Adaptive Cost-Estimating Relationships

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Origin of the Study

- Up to now, CERs have been based on full data sets consisting of all cost and technical data associated with a particular class of products of interest (PoIs) (e.g., components, subsystems or entire systems satellites, ground systems, etc.)
- In this “proof-of-concept” study, we extend the concept of “analogy estimating” to parametric estimating by deriving “adaptive” CERs, namely CERs that are based on specific needs that may not be reflected in the full data set available
- The eventual goal is to be able to apply CERs that have smaller estimating error and narrower prediction bounds

Agenda

- Discussion of the regression idea and extent of confidence in results, including theory of Weighted Least Squares Regression
- Three methods of adapting CERs to particular data sets or estimating needs
  - *A Priori* method: Weighting each point by its quality or confidence in its accuracy
  - Piecewise CER method: Grouping data into separate subsets based on natural divisions
  - "X-Distance" method: Weighting points by distance from a cost-driver value of interest
- Conclusions

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CER Error Sources

- Inability of any CER to account for all influences on cost, no matter how many inputs it allows – too bad, we usually can't do anything about this
- Incorrectness of algebraic CER model to which cost numbers in data base are statistically fit – tough, try another algebraic form
- Location of cost driver value x among parameter values comprising historical cost data base – **this issue is what we will try to resolve in this briefing**
  - If x is located near center of range of parameter values, CER will provide fairly precise estimate of system's cost
  - If x is located far from center of range, CER-based estimate will be considerably less precise

Prior Thoughts on This Issue

Space system study

Pierre Foussier (*3f*)
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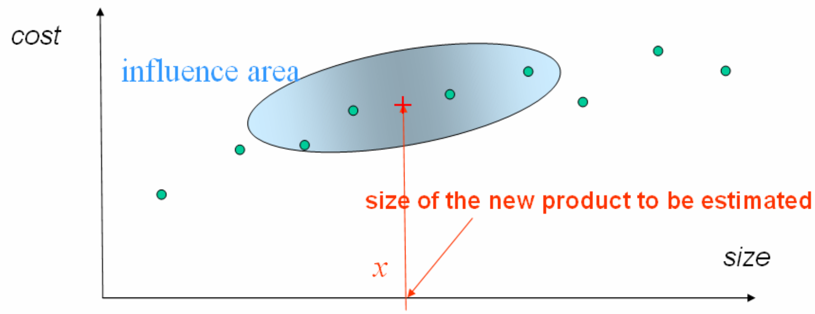
SSCAG. Montréal. May 16, 2007

- How can we improve the cost estimating process in order **to get the best from our data** ?
- I will today only mention five points :
  - ① improving the way we prepare CERs
  - ② using, each time it is possible, the "bayesian approach"
  - ③ using sometimes "non parametric" cost estimating
  - ④ taking into account other industrial variables (which may require the use of highly non linear relationships)
  - ⑤ developing tools for early cost estimating

Some Ideas Toward its Solution

③ Non parametric techniques

- The idea - for cost estimating a product of size x - is to consider only the data points which are in the "**vicinity**" of x.
- The vicinity can easily be defined by a "kernel" K (a function which decreases the weight given to each data point when it becomes distant from x).



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③ Non parametric techniques

- The approach is really "local" : we try to solve the question **only** for the product we have to estimate.
- Several solutions (besides the splines) have been developed :
  - ❶ **local regressions** : we consider only the data in the vicinity of our product. In this vicinity, we prepare a "**local CER**" by minimizing

$$\sum_i w_i \times (b_0 + b_1 \times x_i - y_i)^2$$

where the weight is given by a "kernel" function (here Gaussian), h being the "bandwidth"

$$w_i = \exp\left(-\frac{1}{2}\left(\frac{x - x_i}{h}\right)^2\right)$$

the main difficulty being to have a smooth algorithm when the value of the causal variable x does change.

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Choice of Additive-Error Model

- Normally, the multiplicative-error model is preferred for CERs
  - Typically, data-base cost values range over large intervals – two or three orders of magnitude
  - So the output of the CER will range over a similar interval
  - ±30% is more meaningful as a standard-error metric than ±$30,000
- For what we are considering here, however, CER output will range over a relatively short (or even zero-length) interval
  - The CER will be valid for only a small set of cost-driver values (or even only one cost-driver value)
  - Therefore, a dollar-valued standard-error metric is just as meaningful as a percentage standard-error metric
- We will therefore apply the additive-error model

Ordinary Least Squares

- OLS "best" fits a straight line $y = a+bx$ to set of data points (x_k,y_k) in two-dimensional space
  - x_k is the value of the cost driver
  - y_k is the cost
- The OLS criterion is that the coefficients a and b are selected so that the sum of squares of the differences $d_k = y_k-(a+bx_k) = y_k-a-bx_k$ between the actual costs and their estimates is as small as possible
- The mathematics results in numerical values of a and b that minimize the quantity

$$f(a,b) = \sum_{k=1}^{n} d_k^2 = \sum_{k=1}^{n} (y_k - a - bx_k)^2$$

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Weighted Least Squares

- In "weighted" least squares (WLS), the problem is the same, except that the points are not considered of equal value
- Accompanying each data point (x_k,y_k) is a "weight" w_k, so that the data set consists of "triples" (x_k,y_k,w_k), rather than pairs (x_k,y_k)
- The WLS criterion is that the coefficients a and b are selected so that the sum of squares of the weighted differences $d_k/w_k = (y_k-a-bx_k)/w_k$ is as small as possible
- The mathematics results in numerical values of a and b that minimize the quantity

$$f(a,b) = \sum_{k=1}^{n} \frac{d_k^2}{w_k^2} = \sum_{k=1}^{n} \frac{(y_k - a - bx_k)^2}{w_k^2}$$

- Weights w_k are chosen as follows:
  - Small when the data point is to contribute heavily to the CER
  - Large when the data point is to contribute only in a minor way, if at all, to the CER

The Weighted Least Squares Solution

- Applying some calculus, we can derive explicit formulas for the numerical values of ***a*** and ***b*** that minimize the quantity

$$f(a,b)=\sum_{k=1}^{n}\frac{d_k^2}{w_k^2}=\sum_{k=1}^{n}\frac{(y_k-a-bx_k)^2}{w_k^2}$$

- The resulting expressions for ***a*** and ***b*** are as follows:

$$b=\frac{\left(\sum_{k=1}^{n}w_k^{-2}\right)\left(\sum_{k=1}^{n}x_k y_k w_k^{-2}\right)-\left(\sum_{k=1}^{n}x_k w_k^{-2}\right)\left(\sum_{k=1}^{n}y_k w_k^{-2}\right)}{\left(\sum_{k=1}^{n}w_k^{-2}\right)\left(\sum_{k=1}^{n}x_k^2 w_k^{-2}\right)-\left(\sum_{k=1}^{n}x_k w_k^{-2}\right)^2}$$

$$a=\frac{\left(\sum_{k=1}^{n}y_k w_k^{-2}\right)\left(\sum_{k=1}^{n}x_k^2 w_k^{-2}\right)-\left(\sum_{k=1}^{n}x_k w_k^{-2}\right)\left(\sum_{k=1}^{n}x_k y_k w_k^{-2}\right)}{\left(\sum_{k=1}^{n}w_k^{-2}\right)\left(\sum_{k=1}^{n}x_k^2 w_k^{-2}\right)-\left(\sum_{k=1}^{n}x_k w_k^{-2}\right)^2}$$

Reference: S.A. Book, "Deriving Cost-Estimating Relationships Using Weighted Least-Squares Regression," IAA/ISPA/AIAA Space Systems Cost Methodologies and Applications Symposium, San Diego CA, 10-11 May 1990.

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Why Adapt CERs? – To Add Value

START set of Information

- **Program A**
  - Part a
    - Cost Driver 1
    - Cost Driver 2
    - ⋮
    - Cost Driver n
  - Part b
  - ⋮
  - Part n
- **Program B**
- ⋮
- **Program N**

Part a

Part a

Part a$ = a + b(cost driver)^c

CER Builders

Value added information from programs, parts and cost drivers

CER User

Weighting Information

Part a$ weighted=[a' + b'(cost driver)$^{c'}$]

Adapt \A*dapt"\, v. t. [imp. & p. p. {Adapted}; p. pr. & vb. n. {Adapting}.] [L. adaptare; ad + aptare to fit; cf. F. adapter. See {Apt}, {Adept}.]

To make suitable; to fit, or suit; to adjust; to alter so as to fit for a new use

Weighting Data Points *A Priori*

- Data points supporting CER development may not all be of equal value
  - Some may be known with greater precision than others
  - Some may be more relevant to the estimating task than others
  - Some may be very far from the cost-driver region where estimating is most commonly done
- Should all data points contribute equally to the computation of the CER?

WLS Example*

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost y
1	100	28.04	1,132.20
2	100	118.89	2,314.62
3	100	78.50	18.80
4	50	0.40	383.00
5	60	26.90	708.00
6	3000	80.10	8,771.50
7	80	15.40	1,173.50
8	60	5.50	260.20
9	70	10.50	407.70
10	200	126.36	1,386.90
11	100	22.40	345.30
12	200	82.20	3,260.60
13	50	37.80	1,115.20
14	100	48.00	730.50
Sums =	4,270.00	680.99	22,008.02
a =	408.34		
b =	12.27		

*** From the 1990 paper cited above, where the weights represented sigma values that tracked the uncertainty with which the unit costs were known.**

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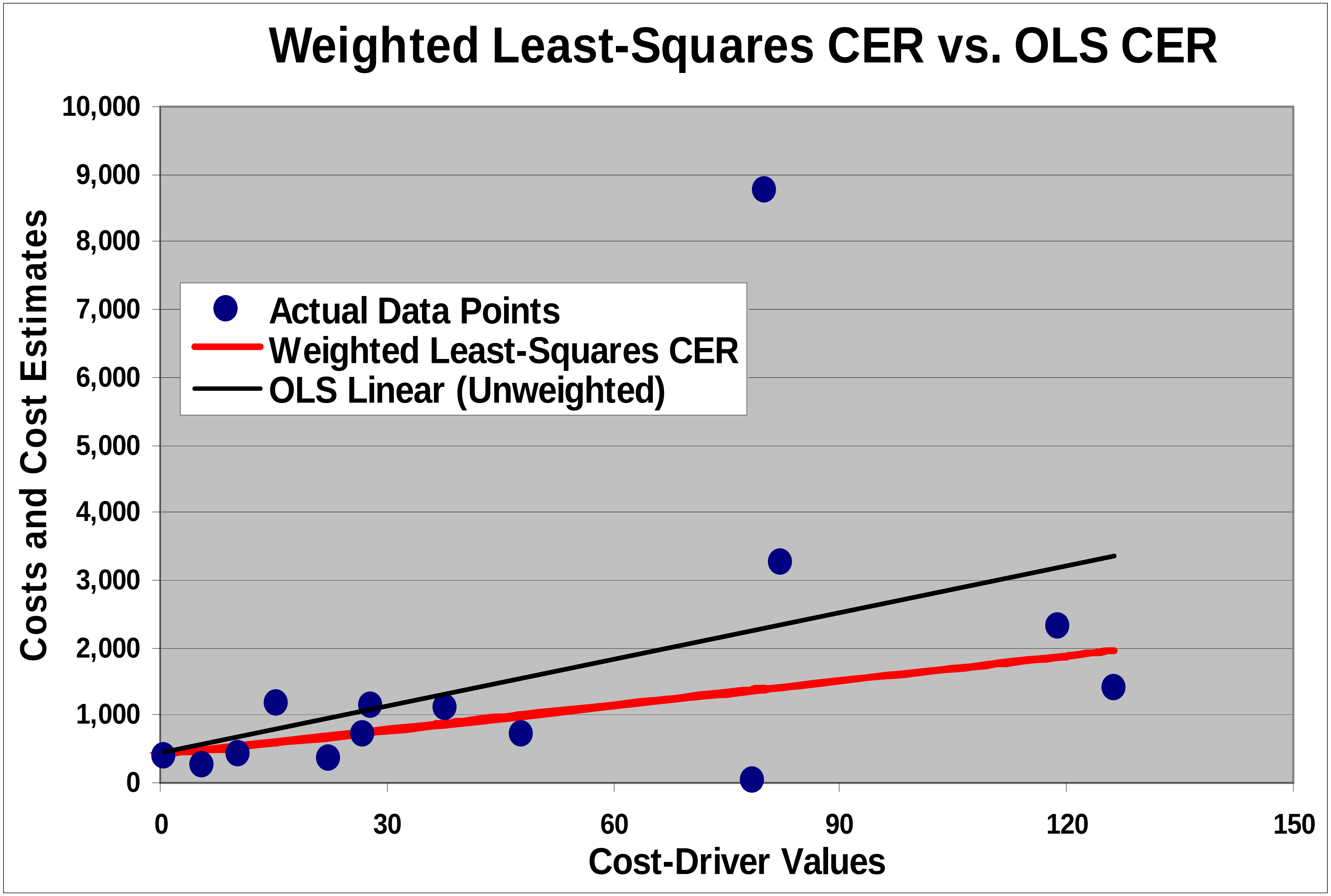
WLS Example Graphics



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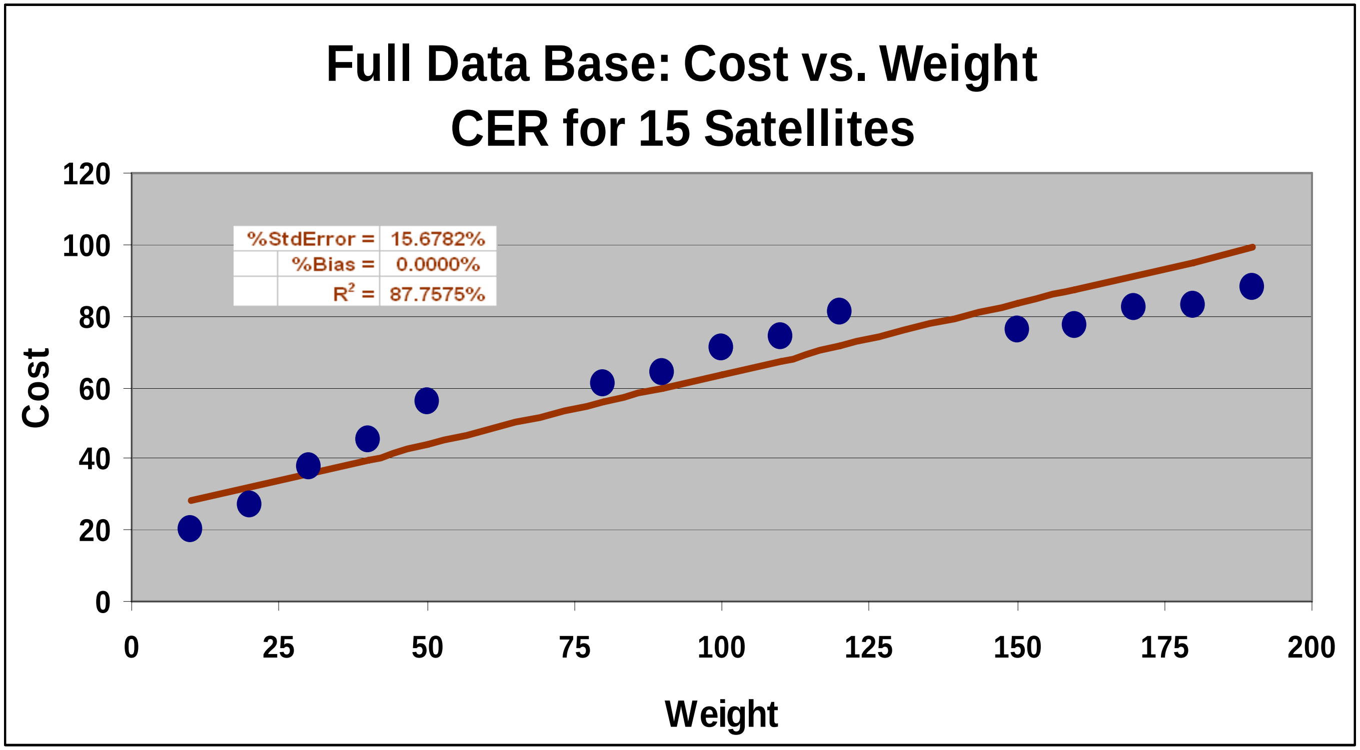
Example: Traditional CER

- Using a full data set, here is the derived CER:

Weight	Cost
10	20
20	27
30	38
40	45
50	56
80	61
90	64
100	71
110	74
120	81
150	76
160	77
170	82
180	83
190	88



- Can we reduce our estimating error by using one of three CERs, each based on the one of three data subsets into which the full data set naturally separates?

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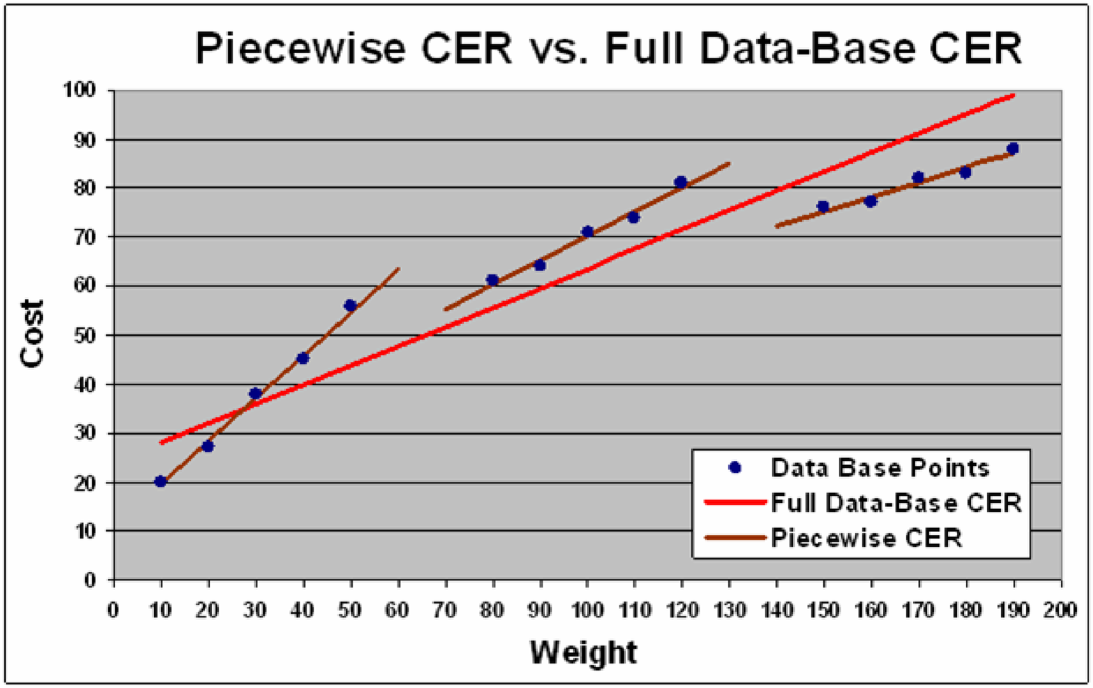
Example: Piecewise CER



- The “Piecewise CER” is composed of three distinct pieces, each of which provides better estimating capability in its region of the data set than does the traditional CER

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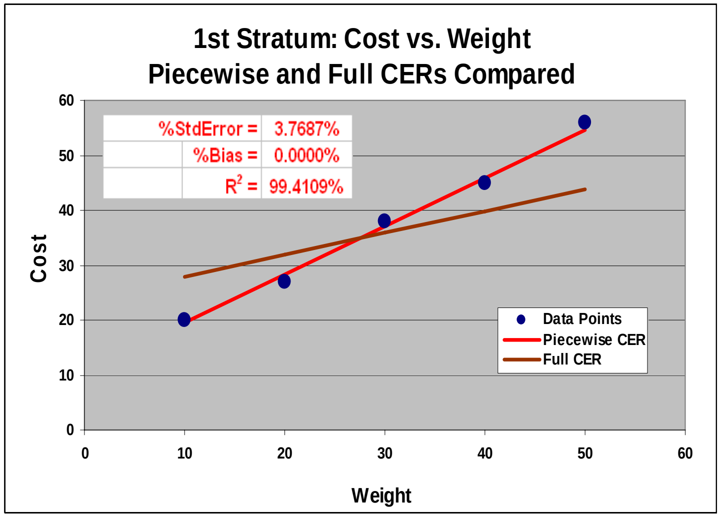
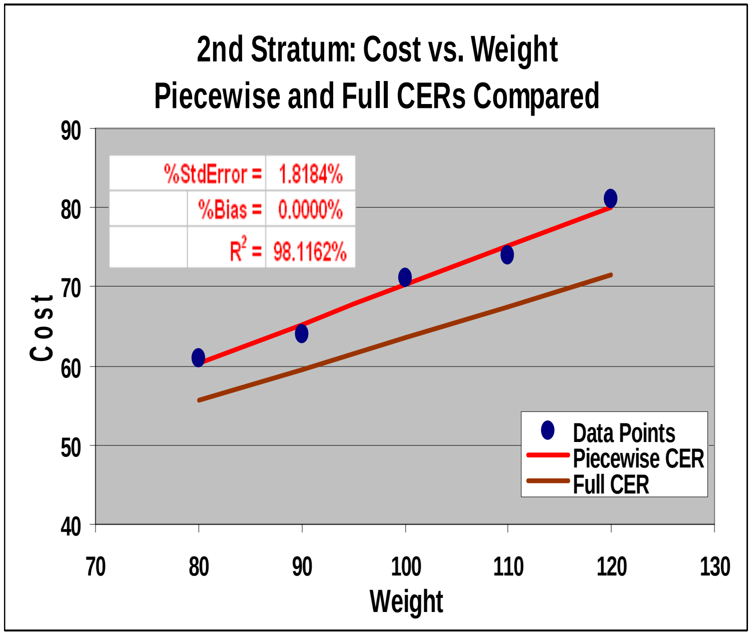
"Pieces" of the Piecewise CER

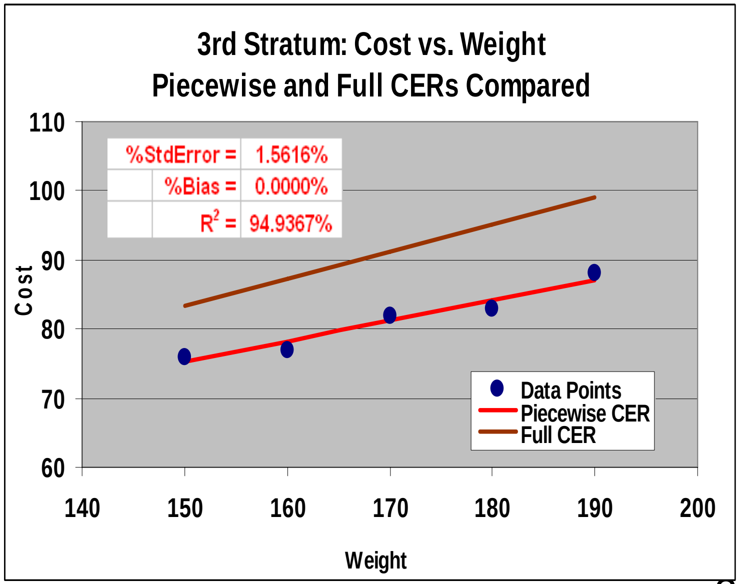
Weight	Cost
10	20
20	27
30	38
40	45
50	56



Weight	Cost
80	61
90	64
100	71
110	74
120	81



Weight	Cost
150	76
160	77
170	82
180	83
190	88



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Quality Metrics of the “Pieces”

- In this example, the traditional CER has standard error around 16.7% and R^2 around 87.8% – not bad, but the three “pieces” each have standard errors not exceeding 3.8% and R^2 values of at least 94.9%
- Therefore there may be substantial merit in seeking CERs based on portions of a data set, rather than on a full data set

A "Real" Data Set (Unweighted)

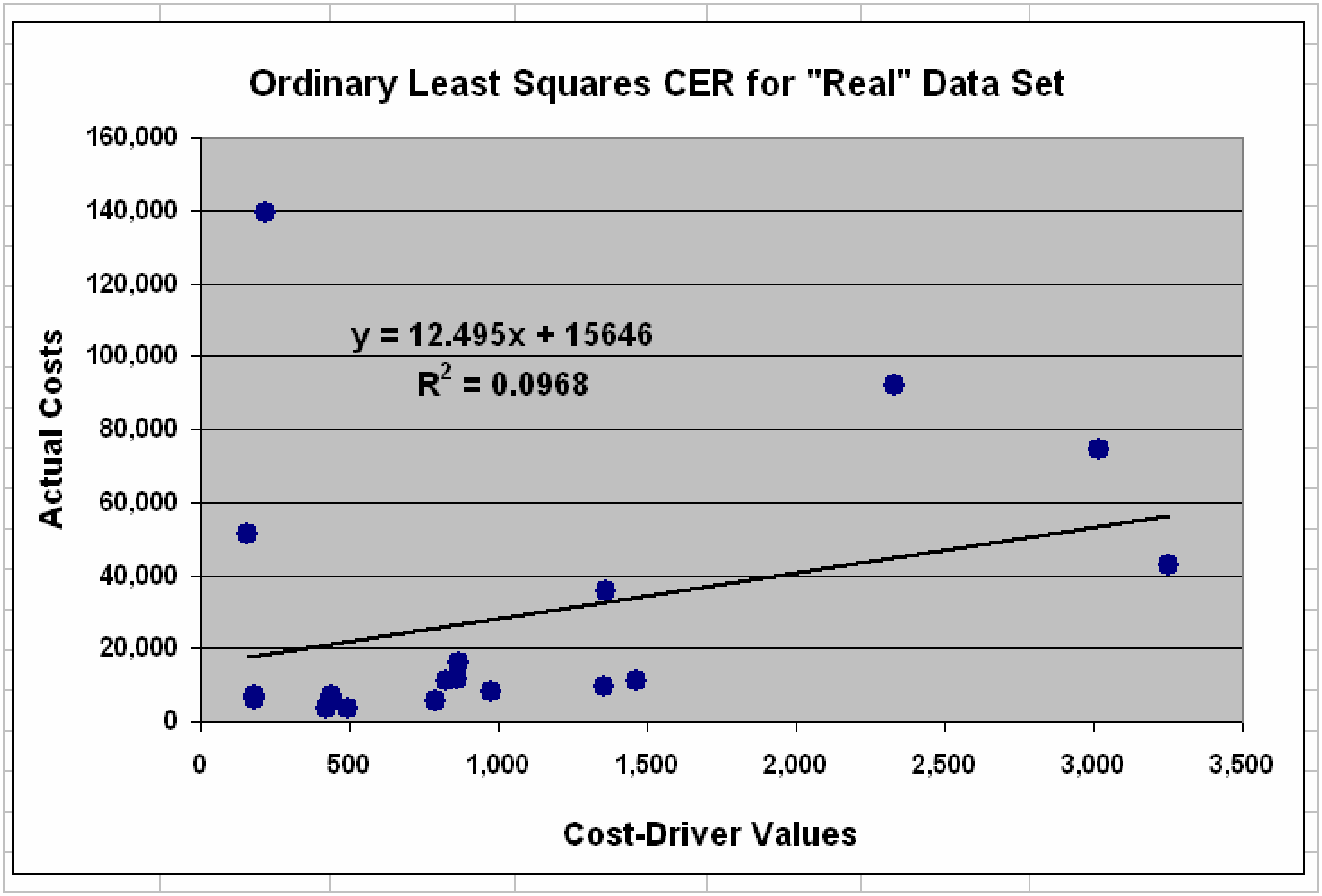
Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	1	156.12	51,367.22	17,596.30	1
2	1	179.40	5,885.00	17,887.18	1
3	1	180.30	7,060.00	17,898.42	1
4	1	217.50	139,483.12	18,363.23	1
5	1	419.14	3,386.00	20,882.67	1
6	1	437.09	6,738.00	21,106.95	1
7	1	440.93	6,812.00	21,154.93	1
8	1	494.45	3,291.34	21,823.65	1
9	1	789.90	5,723.14	25,515.22	1
10	1	826.10	10,992.00	25,967.53	1
11	1	864.30	11,590.00	26,444.83	1
12	1	869.30	15,973.00	26,507.30	1
13	1	976.50	7,970.67	27,846.74	1
14	1	1,355.80	9,524.10	32,586.00	1
15	1	1,360.90	35,927.22	32,649.72	1
16	1	1,463.21	11,238.73	33,928.06	1
17	1	2,332.10	92,059.97	44,784.62	1
18	1	3,017.73	74,649.00	53,351.39	1
19	1	3,253.00	42,915.23	56,291.03	1
Sums =	19.00	19,633.77	542,585.74	542,585.73	19.00

a =	15,645.62
b =	12.49

Std Error =	36,300.49
Bias =	0.00
R^2 =	0.10

Note: This data set is a set of actual cost data; due to proprietary issues, however, the exact descriptions of the data points cannot be revealed.

“Real” Data Set Graphics



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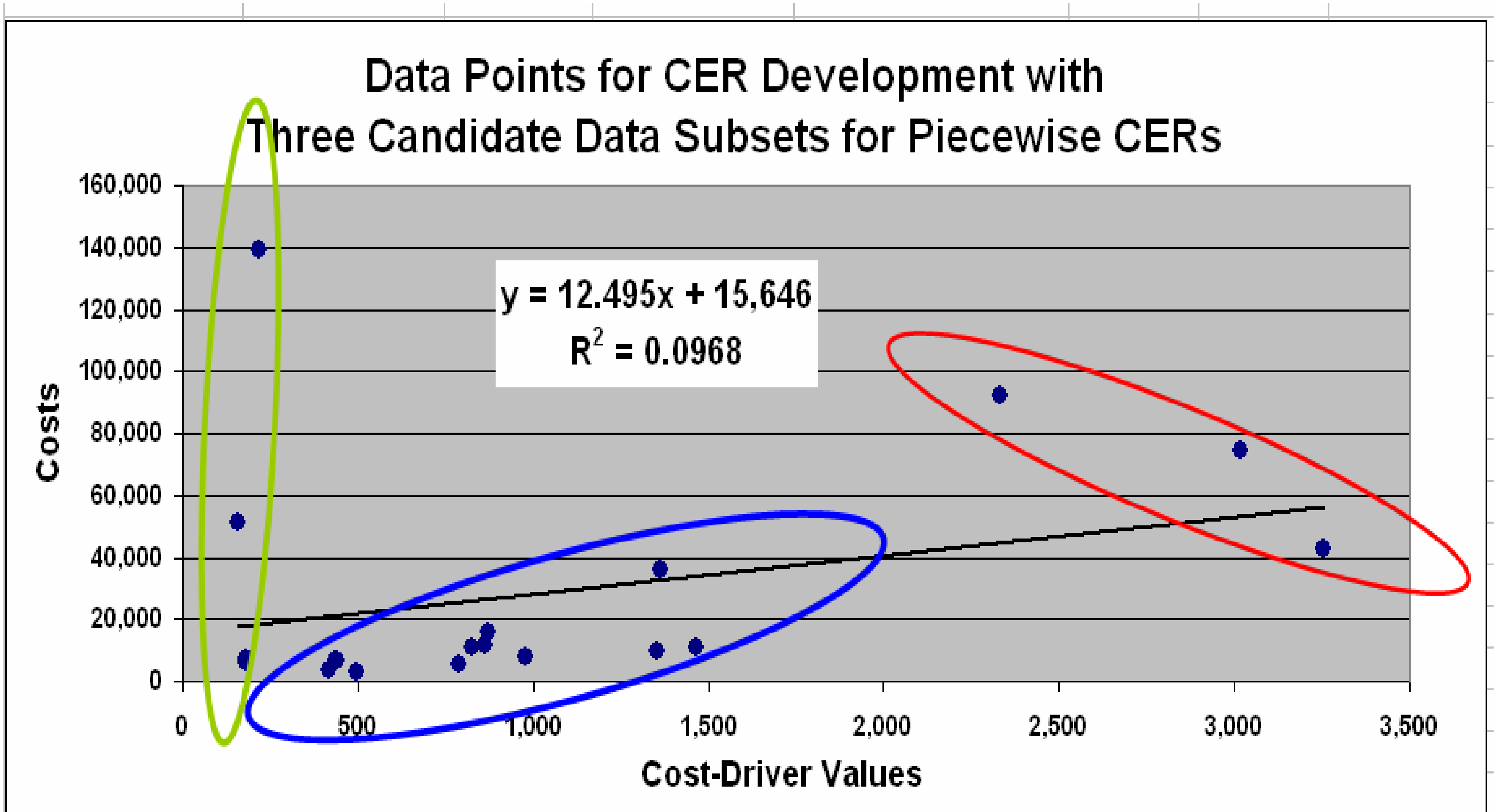
Partitioning the Real Data Set into Separate Subsets



Note: This data set is a set of actual cost data; due to proprietary issues, however, the exact descriptions of the data points cannot be revealed.

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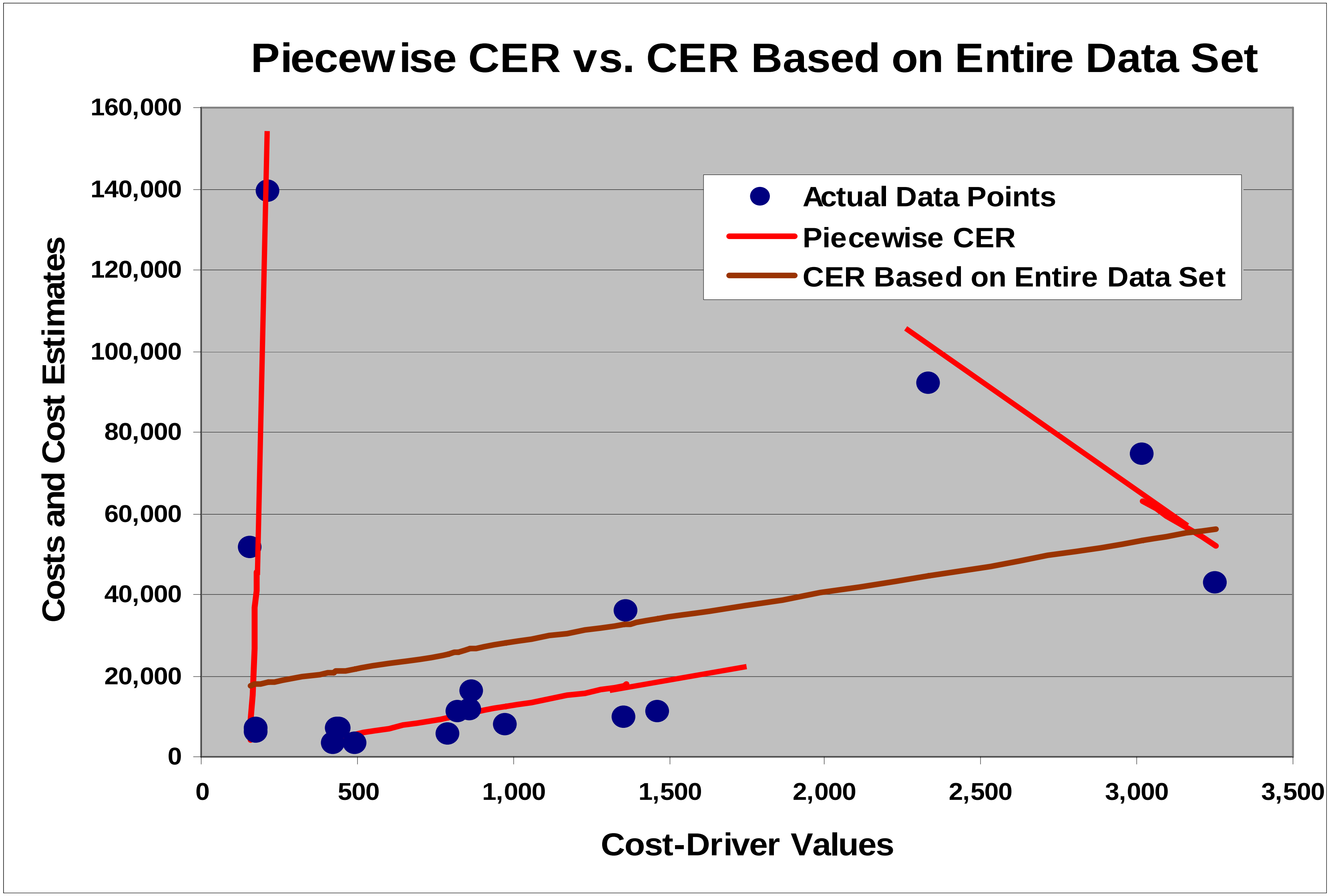
A CER for Each Subset



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Statistical Summary: Piecewise CER vs. CER Based on Entire Data Set

	1st Piece	2nd Piece	3rd Piece	Entire Data Set
b	1,719.92	13.88	-47.08	12.49
a	-264,364.19	-1,147.28	204,874.77	15,645.62
Std Error	55,129.86	7,328.40	15,069.88	36,300.49
Bias	0.0000	0.0000	0.0000	0.0000
R^2	48.47%	35.92%	81.71%	9.68%

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Applying the WLS Process to Calculate a Piecewise CER

- We can apply the WLS process by choosing weights that result in the data points associated with one "piece" being included in the computation and all other data points being excluded.
- The resulting piecewise CERs turn out the be the same as earlier

A Weighting Scheme that Produces the First Piecewise CER

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	0.00001	156.12	51,367.22	4,149.87	10000000000
2	0.00001	179.40	5,885.00	44,189.56	10000000000
3	0.00001	180.30	7,060.00	45,737.48	10000000000
4	0.00001	217.50	139,483.12	109,718.43	10000000000
5	1	423.00	3,386.00	463,161.56	1
6	1	437.09	6,738.00	487,395.20	1
7	1	440.93	6,812.00	493,999.69	1
8	1	494.45	3,291.34	586,049.69	1
9	1	789.90	5,723.14	1,094,199.44	1
10	1	826.10	10,992.00	1,156,460.47	1
11	1	864.30	11,590.00	1,222,161.33	1
12	1	869.30	15,973.00	1,230,760.92	1
13	1	976.50	7,970.67	1,415,136.12	1
14	1	1,355.80	9,524.10	2,067,500.98	1
15	1	1,360.90	35,927.22	2,076,272.56	1
16	1	1,463.21	11,238.73	2,252,237.36	1
17	1	2,332.10	92,059.97	3,746,656.83	1
18	1	3,017.73	74,649.00	4,925,884.15	1
19	1	3,253.00	42,915.23	5,330,529.23	1
Sums =	15.00	19,637.63	542,585.74	28,752,200.88	40000000015
a =	-264,363.72		[redacted]		
b =	1,719.92				

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A Weighting Scheme that Produces the Second Piecewise CER

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	1	156.12	51,367.22	4,149.87	1
2	1	179.40	5,885.00	44,189.56	1
3	1	180.30	7,060.00	45,737.48	1
4	1	217.50	139,483.12	109,718.43	1
5	0.00001	423.00	3,386.00	463,161.56	10000000000
6	0.00001	437.09	6,738.00	487,395.20	10000000000
7	0.00001	440.93	6,812.00	493,999.69	10000000000
8	0.00001	494.45	3,291.34	586,049.69	10000000000
9	0.00001	789.90	5,723.14	1,094,199.44	10000000000
10	0.00001	826.10	10,992.00	1,156,460.47	10000000000
11	0.00001	864.30	11,590.00	1,222,161.33	10000000000
12	0.00001	869.30	15,973.00	1,230,760.92	10000000000
13	0.00001	976.50	7,970.67	1,415,136.12	10000000000
14	0.00001	1,355.80	9,524.10	2,067,500.98	10000000000
15	0.00001	1,360.90	35,927.22	2,076,272.56	10000000000
16	0.00001	1,463.21	11,238.73	2,252,237.36	10000000000
17	1	2,332.10	92,059.97	3,746,656.83	1
18	1	3,017.73	74,649.00	4,925,884.15	1
19	1	3,253.00	42,915.23	5,330,529.23	1
Sums =	7.00	19,637.63	542,585.74	28,752,200.88	1.2E+11
a =	-1,161.86		[redacted]		
b =	13.89				

A Weighting Scheme that Produces the Third Piecewise CER

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	1	156.12	51,367.22	4,149.87	1
2	1	179.40	5,885.00	44,189.56	1
3	1	180.30	7,060.00	45,737.48	1
4	1	217.50	139,483.12	109,718.43	1
5	1	423.00	3,386.00	463,161.56	1
6	1	437.09	6,738.00	487,395.20	1
7	1	440.93	6,812.00	493,999.69	1
8	1	494.45	3,291.34	586,049.69	1
9	1	789.90	5,723.14	1,094,199.44	1
10	1	826.10	10,992.00	1,156,460.47	1
11	1	864.30	11,590.00	1,222,161.33	1
12	1	869.30	15,973.00	1,230,760.92	1
13	1	976.50	7,970.67	1,415,136.12	1
14	1	1,355.80	9,524.10	2,067,500.98	1
15	1	1,360.90	35,927.22	2,076,272.56	1
16	1	1,463.21	11,238.73	2,252,237.36	1
17	0.00001	2,332.10	92,059.97	3,746,656.83	10000000000
18	0.00001	3,017.73	74,649.00	4,925,884.15	10000000000
19	0.00001	3,253.00	42,915.23	5,330,529.23	10000000000
Sums =	16.00	19,637.63	542,585.74	28,752,200.88	30000000016
a =	204,874.77		[redacted]		
b =	-47.08				

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The Method

- **The "X-Distance" Method calls for weighting points by their distance along the x-axis from a cost-driver value of interest**
- **Given:**
  - x_0 **is the value of the cost driver for the Product of Interest (PoI) being investigated**
  - x_k **is the** x **value of** k^{th} **data point**
  - D_k **is the squared (quadratic) distance from the** x_0 **value to the data point** x_k **so that** $D_k^2 = (x_0 - x_k)^2$
- **Then the weight of the data point** x_k **is the reciprocal of its distance, namely** D_k^{-2}
- **Causal factors considered for doing it this way**
  - **Economies of scale**
  - **Relevant physics of design and build**
  - **Better approximation of other variables**

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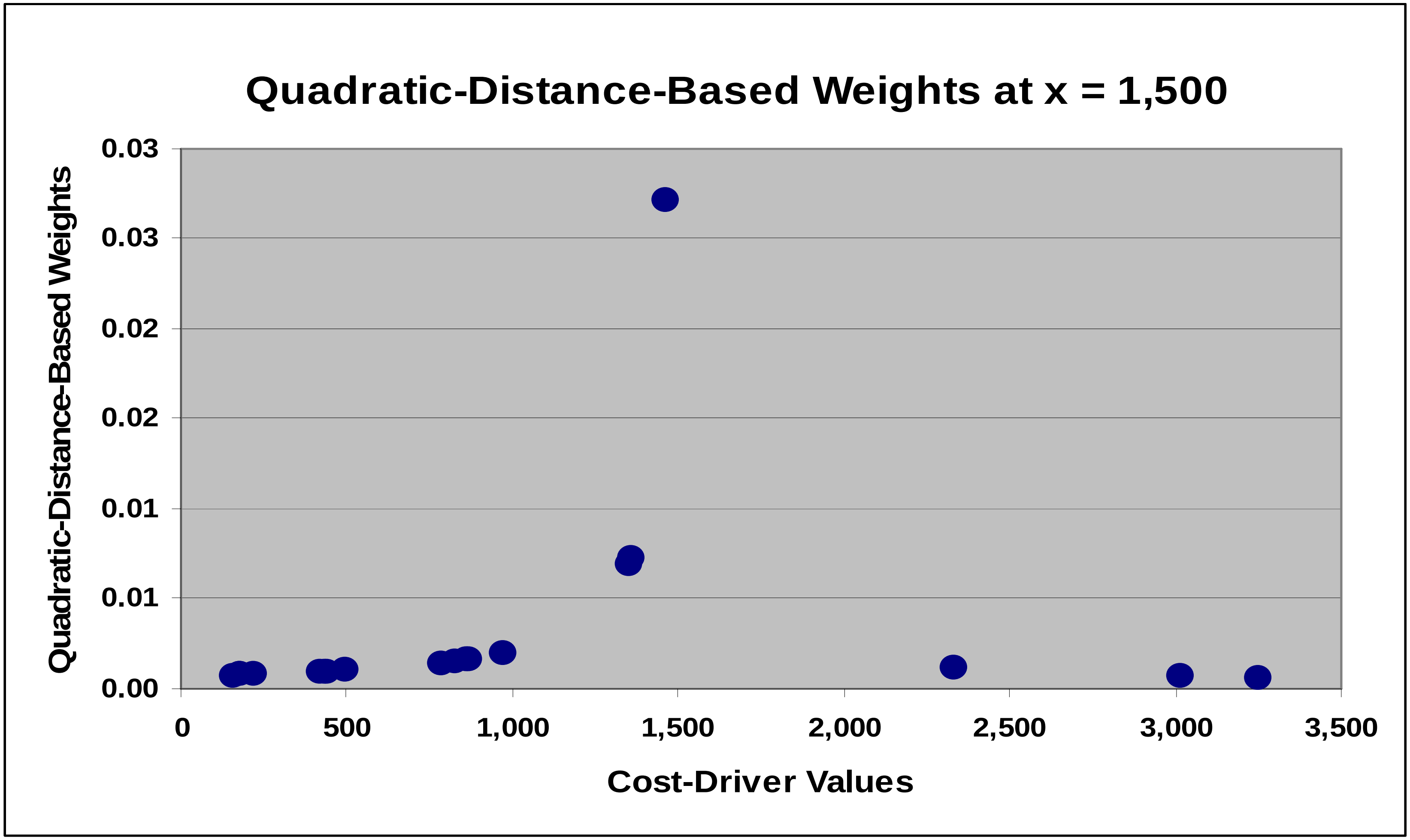
Why D_k^{-2}?

- There are an infinite number of ways to define the weighting as distance from x_0
- We chose the squared (quadratic) distance, because OLS calculation uses the squares of residuals for best fit – This forces the CER to pass through the point $(\bar{x}, \bar{y})$, where $\bar{x}$ is the mean of the cost-driver values and $\bar{y}$ is the mean of the cost-driver values
- However, other weighting schemes can be used

Quadratic-Distance-Based Weights at x = 1,500



Weighting Points by their Squared X-Distance from x = 1,500

- **Quadratic-Distance Weighting anchored at x = 1,500**

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	1343.88	156.12	51,367.22	13,969.64	0.0000005537
2	1320.6	179.40	5,885.00	13,949.82	0.0000005734
3	1319.7	180.30	7,060.00	13,949.06	0.0000005742
4	1282.5	217.50	139,483.12	13,917.39	0.0000006080
5	1080.86	419.14	3,386.00	13,745.72	0.0000008560
6	1062.91	437.09	6,738.00	13,730.44	0.0000008851
7	1059.07	440.93	6,812.00	13,727.17	0.0000008916
8	1005.55	494.45	3,291.34	13,681.61	0.0000009890
9	710.1	789.90	5,723.14	13,430.08	0.0000019832
10	673.9	826.10	10,992.00	13,399.26	0.0000022020
11	635.7	864.30	11,590.00	13,366.74	0.0000024745
12	630.7	869.30	15,973.00	13,362.48	0.0000025139
13	523.5	976.50	7,970.67	13,271.22	0.0000036489
14	144.2	1,355.80	9,524.10	12,948.30	0.0000480916
15	139.1	1,360.90	35,927.22	12,943.96	0.0000516828
16	36.79	1,463.21	11,238.73	12,856.86	0.0007388230
17	832.1	2,332.10	92,059.97	12,117.13	0.0000014443
18	1517.73	3,017.73	74,649.00	11,533.42	0.0000004341
19	1753	3,253.00	42,915.23	11,333.12	0.0000003254
Sums =	17,071.89	19,633.77	542,585.74	251,233.42	0.0008595547
a =	14,102.56		[redacted]		
b =	-0.85				

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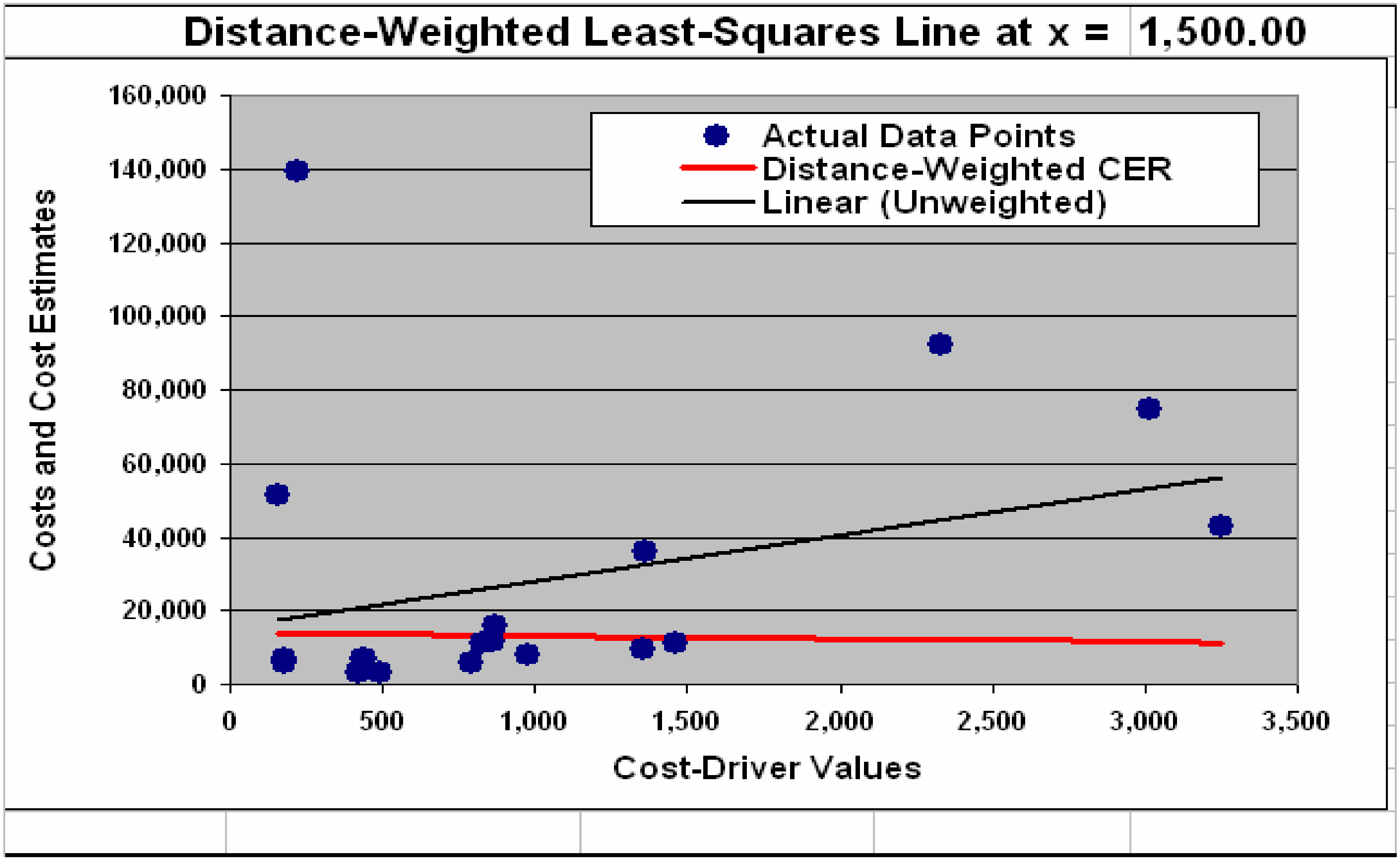
Resulting X-Distance CER for Real Data Set at x = 1,500



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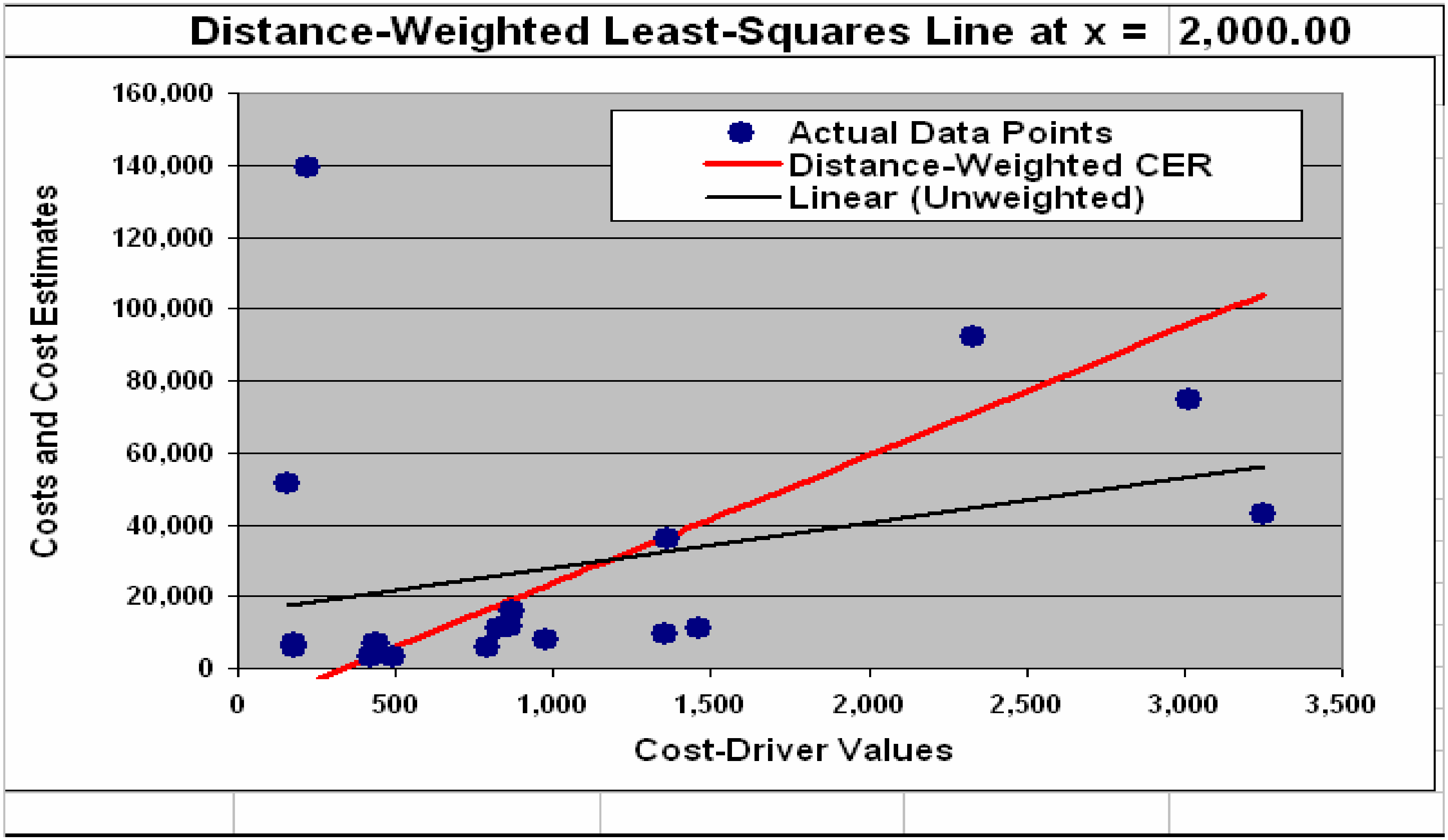
Weighting Points by their Squared X-Distance from x = 2,000

- **Quadratic-Distance Weighting anchored at x = 2,000**

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	1843.88	156.12	51,367.22	-6,410.48	0.0000002941
2	1820.6	179.40	5,885.00	-5,580.04	0.0000003017
3	1819.7	180.30	7,060.00	-5,547.93	0.0000003020
4	1782.5	217.50	139,483.12	-4,220.94	0.0000003147
5	1580.86	419.14	3,386.00	2,971.93	0.0000004001
6	1562.91	437.09	6,738.00	3,612.24	0.0000004094
7	1559.07	440.93	6,812.00	3,749.22	0.0000004114
8	1505.55	494.45	3,291.34	5,658.37	0.0000004412
9	1210.1	789.90	5,723.14	16,197.62	0.0000006829
10	1173.9	826.10	10,992.00	17,488.94	0.0000007257
11	1135.7	864.30	11,590.00	18,851.60	0.0000007753
12	1130.7	869.30	15,973.00	19,029.96	0.0000007822
13	1023.5	976.50	7,970.67	22,853.98	0.0000009546
14	644.2	1,355.80	9,524.10	36,384.30	0.0000024097
15	639.1	1,360.90	35,927.22	36,566.23	0.0000024483
16	536.79	1,463.21	11,238.73	40,215.82	0.0000034705
17	332.1	2,332.10	92,059.97	71,210.71	0.0000090670
18	1017.73	3,017.73	74,649.00	95,668.39	0.0000009655
19	1253	3,253.00	42,915.23	104,060.90	0.0000006369
Sums =	23,571.89	19,633.77	542,585.74	472,760.81	0.0000257931
a =	-11,979.56		[redacted]		
b =	35.67				

Resulting X-Distance CER for Real Data Set at x = 2,000



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MCR

Weighting Points by their Squared X-Distance from x = 100

- **Quadratic- Distance Weighting anchored at x = 100**

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	56.12	156.12	51,367.22	39,455.23	0.0003175153
2	79.4	179.40	5,885.00	38,890.21	0.0001586204
3	80.3	180.30	7,060.00	38,868.36	0.0001550847
4	117.5	217.50	139,483.12	37,965.50	0.0000724310
5	319.14	419.14	3,386.00	33,071.58	0.0000098183
6	337.09	437.09	6,738.00	32,635.93	0.0000088005
7	340.93	440.93	6,812.00	32,542.73	0.0000086034
8	394.45	494.45	3,291.34	31,243.77	0.0000064271
9	689.9	789.90	5,723.14	24,073.03	0.0000021010
10	726.1	826.10	10,992.00	23,194.43	0.0000018967
11	764.3	864.30	11,590.00	22,267.30	0.0000017119
12	769.3	869.30	15,973.00	22,145.95	0.0000016897
13	876.5	976.50	7,970.67	19,544.14	0.0000013017
14	1255.8	1,355.80	9,524.10	10,338.32	0.0000006341
15	1260.9	1,360.90	35,927.22	10,214.54	0.0000006290
16	1363.21	1,463.21	11,238.73	7,731.42	0.0000005381
17	2232.1	2,332.10	92,059.97	-13,357.03	0.0000002007
18	2917.73	3,017.73	74,649.00	-29,997.66	0.0000001175
19	3153	3,253.00	42,915.23	-35,707.79	0.0000001006
Sums =	17,733.77	19,633.77	542,585.74	345,119.93	0.0007482216
a =	43,244.35				
b =	-24.27				

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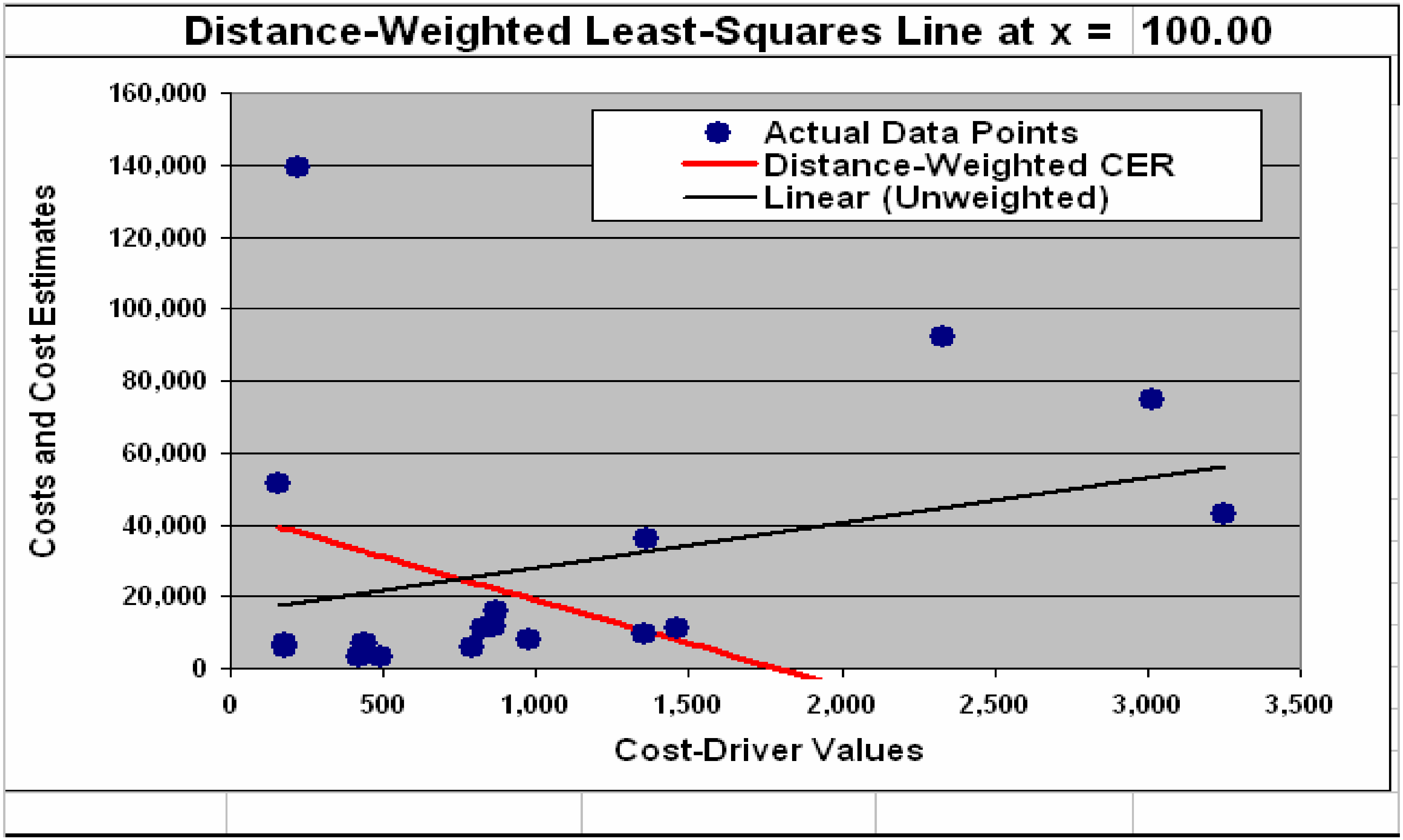
Resulting X-Distance CER for Real Data Set at x = 100



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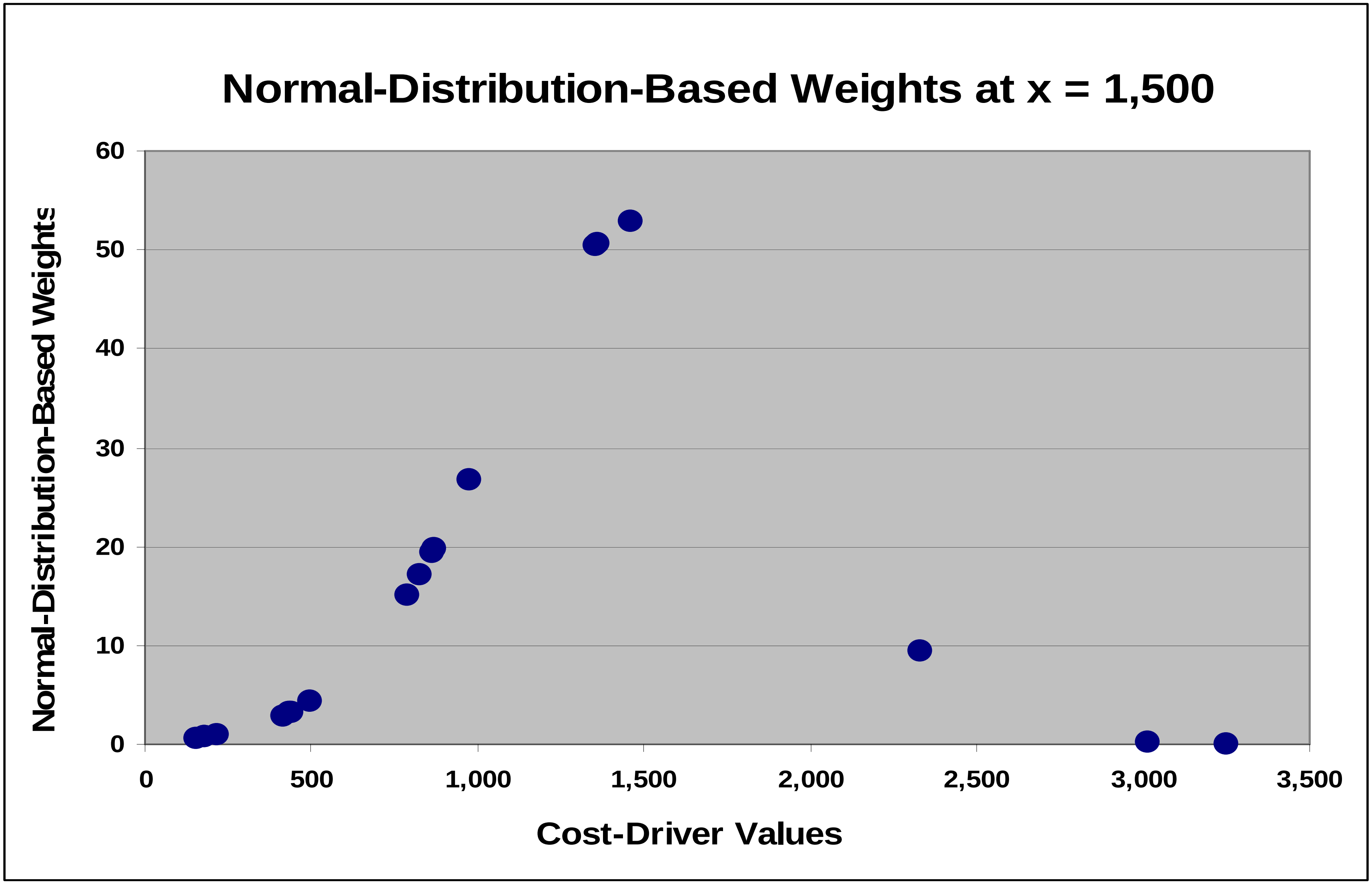
Using Weights Based on the Normal Distribution

- Weighting by the normal distribution using the exponentiated "distance" $(2\pi\sigma)^{½}exp\{(x-x_0)^2/2\sigma^2\}$ **instead of the quadratic distance** $(x-x_0)^2$ **provides a** dramatic fall-off in weights $(1/w)$ as we move from x_0 to the ends of the cost-driver range
- Rate of weighting fall-off is controllable by choice of σ
  - **One reasonable way to define σ is as $1/3$ the minimum half-range of the x values**
  - **Given x_0, we calculate l(smallest x) – x_0l and l(largest x) – x_0l and define the smaller of those numbers of the minimum half-range and consider it to be the 3σ value of the distribution of cost-driver values**
  - **Smaller values of σ increase the rate of fall-off, larger values decrease it**

Normal-Distribution-Based Weights at x = 1,500



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Weighting Points Based on the Normal Distribution at x = 1,500

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights 1/w²
1	1.696741243	156.12	51,367.22	-6,298.97	0.3474
2	1.453759708	179.40	5,885.00	-5,811.44	0.4732
3	1.445177582	180.30	7,060.00	-5,792.60	0.4788
4	1.135454949	217.50	139,483.12	-5,013.55	0.7756
5	0.346327824	419.14	3,386.00	-790.79	8.3373
6	0.314663696	437.09	6,738.00	-414.88	10.0997
7	0.308339441	440.93	6,812.00	-334.46	10.5182
8	0.234129625	494.45	3,291.34	786.36	18.2426
9	0.066212706	789.90	5,723.14	6,973.69	228.0958
10	0.058442153	826.10	10,992.00	7,731.80	292.7842
11	0.051593055	864.30	11,590.00	8,531.78	375.6795
12	0.050785444	869.30	15,973.00	8,636.49	387.7229
13	0.037311968	976.50	7,970.67	10,881.48	718.2964
14	0.019851425	1,355.80	9,524.10	18,824.81	2,537.5618
15	0.019780087	1,360.90	35,927.22	18,931.62	2,555.8985
16	0.018912768	1,463.21	11,238.73	21,074.20	2,795.6950
17	0.105813403	2,332.10	92,059.97	39,270.56	89.3138
18	5.860855149	3,017.73	74,649.00	53,629.07	0.0291
19	39.87109487	3,253.00	42,915.23	58,556.11	0.0006
Sums =	53.09524709	19,633.77	542,585.74	229,371.26	10,030.3505

a = -9,568.45

b = 20.94

[redacted]

Input Cell:

μ = Cost-driver value at which estimate is to be made = 1,500.00

Minimum Half-Range of Cost-Driver Interval = 1,343.88 = 3σ

σ = 447.96

Weight = Height of Normal Density Function at x Value of Data Point

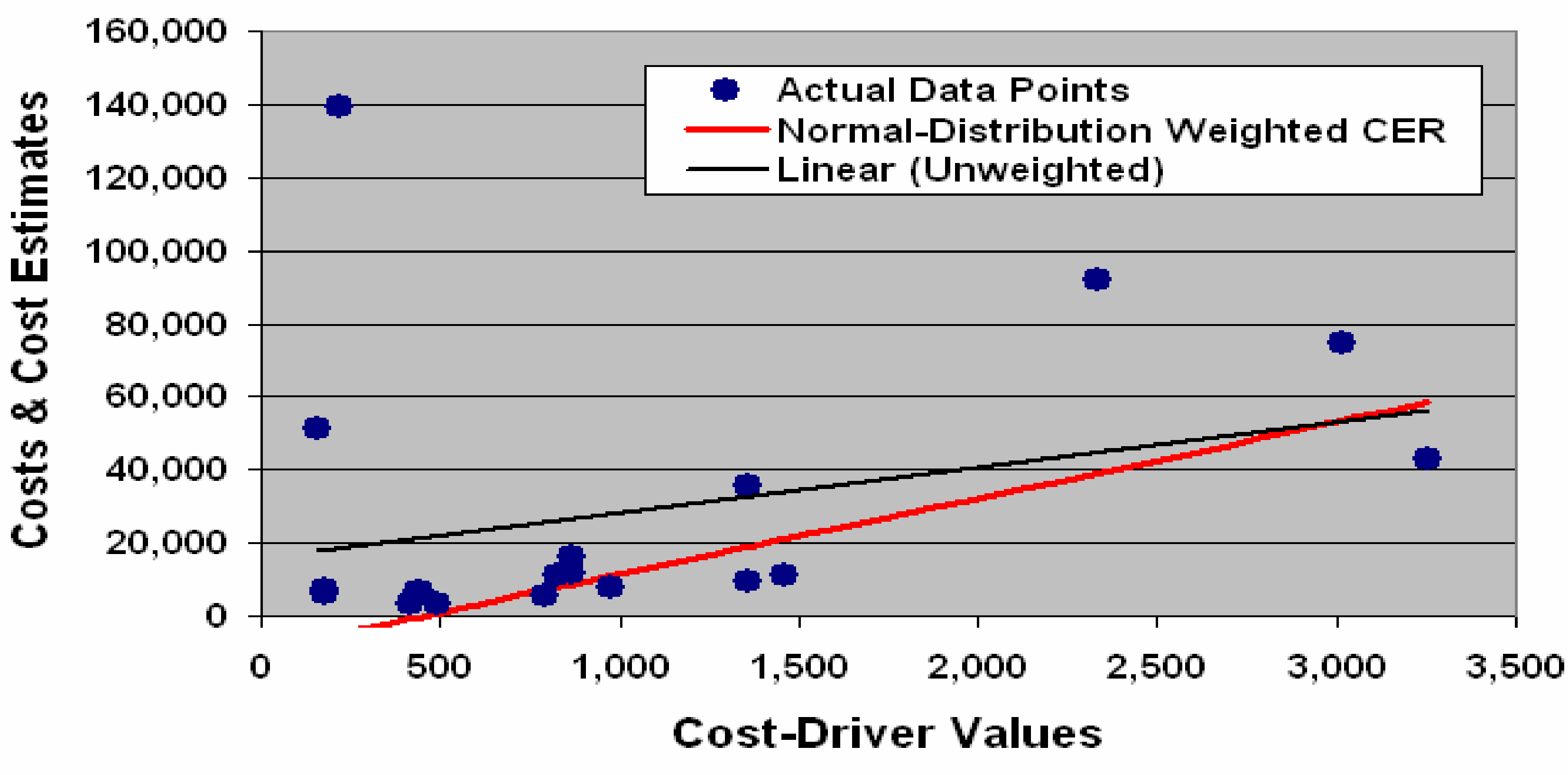
= $(2\pi\sigma)^{1/2}\exp((x-\mu)^2/2\sigma^2)$

THE AEROSPACE CORPORATION

MCR

The CER at x = 1,500 Based on Normal-Distribution Weighting

Normal-Distribution-Weighted Least-Squares Line at x =	1,500.00



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MCR

Weighting Points Based on the Normal Distribution at x = 2,000

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	333.1461611	156.12	51,367.22	-80,040.34	0.0000
2	260.8814801	179.40	5,885.00	-78,211.83	0.0000
3	258.4431316	180.30	7,060.00	-78,141.14	0.0000
4	176.019744	217.50	139,483.12	-75,219.31	0.0000
5	25.19779814	419.14	3,386.00	-59,381.71	0.0016
6	21.43475306	437.09	6,738.00	-57,971.85	0.0022
7	20.71073389	440.93	6,812.00	-57,670.24	0.0023
8	12.94278117	494.45	3,291.34	-53,466.56	0.0060
9	1.298038399	789.90	5,723.14	-30,260.76	0.5935
10	1.013588129	826.10	10,992.00	-27,417.47	0.9734
11	0.787114898	864.30	11,590.00	-24,417.09	1.6141
12	0.761960079	869.30	15,973.00	-24,024.37	1.7224
13	0.393073256	976.50	7,970.67	-15,604.46	6.4722
14	0.064131798	1,355.80	9,524.10	14,187.26	243.1382
15	0.062939966	1,360.90	35,927.22	14,587.83	252.4335
16	0.044583293	1,463.21	11,238.73	22,623.66	503.1016
17	0.026778307	2,332.10	92,059.97	90,869.71	1,394.5490
18	0.380025322	3,017.73	74,649.00	144,721.79	6.9243
19	1.757196487	3,253.00	42,915.23	163,200.82	0.3239
Sums =	1115.366013	19,633.77	542,585.74	-211,636.04	2,411.8581

a =	-92,302.62
b =	78.54

[redacted]

Input Cell:

μ = Cost-driver value at which estimate is to be made = 2,000.00

Minimum Half-Range of Cost-Driver Interval = 1,253.00 = 3σ

σ = 417.67

Weight = Height of Normal Density Function at x Value of Data Point

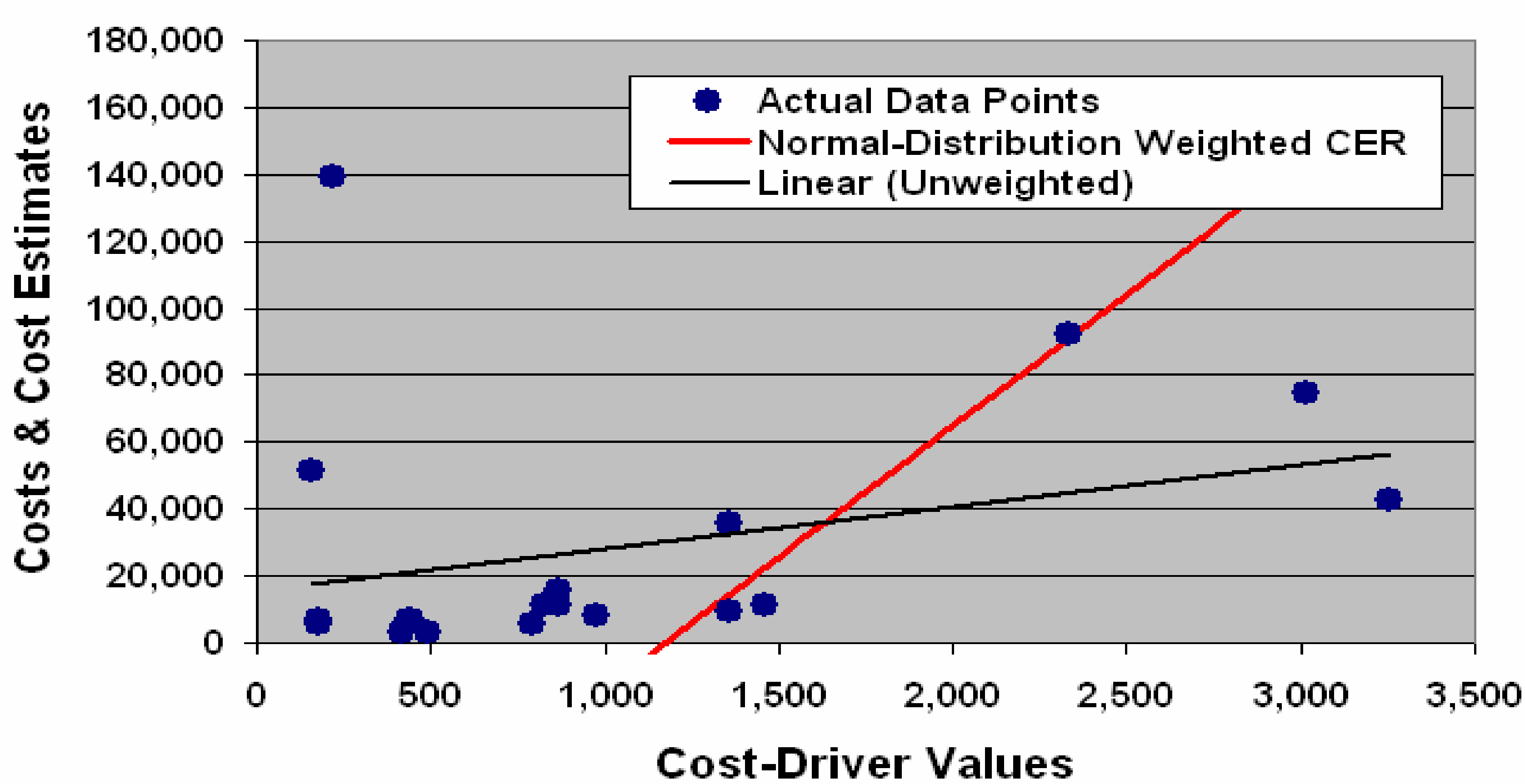
= $(2\pi\sigma)^{1/2}\exp((x-\mu)^2/2\sigma^2)$

THE AEROSPACE CORPORATION

MCR

The CER at x = 2,000 Based on Normal-Distribution Weighting

Normal-Distribution-Weighted Least-Squares Line at x =	2,000.00



THE AEROSPACE CORPORATION

MCR

Weighting Points Based on the Normal Distribution at x = 800

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	2.451282665	156.12	51,367.22	637.78	0.1664
2	1.780851758	179.40	5,885.00	970.38	0.3153
3	1.759404524	180.30	7,060.00	983.24	0.3230
4	1.082799943	217.50	139,483.12	1,514.72	0.8529
5	0.131477141	419.14	3,386.00	4,395.57	57.8495
6	0.113740613	437.09	6,738.00	4,652.02	77.2981
7	0.110368855	440.93	6,812.00	4,706.89	82.0932
8	0.075018502	494.45	3,291.34	5,471.53	177.6901
9	0.027261459	789.90	5,723.14	9,692.65	1,345.5561
10	0.027433387	826.10	10,992.00	10,209.85	1,328.7435
11	0.028481188	864.30	11,590.00	10,755.61	1,232.7750
12	0.028688446	869.30	15,973.00	10,827.05	1,215.0271
13	0.038187308	976.50	7,970.67	12,358.62	685.7438
14	0.778535331	1,355.80	9,524.10	17,777.72	1.6498
15	0.828180872	1,360.90	35,927.22	17,850.58	1.4580
16	3.224755802	1,463.21	11,238.73	19,312.30	0.0962
17	3164586165	2,332.10	92,059.97	31,726.21	0.0000
18	4.16791E+21	3,017.73	74,649.00	41,521.87	0.0000
19	6.30987E+26	3,253.00	42,915.23	44,883.19	0.0000
Sums =	6.30991E+26	19,633.77	542,585.74	250,247.78	6,207.6379

a = -1,592.72

b = 14.29

[redacted]

Input Cell:

μ = Cost-driver value at which estimate is to be made = 800.00

Minimum Half-Range of Cost-Driver Interval = 643.88 = 3σ

σ = 214.63

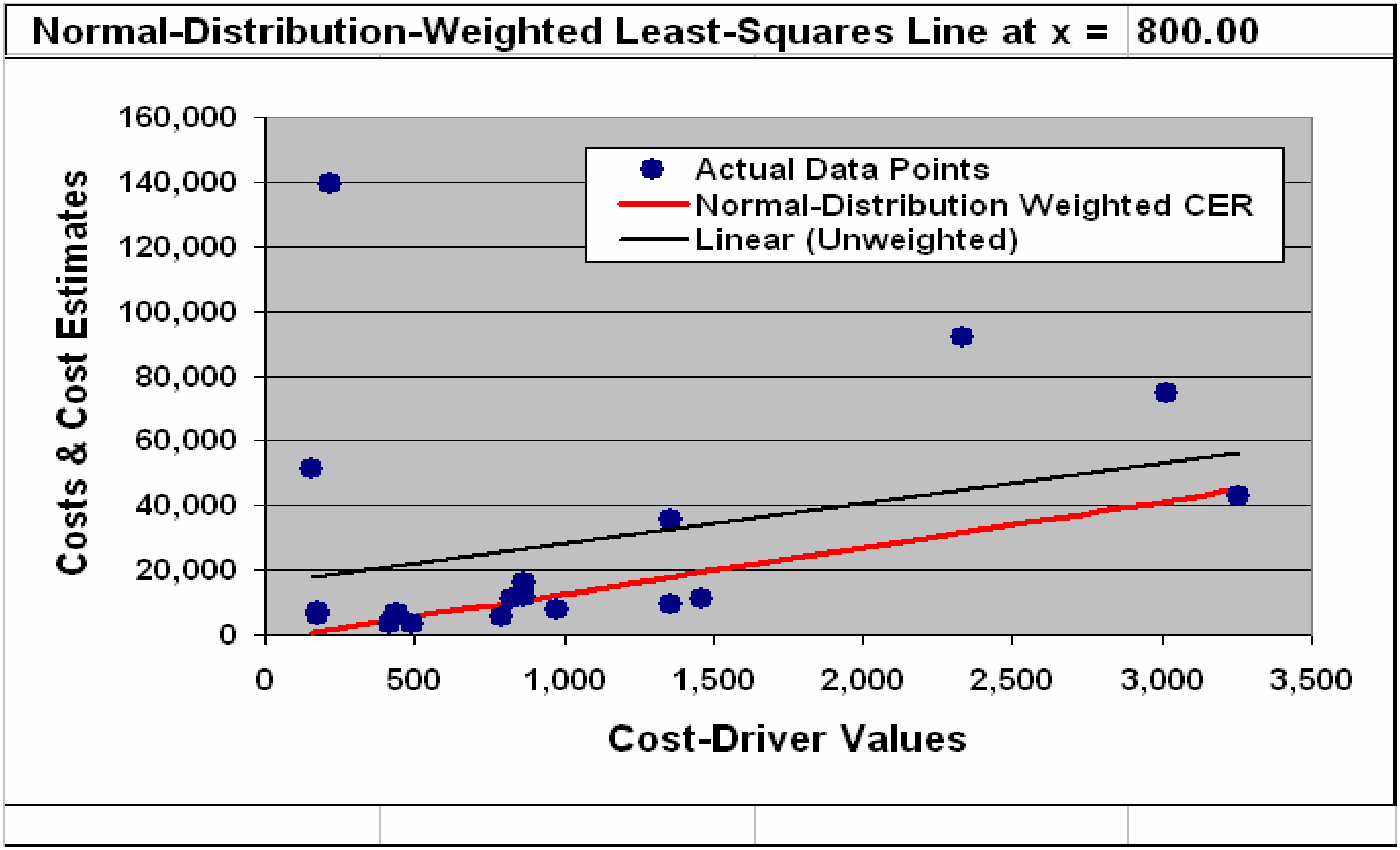
Weight = Height of Normal Density Function at x Value of Data Point

= $(2\pi\sigma)^{\frac{1}{2}}\exp((x-\mu)^2/2\sigma^2)$

THE AEROSPACE CORPORATION

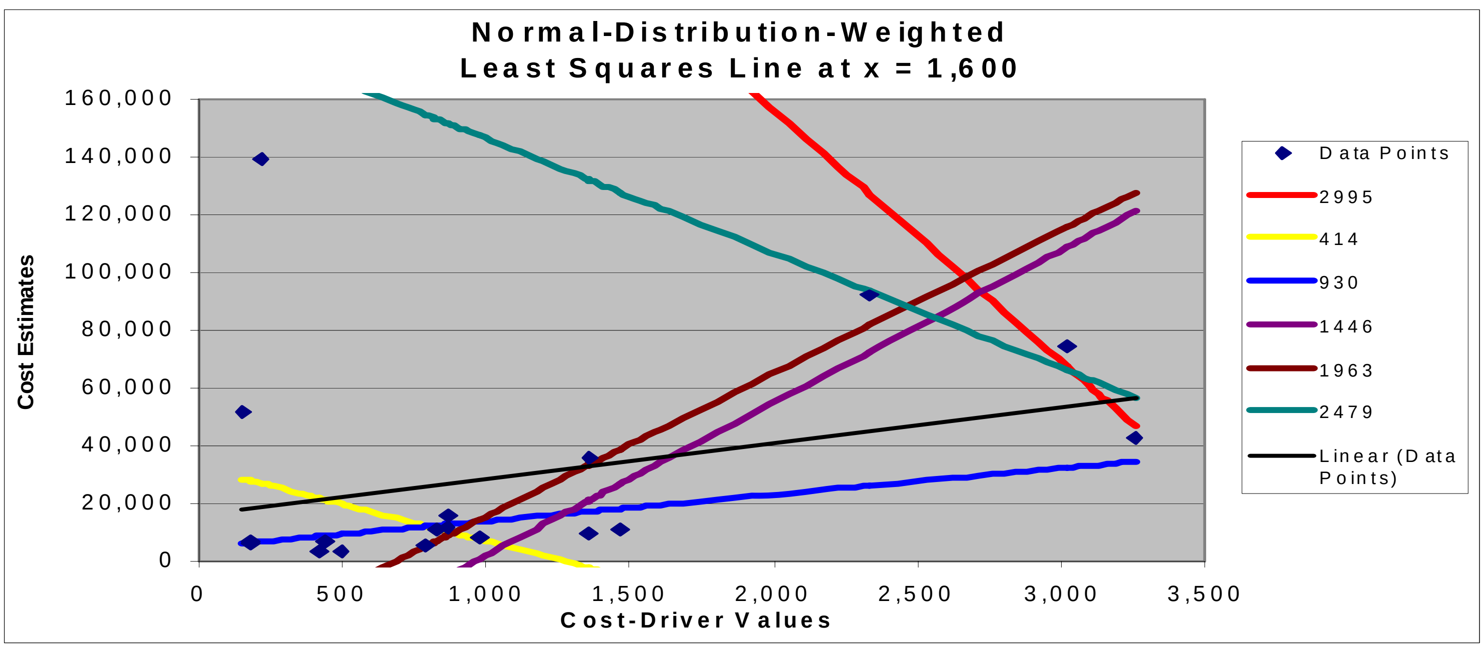
MCR

The CER at x = 800 Based on Normal-Distribution Weighting



Variety of CERs Generated at Different *x* Values

- Range is divided into 6 equal bins, with point x_0 set at the midpoint of each
- Residuals for each data point are multiplied by the normally-distributed weighting values
- A new CER is calculated for each of 6 bins
- These 6 plus the original OLS CER are shown



THE AEROSPACE CORPORATION

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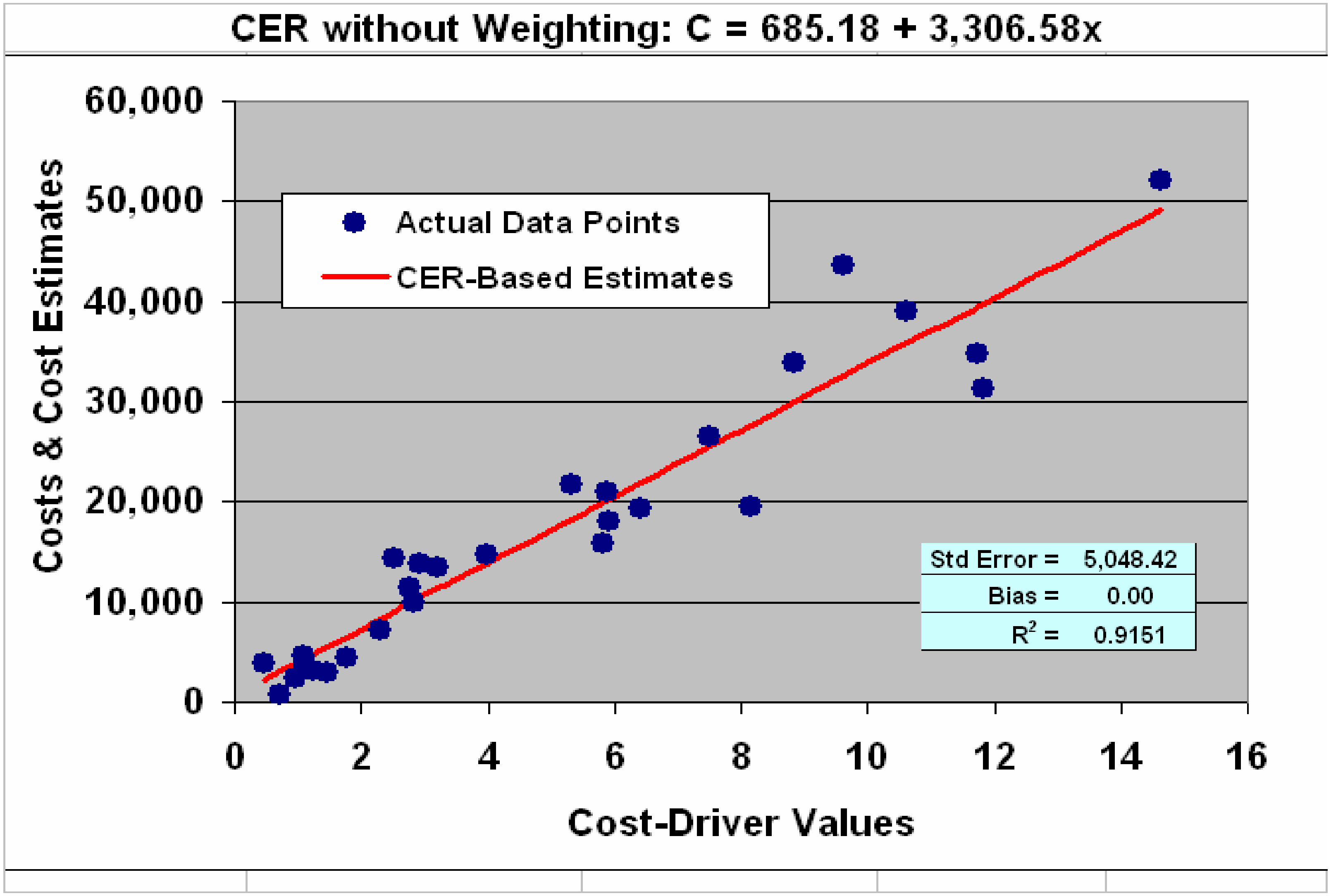
A "Good" Unweighted Data Set

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	1	0.45	3,898.71	2,186.36	1
2	1	0.73	726.14	3,098.98	1
3	1	0.96	2,438.27	3,859.49	1
4	1	1.10	4,688.87	4,309.18	1
5	1	1.14	3,620.85	4,441.45	1
6	1	1.25	3,186.70	4,825.01	1
7	1	1.46	2,870.88	5,512.78	1
8	1	1.76	4,471.57	6,504.75	1
9	1	2.30	7,100.55	8,290.30	1
10	1	2.52	14,413.94	9,024.36	1
11	1	2.78	11,497.04	9,877.46	1
12	1	2.82	9,985.56	10,003.11	1
13	1	2.94	13,892.95	10,405.85	1
14	1	3.22	13,502.21	11,325.74	1
15	1	3.99	14,761.26	13,885.03	1
16	1	5.31	21,707.74	18,236.48	1
17	1	5.82	15,873.60	19,929.45	1
18	1	5.89	20,944.67	20,147.68	1
19	1	5.92	18,107.61	20,246.88	1
20	1	6.41	19,363.30	21,873.72	1
21	1	7.51	26,483.44	25,517.56	1
22	1	8.16	19,557.46	27,666.84	1
23	1	8.85	33,844.69	29,962.26	1
24	1	9.62	43,634.41	32,494.44	1
25	1	10.62	38,964.17	35,801.02	1
26	1	11.72	34,785.12	39,438.25	1
27	1	11.83	31,360.58	39,785.44	1
28	1	14.64	52,047.80	49,080.23	1
Sums =	28.00	141.70	487,730.10	487,730.12	28.00

CER Derived from "Good" Unweighted Data Set



THE AEROSPACE CORPORATION

MCR

Weighting Points of "Good" Data Set by their Squared X-Distance from x = 3

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	2.5460	0.45	3,898.71	3,468.83	0.154271
2	2.2700	0.73	726.14	4,567.88	0.194065
3	2.0400	0.96	2,438.27	5,483.76	0.240292
4	1.9040	1.10	4,688.87	6,025.32	0.275846
5	1.8640	1.14	3,620.85	6,184.60	0.287812
6	1.7480	1.25	3,186.70	6,646.52	0.327278
7	1.5400	1.46	2,870.88	7,474.79	0.421656
8	1.2400	1.76	4,471.57	8,669.41	0.650364
9	0.7000	2.30	7,100.55	10,819.72	2.040816
10	0.4780	2.52	14,413.94	11,703.74	4.376674
11	0.2200	2.78	11,497.04	12,731.11	20.66116
12	0.1820	2.82	9,985.56	12,882.43	30.18959
13	0.0602	2.94	13,892.95	13,367.44	275.9351
14	0.2180	3.22	13,502.21	14,475.25	21.042
15	0.9920	3.99	14,761.26	17,557.37	1.016194
16	2.3080	5.31	21,707.74	22,797.76	0.187728
17	2.8200	5.82	15,873.60	24,836.57	0.125748
18	2.8860	5.89	20,944.67	25,099.39	0.120062
19	2.9160	5.92	18,107.61	25,218.85	0.117605
20	3.4080	6.41	19,363.30	27,178.02	0.0861
21	4.5100	7.51	26,483.44	31,566.26	0.049164
22	5.1600	8.16	19,557.46	34,154.60	0.037558
23	5.8542	8.85	33,844.69	36,918.94	0.029179
24	6.6200	9.62	43,634.41	39,968.40	0.022818
25	7.6200	10.62	38,964.17	43,950.47	0.017222
26	8.7200	11.72	34,785.12	48,330.73	0.013151
27	8.8250	11.83	31,360.58	48,748.85	0.01284
28	11.6360	14.64	52,047.80	59,942.42	0.007386
Sums =	91.29	141.70	487,730.10	610,769.41	358.64

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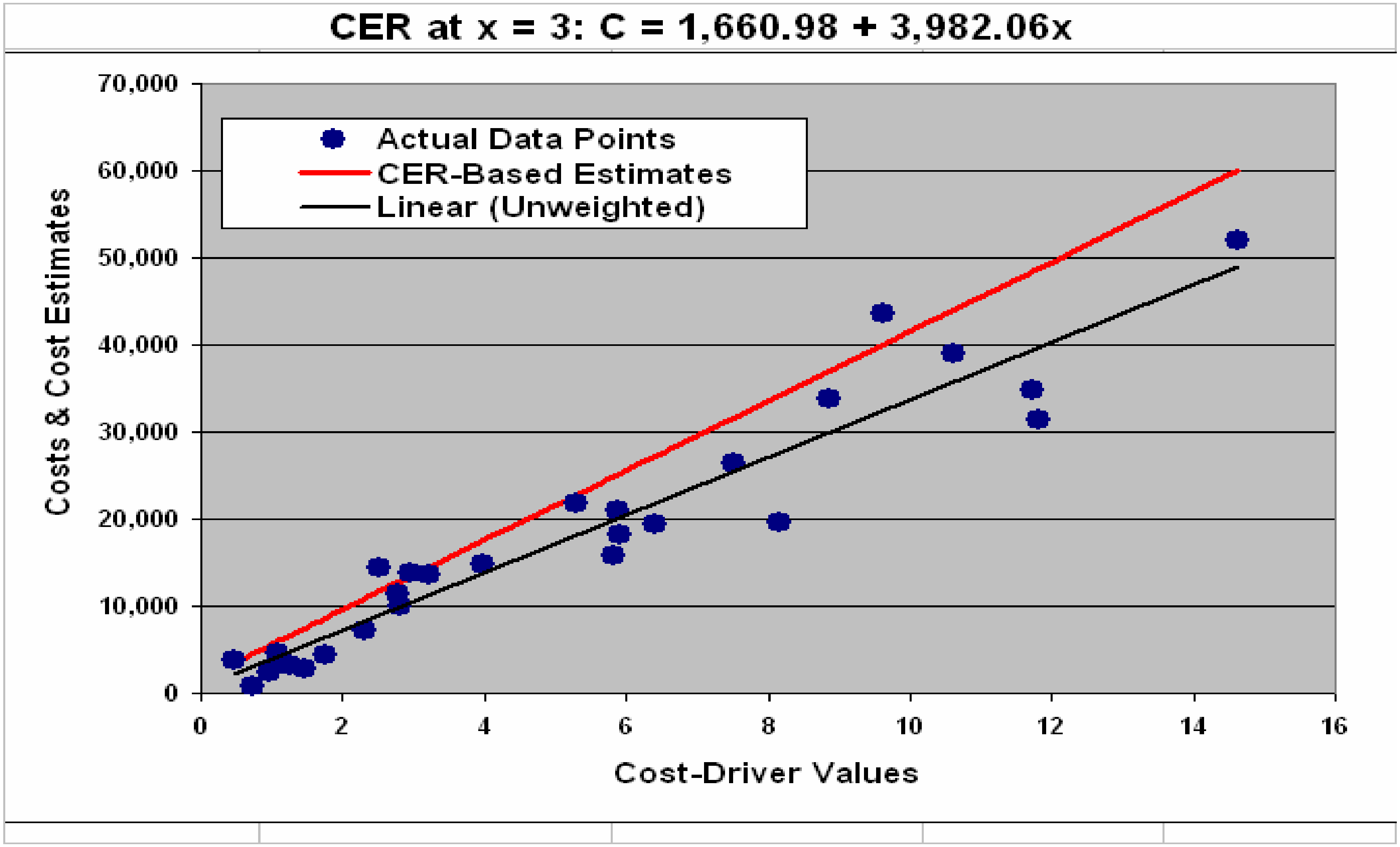
The Resulting X-Distance CER at $x = 3$



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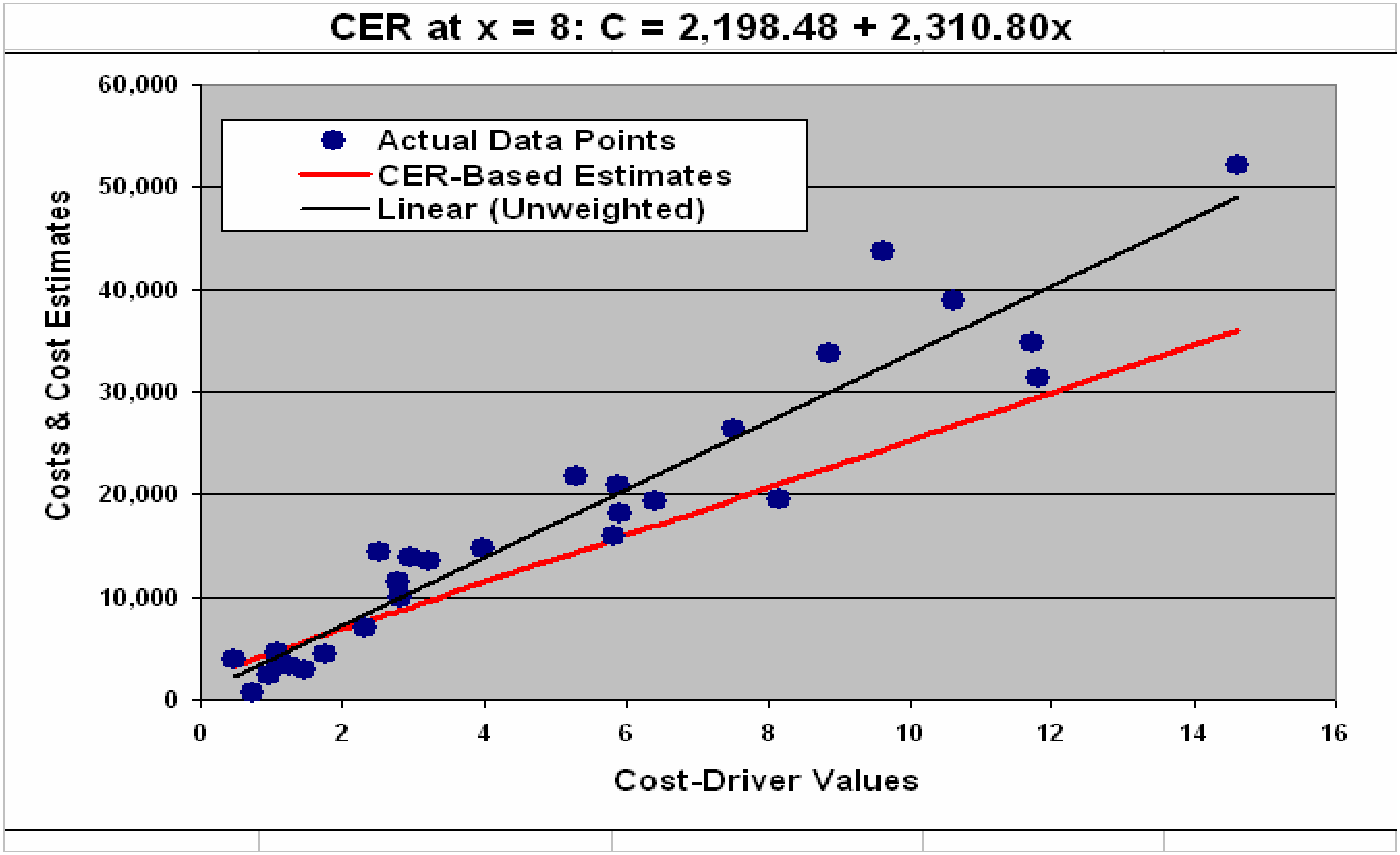
Weighting Points of "Good" Data Set by their Squared X-Distance from $x = 8$

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	7.5460	0.45	3,898.71	3,247.58	0.017562
2	7.2700	0.73	726.14	3,885.36	0.01892
3	7.0400	0.96	2,438.27	4,416.84	0.020177
4	6.9040	1.10	4,688.87	4,731.11	0.02098
5	6.8640	1.14	3,620.85	4,823.55	0.021225
6	6.7480	1.25	3,186.70	5,091.60	0.021961
7	6.5400	1.46	2,870.88	5,572.25	0.02338
8	6.2400	1.76	4,471.57	6,265.49	0.025682
9	5.7000	2.30	7,100.55	7,513.32	0.030779
10	5.4780	2.52	14,413.94	8,026.32	0.033324
11	5.2200	2.78	11,497.04	8,622.50	0.036699
12	5.1820	2.82	9,985.56	8,710.31	0.03724
13	5.0602	2.94	13,892.95	8,991.77	0.039054
14	4.7820	3.22	13,502.21	9,634.64	0.04373
15	4.0080	3.99	14,761.26	11,423.20	0.062251
16	2.6920	5.31	21,707.74	14,464.21	0.137991
17	2.1800	5.82	15,873.60	15,647.34	0.21042
18	2.1140	5.89	20,944.67	15,799.85	0.223764
19	2.0840	5.92	18,107.61	15,869.18	0.230253
20	1.5920	6.41	19,363.30	17,006.09	0.394561
21	0.4900	7.51	26,483.44	19,552.60	4.164931
22	0.1600	8.16	19,557.46	21,054.62	39.0625
23	0.8542	8.85	33,844.69	22,658.78	1.370506
24	1.6200	9.62	43,634.41	24,428.39	0.381039
25	2.6200	10.62	38,964.17	26,739.19	0.145679
26	3.7200	11.72	34,785.12	29,281.07	0.072263
27	3.8250	11.83	31,360.58	29,523.71	0.06835
28	6.6360	14.64	52,047.80	36,019.37	0.022708
Sums =	121.17	141.70	487,730.10	389,000.24	46.94

The Resulting X-Distance CER at $x = 8$



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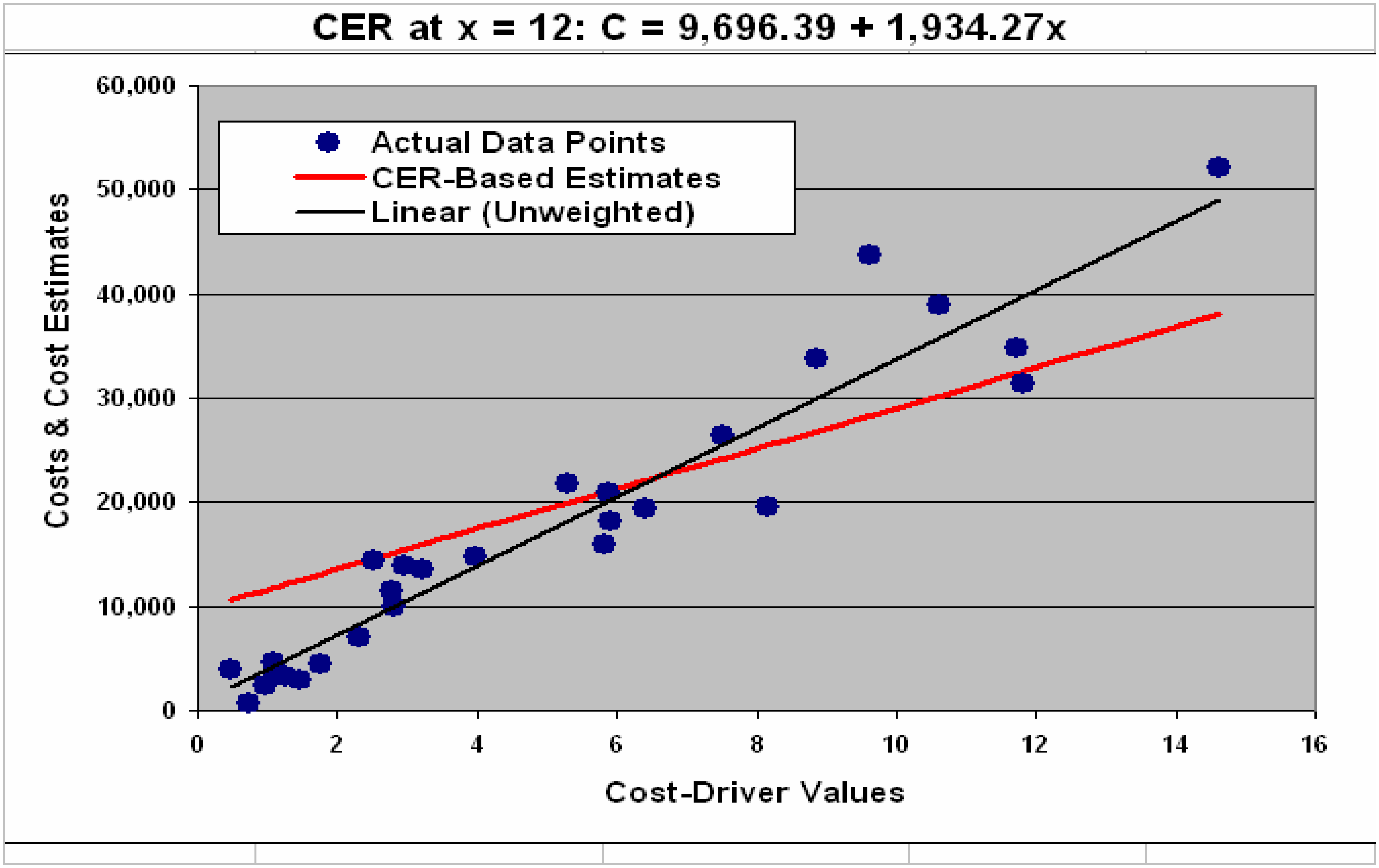
Weighting Points of "Good" Data Set by their Squared X-Distance from x = 12

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	11.5460	0.45	3,898.71	10,574.55	0.007501
2	11.2700	0.73	726.14	11,108.41	0.007873
3	11.0400	0.96	2,438.27	11,553.29	0.008205
4	10.9040	1.10	4,688.87	11,816.35	0.008411
5	10.8640	1.14	3,620.85	11,893.72	0.008473
6	10.7480	1.25	3,186.70	12,118.10	0.008657
7	10.5400	1.46	2,870.88	12,520.43	0.009002
8	10.2400	1.76	4,471.57	13,100.71	0.009537
9	9.7000	2.30	7,100.55	14,145.22	0.010628
10	9.4780	2.52	14,413.94	14,574.62	0.011132
11	9.2200	2.78	11,497.04	15,073.67	0.011764
12	9.1820	2.82	9,985.56	15,147.17	0.011861
13	9.0602	2.94	13,892.95	15,382.76	0.012182
14	8.7820	3.22	13,502.21	15,920.88	0.012966
15	8.0080	3.99	14,761.26	17,418.00	0.015594
16	6.6920	5.31	21,707.74	19,963.51	0.02233
17	6.1800	5.82	15,873.60	20,953.85	0.026183
18	6.1140	5.89	20,944.67	21,081.51	0.026752
19	6.0840	5.92	18,107.61	21,139.54	0.027016
20	5.5920	6.41	19,363.30	22,091.20	0.031979
21	4.4900	7.51	26,483.44	24,222.77	0.049603
22	3.8400	8.16	19,557.46	25,480.05	0.067817
23	3.1458	8.85	33,844.69	26,822.82	0.10105
24	2.3800	9.62	43,634.41	28,304.09	0.176541
25	1.3800	10.62	38,964.17	30,238.36	0.5251
26	0.2800	11.72	34,785.12	32,366.06	12.7551
27	0.1750	11.83	31,360.58	32,569.15	32.65306
28	2.6360	14.64	52,047.80	38,006.39	0.143916
Sums =	199.57	141.70	487,730.10	545,587.20	46.76

The Resulting X-Distance CER at *x* = 12



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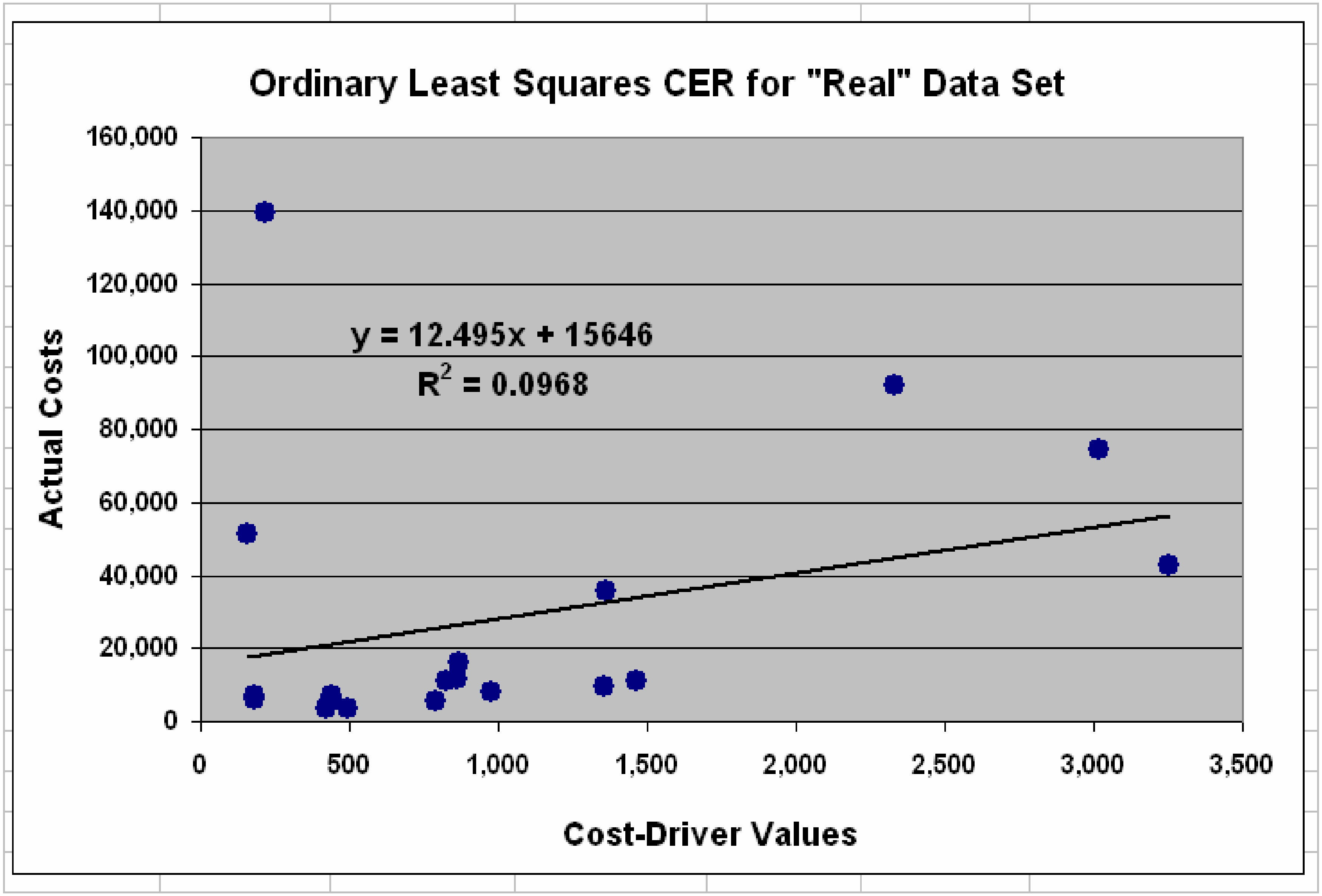
Back to the "Real" Data Set

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	1	156.12	51,367.22	17,596.30	1
2	1	179.40	5,885.00	17,887.18	1
3	1	180.30	7,060.00	17,898.42	1
4	1	217.50	139,483.12	18,363.23	1
5	1	419.14	3,386.00	20,882.67	1
6	1	437.09	6,738.00	21,106.95	1
7	1	440.93	6,812.00	21,154.93	1
8	1	494.45	3,291.34	21,823.65	1
9	1	789.90	5,723.14	25,515.22	1
10	1	826.10	10,992.00	25,967.53	1
11	1	864.30	11,590.00	26,444.83	1
12	1	869.30	15,973.00	26,507.30	1
13	1	976.50	7,970.67	27,846.74	1
14	1	1,355.80	9,524.10	32,586.00	1
15	1	1,360.90	35,927.22	32,649.72	1
16	1	1,463.21	11,238.73	33,928.06	1
17	1	2,332.10	92,059.97	44,784.62	1
18	1	3,017.73	74,649.00	53,351.39	1
19	1	3,253.00	42,915.23	56,291.03	1
Sums =	19.00	19,633.77	542,585.74	542,585.73	19.00

a =	15,645.62		Std Error =	36,300.49	
b =	12.49		Bias =	0.00	
			R^2 =	0.10	

Note: This data set is a set of actual cost data; due to proprietary issues, however, the exact descriptions of the data points cannot be revealed.

Real Data Set Graphics



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The Triad CER Form

- By the "Triad" CER Form is Meant an Algebraic Expression of the form $y = a + bx^c$, where
  - x is the value of the cost driver
  - $a, b,$ and c are coefficients derived from the data
- Our optimization criterion is that the coefficients $a, b,$ and c will be selected so that the sum of squares of the differences $d_k = y_k - a - bx_k^c$ between the actual costs and their estimates is as small as possible
- The mathematics results in numerical values of a and b that minimize the quantity

$$f(a,b,c) = \sum_{k=1}^{n} d_k^2 = \sum_{k=1}^{n} (y_k - a - bx_k^c)^2$$

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Weighting Points of Real Data Set by their Squared X-Distance from x = 100

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	56.12	156.12	51,367.22	20,854.13	0.0003175153
2	79.40	179.40	5,885.00	20,872.37	0.0001586204
3	80.30	180.30	7,060.00	20,873.14	0.0001550847
4	117.50	217.50	139,483.12	20,909.34	0.0000724310
5	319.14	419.14	3,386.00	21,267.35	0.0000098183
6	337.09	437.09	6,738.00	21,313.49	0.0000088005
7	340.93	440.93	6,812.00	21,323.68	0.0000086034
8	394.45	494.45	3,291.34	21,477.60	0.0000064271
9	689.90	789.90	5,723.14	22,751.34	0.0000021010
10	726.10	826.10	10,992.00	22,959.55	0.0000018967
11	764.30	864.30	11,590.00	23,192.16	0.0000017119
12	769.30	869.30	15,973.00	23,223.60	0.0000016897
13	876.50	976.50	7,970.67	23,953.51	0.0000013017
14	1,255.80	1,355.80	9,524.10	27,432.43	0.0000006341
15	1,260.90	1,360.90	35,927.22	27,489.11	0.0000006290
16	1,363.21	1,463.21	11,238.73	28,683.43	0.0000005381
17	2,232.10	2,332.10	92,059.97	43,480.83	0.0000002007
18	2,917.73	3,017.73	74,649.00	61,488.64	0.0000001175
19	3,153.00	3,253.00	42,915.23	69,040.03	0.0000001006
Sums =	17,733.77	19,633.77	542,585.74	542,585.74	0.0007482216

Note: "Estimated Cost EST C" is based on Triad CER.

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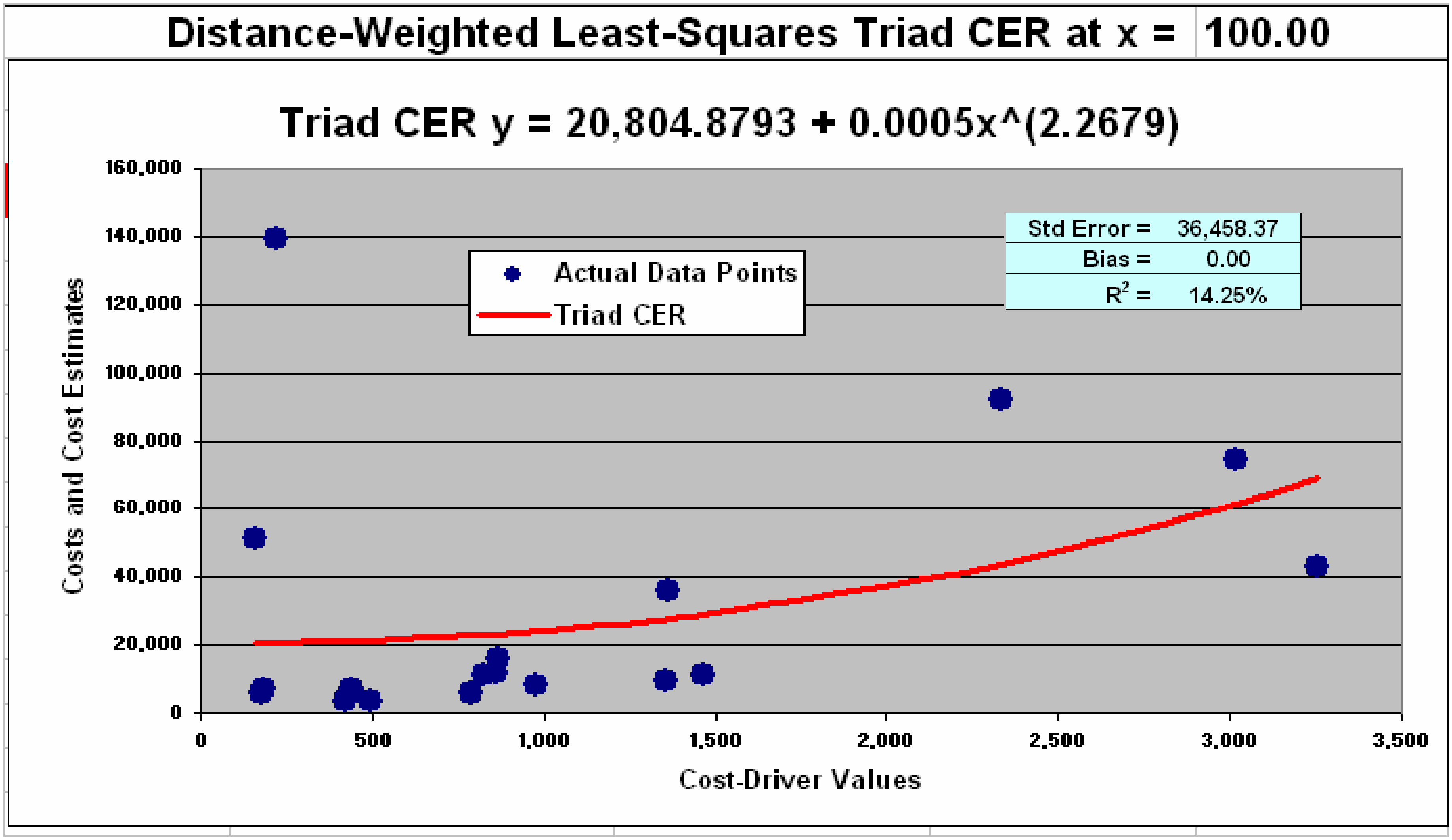
Triad CER Derived from Real Data Set at x = 100



Note: The reported quality metrics refer to the entire unweighted data set.

Weighting Points of Real Data Set by their Squared X-Distance from x = 1,000

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	843.88	156.12	51,367.22	53,493.49	0.0000014042
2	820.60	179.40	5,885.00	50,257.16	0.0000014850
3	819.70	180.30	7,060.00	50,140.44	0.0000014883
4	782.50	217.50	139,483.12	45,755.36	0.0000016332
5	580.86	419.14	3,386.00	30,264.84	0.0000029639
6	562.91	437.09	6,738.00	29,266.40	0.0000031559
7	559.07	440.93	6,812.00	29,058.01	0.0000031994
8	505.55	494.45	3,291.34	26,324.70	0.0000039127
9	210.10	789.90	5,723.14	15,069.83	0.0000226542
10	173.90	826.10	10,992.00	13,986.69	0.0000330675
11	135.70	864.30	11,590.00	12,892.84	0.0000543051
12	130.70	869.30	15,973.00	12,753.17	0.0000585395
13	23.50	976.50	7,970.67	9,933.48	0.0018107741
14	355.80	1,355.80	9,524.10	1,933.82	0.0000078993
15	360.90	1,360.90	35,927.22	1,841.94	0.0000076776
16	463.21	1,463.21	11,238.73	66.39	0.0000046606
17	1,332.10	2,332.10	92,059.97	-11,424.97	0.0000005635
18	2,017.73	3,017.73	74,649.00	-17,833.66	0.0000002456
19	2,253.00	3,253.00	42,915.23	-19,707.77	0.0000001970
Sums =	12,931.71	19,633.77	542,585.74	334,072.18	0.0020198265

Note: "Estimated Cost EST C" is based on Triad CER.

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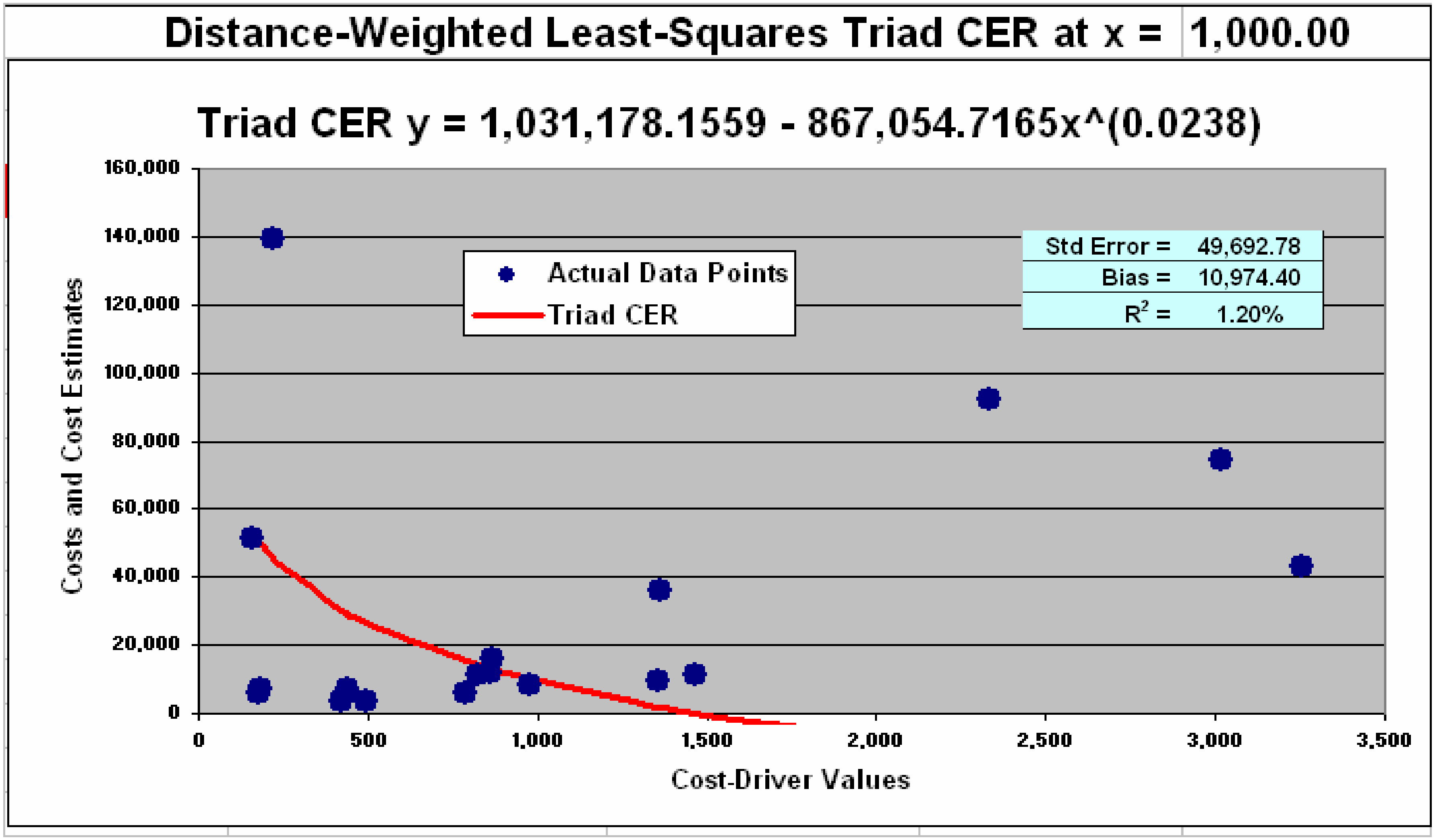
Triad CER Derived from Real Data Set at x = 1,000



Note: The reported quality metrics refer to the entire unweighted data set.

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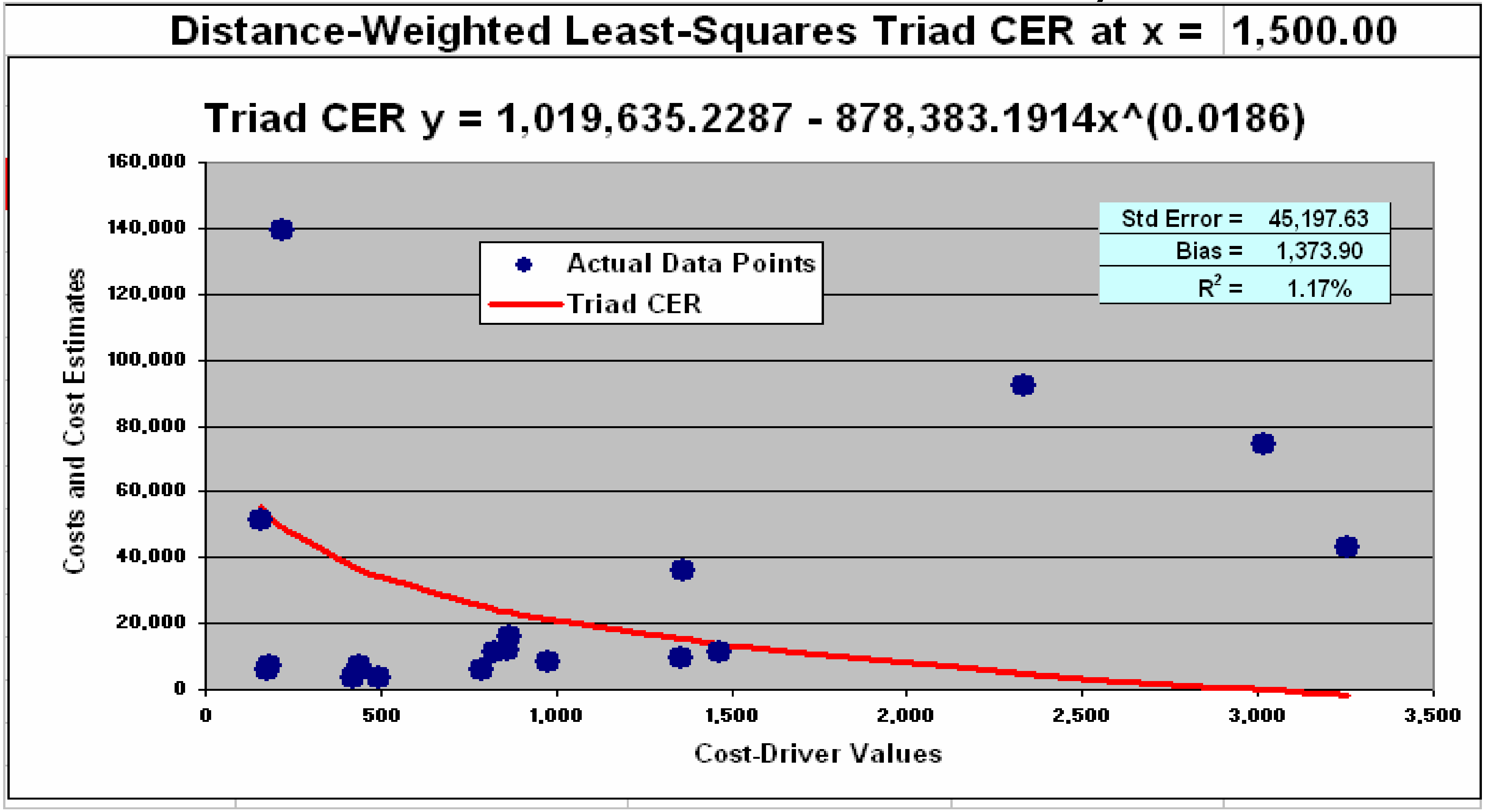
Weighting Points of Real Data Set by their Squared X-Distance from x = 1,500

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	1,343.88	156.12	51,367.22	54,753.58	0.0000005537
2	1,320.60	179.40	5,885.00	52,256.38	0.0000005734
3	1,319.70	180.30	7,060.00	52,166.35	0.0000005742
4	1,282.50	217.50	139,483.12	48,785.72	0.0000006080
5	1,080.86	419.14	3,386.00	36,869.56	0.0000008560
6	1,062.91	437.09	6,738.00	36,102.89	0.0000008851
7	1,059.07	440.93	6,812.00	35,942.89	0.0000008916
8	1,005.55	494.45	3,291.34	33,845.02	0.0000009890
9	710.10	789.90	5,723.14	25,219.70	0.0000019832
10	673.90	826.10	10,992.00	24,390.72	0.0000022020
11	635.70	864.30	11,590.00	23,553.74	0.0000024745
12	630.70	869.30	15,973.00	23,446.89	0.0000025139
13	523.50	976.50	7,970.67	21,290.31	0.0000036489
14	144.20	1,355.80	9,524.10	15,179.02	0.0000480916
15	139.10	1,360.90	35,927.22	15,108.89	0.0000516828
16	36.79	1,463.21	11,238.73	13,753.90	0.0007388230
17	832.10	2,332.10	92,059.97	4,996.64	0.0000014443
18	1,517.73	3,017.73	74,649.00	121.90	0.0000004341
19	1,753.00	3,253.00	42,915.23	-1,302.41	0.0000003254
Sums =	17,071.89	19,633.77	542,585.74	516,481.70	0.0008595547

Note: "Estimated Cost EST C" is based on Triad CER.

Triad CER Derived from Real Data Set at x = 1,500



Note: The reported quality metrics refer to the entire unweighted data set.

Weighting Points of Real Data Set by their Squared X-Distance from x = 2,000

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	1,843.88	156.12	51,367.22	-25,485.56	0.0000002941
2	1,820.60	179.40	5,885.00	-20,806.10	0.0000003017
3	1,819.70	180.30	7,060.00	-20,638.10	0.0000003020
4	1,782.50	217.50	139,483.12	-14,364.10	0.0000003147
5	1,580.86	419.14	3,386.00	7,221.92	0.0000004001
6	1,562.91	437.09	6,738.00	8,583.16	0.0000004094
7	1,559.07	440.93	6,812.00	8,866.83	0.0000004114
8	1,505.55	494.45	3,291.34	12,573.08	0.0000004412
9	1,210.10	789.90	5,723.14	27,558.38	0.0000006829
10	1,173.90	826.10	10,992.00	28,977.53	0.0000007257
11	1,135.70	864.30	11,590.00	30,406.69	0.0000007753
12	1,130.70	869.30	15,973.00	30,588.87	0.0000007822
13	1,023.50	976.50	7,970.67	34,253.01	0.0000009546
14	644.20	1,355.80	9,524.10	44,504.80	0.0000024097
15	639.10	1,360.90	35,927.22	44,621.33	0.0000024483
16	536.79	1,463.21	11,238.73	46,867.85	0.0000034705
17	332.10	2,332.10	92,059.97	61,164.42	0.0000090670
18	1,017.73	3,017.73	74,649.00	68,958.89	0.0000009655
19	1,253.00	3,253.00	42,915.23	71,214.62	0.0000006369
Sums =	23,571.89	19,633.77	542,585.74	445,067.51	0.0000257931

Note: "Estimated Cost EST C" is based on Triad CER.

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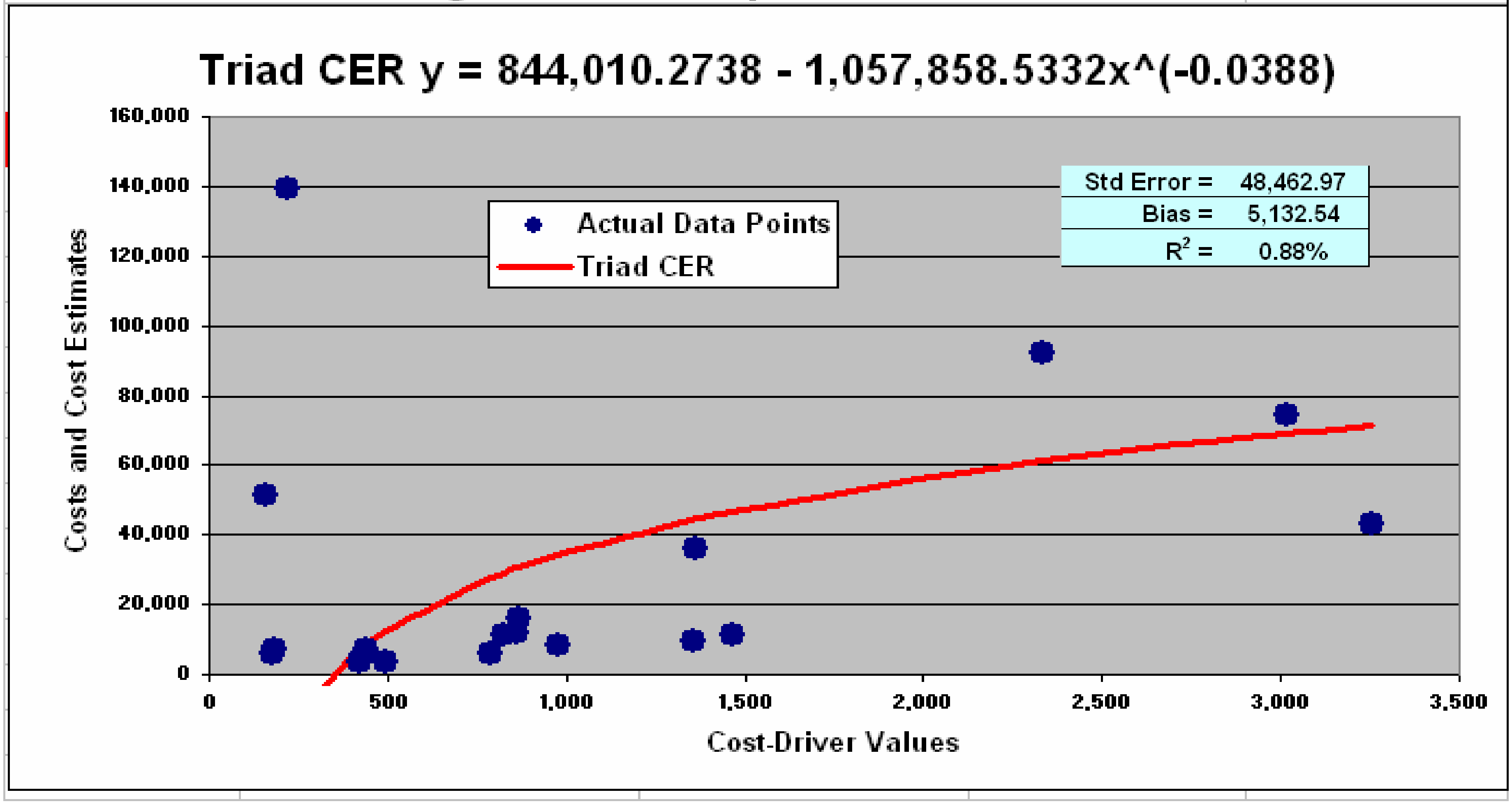
Triad CER Derived from Real Data Set at x = 2,000

Distance-Weighted Least-Squares Triad CER at x =	2,000.00



Note: The reported quality metrics refer to the entire unweighted data set.

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Weighting Points of Real Data Set by their Squared X-Distance from x = 2,500

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	2,343.88	156.12	51,367.22	-56,094.90	0.0000001820
2	2,320.60	179.40	5,885.00	-48,560.63	0.0000001857
3	2,319.70	180.30	7,060.00	-48,290.67	0.0000001858
4	2,282.50	217.50	139,483.12	-38,236.04	0.0000001919
5	2,080.86	419.14	3,386.00	-4,042.90	0.0000002309
6	2,062.91	437.09	6,738.00	-1,907.49	0.0000002350
7	2,059.07	440.93	6,812.00	-1,462.82	0.0000002359
8	2,005.55	494.45	3,291.34	4,337.26	0.0000002486
9	1,710.10	789.90	5,723.14	27,600.33	0.0000003419
10	1,673.90	826.10	10,992.00	29,787.75	0.0000003569
11	1,635.70	864.30	11,590.00	31,987.83	0.0000003738
12	1,630.70	869.30	15,973.00	32,268.10	0.0000003761
13	1,523.50	976.50	7,970.67	37,895.29	0.0000004308
14	1,144.20	1,355.80	9,524.10	53,542.98	0.0000007638
15	1,139.10	1,360.90	35,927.22	53,720.03	0.0000007707
16	1,036.79	1,463.21	11,238.73	57,129.61	0.0000009303
17	167.90	2,332.10	92,059.97	78,666.94	0.0000354731
18	517.73	3,017.73	74,649.00	90,291.69	0.0000037307
19	753.00	3,253.00	42,915.23	93,640.40	0.0000017636
Sums =	30,407.69	19,633.77	542,585.74	392,272.77	0.0000470076

Note: "Estimated Cost EST C" is based on Triad CER.

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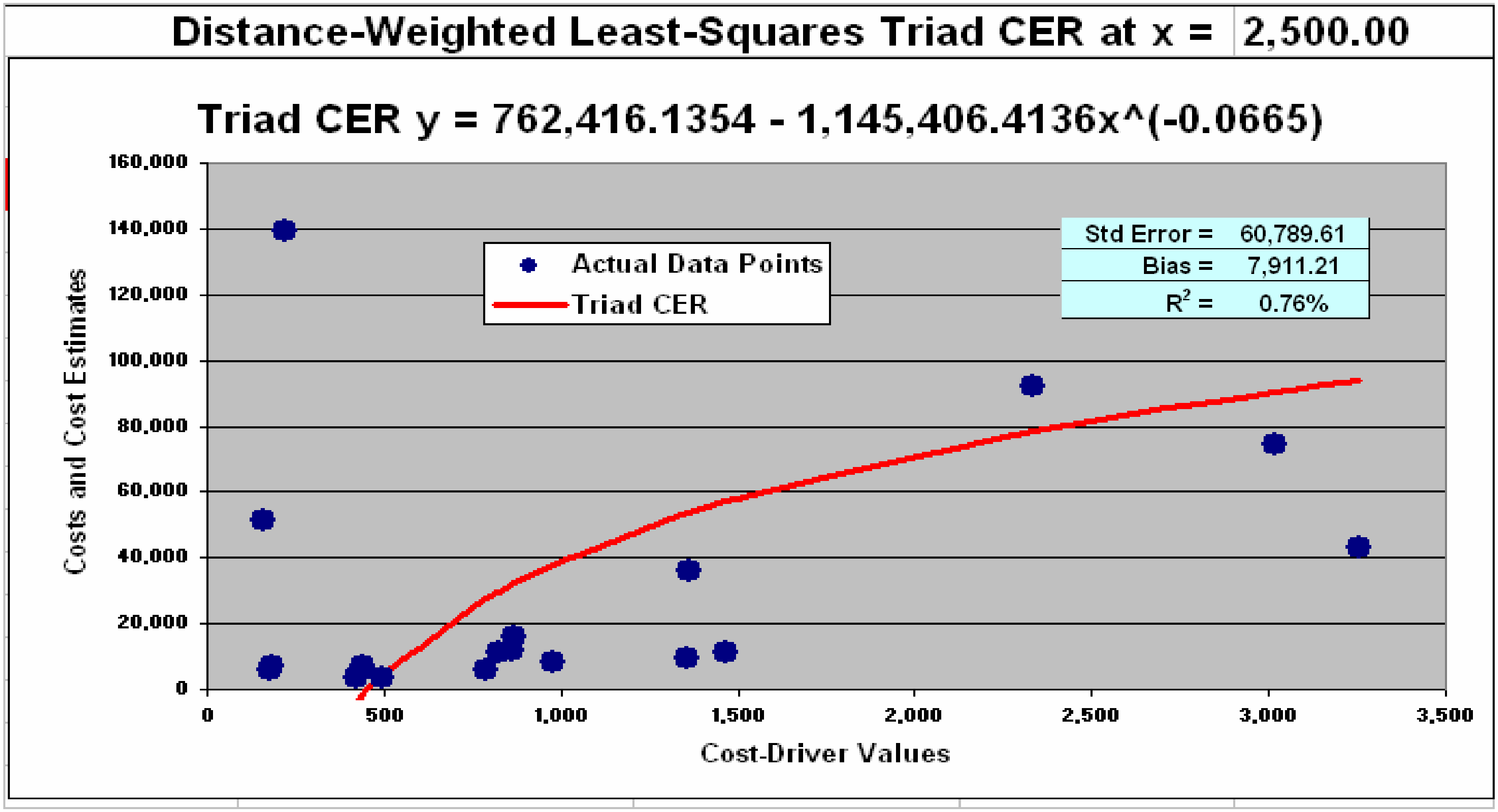
Triad CER Derived from Real Data Set at x = 2,500



Note: The reported quality metrics refer to the entire unweighted data set.

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Weighting Points of Real Data Set by their Squared X-Distance from *x* = 3,000

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	2,843.88	156.12	51,367.22	-49,899.85	0.0000001236
2	2,820.60	179.40	5,885.00	-43,623.83	0.0000001257
3	2,819.70	180.30	7,060.00	-43,398.74	0.0000001258
4	2,782.50	217.50	139,483.12	-35,005.13	0.0000001292
5	2,580.86	419.14	3,386.00	-6,306.11	0.0000001501
6	2,562.91	437.09	6,738.00	-4,505.71	0.0000001522
7	2,559.07	440.93	6,812.00	-4,130.67	0.0000001527
8	2,505.55	494.45	3,291.34	764.96	0.0000001593
9	2,210.10	789.90	5,723.14	20,473.96	0.0000002047
10	2,173.90	826.10	10,992.00	22,333.34	0.0000002116
11	2,135.70	864.30	11,590.00	24,204.57	0.0000002192
12	2,130.70	869.30	15,973.00	24,443.03	0.0000002203
13	2,023.50	976.50	7,970.67	29,234.43	0.0000002442
14	1,644.20	1,355.80	9,524.10	42,596.10	0.0000003699
15	1,639.10	1,360.90	35,927.22	42,747.61	0.0000003722
16	1,536.79	1,463.21	11,238.73	45,666.74	0.0000004234
17	667.90	2,332.10	92,059.97	64,169.80	0.0000022417
18	17.73	3,017.73	74,649.00	74,203.50	0.0031811381
19	253.00	3,253.00	42,915.23	77,100.09	0.0000156228
Sums =	37,907.69	19,633.77	542,585.74	281,068.07	0.0032023869

Note: "Estimated Cost EST C" is based on Triad CER.

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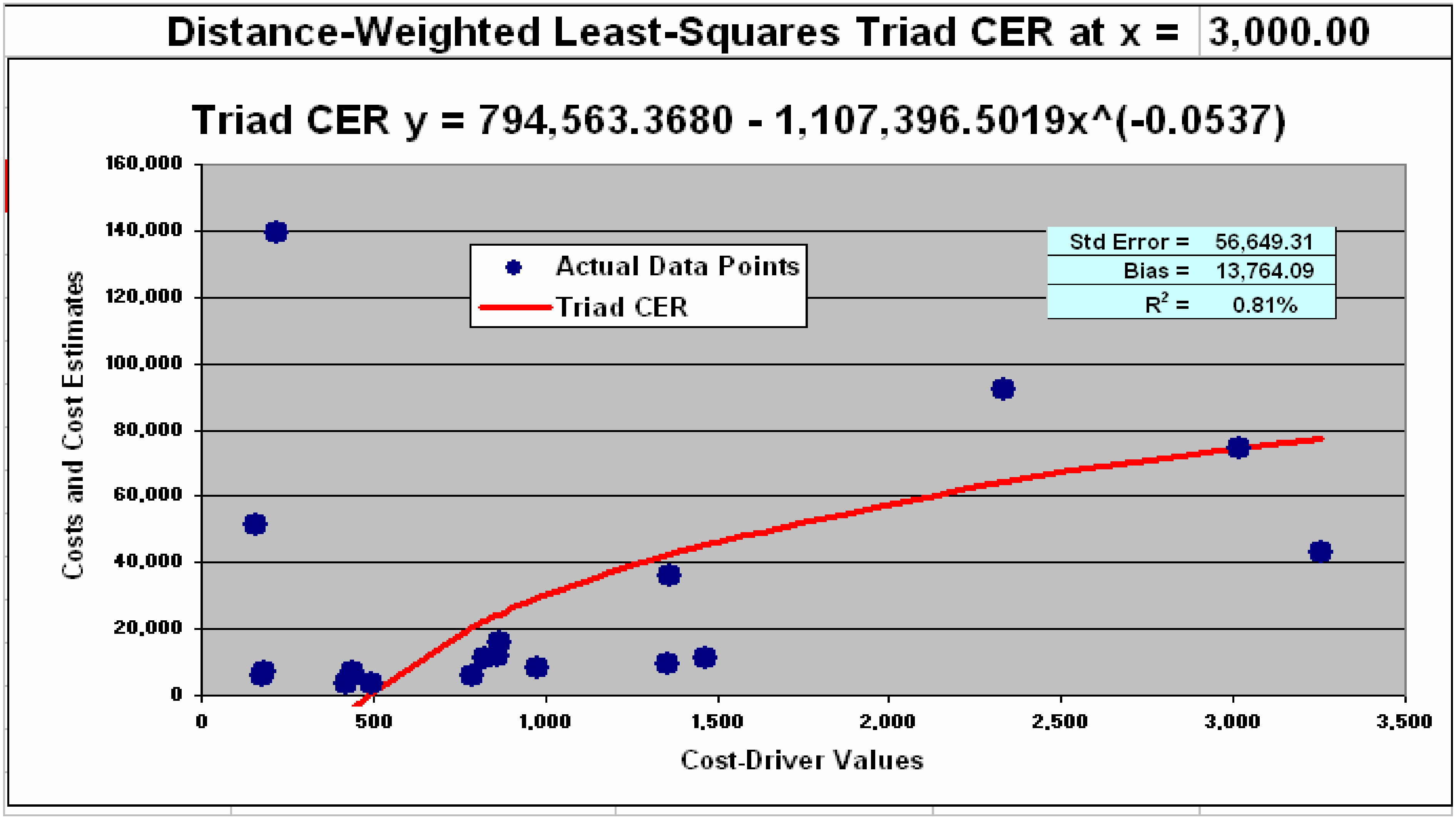
Triad CER Derived from Real Data Set at x = 3,000



Note: The reported quality metrics refer to the entire unweighted data set.

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Weighting Points of Real Data Set by their Squared X-Distance from *x* = 3,500

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	3,343.88	156.12	51,367.22	14,902.11	0.0000000894
2	3,320.60	179.40	5,885.00	16,533.57	0.0000000907
3	3,319.70	180.30	7,060.00	16,592.26	0.0000000907
4	3,282.50	217.50	139,483.12	18,789.18	0.0000000928
5	3,080.86	419.14	3,386.00	26,430.58	0.0000001054
6	3,062.91	437.09	6,738.00	26,916.84	0.0000001066
7	3,059.07	440.93	6,812.00	27,018.23	0.0000001069
8	3,005.55	494.45	3,291.34	28,345.15	0.0000001107
9	2,710.10	789.90	5,723.14	33,750.74	0.0000001362
10	2,673.90	826.10	10,992.00	34,266.09	0.0000001399
11	2,635.70	864.30	11,590.00	34,785.66	0.0000001439
12	2,630.70	869.30	15,973.00	34,851.94	0.0000001445
13	2,523.50	976.50	7,970.67	36,187.05	0.0000001570
14	2,144.20	1,355.80	9,524.10	39,944.02	0.0000002175
15	2,139.10	1,360.90	35,927.22	39,986.91	0.0000002185
16	2,036.79	1,463.21	11,238.73	40,814.56	0.0000002411
17	1,167.90	2,332.10	92,059.97	46,118.35	0.0000007331
18	482.27	3,017.73	74,649.00	49,037.19	0.0000042995
19	247.00	3,253.00	42,915.23	49,885.56	0.0000163910
Sums =	46,866.23	19,633.77	542,585.74	615,156.01	0.0000236155

Note: "Estimated Cost EST C" is based on Triad CER.

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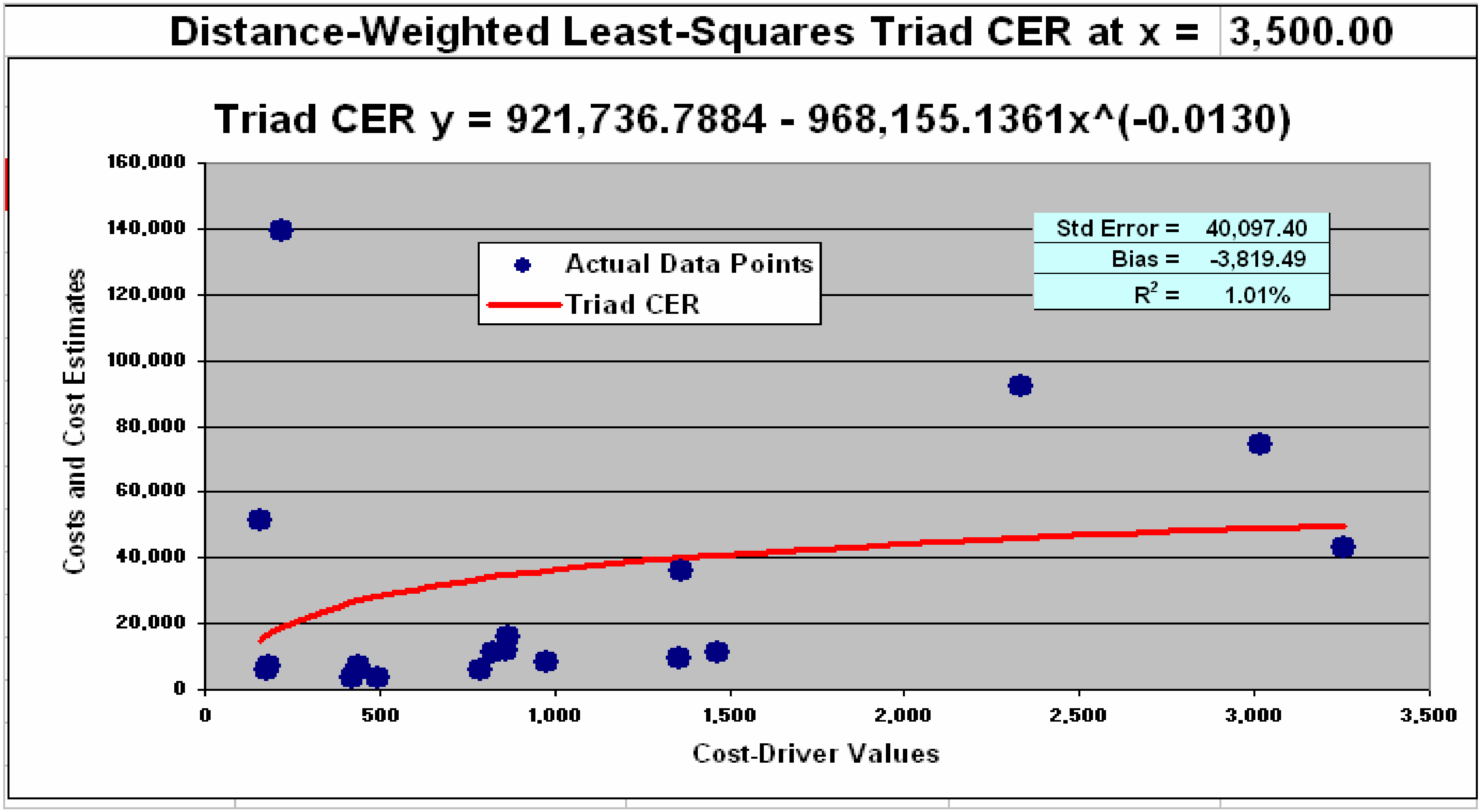
Triad CER Derived from Real Data Set at x = 3,500



Note: The reported quality metrics refer to the entire unweighted data set.

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Weighting Points of Real Data Set by their Squared X-Distance from x = 5,000

Program	Statistical Weight w	Cost-Driver Value x	Unit Cost C	Estimated Cost EST C	Squared Weights $1/w^2$
1	4,843.88	156.12	51,367.22	8,019.01	0.0000000426
2	4,820.60	179.40	5,885.00	10,022.54	0.0000000430
3	4,819.70	180.30	7,060.00	10,094.59	0.0000000430
4	4,782.50	217.50	139,483.12	12,791.17	0.0000000437
5	4,580.86	419.14	3,386.00	22,158.58	0.0000000477
6	4,562.91	437.09	6,738.00	22,754.04	0.0000000480
7	4,559.07	440.93	6,812.00	22,878.20	0.0000000481
8	4,505.55	494.45	3,291.34	24,502.69	0.0000000493
9	4,210.10	789.90	5,723.14	31,114.75	0.0000000564
10	4,173.90	826.10	10,992.00	31,744.62	0.0000000574
11	4,135.70	864.30	11,590.00	32,379.59	0.0000000585
12	4,130.70	869.30	15,973.00	32,460.58	0.0000000586
13	4,023.50	976.50	7,970.67	34,091.75	0.0000000618
14	3,644.20	1,355.80	9,524.10	38,678.78	0.0000000753
15	3,639.10	1,360.90	35,927.22	38,731.12	0.0000000755
16	3,536.79	1,463.21	11,238.73	39,741.00	0.0000000799
17	2,667.90	2,332.10	92,059.97	46,207.34	0.0000001405
18	1,982.27	3,017.73	74,649.00	49,762.10	0.0000002545
19	1,747.00	3,253.00	42,915.23	50,794.77	0.0000003277
Sums =	75,366.23	19,633.77	542,585.74	558,927.19	0.0000016115

Note: "Estimated Cost EST C" is based on Triad CER.

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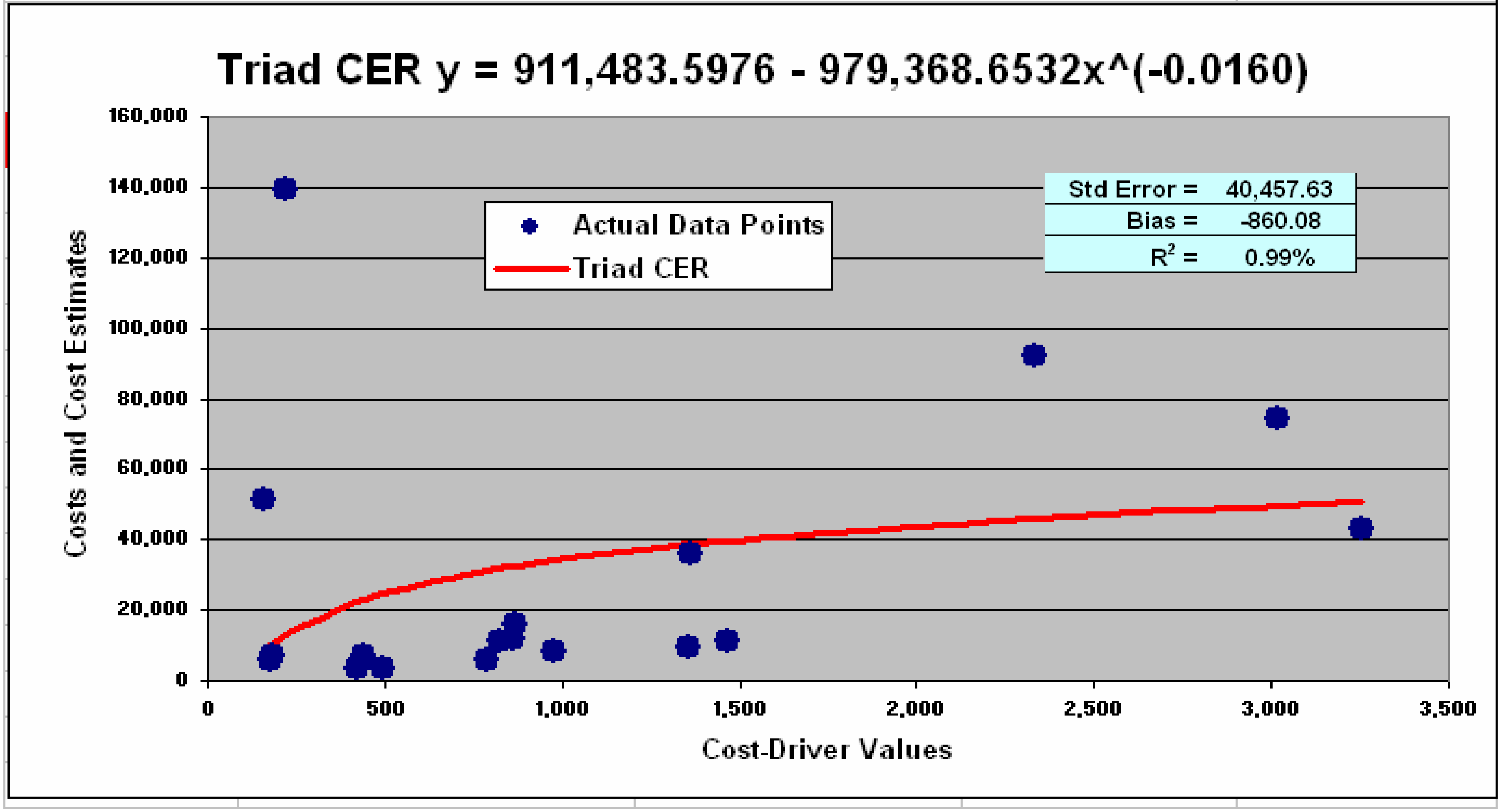
Triad CER Derived from Real Data Set at x = 5,000

Distance-Weighted Least-Squares Triad CER at x =	5,000.00



Note: The reported quality metrics refer to the entire unweighted data set.

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Agenda

- Discussion of the regression idea and extent of confidence in results including the theory of Weighted Least Squares Regression
- Three methods of adapting CERs to particular data sets or estimating needs
  - *A Priori* method: Weighting each point by its quality or confidence in its accuracy
  - Piecewise CER method: Grouping data into separate subsets based on natural divisions
  - "X-Distance" method: Weighting points by distance from a cost-driver value of interest
- Conclusions

Summary

Method	Advantages	Disadvantages
A Priori Method	Produces one new CER that can be distributed without the data	Requires knowledge about some or all of the data points
Weighting each point		
Piecewise CER Method	Produces small set of CERs more responsive to *x* value that can be distributed without data	Arbitrary decision about how to do `piecewise grouping is required
Weighting by grouping data into separate pieces		
"X-Distance" Method	Method provides analogy-like estimating near *x* value chosen	CERs cannot be generated without having all the data points available
Weighting points by distance from cost-driver value		
Square of Distance	Produces "good" fitting weighted CER across data set	
Normal-Distribution Weighting	Impact of weights is adjustable by choice of sigma value	Can generate piecewise-type CERs as well

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Concluding Remarks

- CERs are the mainstay of parametric cost estimating - Their major drawback is the uncertainty of applicability to the PoI in any particular estimating situation
- Weighting techniques
  - … add value by taking advantage of more specific information on the use of an existing CER or by adding CERs to the estimator’s toolkit
  - … (intuitively) reduce estimating uncertainty, but a formal proof of this has not yet been shown in this paper – we hope to be able to establish this as a fact soon
- Deriving adaptive CERs requires more work that deriving full data set CER, but it increases their usefulness and applications