PR slope percentages over the 29 years of production indicate that the URF costs (predicted by the JPO) are driven by both the production rate and cost improvement due to learning.[5] It is interesting to note that the best-fit production rate slope coincides with the historical production rate slope observed by RAND, while the best-fit learning slope is slightly more favourable than what was historically observed (see Table 3).

[5] The quantity effects model including both cost improvement and production rate parameters was found to better fit the data than a learning curve model (without production rate extension).

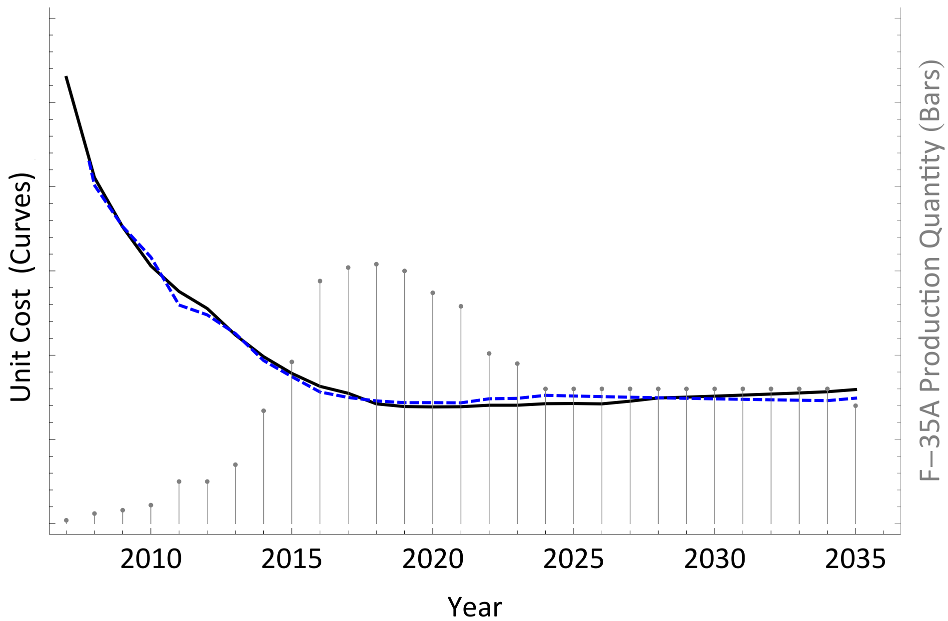

Figure 3: Quantity effects curve (dashed line) fitted to the F-35A CTOL costs projected by JPO (solid line).

While the JPO has documented observed learning rates for initial production lots that support the 94% learning slope, the argument can be made that the best-fit learning slope may be too optimistic to sustain for all 29 years of production. Hartley [10] claims that gradually ‘economies of learning’ are reaped at a decreasing rate, to such an extent that it is conceivable that at some point learning will cease, and average direct labour input per aircraft will tend to become constant. Reaching a learning saturation point is plausible since the JSF production run is 29 years.[6]

To provide further insight into the JPO’s anticipated learning and production rate slopes, the quantity effects regression models were fitted to subsets of the JPO planning profile and cost projections: the first 10 lots, and the first 19 lots. The goal was to reverse-engineer the anticipated learning/production rate slopes for early years (rather than for the entire 29 year production plan). These results are synthesized in Table 5 and indicate that the JPO projects that learning will be more of a factor in early production lots, the best fit learning slope percentages are 90% and 91% respectively. It should be noted that there is high correlation (exceeding RAND’s limit of 0.6) between lot midpoints and production rates when considering only the first 18 lots or less. This indicates that there is possibility that the regression results when considering only the first 10 lots are not statistically reliable (the results generated for 19 and 29 lots are indeed reliable, however).

[6]Annex E, “Follow-on Development Process”, of the JSF Production, Sustainment and Follow-On Development Memorandum of Understanding [12] indicates that the JSF production will follow an evolutionary acquisition approach that is designed to deliver new capabilities (software and mission systems) in a time-phased implementation (approximately every two years). Literature (see Stocker [20] and references therein) shows that evolutionary acquisition tends to affect the life-cycle cost of a platform, but not so much the acquisition cost.

Table 5: Parameter estimates of the quantity effects curve fitted to subsets of the F-35A CTOL costs projected by JPO.

	Lots 1-10	Lots 1-19
CI Slope %	0.90	0.91
PR Slope %	0.93	0.92
Correlation	0.99	0.57

4.2 An Independent Estimate of Canada's Average URF Cost

The production profile and EAC costs (normalized to a common base year) for the first three lots represent the data for which the quantity effects regression model can be applied to project cost estimates for future production lots, including the lots from which Canada is scheduled to procure its F-35A jets. Ideally the quantity effects regression model would be used to estimate the production rate and cost improvement slopes simultaneously. However, for the limited actual F-35A production history (LRIPs 1-3), the production rate and lot midpoint values are highly correlated, with a correlation of 0.94, suggesting that such an approach is invalid as it can lead to statistically misleading results. Even if the correlation was acceptable, Equation (4) cannot be used for systems with fewer than four data points as three parameters need to be estimated.

The RAND Corporation encountered the same hurdle when applying the quantity effects model to initial F-22A production data [23]. To increase the model's degrees of freedom, RAND fixed the production rate parameter, c, to reflect the mean production rate slope percentage of 89% observed by RAND on historical U.S. military aircraft (see Table 3). Similarly, a production rate slope percentage of 97% was fixed for propulsion system cost estimation. Quantity effects models were then used to fit the best cost improvement slopes.

Following the RAND approach, the quantity effects model is applied several times, each fixing one of the parameters:

- The propulsion system production rate slope percentage is fixed to reflect the mean historical production rate slope percentage of 97%.
- The air vehicle production rate slope percentage is fixed to 89% as observed by RAND on historical U.S. military aircraft programs.
- The air vehicle production rate slope percentage is fixed to 78% as observed by RAND on historical U.S. fighter jet programs.
- The air vehicle cost improvement slope percentage is fixed to 97% as observed by RAND on historical U.S. military aircraft programs.
- The air vehicle cost improvement slope percentage is fixed to 93% as observed by RAND on historical U.S. fighter jet programs.

The LRIP completion rates presented in Table 1 were used as data weights to indicate the relative amount of influence over the parameter estimates in the quantity effects regression model.

4.2.1 F135 Engine Cost Estimation

Figure 4 graphs the propulsion system quantity effects curve derived from the weighted regression models fixing the production rate slope at 97%. Table 6 presents the parameter estimates, listing the best fit estimate, the standard error, the 95% confidence interval, and statistical tests. The results indicate a 93% learning slope. It is interesting to note that although the F135 is a derivative of the F119 engine (built for the F-22), the early LRIP data indicates a cost improvement effect. To project future F135 costs, the production quantities of both the F-35A and F-35C variants were considered since the propulsion system is the same.[7]

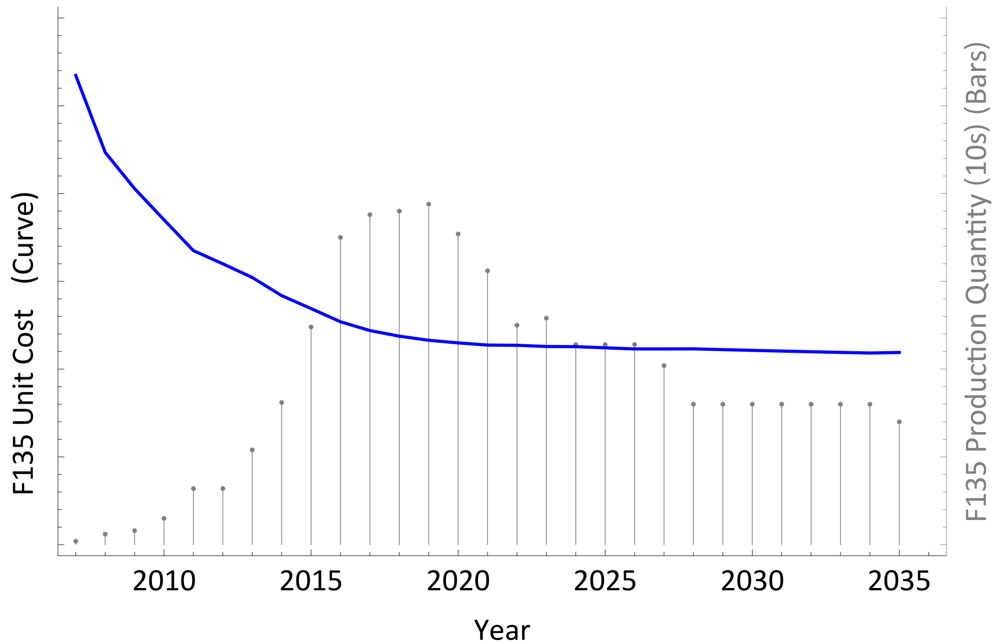

Figure 4: Quantity effects curve fitted to the F135 propulsion system EAC costs.

Table 6: Parameter estimates of the quantity effects curve fitted to the F135 engine EAC costs projected by JPO.

Parameter	Estimate	Standard Error	95% Confidence Interval	*t*-Statistic	*P*-Value
T_1	28.4	1.02	(15.5, 41.3)	27.9	0.023
CI Slope %	0.93	0.02	(0.74, 1.13)	60.8	0.010

[7]The JPO noted that the F-35A propulsion system is a subset of the F-35B propulsion system (i.e., 100% commonality from an F-35A perspective). Revised results (not presented here) were produced. Counter-intuitively, the projected engine costs increased as the additional F-35B aircraft produced in LRIP 1-3 have an averse impact on the quantity effects—additional quantities in later years are not enough to compensate.

4.2.2 F-35A Air Vehicle Cost Estimation

Figure 5 graphs the four air vehicle quantity effects curves derived from the weighted regression models fixing individual parameters. Table 7 presents the parameter estimates for the four quantity effects models employed. When applicable, each table entry lists the best fit estimate, the standard error, and the 95% confidence interval. The results indicate fitted learning slopes of 95% (PR set to 89%) and 103% (PR set to 78%), and production rate slopes of 86% (CI set to 97%) and 91% (CI set to 93%). Out of the four curves generated, the curve generated based on historical production rate slopes of U.S. fighter jets (78%) differs the most. Consistent with RAND's studies, the quantity effects model with an air vehicle production rate parameter fixed to reflect the mean historical production rate slope percentage of 89% was selected as the primary estimation model.[8]

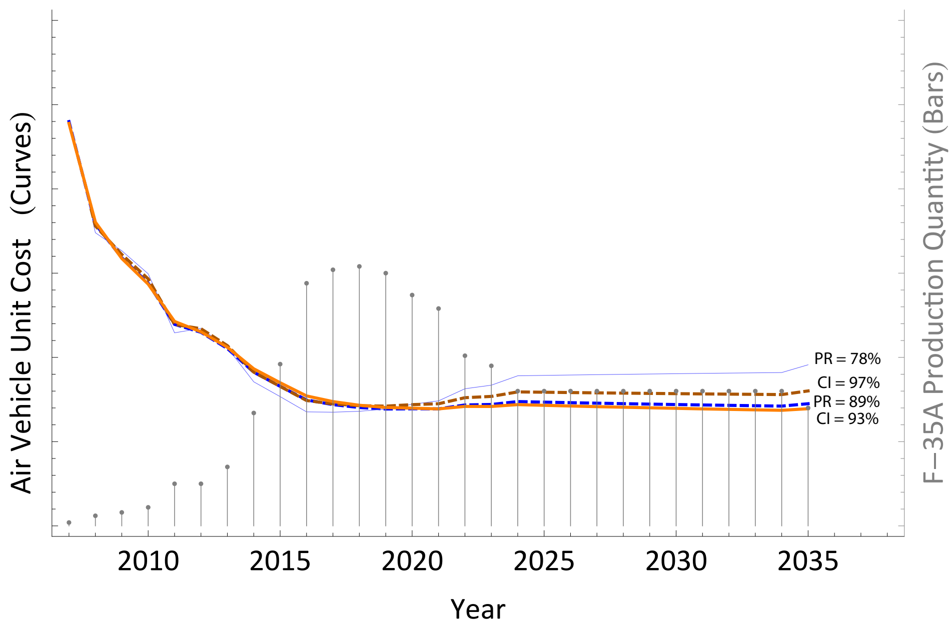

Figure 5: Quantity effects curves fitted to F-35A air vehicle EAC costs (LRIP 1-3).

Table 7: Parameter estimates (including standard error and 95% confidence intervals) of the quantity effects curves fitted to F-35A CTOL EAC costs (LRIP 1-3).

Parameter	PR = 89%	PR = 78%	CI = 97%	CI = 93%
T_1	276.5, 7.4, (182.8, 370.1)	306.3, 20.9, (40.0, 572.5)	282.1, 15.2, (88.3, 475.8)	269.8, 7.2, (178.7, 360.8)
PR slope %	0.89	0.78	0.86, 0.02, (0.60, 1.12)	0.91, 0.01, (0.78, 1.05)
CI Slope %	0.95, 0.01, (0.81, 1.08)	1.03, 0.03, (0.65, 1.42)	1.03	0.93

[8]The *t*-Statistic for T_1 and CI slope percentage is 37.5 and 86.7 respectively. The *p*-Value for T_1 and CI slope percentage is 0.017 and 0.007 respectively.

4.2.3 F-35A (Air Vehicle and Engine) Cost Estimation

Figure 6 graphs the four derived air vehicle quantity effects curves combined with the F135 engine quantity effects model. For comparison, the JPO cost projection curve is also graphed (gray dotted line).

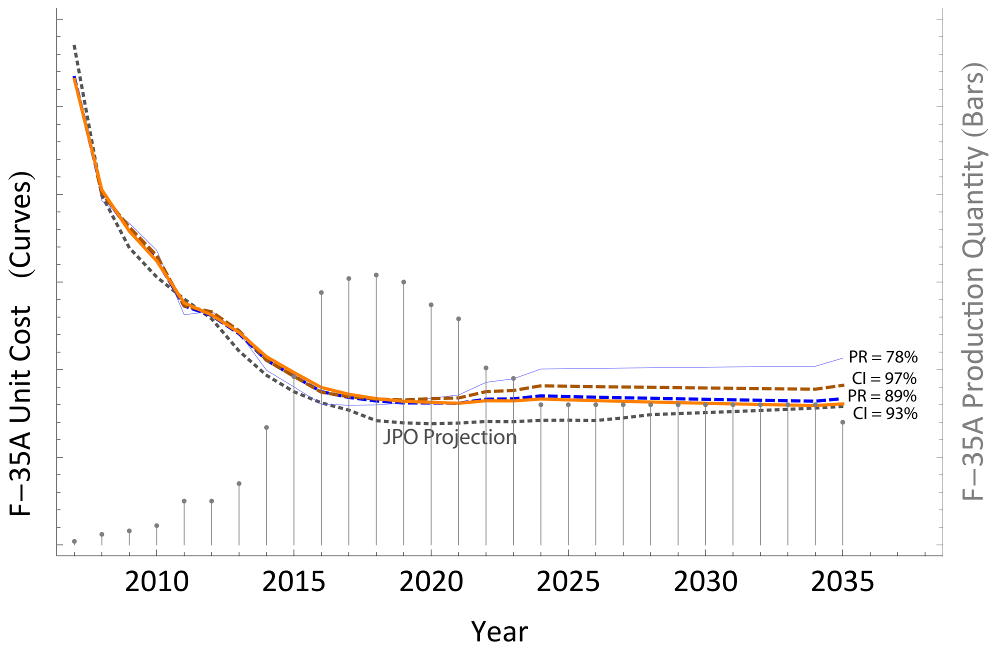

Figure 6: Quantity effects curves fitted to F-35A URF EAC costs (LRIP 1-3) compared to JPO projection (gray dotted line).

The model with an air vehicle production rate parameter fixed to reflect the mean historical production rate slope percentage of 89% is deemed to be the primary estimation model. The best-fit cost improvement (learning) slope is 93% for the engine and 95% for the air vehicle. For clarity, Figure 7 depicts the projected F-35A costs based on the best-fit quantity effects model (dashed line) in comparison to the JPO projections (dotted line). (Figure 7 is contained in Figure 6.)

Assuming the (June 2011) JPO production planning profile, presented in Table 2, Canada will procure aircraft between 2015 and 2021. Based on the F-35A cost estimates obtained by using best-fit quantity effects models, the expected average unit recurring flyaway (air vehicle and engine included) price for Canada is 13.5% higher than the JPO estimate. This is a secondary, independent estimate. This baseline estimate is void of any risk or uncertainty considerations and is valid if the program runs without any improvements or hitches, no withdrawals/downsizing/changes to orders, etc. Although the baseline estimate is higher than the JPO projection (computed using the June 2011 JPO cost estimates and production planning profile), it should be considered complementary given the cost estimate is top-down and lacks the more detailed granularity of JPO's cost estimate.

5 RISK AND UNCERTAINTY ANALYSIS

5.1 Model Uncertainty

There is statistical uncertainty in the parameter estimates (cost improvement and production rate slopes) of the quantity effects cost models for the air vehicle and engine. The statistical uncertainty is driven by the

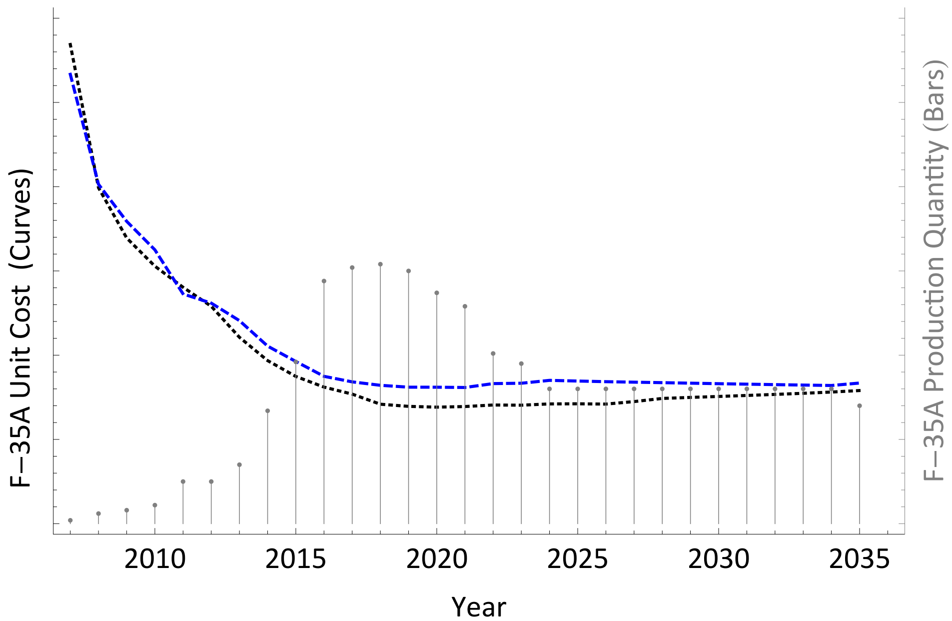

Figure 7: Quantity effects model (dashed curve) and JPO cost projections (dotted curve).

input data. The statistical prediction error of the cost estimates are represented by continuous probability distributions. For regression models, prediction bands provide a measure of confidence concerning where the true function lives. A higher level of confidence requires wider bands. Figure 8 illustrates the 50% and 80% single prediction bands of the F-35 air vehicle quantity effects regression model (single prediction bands incorporate both the variation in parameter estimates and the overall variation in response values). Figure 8a presents the bands between 2007 and 2011. It shows how the input data (EAC LRIP 1-3 costs) are fitted. Figure 8b presents the bands for the entire 29-year production plan. The prediction error increases the further the model extrapolates.

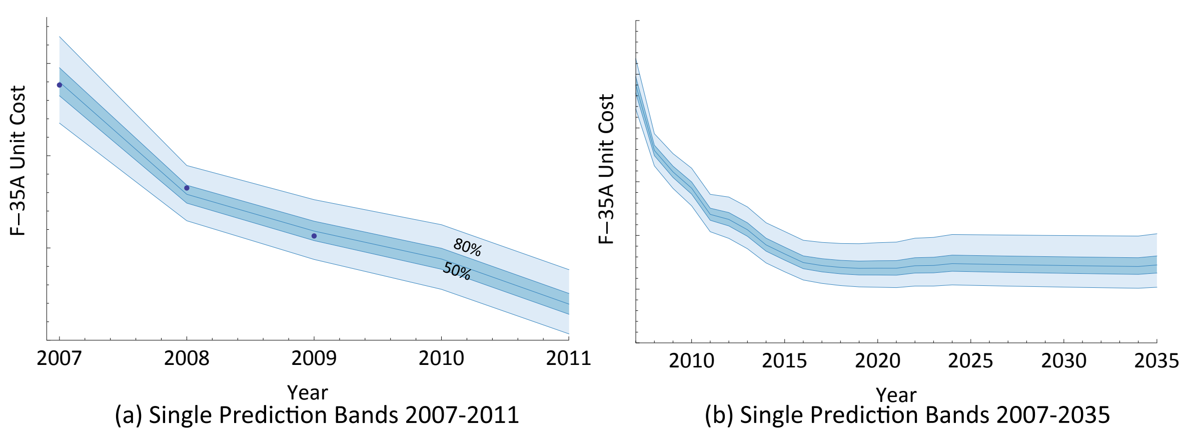

Figure 8: Single prediction bands of the F-35A air vehicle quantity effects regression model: (a) 2007 to 2011 (b) 2007-2035.

The prediction uncertainty associated with the best-fit quantity effects model provides marginal value when attempting to quantify the real risks and uncertainties of the JSF program. Cost improvement and production rate slope percentages are indeed uncertainties of the JSF program, and sensitivity analysis of these parameters (CI and PR) provides a means to observe changes in the prediction, i.e., quantity effects model outputs, when these parameters change. Table 8 lists Canada's projected average URF cost (percent change compared to a PR of 90% and a CI of 94%) as a function of various air vehicle cost improvement and production rate slope percentages. It is interesting to focus on the 92%-96% CI range and 88%-92% PR range. This range includes the best-fit quantity effects model CI and PR percentages as well as the reverse-engineered JPO CI and PR percentage projections. A very rough calculation shows that, in this range, a ±1% point deviation in the air vehicle PR slope percentage results in roughly ±5% change to the URF cost, a ±1% point deviation in the air vehicle CI slope percentage results in roughly ±7% change to the URF cost.

Table 8: Sensitivity of Canada's average URF cost subject to changes in air vehicle cost improvement and production rate slopes.

	Production Rate Slope											
CI Slope	80%	82%	84%	86%	88%	90%	92%	94%	96%	98%	100%	102%
80%	-71%	-69%	-67%	-65%	-62%	-60%	-56%	-53%	-49%	-45%	-41%	-36%
82%	-68%	-66%	-64%	-61%	-58%	-54%	-51%	-47%	-42%	-37%	-32%	-26%
84%	-65%	-62%	-59%	-56%	-52%	-48%	-44%	-39%	-34%	-28%	-22%	-15%
86%	-61%	-58%	-54%	-50%	-46%	-41%	-36%	-30%	-24%	-17%	-10%	-2%
88%	-57%	-53%	-49%	-44%	-39%	-33%	-27%	-20%	-12%	-4%	4%	14%
90%	-52%	-47%	-42%	-36%	-30%	-23%	-16%	-8%	1%	10%	21%	32%
92%	-45%	-40%	-34%	-28%	-20%	-13%	-4%	6%	16%	27%	39%	52%
94%	-39%	-32%	-25%	-18%	-9%	0%	10%	21%	33%	46%	61%	76%
96%	-31%	-23%	-15%	-6%	4%	14%	26%	39%	53%	68%	85%	103%
98%	-22%	-13%	-4%	7%	18%	31%	45%	59%	76%	94%	113%	134%
100%	-11%	-2%	9%	22%	35%	49%	65%	83%	102%	123%	145%	170%
102%	0%	12%	24%	38%	54%	71%	89%	109%	131%	155%	181%	210%

5.2 JSF Project Risk and Uncertainty

It is also of interest to capture the other JSF program risks and uncertainties, e.g., withdrawal of international participation, higher-than-expected inflation, increased overhead and labour rates, setbacks in software development, intellectual property issues, requirements above and beyond planned engineering changes, etc. Capturing and quantifying these risks and uncertainties is a daunting task (as of 26 July 2011 the JPO had yet to complete such analysis).

An in-depth analysis and understanding of the JSF program's risks and uncertainties is outside the scope of the research presented in this paper. While there is no true substitute for this analysis, as a proxy a top-down risk/uncertainty model developed by the U.S. Naval Center for Cost Analysis (NCCA) [7] was applied. NCCA analyzed Selected Acquisition Reports (SARs) on the cost outcomes of one hundred U.S. Department of Defense major acquisition programs. SARs are tied to program milestones (MS) and present total program acquisition costs. MS are points at which a recommendation is made and approval sought regarding starting or continuing an acquisition program, i.e., proceeding to the next phase. The U.S. Department of Defense established milestones are: MS A that approves entry into the Technology Development phase; MS B that approves entry into the Engineering and Manufacturing Development phase; and MS C that approves entry into the Production and Deployment phase. For a given program, a SAR provides two estimates of cost: a baseline estimate (BE) when a project nears a milestone, and a current estimate (CE) based on the latest information (the SAR CE of a completed acquisition program is regarded as the actual cost of the program). The ratio of a program's CE to BE (quantity and inflation corrected) is defined as its Cost Growth Factor (CGF). NCCA computed CGFs for one hundred completed programs (including aircraft, helicopters, missiles, ships, etc.) and estimated the mean and standard deviation of acquisition program cost outcomes. Table 9 lists the NCCA results for programs at milestones B and C.

Table 9: NCCA analysis of historical cost growth factors.

	Cost Growth Factor	
	MS B	MS C
Mean	1.36	1.10
Standard Deviation	0.69	0.28

NCCA uses the standard deviation of the historical CGFs as a proxy for the spread of a program's cost estimate. To enable this, NCCA computes the historical coefficients of variation (CV)—CV is a normalized measure of dispersion of a probability distribution, and it is defined as the ratio of the standard deviation to the mean:

$$cv = \frac{\sigma}{\mu}. \tag{5}$$

NCCA derived the historical CVs for MS B and C programs as 0.51 and 0.26 respectively. Using statistical tests, Flynn and Garvey [7] show strong support for platform homogeneity (CGFs, CVs are equivalent for aircraft, ships, and other platform types). NCCA's findings are consistent with a 2006 RAND study [2] which analyzed a sample of 68 completed programs and determined that the average total cost growth was 46 percent over the baseline estimate made at MS B and 16 percent over the baseline estimate made at MS C.

There are two ways to generate a probability distribution capturing program risks/uncertainties using NCCA's results. The first is to just apply a historical CV (either MS B or C) to determine the spread for the baseline point estimate. The second approach is to multiply the point estimate by NCCA's CGF (either MS B or C) and then use the corresponding historical CV to determine the spread of the distribution. The decision as to which approach to use depends on the confidence in the baseline point estimate: NCCA advises to use the first approach if "you believe your risk, but your uncertainty is off", and the second if "your risk and uncertainty are both off" [4]. As the baseline estimate is projected from early EAC costs, it is clear that neither risk nor uncertainty is considered. Applying the second approach (CGF and CV) is suitable.

Table 10 lists the results of the risk and uncertainty analysis of Canada's URF F-35A cost (quantity effects model prediction) based on NCCA's historical CGF and CV methodology. Historical CGFs and CVs of both MS B and C programs were considered. Although the JSF program passed MS B on October 24, 2001, on March 20, 2010, the U.S. Department of Defense formally announced that the JSF program had exceeded the cost increases limits specified in the Nunn-McCurdy cost containment law [19]. As a result, the U.S. Under Secretary of Defense for Acquisition, Technology and Logistics issued an Acquisition Decision Memorandum rescinding the MS B of the F-35 Program. The JSF program must now regain MS B approval, although it is already proceeding with the production phase and in reality is closer to MS C (approval for entry into the Production and Deployment phase). For this reason the risk and uncertainty analysis is presented considering both milestones. The table lists the mean, mode (most likely), median (50th percentile), 20th and 80th percentile cost estimates of the underlying probability distributions. Each of the entries are expressed as a percentage of the baseline estimate.

Table 10: Risk and uncertainty analysis of Canada's average URF F-35A cost (% of baseline).

	Based on Historical MS B	Based on Historical MS C
Mean	152%	114%
Mode	108%	103%
Median	136%	110%
20th percentile	91%	89%
80th percentile	203%	136%

The table entries were calculated as follows. The baseline estimate was multiplied by the CGF (1.36 for MS B, 1.1 for MS C) to yield the median value (or 50th percentile) of a log-normal probability distribution function.[9] This effect is explained by Goldberger [9]: when a power-function form is used for a cost estimating relationship, attention shifts from the mean to the median as a measure of central tendency; the cost estimating relationship yields an estimate of the median value rather than the mean. The CV (0.51 for MS B, 0.26 for MS C) is used to determine the spread of the log-normal distribution. The mean, mode, and other percentiles are then determined from the respective distributions.

Figure 9 depicts the probability density functions and cumulative distribution functions of the two probability distributions generated (MS B and MS C).

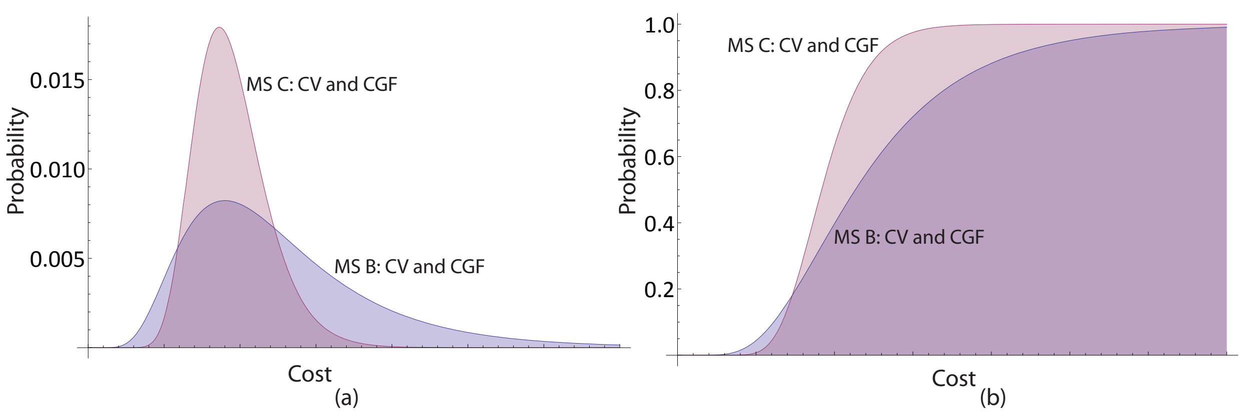

Figure 9: Estimate of Canada's average F-35A URF cost: (a) probability density functions and (b) cumulative distribution functions of risk/uncertainty probability distributions generated based on historical cost growth.

As the JSF program is more than three years into initial production, it can be argued that the mean cost estimate of the risk/uncertainty model, based on historical cost growth (incorporating CGF and CV) of U.S. Department of Defense major acquisition programs approaching milestone C, should be considered the primary risk-adjusted estimate. Applied to the secondary, independent estimate generated herein, the mean cost estimate—taking into account program risk and uncertainty—of a single Canadian F-35A is 13.5%

[9] As detailed in Section 3.2, the log-normal distribution is suitable for modelling positive costs and typically used for cost estimates of weapon systems.

more than the baseline estimate.[10] The U.S. Air Force Cost Analysis Agency also use the risk-adjusted mean as it brings the "natural benefit of a higher confidence level for riskier programs". Using the mean is also best practice as per the U.S. Department of the Navy Cost Estimating Standard. The mean represents the expected value accounting for risk and uncertainty.

The selected probability density function and cumulative probability distribution function are depicted in Figure 10. The vertical lines indicate the mean and 80th percentile estimates of the probability distributions. Table 11 presents the probability distribution percentiles (in increments of 5%) of the risk/uncertainty adjusted estimated average unit recurring flyaway cost of Canada's F-35 (as a percent increase/decrease from the baseline estimate).

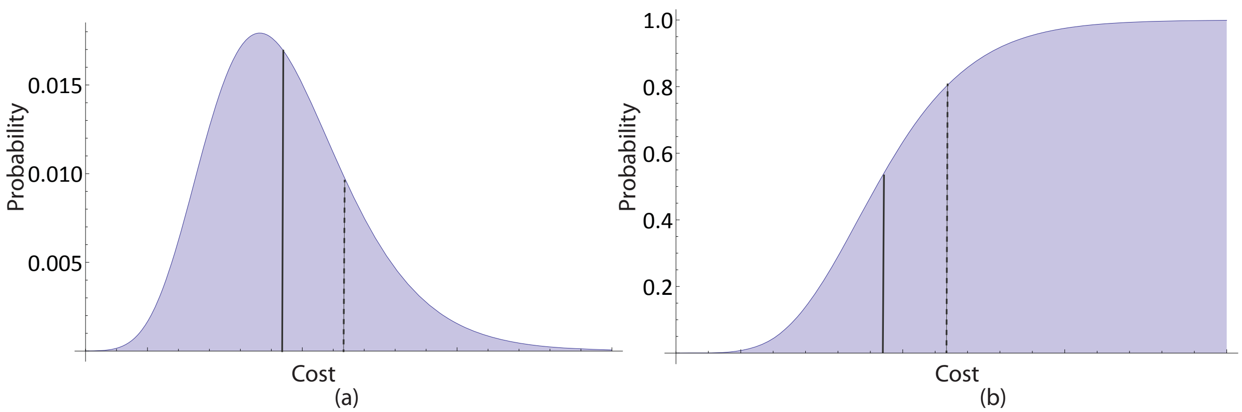

Figure 10: (a) Probability density function and (b) cumulative distribution function of the risk & uncertainty probability distributions generated based on historical cost growth of MS C programs.

Table 11: Probability distribution percentiles of the estimated average unit recurring flyaway cost of Canada's F-35 (compared to baseline).

Percentile	Change in Cost	Percentile	Change in Cost
5th	-27%	55th	14%
10th	-20%	60th	17%
15th	-15%	65th	21%
20th	-11%	70th	26%
25th	-7%	75th	30%
30th	-4%	80th	36%
35th	0%	85th	43%
40th	3%	90th	52%
45th	7%	95th	66%
50th	10%		

In the United States public law (the 2009 Weapon System Acquisition Reform Act [1]) stipulates that the confidence level for a cost estimate for a major defence acquisition program should be presented and justified

[10] Applying the method to JPO's baseline estimate also yields a risk-adjusted mean value that is 13.5% more.

if the confidence level selected is less than 80% (indicating the percent chance that the program's true cost will not exceed the estimate). Applying the NCCA risk/uncertainty model to the secondary, independent estimate generated herein, there is a 20% chance that the average F-35A will cost 36% more than the baseline. Using the quantity effects models, one generated example scenario where Canada's URF cost might exceed this amount is as follows: production rate efficiencies are not realized (a 5% increase in the air vehicle predicted production rate slope percentage) combined with the withdrawal of two major international partners.

As indicated, there is no true substitute for a proper risk and uncertainty analysis of the Joint Strike Fighter program. The NCCA CGFs and CVs provide a proxy enabling top-level analysis. It is recommended that PMO NGFC (in conjunction with the JPO) examines risk and uncertainty of specific program elements, particularly important cost drivers such as inflation, labour, equipment, and materiel.

5.2.1 Example: Risk of Withdrawal of International Participants

A pillar of the JSF program is affordability, largely driven by efficiencies resulting from high production rates. One of the highly publicized risks of the program is the potential withdrawal of international participants. In order to give Canadian decision makers an appreciation of this risk, the best-fit quantity effects models determined in Section 4.2 was applied on hypothetical production profiles with reduced F-35A CTOL orders. In particular, the following scenarios were considered (in order of increasing risk):

- Scenario A: A major European partner withdraws (reduction by ~100 aircraft);
- Scenario B: Two major partners withdraw (reduction by ~200 aircraft);
- Scenario C: All countries except for the U.S. and Canada withdraw (reduction by ~450 aircraft);
- Scenario D: The United States downsizes to 75% of its original order, Canada the only remaining international participant; and,
- Scenario E: The United States downsizes to 50% of its original order, Canada the only remaining international participant.

To generate each scenario, the official JPO production planning profile was adjusted. Downsizing was assumed to be uniform (across all years) and unaffected order quantities were unchanged. F-35C orders (used for propulsion system cost projection) were unchanged. Table 12 presents the F-35A cost estimates (percent change) for the five scenarios. The results are presented for each lot (buy year). Combined air vehicle and engine cost totals are presented.

Assuming Canada will procure aircraft between 2015 and 2021, based on the F-35A cost estimates presented in Table 12, Canada would expect to pay 2% more should a major European partner withdraw, 4% more should two major partners withdraw, 12% more should all countries except for the U.S. and Canada withdraw, 18% more if the U.S. downsizes to 75% and all other countries withdraw, and 28% more if the U.S. downsizes to 50% and all other countries withdraw.

Table 12: F-35A URF cost estimates upon withdrawal/downsizing of international participation (as a percentage change).

		Estimated Cost (as a % of the baseline estimate)				
Lot	Year	Scenario A	Scenario B	Scenario C	Scenario D	Scenario E
4	2010	0%	0%	2%	8%	19%
5	2011	0%	0%	0%	7%	17%
6	2012	0%	1%	5%	12%	22%
7	2013	3%	3%	7%	14%	26%
8	2014	2%	3%	10%	18%	29%
9	2015	3%	4%	12%	19%	30%
10	2016	2%	4%	13%	20%	30%
11	2017	2%	4%	13%	20%	30%
12	2018	2%	4%	12%	19%	28%
13	2019	2%	4%	11%	18%	27%
14	2020	2%	4%	10%	17%	26%
15	2021	2%	4%	11%	18%	28%
16	2022	1%	2%	7%	15%	25%
17	2023	1%	2%	6%	13%	24%
18	2024	1%	1%	3%	10%	21%
19	2025	0%	1%	3%	10%	20%
20	2026	0%	1%	3%	10%	20%
21	2027	0%	1%	3%	10%	20%
22	2028	0%	1%	3%	10%	20%
23	2029	0%	1%	2%	9%	20%
24	2030	0%	1%	2%	9%	20%
25	2031	0%	1%	2%	9%	20%
26	2032	0%	1%	2%	9%	20%
27	2033	0%	1%	2%	9%	19%
28	2034	0%	1%	2%	9%	20%
29	2035	0%	1%	2%	9%	19%

6 CONCLUSION

The models presented herein facilitate sensitivity analysis such as determining the fiscal impact on Canada should international partners cancel or downsize their F-35 orders. Furthermore, risk and uncertainty analysis of the prediction is provided to facilitate the selection of an appropriate level of confidence (in the baseline estimate) for contingency planning. In 2011 and early 2012 The Canadian Department of National Defence employed to model to estimate the cost implications of various F-35A production plan changes.

The cost estimation methodology used is a *quantity effects* model employed by the Research and Development (RAND) Corporation that combines cost improvement and production rate effects. Using the latest (June 2011) estimate-at-completion (EAC) cost data based on actual data from completed or partially-completed F-35A LRIP lots, the model is used to independently estimate the URF costs for future F-35A production lots, including the lots from which Canada expects to buy its F-35As. The EAC cost data for completed LRIP lots represent the actual production costs incurred for each lot. EAC costs are not settlement costs—all cost overruns are included (as are any contractor incentive fees).

A top-down risk/uncertainty model—developed by the U.S. Naval Center for Cost Analysis (NCCA) which analyzed U.S. Selected Acquisition Reports of one hundred U.S. Department of Defense major acquisition programs to determine expected cost growth factors—was applied.

6.1 Results

The principal contribution of this report is an F-35A URF cost estimation model facilitating secondary, independent cost estimates and sensitivity analysis. The basis for results provided below are mathematical models estimating costs and predicting risk/uncertainty distributions given the study's assumptions presented in Section 1.3.

- The quantity effects model projections support DND's claim that Canada will buy F-35As during lowest-cost production years. The best-fit (to actual EAC data) learning and production rate slope percentages are within 2% of those reverse-engineered from JPO predictions.

- **Baseline estimate:** The mathematical best-fit model was used to project the average unit recurring flyaway price (engine included) that Canada could anticipate to pay. This is a secondary, independent estimate. Although the baseline estimate is more than the JPO projection (computed using the June 2011 JPO cost estimates and production planning profile), it was considered complementary given the limiting assumptions that were made. Both estimates are void of any risk, opportunity, or uncertainty considerations (valid if the program runs without any improvements/hitches, no withdrawals/downsizing/changes to orders, etc.).

- **Contingency:** Contingency is required to achieve a risk-adjusted mean cost estimate that takes into account program risk and uncertainty (to achieve 55% confidence). Inflation and currency exchange rate fluctuations will affect contingency and need to be analyzed in more detail.

- **Sensitivity analysis:**

  - The quantity effects model was employed to estimate the cost implications of international withdrawal, downsizing, or production schedule change. Scenarios considered included the withdrawal of a major European partner, two major partners withdrawing, all other non-U.S. partners withdrawing, and U.S. downsizing its planned order and all other countries withdrawing.
  - A $\pm1\%$ point deviation in the anticipated air vehicle production rate slope percentage results in roughly $\pm5\%$ change to Canada's URF cost; a $\pm1\%$ point deviation in the anticipated air vehicle learning curve slope percentage results in roughly $\pm7\%$ change to Canada's URF cost.

  The risk-adjusted mean estimate (contingency) is meant to capture some of these risks.

- **Worst-case planning:** Based on the NCCA risk/uncertainty model and resulting cost probability distribution, there is a 20% chance that the average Canadian F-35A will exceed a 36% cost overrun. One example scenario where Canada's URF cost might exceed this amount is as follows: production rate efficiencies are not realized (a 5% increase in the predicted air vehicle production rate slope percentage) combined with the withdrawal of two major international partners.

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A THEORY

In Section A.1 the mathematical basis for the cost improvement effect, historically referred to as the learning curve effect, is exposed.[11] Section A.2 specializes the model for production by lots and Section A.3 augments it to model the production rate effect. These subsections are included for rigour and to make the paper self-contained. They do not represent original research and should not be attributed to the author.

A.1 History of Learning Curves

In 1936, at Wright-Patterson Air Force Base in the United States, Wright [22] was the first to study the aircraft production quantity effect. He noted that the more times a task was performed, the less time was required on each subsequent iteration. Wright observed that every time that total aircraft production doubled, the required labour time decreased by 10 to 15 percent. In general, the Wright learning curve is formulated as follows:

$$CAC(Q) = A_1 \times Q^b. \tag{6}$$

$CAC(Q)$ is the cumulative average cost associated with producing the first Q units, A_1 is the cost associated with the first unit produced, and b is the learning slope. Computing 2^b yields the learning slope percentage.

Wright's learning curve for aircraft production has been generalized to other industries and to resources other than labour time (see Henderson's experience effect [11]). Variant learning curve models have also been developed. The Management and Accounting Web [16] maintains an up-to-date bibliography on learning and experience curves. The Crawford model [15] is the most commonly applied learning curve. It expresses the cost of unit Q—the marginal cost—as

$$MC(Q) = T_1 \times Q^b. \tag{7}$$

In Equation (7) T_1 and b are parameters to be estimated.

Goldberg and Touw [8] and Lee [14] show that the Wright and Crawford learning curve models are related. Multiplying Wright's cumulative average cost, $CAC(Q)$, by the quantity Q yields the total cost, $TC(Q)$, of producing the first Q units,

$$TC(Q) = Q \times CAC(Q) = A_1 \times Q^{1+b}, \tag{8}$$

[11] The terms cost improvement and learning are used interchangeably.

and the marginal cost is the derivative of total cost with respect to cumulative quantity:

$$MC(Q) = \frac{d\,TC(Q)}{dQ} = A_1 \times (1+b) \times Q^b. \tag{9}$$

Given Wright's parameter A_1, setting $T_1 = A_1 \times (1+b)$ yields Crawford's learning curve model.

A.2 Lot-Midpoint Learning Curves

The Lockheed Martin Corporation is producing F-35 aircraft in lots. The fundamental defining features of a lot are the number of units that the lot comprises and the (incremental) total cost of the lot. The average cost per aircraft in a given lot can be computed by simple division for the fixed number of aircraft. However, an average aircraft cost per lot does not necessarily mean that all units are equally costly, nor that increasing (or decreasing) the lot size maintains the average cost per aircraft. In the case of production by lots, the lot-midpoint learning curve model exposed by Goldberg and Touw [8] is suitable:

$$LAC_i = T_1 \times [\bar{Q}_i(b)]^b. \tag{10}$$

In Equation (10), LAC_i represents the average cost of an aircraft from lot i, T_1 the cost of the first aircraft produced (of the entire production), $\bar{Q}_i(b)$ the midpoint of the ith lot, and b the learning slope. The lot midpoint, $\bar{Q}_i(b)$, is a function of b. In order to determine a lot-midpoint the learning slope must be known. This is because the lot midpoint is defined as the (generally non-integer) quantity whose marginal cost is equal to the lot average cost.

Using the theory of continuous learning curves, $\bar{Q}_i(b)$ can be computed in usable form. The incremental total lot cost of lot i is approximated by the integral under the marginal cost curve ($MC(Q)$). Define Q_i as the cumulative quantity of units produced up to and including lot i. Then the ith lot begins at unit $Q_{i-1}+1$ and ends with unit Q_i. $TC(Q_i) - TC(Q_{i-1})$ is the incremental total lot cost of lot i:

$$TC_i - TC_{i-1} \approx \int_{z=Q_i+0.5}^{z=Q_{i-1}+0.5} T_1\, z^b\, dz = \frac{T_1}{1+b} \times \left[(Q_i+0.5)^{1+b} - (Q_{i-1}+0.5)^{1+b}\right]. \tag{11}$$

Dividing the right-hand-side of Equation (11) by the size of lot i yields the average incremental lot cost, LAC_i:

$$LAC_i \approx \frac{\frac{T_1}{1+b} \times \left[(Q_i+0.5)^{1+b} - (Q_{i-1}+0.5)^{1+b}\right]}{Q_i - Q_{i-1}}. \tag{12}$$

(The symbol $\approx$ means approximately equal to.) Setting the marginal cost $T_1 \times [\bar{Q}_i(b)]^b$ to LAC_i and solving for the lot midpoint yields:

$$\bar{Q}_i(b) \approx \left(\frac{\left[(Q_i+0.5)^{1+b} - (Q_{i-1}+0.5)^{1+b}\right]}{(1+b) \times (Q_i - Q_{i-1})}^{\frac{1}{b}}\right). \tag{13}$$

A.3 Production Rate Effect

A production rate effect measures the effect of producing different quantities in a given year (or time period). The unit cost is affected through operating efficiency and the spreading of fixed costs over the number of units. The cost improvement curve (learning curve) is augmented as follows to model the production rate effect:

$$LAC_i = T_1 \times [\bar{Q}_i(b)]^b \times r_i^c. \tag{14}$$

In Equation (14) r_i is the production rate of lot i. Goldberg and Touw show that to properly handle the production rate effect r_t would ideally be the number of units produced in annual buy t, and LAC_t would be the cost of the unit at the midpoint, $\bar{Q}_t(b)$, of annual buy t. In reality several F-35 lots are in progress concurrently. However the data provided by the JPO and in U.S. Selection Acquisition Reports does not detail which lots are produced at which plants (including sub-contractors) at which stages of assembly. Lot size is often used as a proxy for production rate—Goldberg and Touw state that the lot size provides a "serviceable approximation" to the true production rate.

A second issue regarding modelling the production effect occurs when the production rate is highly correlated with the cumulative quantity. Several authors (e.g., Large *et al.* [13]) have shown that results (estimation of parameters b, c, and T_1) are unreliable when this is the case.