



An Elicitation Method to Generate Minimum-Bias Probability Distributions

Presented by:

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Risk, Uncertainty & Estimating

***“It is better to be approximately right
rather than precisely wrong.*”**

Warren Buffett



Outline

- Purpose of Presentation
- Background
 - The Uncertainty Spectrum
 - Expert Judgment Elicitation (EE)
 - Continuous Distributions
 - More details on Triangular, Beta & Beta-PERT Distributions
- Five Expert Elicitation (EE) Phases
- Example: Estimate Morning Commute Time
 - Expert Elicitation (EE) to create a Triangular Distribution
 - With emphasis on Phase 4's Q&A with Expert (2 iterations)
 - Convert Triangular Distribution into a Beta-PERT
- Conclusion & Potential Improvements



Purpose of Presentation

Adapt / combine known methods to demonstrate an expert judgment elicitation process that ...

1. Models expert's inputs as a triangular distribution

- 12 questions to elicit required parameters for a bounded distribution
- Not too complex to be impractical; not too simple to be too subjective

2. Incorporates techniques to account for expert bias

- A repeatable Q&A process that is iterative & includes visual aids
- Convert Triangular to Beta-PERT (if overconfidence was addressed)

3. Is structured in a way to help justify expert's inputs

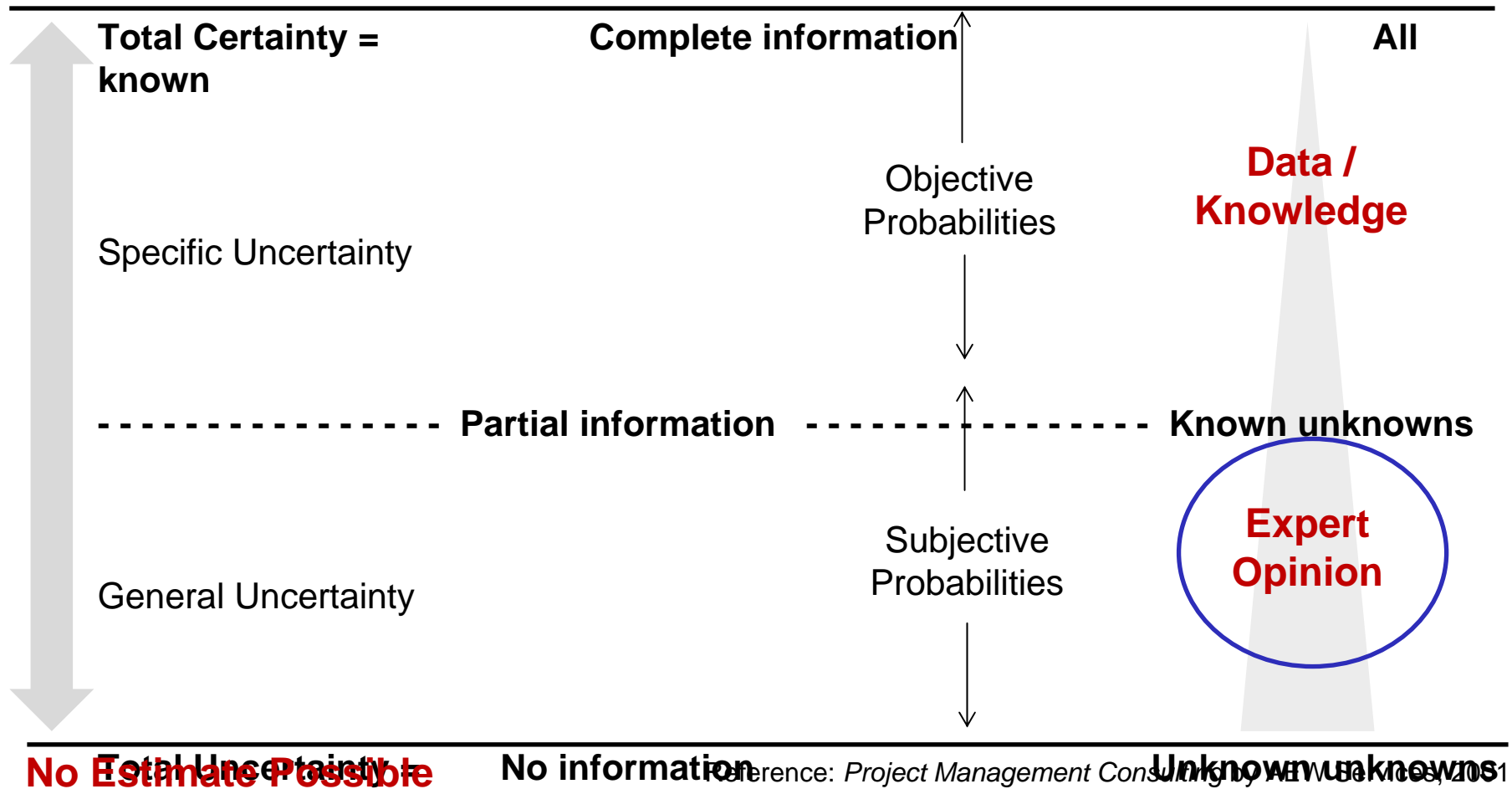
- Expert must provide rationale for each of his/her responses
- Using Risk Breakdown Structure, expert specifies each risk factor's relative contribution to a given uncertainty (of cost, duration, reqt, etc.)

This paper will show one way of “extracting” expert opinion for estimating purposes. Nevertheless, as with most subjective methods, there are many ways to do this.



The Uncertainty Spectrum

No Estimate Required

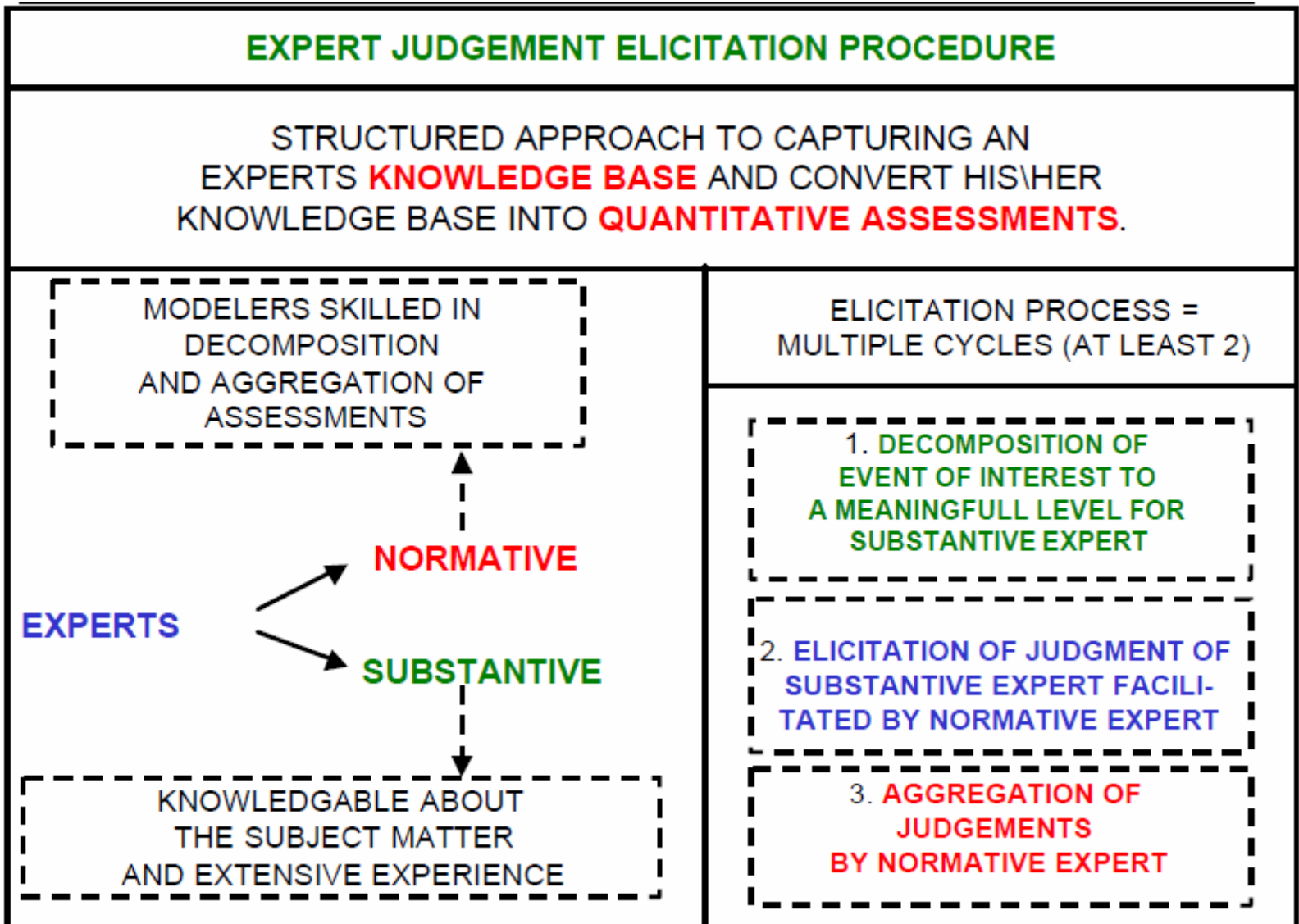


Reference: Project Management Consulting by W. W. Rouse Ball, 2001



Expert opinion is useful when little information is available for system requirements, system characteristics, durations & cost

Expert Judgment Elicitation (EE)



Source: Making Hard Decisions, An Introduction to Decision Analysis by R.T. Clemen

Continuous Distributions

Bounded

- **Triangular & Uniform**
- **Histogram**
- **Discrete & Cumulative**
- **Beta & Beta-PERT**

Unbounded

- **Normal & Student-t**
- **Logistic**

Left bounded

- **Lognormal**
- **Weibull & Gamma**
- **Exponential**
- **Chi-square**

Non-Parametric Distributions: Mathematics defined by the shape that is required. Empirical, intuitive and easy to understand.

Parametric Distributions: Shape is born of the mathematics describing theoretical problem. Model-based. Not usually intuitive.

Of the many continuous distributions out there, **Triangular & Beta-PERT** are among the most popular used for expert elicitation



Triangular Distribution

- **Used in situations where there is little or no data**
 - Just requires the lowest (L), highest (H) and most likely values (M)

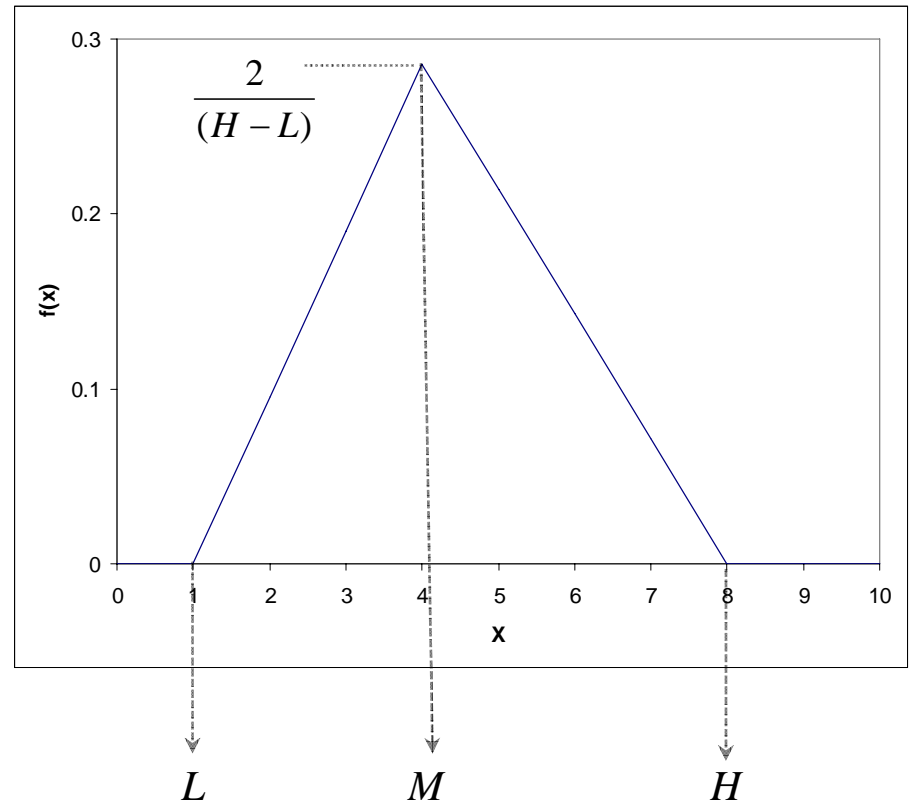
Each x -value has a respective $f(x)$, sometimes called “Intensity” that forms the following

$$\begin{aligned} \text{PDF: } f(x) &= \frac{2(x-L)}{(M-L)(H-L)}, & L \leq x < M \\ &= \frac{2(H-x)}{(H-M)(H-L)}, & M \leq x < H \\ &= 0, & \text{otherwise} \end{aligned}$$

L , M & H are all that's needed to calculate the Mean and Standard Deviation:

$$\mu = \frac{(L + M + H)}{3}$$

$$\sigma = \sqrt{\frac{(L^2 + M^2 + H^2 - L * M - L * H - M * H)}{18}}$$



Beta Distribution

- **Used in situations where there is little or no data.**
 - Bounded on [0,1] interval, scale to any interval & very flexible shape

$$f(x) = \left(\frac{1}{H-L} \right) \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \left(\frac{x-L}{H-L} \right)^{\alpha-1} \left(\frac{H-x}{H-L} \right)^{\beta-1} \quad L < x < H$$

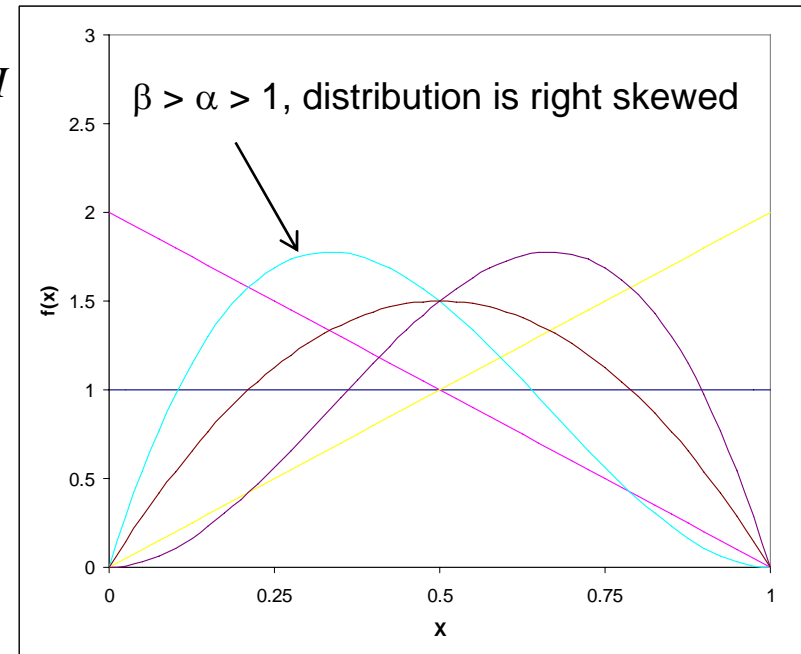
= 0 otherwise

$$\Gamma(\alpha + \beta) = \text{EXP}[\text{GAMMALN}(\alpha + \beta)]$$

$$\Gamma(\alpha) = \text{EXP}[\text{GAMMALN}(\alpha)]$$

$$\Gamma(\beta) = \text{EXP}[\text{GAMMALN}(\beta)]$$

Shape Parameters : $\alpha > 0, \beta > 0$



Most schedule or cost estimates follow right skewed pattern. But how do we know α and β ? Answer: Beta-PERT Distribution.



Sources: 1. Dr. Paul Garvey, *Probability Methods for Cost Uncertainty Analysis*, 2000
2. LaserLight Networks, Inc, "Beta Modeled PERT Schedules"

Beta-PERT Distribution

- **A Beta distribution for a PERT-based schedule, cost, etc.**

- Like Triang, requires lowest (L), highest (H) and most likely values (M)

To calculate shape parameters α and β ...
$$\alpha = \frac{(\mu - L)}{(H - L)} * \frac{(\mu - L)(H - \mu)}{\sigma^2} - 1$$

$$\beta = \frac{(H - \mu)}{(\mu - L)} * \alpha \quad \text{where } \alpha > 0, \beta > 0$$

L, M & H are not only used to calculate Mean & Standard Deviation, also used to compute the Beta Probability Density:

α and β are needed to define the Beta Function & compute the Beta Probability Density:

$$\mu = \frac{(L + 4 * M + H)}{6}$$

$$\sigma = \frac{(H - L)}{6}$$

$$f(x) = \left(\frac{1}{H - L} \right) \left(\frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \right) \left(\frac{x - L}{H - L} \right)^{\alpha - 1} \left(\frac{H - x}{H - L} \right)^{\beta - 1} \quad L < x < H$$

Elements of the Beta Function

$$\begin{cases} \Gamma(\alpha + \beta) = \text{EXP}[\text{GAMMALN}(\alpha + \beta)] \\ \Gamma(\alpha) = \text{EXP}[\text{GAMMALN}(\alpha)] \\ \Gamma(\beta) = \text{EXP}[\text{GAMMALN}(\beta)] \end{cases}$$



Sources: 1. Dr. Paul Garvey, *Probability Methods for Cost Uncertainty Analysis*, 2000
 2. LaserLight Networks, Inc, "Beta Modeled PERT Schedules"

Expert Elicitation (EE) Phases

Expert Elicitation consists of five phases:

(note that Phases 4 & 5 are iterative)

1. Motivating the expert
2. Training (conditioning) the expert
3. Structuring objective, assumptions & process
4. Assessing (encoding) expert's responses
 - Q&A – Expert's technical opinion is elicited
 - Quantitative results w/ documented rationale
5. Verifying encoded values & documentation

Our Example will emphasize the Phase 4 Q&A



Example: Estimate **Commute Time**

- **Why this example?**
 - Fairly easy to find a subject matter expert
 - It is a parameter that is measurable
 - Most experts can estimate a most likely time
 - Factors that drive uncertainty can be readily identified
 - People general care about their morning commute time!

Let's begin with Phase I ... Motivating the Expert:

I. Motivating the expert

- Explain the importance & reasons for collecting the data
- Explore stake in decision & potential for motivational bias



EE Phase 2: **Commute Time**

2. Structuring objective, assumptions & process

- Explain the importance & reasons for collecting the data
- Explore stake in decision & potential for motivational bias

The Interviewer should have worked with you to develop the Objective and up to 5 Major Assumptions in the table below

- *Please resolve any questions or concerns about the Objective and/or Major Assumptions prior to continuing to "Instructions".*

Objective: Develop uncertainty distribution associated with time (minutes) it will take for your morning commute starting 1 October 2014.

Assumption 1: Your commute estimate includes only MORNING driving time

Assumption 2: The commute will be analogous to the one you've been doing

Assumption 3 Period of commute will be from 1 Oct 2014 thru 30 Sep 2015

Assumption 4 Do not try to account for extremely rare & unusual scenarios

Assumption 5: Unless you prefer otherwise, time will be measured in minutes



EE Phase 3: **Commute Time**

3. Training (conditioning) the expert

- Go over instructions for Q&A process
- Emphasize benefits of time constraints & 2 iterations

Instructions: This interview is intended to be conducted in two iterations.
Each iteration should take no longer than 30 minutes.

- A. Based on your experience, answer the 12 question sets below.
- B. Once you've completed the questions, review them & take a 15 minute break.
- C. Using the triangular graphic to assist you, answer all of the questions again.

Notes:

- A. The 2nd iteration is intended to be a refinement of your 1st round answers.
- B. Use lessons-learned from the 1st iteration to assist you in the 2nd iteration.
- C. Your interviewer is here to assist you at any point in the interview process.



EE Phase 3: Commute Time (cont'd)

3. Training the expert (continued)

For 3 Questions, you'll need to provide your assessment of likelihood:

Descriptor	Explanation
Absolutely Impossible	No possibility of occurrence
Extremely Unlikely	Nearly impossible to occur; very rare
Very Unlikely	Highly unlikely to occur; not common
Unlikely	Indifferent between "Somewhat" & "Very" Unlikely
Somewhat Unlikely	Slightly less than 50-50 chance
Even chance	50/50 chance of being higher or lower
Somewhat Likely	Slightly more than 50-50 chance
Likely	Indifferent between "Somewhat" & "Very" Unlikely
Very Likely	Highly likely to occur; common occurrence
Extremely Likely	Nearly certain to occur; near 100% confidence
Absolutely Certain	100% Likelihood

Example: Assume you estimated a "LOWEST" commute time of 20 minutes.

Your place a value = 10.0% as the probability associated with "Very Unlikely."

Therefore:

- You believe it's "VERY UNLIKELY" your commute time will be less than 20 minutes, and
- This is equal to a 10.0% chance that your commute time would be less than 20 min.

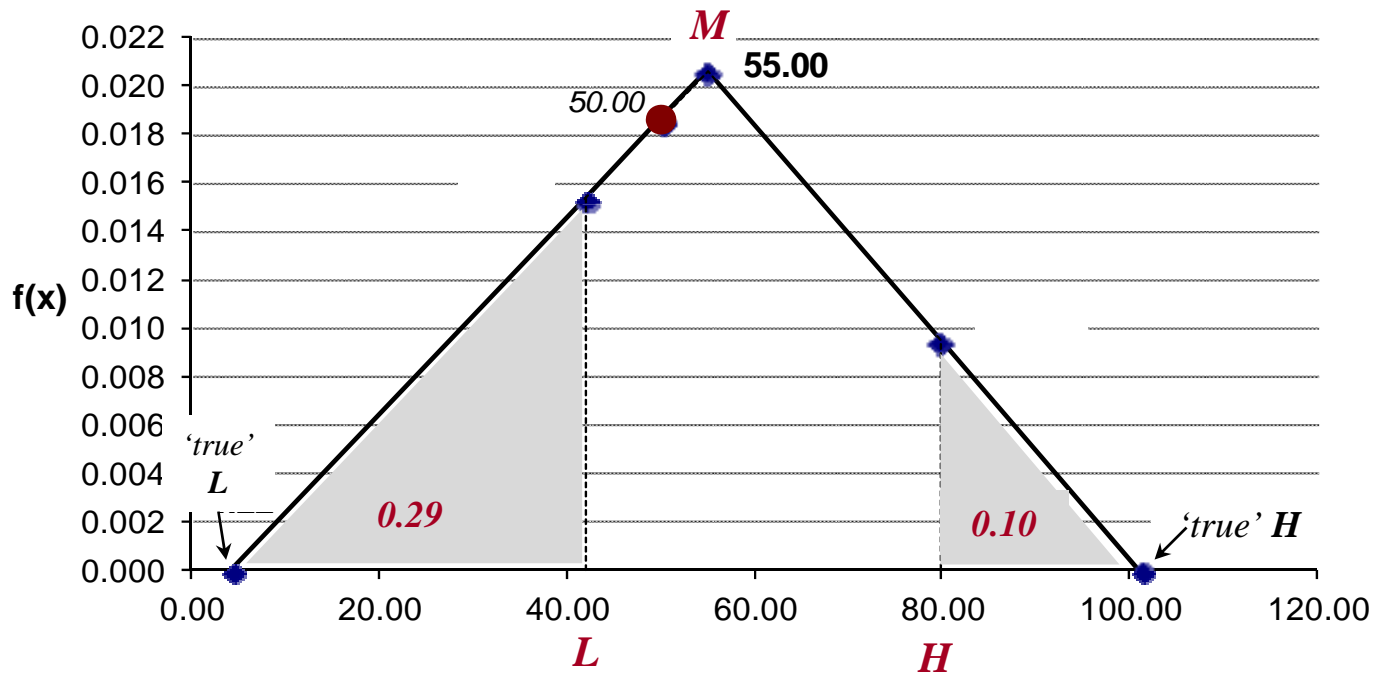


EE Phase 4: Commute Time (iteration 1)

4. Assessing expert's responses (Q&A)

User-Provided Distribution for Commute Time

Red dot depicts unadjusted point estimate. Dashed lines depict unadjusted lowest & highest



PDF created based upon Expert's responses to Questions 1 through 8.

Given from Expert: $L=42, M=55, H=80, p(x<L)=0.29$ and $p(x>H)=0.10$

Calculation of 'true' L and H (a): $L = 4.22$ and $H = 101.15$... Do these #'s appear reasonable?



(a) Method to solve for L and H presented in "Beyond Beta," Ch1 (The Triangular Distribution)

EE Phase 4: **Commute Time** (Iteration 1)

Given the objective and assumptions:

1. Characterize input parameter (e.g. WBS4: Commute Time)
2. What's the Most Likely value, M ?
3. Adjust M (if applicable)
4. What's the chance the actual value could exceed M ?
5. What's the Lowest value, L
6. What's the chance the actual value could be less than L ?
7. What's the Highest value, H
8. What's the chance the actual value could be higher than H ?

This 1st iteration tends to result in anchoring bias on M , over-confidence on L and H , and poor rationale



EE Phase 4: Commute Time (iteration 1)

Question 9: Expert creates “value-scale” tailored his/her bias ...

What probability would you assign to a value that's "Somewhat Unlikely"

What probability would you assign to a value that's "Very Unlikely"

What probability would you assign to a value that's "Extremely Unlikely"

Available Selection of Values to the Expert (shaded cells were selected by expert):

SOMEWHAT LIKELY	SOMEWHAT UNLIKELY	VERY LIKELY	VERY UNLIKELY	EXTREMELY LIKELY	EXTREMELY UNLIKELY
51.0%	49.0%	80.0%	20.0%	96.0%	4.0%
52.0%	48.0%	82.5%	17.5%	97.0%	3.0%
52.5%	47.5%	85.0%	15.0%	98.0%	2.0%
55.0%	45.0%	87.5%	12.5%	98.5%	1.5%
57.5%	42.5%	90.0%	10.0%	99.0%	1.0%
60.0%	40.0%	92.5%	7.5%	99.5%	0.5%
62.5%	37.5%	95.0%	5.0%	99.9%	0.1%
To help estimate where mode is positioned on triangular distribution		To help estimate where Low & High are on triangular distribution		To help adjust where Low & High are on triangular distribution	



EE Phase 4: Commute Time (iteration 1)

Recall Question 9: Expert creates “value-scale” tailored his/her bias ...

What probability would you assign to a value that's "Somewhat Unlikely"

What probability would you assign to a value that's "Very Unlikely"

What probability would you assign to a value that's "Extremely Unlikely"

Descriptor	Explanation	Probability
Absolutely Impossible	No possibility of occurrence	0%
Extremely Unlikely	Nearly impossible to occur; very rare	1.0%
Very Unlikely	Highly unlikely to occur; not common	10.0%
Unlikely	Indifferent between "Somewhat" & "Very" Unlikely	29.0%
Somewhat Unlikely	Slightly less than 50-50 chance	48.0%
Even chance	50/50 chance of being higher or lower	50%
Somewhat Likely	Slightly more than 50-50 chance	52.0%
Likely	Indifferent between "Somewhat" & "Very" Unlikely	71.0%
Very Likely	Highly likely to occur; common occurrence	90.0%
Extremely Likely	Nearly certain to occur; near 100% confidence	99.0%
Absolutely Certain	100% Likelihood	100%

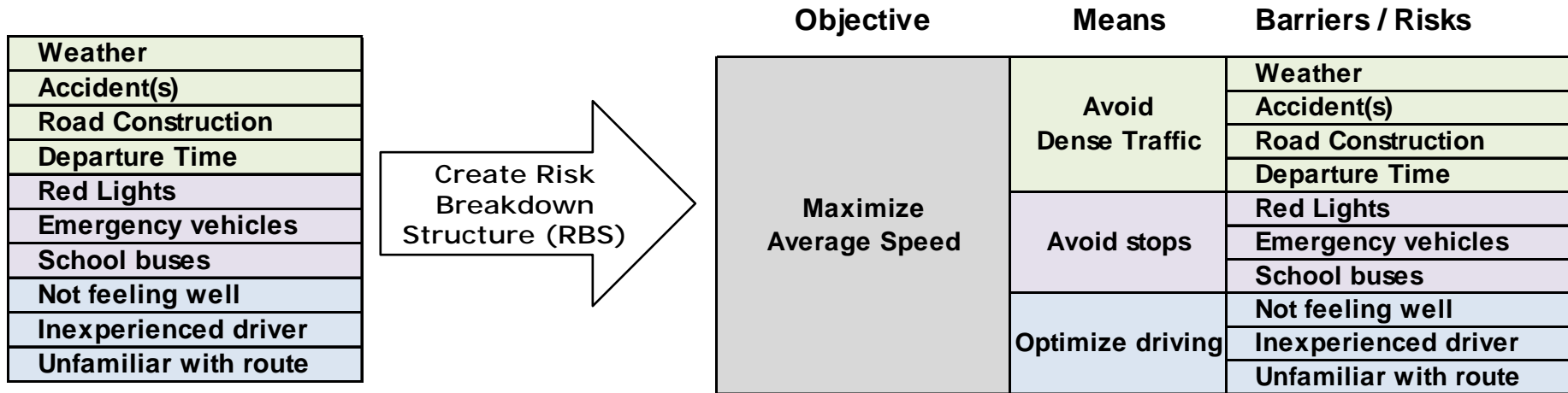
Only 3 probabilities needed to be elicited in order to create a Value-Scale that has 11 categories!



EE Phase 4: Commute Time (iteration 1)

Question 10: Expert & Interviewer brainstorm risk factors ...

What risk factors contributed to the uncertainty in your estimate?



Question 11: Expert selects top 6 risk factors ...

What are the top 6 risk factors that contributed to your estimate uncertainty?

User Input

Examples or Justification:

Weather	Rain, snow & especially ice, have caused major delays in the past; I expect similar impacts in 2014.
Accident(s)	Accidents occasionally occur. In some cases, these have added 60 minutes to my commute!
Road Construction	Sometimes road crews shut down 1 or 2 lanes; typically adding 10 - 20 minutes to my commute.
Departure Time	I try to leave 1 hour before rush hour. Leaving later can add 10-15 minutes to my commute.
Not Feeling Well	If I'm not feeling well, I'll drive more slowly or even make a wrong turn! Can add 5 min to commute.
Red Lights	I tend to "catch" the same lights every day so this factor could add 1-2 minutes to my commute.



EE Phase 4: Commute Time (iteration 1)

Question 12: Expert scores each risk factor's contribution to uncertainty ...

Score each risk factor a value based upon the following instruction:

If the specified risk factor: *		
Is the largest contributor causing your ML to go up by	184%	then score it a 5
Is a significant contributor causing your ML to go up by	184%	then score it a 4
Has a moderate effect in causing your ML to go up by	184%	then score it a 3
Has a small effect in causing your ML to go up by	184%	then score it a 2
Is the smallest contributor causing your ML to go up by	184%	then score it a 1

* Note: You can have 2 or more risk factors with a score of 5 (or score of 1).

<u>Risk Factor</u>	<u>Score</u>
Weather	5
Accident(s)	5
Road Construction	3
Departure Time	4
Not Feeling Well	1
Red Lights	1

Expert provides a score for each risk factor (rationale not shown).

The 1st iteration of Q&A is complete. Recommend the expert take a 15 minute break before re-starting Q&A

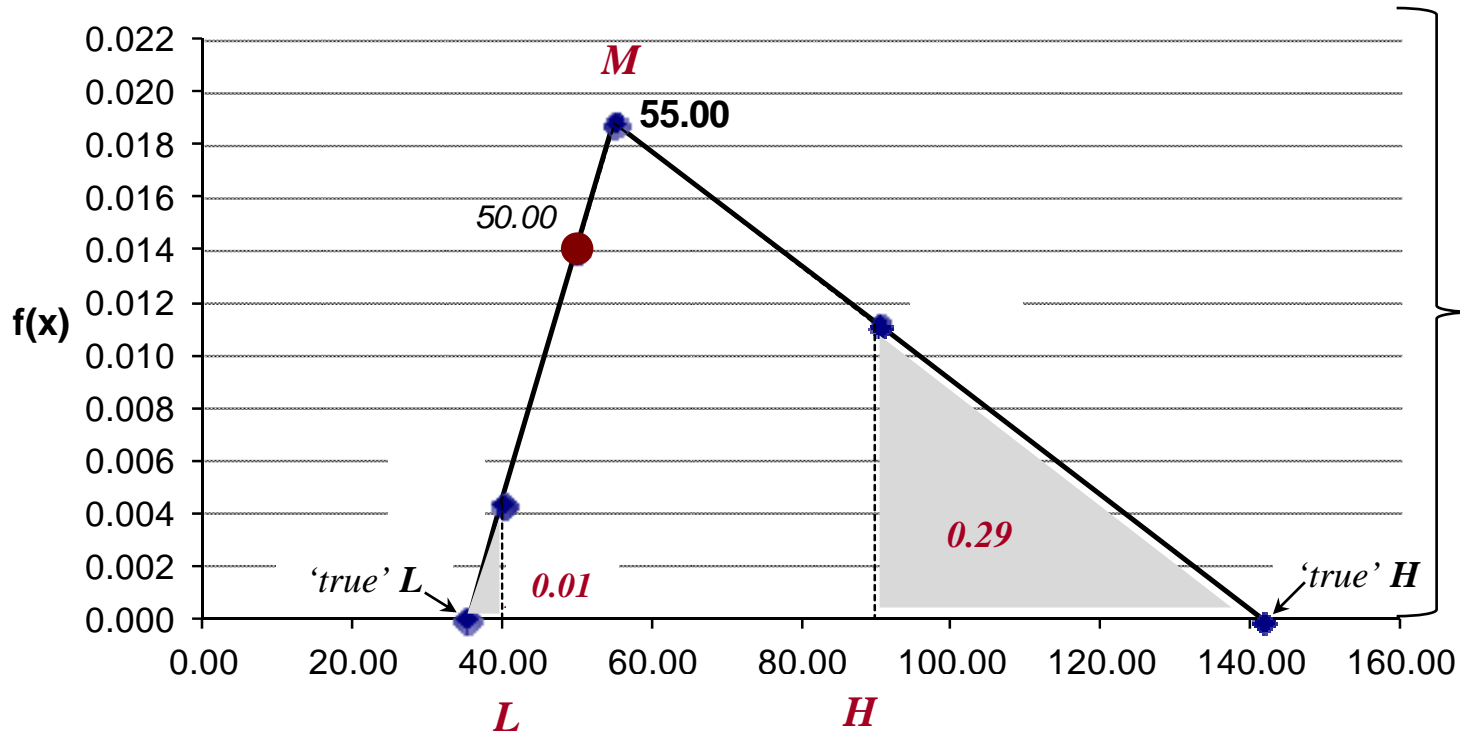


EE Phase 4: Commute Time (iteration 2)

4. Assessing expert's responses (Q&A)

User-Provided Distribution for Commute Time

Red dot depicts unadjusted point estimate. Dashed lines depict unadjusted lowest & highest



PDF created based upon Expert's responses to Questions 3 through 8.

Given from Expert: $L=40$, $M=55$, $H=90$, $p(x < L)=0.10$ and $p(x > H)=0.29$

Calculation of 'true' L and H (a): $L = 35.44$ and $H = 141.67$... *Do these #'s appear reasonable?*



EE Phase 4: **Commute Time** (Iteration 2)

Given the objective, assumptions & input parameter (WBS4):

3. Do you want to adjust your Most Likely Value, M ?

4. What's the chance the actual value could exceed M ?

Assuming best case: weather, accidents, road const, departure time, etc.:

5. What's the Lowest value, L

6. What's the chance the actual value could be less than L ?

Assuming worst case: weather, accidents, road const, departure time, etc.:

7. What's the Highest value, H

8. What's the chance the actual value could be higher than H ?

This 2nd iteration helps “condition” expert to reduce anchoring bias on M , counter over-confidence on L and H , calibrate ‘values’ & improve rationale.



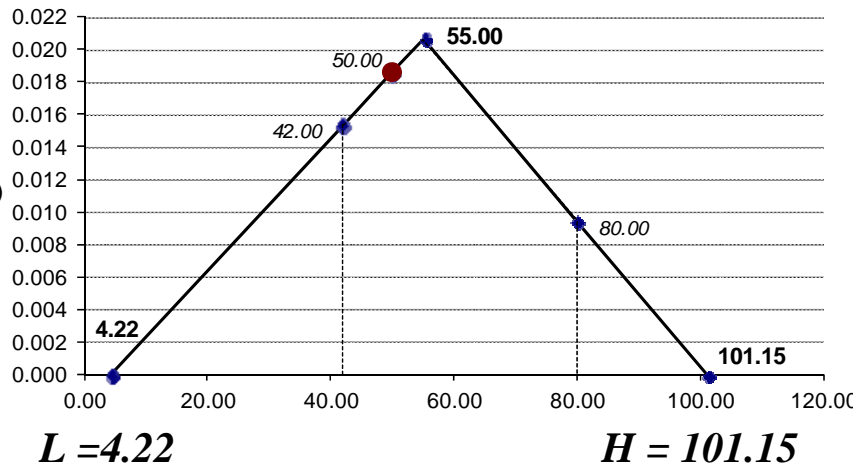
EE Phase 5: Commute Time (iteration 2)

5. Verifying encoded values & documentation

Triangular PDF from Iteration 1

User-Provided Distribution for Commute Time

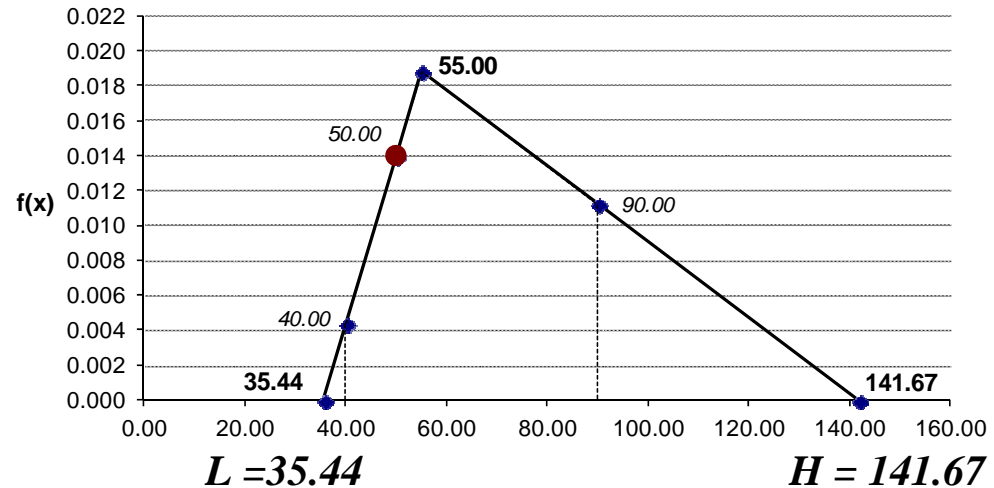
Red dot depicts unadjusted point estimate. Dashed lines depict unadjusted lowest & highest



Triangular PDF from Iteration 2

User-Provided Distribution for Commute Time

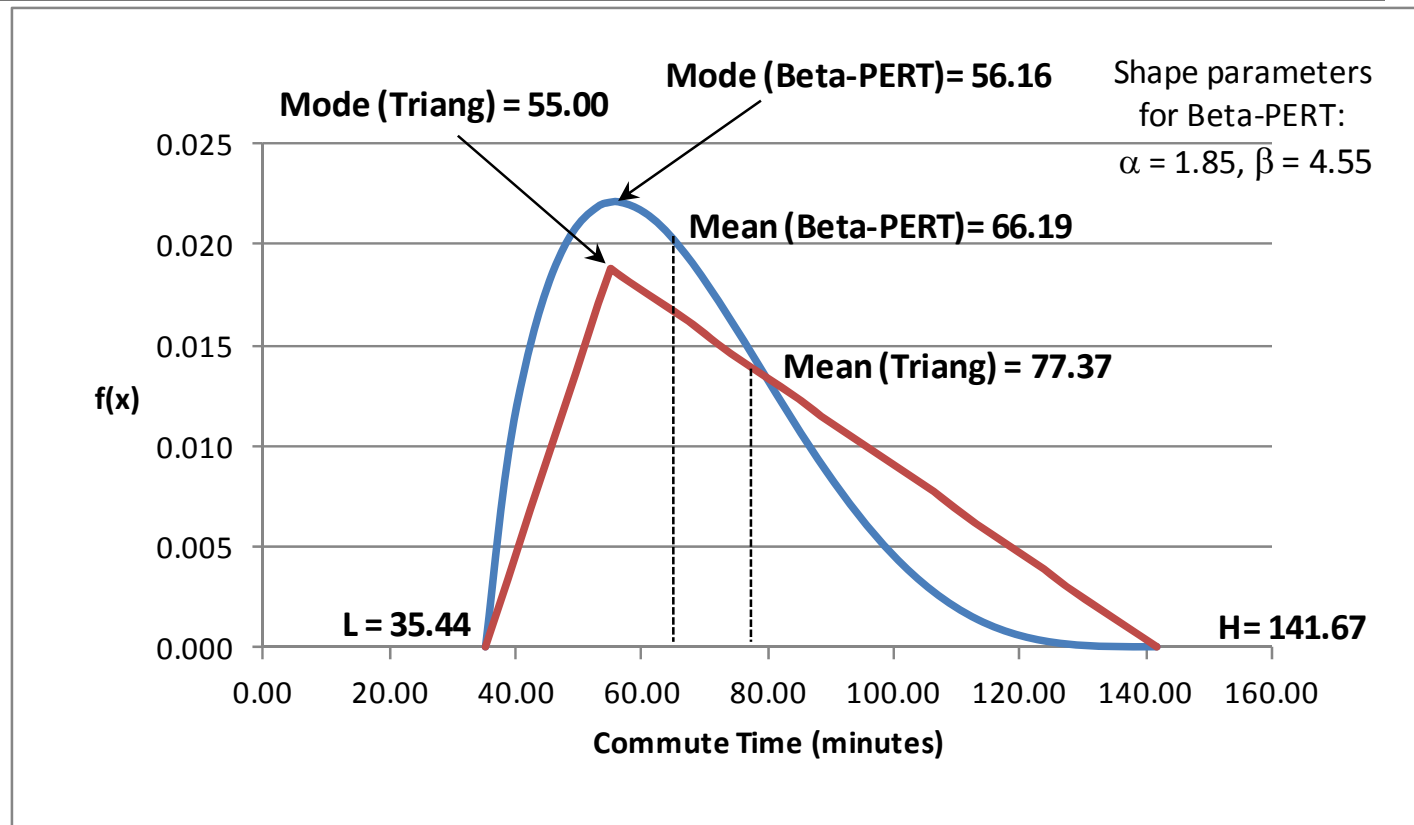
Red dot depicts unadjusted point estimate. Dashed lines depict unadjusted lowest & highest values



The 2nd iteration helped elicit an *L* that seems feasible and an *H* that accounts for worst-case risk factors



Results (Triangular & Beta-PERT)



- **In most cases, Beta-PERT is preferred (vs triangular)**
 - Beta-PERT's mean is only slightly greater than its mode
- **However, triangular would be preferred (vs Beta-PERT) if elicited data seems to depict over-confidence (i.e. overly optimistic)**
 - Triangular PDF compensates for this by 'inflating' the mean



Conclusion

We provided an expert elicitation overview that ...

1. Demonstrated a way to model expert opinion as a triangular distribution

- 12 questions to elicit required parameters for a bounded distribution
- Not too complex to be impractical; not too simple to be too subjective

2. Incorporated techniques to account for expert bias

- A repeatable Q&A process that is iterative & includes visual aids
- 2nd iteration was more robust to help improve 1st iteration results
- Convert Triangular to Beta-PERT (if overconfidence was addressed)

3. Structured the process to help justify expert's inputs

- Expert must provide rationale for each of his/her responses
- Using Risk Breakdown Structure, expert specifies each risk factor's relative contribution to a given uncertainty (of cost, duration, reqt, etc.)



Potential Improvements

- More Upfront work on “Training” Expert
- Include questions to create Modified Beta-PERT
- Improve scaling tables for expert opinion
 - Example: Increase # of categories, have a non-linear scale, etc.
- Create “starter” Risk Breakdown Structures”
 - Facilitates brainstorming process of possible risk factors
- Improve method of weighting risk factors
 - Example: use pair-wise comparison
- Add other distributions such as Weibull & LogNormal
- Incorporate methods to combine expert opinions

So ... hopefully ... this adds to the conversation on how best to leverage expert opinion in the cost community ...



Intuition versus Analysis

Quickly answer the question:

***“A bat and a ball cost \$ 1.10 in total.
The bat costs \$1 more than the ball.
How much does the ball cost?”***



Sources not Referenced in Presentation

1. Liu, Y., “Subjective Probability,” Wright State University.
2. Kirkebøen, G., “Decision behaviour – Improving expert judgement, 2010
3. Vose, D., *Risk Analysis (2nd Edition)*, John Wiley and Sons, 2004
4. “Expert Elicitation Task Force White Paper,” US EPA, 2009
5. Clemen, R.T. and Winkler, R.L. (1990) Unanimity and compromise among probability forecasters. *Management Science* 36 767-779



An Elicitation Method to Generate Minimum-Bias Probability Distributions

Questions?

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Reasons For & Against Conducting EE

Reasons for Conducting an Expert Elicitation

- The problem is complex and more technical than political
- Adequate data (of suitable quality and relevance) are unavailable or unobtainable in the decision time framework
- Reliable evidence or legitimate models are in conflict
- Qualified experts are available & EE can be completed within decision timeframe
- Finances and expertise are sufficient to conduct a robust & defensible EE

Reasons Against Conducting and Expert Elicitation

- The problem is more political than technical
- A large body of empirical data exists with a high degree of consensus
- Findings of an EE will not be considered legitimate or acceptable by stakeholders
- Information that EE could provide is not critical to the assessment or decision
- Cost of obtaining EE info is not commensurate with its value in decision-making
- Finances and/or expertise are insufficient to conduct a robust & defensible EE
- Other acceptable methods or approaches are available for obtaining the needed information that are less intensive and expensive



Sources of Cost Uncertainty

Source	How Addressed
Knowns	Identify Estimation Uncertainty
"I Forgot"s	Standard WBS Templates & Checklists
Known Unknowns	Risk Lists Risk Assessment
Unknown Unknowns	Design Principle Reserve %

Best Practices

Focus of Cost Risk Estimation



Source: "Incorporating Risk," presentation by J. Hihn, SQI, NASA, JPL, 2004

Classic “I Forgets”

- Review preparation
- Documentation
- Fixing Anomalies and ECR’s
- Testing
- Maintenance
- Basic management and coordination activities
- CogE’s do spend time doing management activities
- Mission Support Software Components
- Development and test environments
- Travel
- Training



Source: “Incorporating Risk,” presentation by J. Hihn, SQI, NASA, JPL, 2004

Some Common Cognitive Biases

- **Availability**
 - Base judgments on outcomes that are more easily remembered
- **Representativeness**
 - Base judgments on similar yet limited data and experience. Not fully considering other relevant, accessible and/or newer evidence
- **Anchoring and adjustment**
 - Fixate on particular value in a range and making insufficient adjustments away from it in constructing an uncertainty estimate
- **Overconfidence (sometimes referred to as Optimistic bias)**
 - Strong tendency to be more certain about one's judgments and conclusions than one has reason. Tends to produce optimistic bias.
- **Control (or "Illusion of Control")**
 - SME believes he/she can control or had control over outcomes related to an issue at hand; tendency of people to act as if they can influence a situation over which they actually have no control.

