A REFRESHER ON ENGINEERING ECONOMICS

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. . . Tom and me found the money that the robbers hid in the cave, and it made us rich. We got six thousand dollars apiece – all gold. It was an awful sight of money when it was piled up. Well, judge Thatcher, he took it and put it out at interest, and it fetched us a dollar a day apiece, all year round – more than a body could tell what to do with.

Mark Twain in *Adventures of Huckleberry Finn*

INTRODUCTION

This chapter covers the most common analytical techniques used in discounted cash flow studies to account for the time value of money when choosing between alternatives. The underlying logic of the concepts is explained and the techniques of present value, equivalent annual amount, and internal rate of return are presented. Special coverage is included for cases involving the choice between multiple alternatives, the relationship between interest and inflation, and the choice of the proper discount rate.

The Time Value of Money Concept and When It Applies

All money has time value. Borrowed money has the time value equal to the interest payments made on the loan, whereas invested money has a time value equal to the returns or income that accrue from the investment. Money that is held as cash or in non-interest bearing accounts has time value because the money is forgoing either profits or interest that could have been earned if it had been invested in some other way.

Whenever an organization is contemplating an investment that involves more than one option, the time value of money will potentially affect the decision. This is true regardless of the source of the funding. It is not necessary that borrowed funds be involved, or is it necessary that the funds are visibly forgoing interest because they re being taken out of some interest bearing account. Equity money raised by the sale of stock has time value even though there is no guarantee made to the stockholder that a return will be earned or even that the original investment can be retrieved intact. The stockholder’s investment has a time value because there are other uses to which the money could be put that would earn a return. By forgoing these alternate uses, the money is given a time value.

Thus it is advisable for the cost analyst to consider the time value of money whenever the costs of two or more alternatives are being compared that have disbursements and/or receipts distributed over time. Only if one alternative has lower disbursements and higher receipts in each period of the analysis, or in studies involving first costs only with no differences in cost or receipts in later periods, can the time value of money be safely ignored. In both cases there is still a time value associated with the money but not one that would affect the choice.

Before the time value of money can be considered it is necessary to first estimate the life cycle cost and revenues of options being compared excluding any interest effects.

Stopping here, however, and comparing the total life cycle value of alternatives without considering interest effects may lead to a selection that is not the most cost-effective choice. Similarly, relying solely on payback period or return on investment (ROI) criteria, which, as normally calculated, do not recognize the time value of money, is inadvisable.

Cash Flows

The techniques for analyzing the time value of money are generally referred to as discounted cash flow analysis techniques, a cash flow being the expected life cycle costs and revenues of a contemplated investment presented as a time series of costs and revenues of a contemplated investment presented as a time series of dollar disbursements and receipts. Many analysts utilize a cash flow diagram to visually present the flow of dollars, Figure 1 shows two cash flow diagrams for a study comparing the cash flows of two machines. Machine A is an existing machine with 3 years left in its design life of 5 years, an annual operating cost of $67,000, and a residual value (say, in this case, the scrap value after deducting the cost of dismantling the machine) of $500, and a value of $3000 if sold now. Machine B is a proposed replacement costing $30,000, with an annual operating cost of $60,000, and a residual value of $3000 at the end of its operational life of 8 years. Because Machine A has 3 years of service left and Machine B would last 8 years, the assumption is made that Machine A would be replaced with an identical machine at the end of the 3 years which would give 5 more years of service, thus equalizing the service of the two machines at 8 years each. The cash flow diagrams of Figure 1 follow the convention of using down arrows to represent cash outflows such as investments and operating costs and up arrows to present cash inflows such as residual values. The $5000 replacement cost of Machine A in year 3 is offset to some extent by the cash inflow of $500 residual value, as shown in the diagram. The yearly operating cost of $67,000 is shown as cash outflows, and in year 8 another $500 cash inflow for residual value is shown for Machine A. Machine B's purchase cost of $30,000 is shown in the diagram reduced by $3000 which represents the cash inflow which would accrue from selling Machine A if Machine B is installed. The $63,000 annual cost of Machine B is shown over the 8 years with a residual value cash inflow in year 8 of $3000.

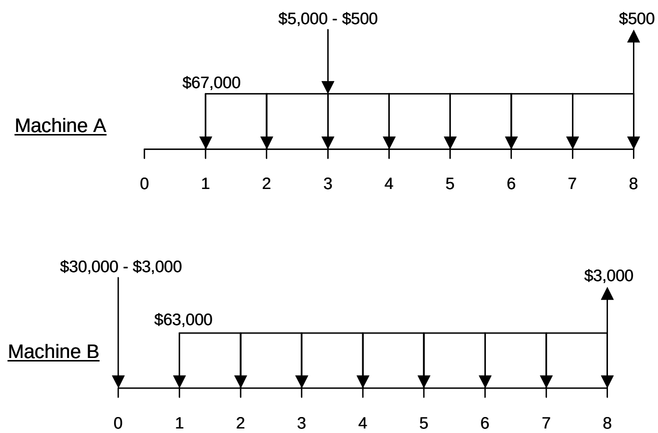
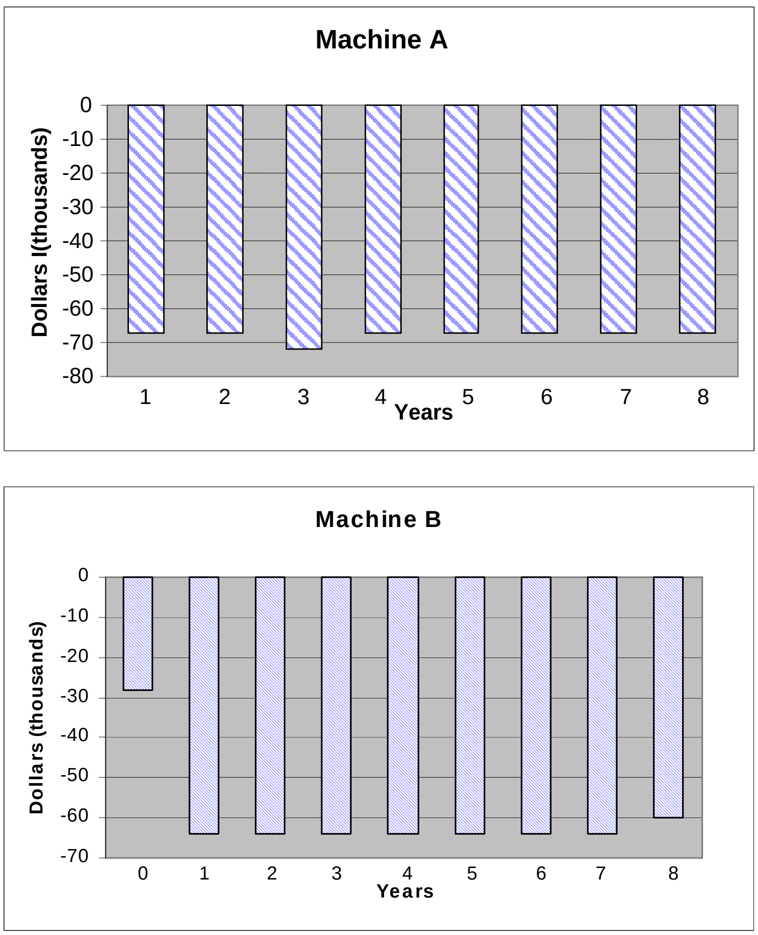


Figure 1 Cash flow diagrams

It is useful to recognize that discounted cash flow analysis applies to several different cash flow situations the cost analyst might encounter. The problem may involve only cash outflows such as an equipment selection, which is comparing alternatives that have identical output capability (like our preceding example). Because the revenue is the same for all options, only the life cycle costs need by considered in the time value of money analysis. Sometimes the problem might be an investment decision analysis between options that have not only different initial purchase and operating costs but also have different expected revenues as well. In such cases the cash flows carried into the time value of money analysis will need to include these revenue differences.

All such situations, if they are capable of being represented by a time series of dollar flows over time, are capable of being analyzed using the same basic time value of money concepts. The first situation (comparing the cost of equipment options with identical revenues) can be represented by an all-negative cash flow for each of the options under consideration. (Here we follow the convention of assigning a negative algebraic sign to costs and any other cash outflows or disbursements and a positive sign to cash receipts, revenues, savings, or other cash inflows.) Figure 2 represents the same



cash

Figure 2. Cash flows.

flows shown in the diagrams of Figure 1 but in the form of bar charts using our sign flow convention. The discounted cash flow analysis would be concerned with determining which of the options has the *least* negative cost after the time value of money is taken into account.

For cases in which the cash flows of each alternative include different revenue effects, the identical comparative techniques are applied, but the analyst would be interested in discovering which alternative resulted in the *most positive* result after taking into account the time value of money (presuming, of course, that the revenues or other cash inflows of the investment exceed the costs).

It is equivalent, and the preference of some analysts, to work with net cash flows. The net cash flow between two alternatives is simply the difference between the two cash flows. Net cash flows will typically show a differential initial investment (negative cash flows) followed by later differential returns (positive cash flows). The net cash flow of our example comparison of two machines is bar charted in Figure 3. The extra cost, which would be incurred in year zero, to purchase Machine B ($27,000) gives us our net investment, while the lower operating cost of Machine B and the fact that Machine A must be replaced in year 3 are responsible for our net savings. Using the net cash flow approach, the cost analyst would be interested in determining if the value of the cash flow was at all positive after taking into account the time value of the money. Any net positive value would indicate that the savings of Machine B justify the extra cost of Machine B.

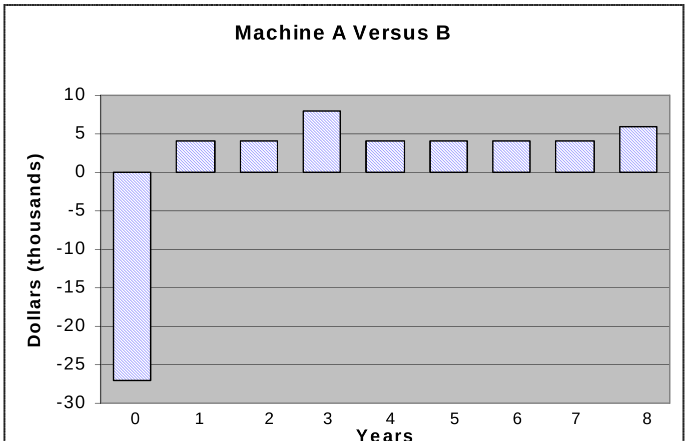


Figure 3

Commonly, therefore, the analysis will involve choosing between (1) one or more alternatives that have relatively high initial costs followed by relatively high savings or revenues later in the life cycle compared to (2) one or more alternatives that have more modest initial costs but also have lower savings or revenues later (usually including the existing method). In cases where one option has lower negative cash flows and higher positive cash flows than any of the options in each and every time period, then it is clear that this option is the most desirable (all other things being equal), and no time value of money considerations are necessary. Otherwise, when cash flow data that will be used to decide among alternatives have different magnitudes over time, then it is necessary for the estimator to consider the time value of money before the estimating job is truly complete.

STRUCTURING THE DISCOUNTED CASH FLOW ANALYSIS

Use of the discounted cash flow analysis method requires that there be at least two alternatives. Frequently there will be several. Often one of the alternatives in an investment analysis is to do nothing; instead, simply continue with the present system.

Other options may involve minor modifications to the present method to make it more efficient. There may be alternatives that require major modifications to the present system and other options that involve replacement of the existing method with totally new methods. Discounted cash flow analysis is a tool for selecting the best from among those cash flows defined. It is obviously important that all viable options be considered. Additional discussion concerning the proper structuring of the cash flow analysis is provided in references [1 and 2].

Multiple Levels of Investment

Some of the options may have multiple levels of investment with incremental costs bringing incremental levels of performance. To the extent practical, such options should be broken down such that each marginal investment can be evaluated against its marginal return. There are sometimes alternatives that have attractive cash flows when compared to the competing alternatives but that actually contain sub-elements that, if separately analyzed based on the sub-element's incremental investment versus incremental return, might be unattractive. The elimination of such sub-elements will enhance the overall alternative's performance. Although it is desirable to identify individually each of the potentially viable alternative courses of action, this must be balanced against the analytical advantages of holding the number of alternatives to the minimum necessary. It is perfectly acceptable to eliminate any options that may be determined to be nonviable based on preliminary analyses, and then concentrate on relatively fewer alternatives in a detailed analysis.

Exclusion of Common Cash Flows and Sunk Costs

It is only necessary to quantify and consider the cash flow differences between alternatives. Cash flows common to all options should be excluded. Likewise, sunk costs are of no consequence. The fact that one option (usually the existing method) has had a large previous sum of money invested in it should not bias the analysis. What is important is to identify, from the current moment onward, which of the alternatives is most economical.

Equal Capabilities Between Alternatives

The cash flow analysis should be structured so that all alternatives are compared on an equal basis. The job of the cost estimator often includes making some analytical adjustments in the raw cash flows of the alternatives in such a manner as to compensate for non-equalities in capability. For example, it is usually the case that newer alternatives being considered have a higher performance than the old method. If this higher performance manifests itself as lower operating costs or higher revenues, then it will be taken into account when the cash flows are estimated for alternative methods. Sometimes however, synthetic adjustments must be made to cash flows to equalize the capability between alternatives. One alternative may have certain capabilities that other alternatives do not possess. In such cases, the scope of the study can be expanded to include the services provided by the most capable alternative. Augmentations are then defined and costed that would bring the less capable alternatives up to the higher level of performance. Such augmentations do not necessarily need to be actions that are actually planned for the less capable alternatives in the event of their selection; however, they do need to be reasonable and viable augmentations that, to the extent possible, reflect the accurate worth of the additional capability.

Although it is desirable to assure that all expected and relevant cash inflows and outflows are captured in the analysis, and that any inequalities in capability are normalized by adjustments to the scope of the analysis and cash flows as discussed, there are sometimes fundamental differences in capability that cause inequalities that are extremely difficult to capture in the cash flows. At some point, adjustments to the cash flows of the systems being compared reach the point where any remaining differences must be treated in a non-quantitative manner. Any such considerations should be clearly documented and explicitly presented in the cash flow analysis. Not only does such documentation help decision makers in understanding the cash flow analysis structure, but also it becomes a potential discriminating factor in cases where quantifiable differences only in the cash flow analysis are not consequential enough to allow a confident selection.

Equal Economic Lifetimes

Alternatives with unequal economic lifetimes represent another common situation in discounted cash flow analyses because it is often the case that a new method will have a longer useful life than the old method. This can be compensated for by assuming that the short-lived options are replaced at the end of their lifetimes with an identical replacement. Thus an analysis comparing an alternative with a 5-year life to one with a 10-year life would require a 10-year cash flow and the inclusion of the replacement cost at the end of year 5 in the cash flow for the 5-year option. However, it still may be difficult to construct cash flows that end simultaneously for all options. Consider the case of an alternative with a 9-year life compared to an alternative with an 11-year life. The least common multiple of life cycles would be 99 years - an unwieldy length for a cash flow. In such cases a residual value approach can be used wherein the remaining value is included as a positive cash flow for any options that are not at the end of their lifetimes when the cash flow analysis terminates.

Income Tax Considerations

If the alternatives under consideration do not all have the same tax impact on the firm, then the cash flow analysis should be structured as an after-tax study by quantifying the tax differences in the cash flows in the appropriate time periods. Note that it is only necessary to calculate the *tax differences*, not the total tax effects of each alternative. Although it would involve a major accounting study to precisely quantify the different tax consequences among alternatives, it will often suffice to consider the differences in taxes due to the net deduction on capitalized (depreciable) assets and the net deduction due to operating expenses. For example, consider a comparison between option A, an existing manufacturing operation with an annual operating cost of $10,000, and option B, a 10-year lifetime capital equipment improvement that would cost $16,000 but reduce the cost of operations by $3000 annually. The existing operation can be assumed to result in a yearly operating expense tax deduction of $10,000 which, after taxes, at a rate of 40%, would yield $4000 of tax savings. Option B would result in a depreciation deduction of $1600 per year (assuming straight-line depreciation), which would convert to a tax savings of $640, and an operating expense deduction of $7000, which would be worth $2800, for a total annual tax savings of $3440. Therefore option B would result in $560 additional tax each year, and this amount should be included in either option B's annual cash flow as a cost (negative) or included in option A's cash flow as a savings (positive). For studies in which the alternatives involve different expected revenues, then the cash flows must include these revenues and the estimated tax on the revenues. Perhaps only one of the alternatives being considered has an eligibility for a specific tax credit. Then this credit should be included as a savings for that alternative and included in the cash flow in the time period when the credit would be expected.

The preceding use of straight-line depreciation was a simplifying assumption for illustrative purposes which, in actual practice, should be replaced by whatever depreciation method (e.g., ARCS) is appropriate for the capital equipment being considered. Also, in light of some tendency among cost estimators to occasionally think otherwise, perhaps it should be stated explicitly that depreciation is not part of cash flow to an organization - it is simply an intermediate calculation leading to the calculation of taxable income. The inclusion of the purchase cost itself in the cash flow in the time period of its purchase fully accounts for the initial cost of the capital equipment. This point is well presented in reference [3].

Disregard Payment Schedules Due to Financing Arrangements

A point of confusion in cash flow analysis similar to the erroneous inclusion of depreciation in the cash flow is that of formulating cash flows that correspond to the payment schedule on the loan for an investment, as opposed to constructing cash flows that correspond to the cash flow obligation. Whether an investment that is being analyzed is to be financed by borrowed money or paid for either totally or in part by cash is not relevant to the timing of money in the cash flow analysis. The fact that the organization is financing an investment and will be making principal and interest payments over a period of time will be implicitly accounted for by the time value of money techniques to be introduced. The cash flow should therefore include the total purchase cost in the period or periods in which the purchase is expected to be made. This is usually the beginning time period in the cash flow. In instances in which investment is expected to be made in several increments (as in the case of progress payments to a contractor), such payments do represent actual cash flows and should be timed accordingly in the analysis.

Uncertainties and Risk

Differences in perceived risk (both technical risks, such as the probability of a new method working as predicted, and economic risks, such as the probability of cost overruns or market uncertainties) can be handled in several ways.

The most straightforward way to account for risks is to add contingencies to the cost estimates of the alternatives. The alternatives with the highest risk would receive the highest contingency. It is sometimes difficult to decide the relative risk among options with sufficient confidence to establish the appropriate level of contingency. Also, the contingency method is limited in reflecting uncertainties in the revenue side of the cash flow.

Another method used to analyze risk is sensitivity analyses. In this approach, the cost analyst identifies the variables in the cash flow that are the least certain and then calculates the effects on the cash flows of allowing these variables to take on either lower or higher values.

A third common approach to the problem of risk is to include an allowance for risk in the discounted rate used. As is shown later, the choice of the discount rate is an important consideration in the cash flow analysis, and the higher the discount rate used, the more difficult it is for new methods that require new expenditures to demonstrate cost-effectiveness against existing systems which require more modest up-front expenses. When there is some risk that the proposed new method may not mature as handsomely as projected, including a risk premium in the discount rate is a way to raise the minimum acceptable rate of return and make it more difficult for contending proposals to win.

Two more statistical, although also more complicated, approaches to risk analysis are Monte Carlo simulation and decision tree analyses. The simulation approach requires a cash flow model that will accept probability distributions for each variable in the analysis instead of single deterministic values. These distributions are then sampled a large number of times to build up a statistical data base that gives the cash flow as a function of probability. Decision makers can thus be presented not only the estimated cash flow but also the confidence associated with the estimated cash flow as well. Decision tree analysis is an approach used to analyze the uncertainties in investment analysis by laying out (usually in a tree-oriented structure) the various alternatives available to the decision maker. The probabilities of the events along the paths are estimated, and statistical methods are used to calculate the overall economic expectations of the investment.

Decision Criteria

Once cash flows have been developed for each alternative, there re several time values of money decision criteria available to the cost analyst to apply to the problem of choosing between cash flows. These include present value comparisons (sometimes called net present value analysis or present worth analysis), equivalent annual amount comparisons (also called uniform annual amount and other similar names), and internal rate of return (sometimes called discounted rate of return, interest rate of return, and other names). As we show, these techniques, properly applied, are essentially equivalent and give consistent results.

Present Value

The most fundamental of these criteria, and probably the most commonly used, is the present value. The present value of a series of future cash flows is the value that it would be necessary to invest at the present time at a specified interest rate to be able just to make the future cash disbursements and receipts of the cash flow and completely exhaust the investment balance. The present value of $1000 one year from the present time at 10% interest is $909.09. That is, if one had $909.09 earning 10% for year, then a $1000 payment could be made leaving a zero balance. This is easily verified because $909.09 invested at 10% for one year will have a future value of $909.09 multiplied by 1.10, which does indeed equal $1000. For this simple cash flow, the value $909.09 is the present value of the cash flow, and $1000 is the future value of the cash flow; both values represent the very same economic worth at 10% interest, and, other things being equal, a rational person would be totally indifferent in choosing one over the other. Thus present value is the reciprocal of future value and is found by dividing the future value by one plus the periodic interest rate raised to the power *n* where *n* equals the number of periods separating the present and the period of the amount. The present value of $1000 ten years from now at 10% interest is $385.54 ($1000 divided by 1.10 to the power of 10). Again, both values, $385.54 now and $1000 ten years from now are exactly equivalent at 10% interest.

The present value of a *series* of cash flows is calculated by summing the present values of each of the individual present values of the cash flow. For example, the present value of the following cash flow:

Year 0	Year 1	Year 2
-$1000	$800	$800

can be calculated as:

$$PV = [-1000/(1.10)^0] + [800/(1.10)^1] + [800/(1.10)^2]$$

$$= -1000 + 727.27 + 661.16$$

$$= 388.43$$

In this example, $388.43 is said to be the discounted present value of the given cash flow. The word *discount* relates to the fact that dollars in the future are not worth as much as dollars now, and the value of future dollars must be discounted both as a function of the interest rate and as a function of how far they are into the future.

In the context of present worth, the fact that future dollars are worth less than current dollars has nothing whatsoever to do with price inflation - a separate economic phenomena that is discussed in this chapter, “Relationship Between Interest and Inflation.” Future dollars have less worth only because they have less time to draw returns. A dollar in hand can be invested today, whereas a dollar not in hand until a later period forgoes the potential to earn its owner returns until it is in hand. The future $800 in year 2 in the previous example is only worth $661.16 at the present time. Because having $661.16 today to invest at 10% interest is equivalent to waiting 2 years and receiving $800, one is theoretically indifferent about the choice.

Mathematically a cash flow is discounted to its present value by calculating the present worth of each of its periodic amounts at the time selected as the present. It does not matter what instant in time is selected as "the present" as long as each cash flow being compared is discounted to the same "present." The present may be defined as the year 1914. Or it can be defined just as well as the year corresponding to the first cash flow, which is the normal convention in discounted cash flow analyses. It could also be defined as some "future present" such as the year 2121 or, say, the year of the last cash flow in the analysis. In this case the calculations would utilize negative exponents. Let us say we want to repeat our discounting example but define the present to be the end of year 2. The resulting present value would be:

$$\begin{aligned} & \quad\text{Year 0} \qquad\qquad \text{Year 1} \qquad\qquad \text{Year 2} \\ PV &= [-1000/(1.1)^{-2}] + [800/(1.10)^{-1}] + [800/(1.10)^{-0}] \\ &= -1000\,(1.10)^{2} + 800\,(1.10)^{1} + 800\,(1.10)^{0} \\ &= -1210 + 880.00 + 800.00 \\ &= 470.00 \end{aligned}$$

This \$470.00 value is the present value of our cash flow because we have temporarily defined the end of year 2 to be the present. It should be apparent from the second line of this calculation that we are performing an operation that is equivalent to what we would do if asked to calculate the future value of the cash flow at the end of year two. That is, \$470.00 is also the future value of the cash flow, or the worth of the cash flow at 10% interest at the end of year 2. The same result for future value could be obtained by calculating the future value of the \$388.43 that we initially calculated as the year zero present value. The future value of \$388.43 two years hence is

$$FV = \$388.43\,(1.10)^{2} = \$470.00$$

Thus, the present value at the end of year zero of \$388.43 is equivalent to the future value of \$470.00 at the end of year two, both of which are equivalent to the original 3-year cash flow of -\$1000, \$800, \$800. In fact, there are an infinite number of other equivalent values because there are an infinite number of periods that we could define as present or future. Thus present value and future value are equivalent concepts, both of which collapse a time series of dollar amounts into a single dollar amount. This amount represents the worth of the entire cash flow it replaces taking into account the time value of money.

Therefore, the general equation for present value is

$$P = F \frac{1}{(1 + i)^{n}}$$

where P is present value, F is the future cash flow amount, i is the discount rate decimal equivalent (e.g., 0.10 for 10%), and n is the number of periods separating the present and the future time periods. Most textbooks on engineering economics contain tables of the function for discount factors at various interest rates for any number of years. Today, most discounted cash flow analyses are performed on computers, which simplifies the mathematics of discounting cash flows. Many of the software packages that are popular among cost estimators (such as spreadsheets) have time value of money functions such as present value.

In comparing two alternatives, using present value as the decision criterion in discounted cash flow analysis, the alternative that enjoys the highest positive present value or the lowest negative present value is the preferred option (still following the algebraic sign convention of negative dollars for costs and positive dollars for savings, revenues, and other cash inflows). As was mentioned previously, the instant in time chosen as the present in the discounting procedure does not matter at all as long as each alternative is computed to the same present. It is common to define the present as the period where the cash flow with the earliest outlays begins. Some cost analysts prefer to use future value as a decision criterion. This is equivalent to defining some future time period as the present and is perfectly valid. Instead of calculating the value of a cash flow at a period of time close to the outset of the activity, the analyst calculates the value of all the cash flows closer to the end period of the cash flows. The equation, which again is in tables in most engineering economy textbooks, is just the reciprocal of present value:

$$F = P(1 + i)^n$$

The avowed advantage in using future instead of present value in discounted cash flow analyses is that future value is easier to explain because it is the same as calculating the ultimate balance if one borrowed each invested (negative) amount in a series of future cash flows and continually refinanced both principal and interest until the time of the last cash flow (e.g., an arrangement like a construction loan for a new house) and *also* reinvested each (positive) return and its associated interest until the time of the last cash are more capable of identifying with the future worth of a cash flow than the discounted present worth. In any event, both present value and future value will give consistent results when they are used as decision criteria to choose between alternatives.

Equivalent Annual Amount

Another technique used as a criterion for selection in discounted cash flow analysis is the equivalent annual amount. In this approach, the cash flows being compared are all converted to a constant annual amount over a specified time period that has the same present value as the original cash flow. In other words, the original cash flow, which may have periodic amounts varying in magnitude from period to period, is converted to a uniform cash flow (one with the same dollar magnitude in each period) that has a present value equivalent to the original cash flow. Mathematically, the procedure is composed of two steps: The first calculates the present value of the series of cash flows just as in the present value technique. The second step uses what is generally called a capital recovery factor to calculate what constant amount of money spread over n periods at i interest rate would have the original present value. Therefore, to calculate the annual equivalent over n periods at i interest of a cash flow with a present value of P, use:

$$\frac{P[i(1 + i)^n]}{(1 + i)^n - 1}$$

where A is the equivalent annual amount, and the parenthetical expression being multiplied by P is the capital recovery factor. For example, a cash flow that has been found to have a present value of $10,000 can also be represented over 5 years at 10% interest with an equivalent annual amount of $2637.97. The capital recovery factor equation is the same equation that lenders use to calculate the payment on a loan; $2637.97 is the same value one would be quoted by a banker as the annual payment on a $10,000.00 loan for 5 years (while for monthly payments *n* would be entered as the number of months over which the loan was to be financed and *i* would be entered as one-twelfth of the annual interest rate). Once two or more cash flows have been thus annualized, the preferred choice is the one with the highest positive present value or lowest negative present value.

One advantage of the equivalent annual amount approach is that in applications where the analysis is choosing the most cost effective production method for a product, the equivalent annual amount can be calculated over a period of time corresponding to the product's revenue life cycle, and the results of the analysis can be presented as a unit cost for the product. For example, if the production is 1000 units per year, the cost per unit in the preceding example could be quoted at $2.64 per unit. Said another way, to realize a 10% rate of return, the products must be sold for $2.64 apiece. For the next alternate production method, the cash flow could also be converted to a cost per unit and compared to the $2.64 value. A lower cost would cause the corresponding method to be selected whereas a higher cost per unit would cause its rejection.

A second advantage to the equivalent annual amount technique is that in many cost analyses the recurring annual cost (of a production method for example) is known. Since this cost is already "annualized" there is no need to perform any other time value of money calculations on these amounts. All that is required is to annualize any costs that are not on an annual basis (nonrecurring capital cost for example) using the appropriate capital recovery factor and to add the result to the know annual cost. Let us say that a proposed alternative method to produce our product involves the purchase of a $5000 machine and a $1000 annual operating cost made up of materials, labor, and all other recurring production costs. Assuming a 5-year life for our machine, the same capital recovery factor as used before would be applied to get a $1318.99 equivalent annual amount for the machine cost (0.2638 X $5000). This could be added directly to the other known annual costs of $1000 to get the total equivalent annual amount of $2318.99, which works out to $2.32 per unit. Since this is less than the $2.64 cost per unit of the former method, the new method is preferred.

Assumption of an Infinite Horizon

Although most cash flow analyses do set a limit on the length of the life cycle that is considered, limiting the economic horizon to one or more multiples of the service lives actually understates the value of the preferred alternative. The justification for this statement is that if two or more alternatives for future investment are compared in a discounted cash flow analysis, and one alternative is chosen because it demonstrates economic benefits over the other alternative is chosen because it demonstrates economic benefits over the other alternatives, then this benefit stream will likely extend indefinitely into the future. Why indefinitely and not just to the end of the alternative's useful life? Because at the end of the useful like of the chosen alternative, the alternative will either be replaced with yet another alternative that is at least as, and probably more, cost-effective. That is, a rational decision-making organization will never choose to return to any of the original contending alternatives which were proven to be less cost-effective, or will they ever again accept any alternative that is less cost-effective than the chosen alternative - instead they will either continue forever with the exact replacements of the chosen alternative or something even more cost-effective. Thus the benefits of the chosen alternative will continue forever at a level as great as that shown for it in the original analysis.

This concept, also called perpetual worth or capitalized costs, can be used in the comparison of alternatives by first calculating the equivalent annual amount of each alternative and then dividing the result by the interest rate. For out preceding cash flow, with an equivalent annual amount of $2637.97 over 5 years at an interest rate of 8%, the present value of an infinite horizon annual amount would be $2637.97 divided by 0.08 or $32,975. That is, the present value of a series of $2637.97 annual cash flows stretching into the future forever is $32,975. Although this idea overwhelmed Huck Finn, a moment's reflection will illuminate the principle because $32,975 put into an 8% bearing investment now would yield an annual interest income of $2637.97 forever without ever touching the principal. Incidentally, Huck's and Tom Sawyer's $6000 put out at interest by Judge Thatcher earning a dollar a day works out to be about 6% interest, apparently the going rate in the time of the novel (the mid-1800s).

Once an infinite horizon present worth has been calculated for two or more alternatives, the one with the most positive or least negative value is the preferred choice. The difference between two such infinite horizon present worths represents the preferred alternative's economic benefits with infinite horizon when compared to the other alternative.

Introducing infinite horizon into a comparison of alternatives will not change the choice that would have been made with either the present value or the equivalent annual amount methods using a finite horizon. It is simply another equivalent method. However, infinite horizon assigns a more encompassing measure to the ultimate economic benefits that will be obtained from an investment. Thus it is sometimes favored in applications such as those dealing with the justification of research and development funds when it is considered useful to remind decision makers that the approval of such investments might lead to far greater ultimate benefits than are indicated by simply calculating the benefits through the first life cycle of some potential improvement made possible by the R& D funding.

For discounted cash flow analyses of relatively long life cycles (say 30 years or longer) and/or relatively high interest rates (say 15% or more), the assumption of infinite horizon may not yield present value quantities significantly higher than those that would have been obtained without infinite horizon. This is because discount factors decrease over time and do so more rapidly at higher discount rates, as shown in Figure 4. For all practical purposes, discount factors approach zero after three decades or so for interest rates above 15%. Any cash flow extending beyond this (including one going to infinity) is essentially zeroed out by the discounting process.

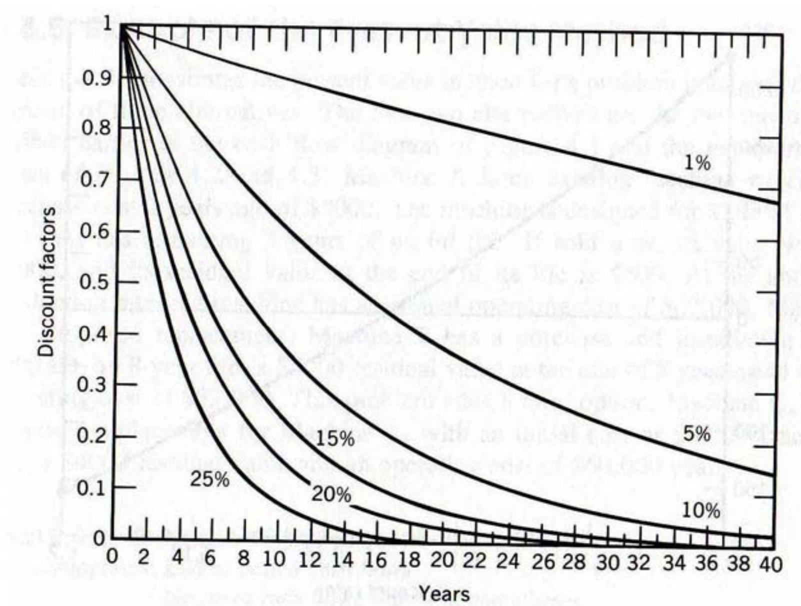


Figure 4.4. Discount factors decrease over time.

Internal Rate of Return

Present values, future values, and equivalent annual amounts are all really just extensions of the same basic concept. Properly applied, they all give consistent and reliable results when choosing among alternatives. A fourth technique often used as a decision criterion in discounted cash flow analyses is the internal rate of return (IRR). Assuming that a new method is being compared to an existing method, in a non-discounted analysis one would say that an incremental investment of, say, $1000 that results in returns of $200 per year is an 20% rate of return. This simple return on investment (ROI) calculation fails to take into account that each dollar in the $200 returns flowing in each year are not worth, in the time value of money sense, the same as the dollars in the $1000 invested at time zero. The internal rate of return is a discounted rate of return that *does* correct for the differing time values of the dollars in the cash flows.

The internal rate of return for a given cash flow is defined as the discount rate that results in a present value of zero. Thus the IRR method finds the interest rate that equalizes the present value of the investment and the present value of the returns. For our ROI cash flow example, the IRR is 15.098%. (This can be verified by discounting the cash flow - a negative $1000 followed by 10 positive $200 amounts - by 15.098%. A net present value of zero will be found.) Since the simple ROI result of 20% does not take into account the time value of money, it is to be expected in this case that IRR should be less than ROI.

The usual method for calculating the IRR is a trial and error procedure to discover just what discount rate will yield a zero present value for the cash flow. Sometimes this is combined with graphic interpolation. Figure 5 shows the net present value of the above cash flow over a range of discount rates. Because net present value goes negative at rates just above the internal rate of return, the IRR can be surmised form the graph to be very near the value of 15%. The use of a computer and some iterative programming logic obviously facilitates such a procedure. Again, many popular software packages such as spreadsheets have built in IRR functions that make the calculations easy, fast, accurate, and eliminate graphic interpolation.

Figure 5

Once IRR is known for a cash flow, it is compared to the minimum acceptable rate of return, and the investment is either accepted or rejected accordingly. Considerations involved in specifying the minimum acceptable rate of return are discussed in this chapter, “Choosing A Discount Rate.” Presuming that the minimum acceptable rate of return is known, the use of IRR as an investment decision criterion has a certain appeal because of this straightforward manner in which the investment decision is made - as long as the contemplated investment meets the organization’s minimum rate of return requirement, then it is considered acceptable.

A potential problem with the use of the IRR as an investment decision criterion can occur in certain circumstances. Because the calculation of IRR involves finding a solution to a complex function, certain cash flows that are not well behaved may cause either multiple solutions or no solutions. A cash flow that begins negative and then turns positive will generally cause no problems in the IRR calculation. Cash flows with multiple sign changes can, however, result in multiple mathematically valid solutions, a disturbing outcome in investment analysis. In such an event, the cost estimator can rely on the criteria of present value and/or equivalent annual amount.

Example of the Present Value Method

Table 1 demonstrates the present value method for a problem involving the comparison of three alternatives. The first two alternatives are the two machines described earlier in the cash flow diagram of Figure 1 and the bar-charted cash flows of Figures 2 and 3. Machine A is an existing machine which had a purchase cost 2 years ago of $5000. The machine is designed for a life of 5 years, and thus has remaining 3 years of useful life. If sold now, its value would be $3000, and its residual value at the end of its life is $500. At the anticipated production rate, the machine has an annual operating cost of $67,000. Machine B is a proposed replacement. Machine B has a purchase and installation cost of $30,000, and 8-year life, a $3000 residual value at the end of 8 years, and a yearly operating cost of $63,000. This problem adds a third option, Machine C, another proposed replacement for Machine A, with an initial cost of $40,000, and 8-year life, a $4000 residual value and an operating cost of $60,000 yearly.

Table 1. Example of the Present Value Method

Assumptions: End of period cash flows
Negative cash flows shown in parentheses
Discount rate used 10%

		Cases		
		A Existing Machine A	B Proposed Machine B	C Proposed Machine C
Purchase cost		$5,000	$30,000	$40,000
Design life		5	8	8
Remaining life (years)		3	8	8
Value if sold now		$3,000	NA	NA
Residual value at end of life		$500	$3,000	$4,000
Annual operating cost		$67,000	$63,000	$60,000
	Year	Total Cash Flows		
(Present value at end of year 0)	0		($27,000)	($37,000)
	1	($67,000)	($63,000)	($60,000)
	2	($67,000)	($63,000)	($60,000)
	3	($71,500)	($63,000)	($60,000)
	4	($67,000)	($63,000)	($60,000)
	5	($67,000)	($63,000)	($60,000)
	6	($67,000)	($63,000)	($60,000)
	7	($67,000)	($63,000)	($60,000)
	8	($66,500)	($60,000)	($56,000)
Undiscounted total		($540,000)	($528,000)	($513,000)
Present value		($360,588)	($361,701)	($355,230)

This problem is the classic case of proposed new alternatives with incremental initial investments that promise later savings in operational costs. Machines B and C cost $30,000 and $40,000, respectively, but offer $4000 and $7000 savings, respectively, when compared to the existing machine. The total cash flows are shown in Table 1 (negative cash disbursements shown in parentheses) based on the data for each of the three alternatives. Table 1 (and all further examples in this chapter) follow the convention of assuming end of year cash flows. That is, it is assumed that each cash disbursement or receipt occurs on December 31. Thus, in Table 1 the present is defined December 31 of year zero. Both Machine B and Machine C are placed in service on December 31 of year 0 with their purchase costs entered in the cash flow at that time and their first annual operating cost placed at the end of year 1. Notice that the $5000 original cost for Machine A is a sunk cost and is not included in year zero. Years one through 8 contain the respective operating cost for each of the machines. Because Machine A has only 3 years of remaining life it must be assumed that Machine A must be replaced at the end of year 3 to provide the continuing service of the two proposed alternatives. Therefore year 3's cash flow for Machine A contains an additional negative cash flow of $5000 for replacement. In year 8 of the cash flows each alternative is given credit for its residual value (a positive value which reduces the net cash outflow as shown).

The discount rate for the problem of Table 1 is assumed to be 10%. The last row of Table 1 gives the discounted present value of each of these three cash flows. These happen to be present values as of December 31 in year zero, but as discussed earlier, they could just as well have been calculated at any other time defined as present provided that all three alternatives were discounted to the same time period. Because Machine C has the least present value of costs ($355,230) at our specified discount rate of 10%, it is the preferred alternative. The present values were calculated by summing the discounted values for each year's cash flow using our equation for present value:

$$P = F\frac{1}{(1+i)^n} \qquad (7)$$

which for year zero's cash flow for Machine C would be

$$P = \$37{,}000\left[\frac{1}{(1.10)^0}\right] \qquad (8)$$
$$= \$37{,}000$$

and for year 1's cash flow for Machine C would be

$$P = \$60{,}000\left[\frac{1}{(1.10)^1}\right] \qquad (9)$$
$$= \$54{,}545$$

and for subsequent years would be:

$$P = \$60{,}000\left[\frac{1}{(1.10)^2}\right] = \$49{,}586 \text{ for year 2}$$

$$P = \$60{,}000\left[\frac{1}{(1.10)^3}\right] = \$45{,}079 \text{ for year 3}$$
$$P = \$60{,}000\left[\frac{1}{(1.10)^4}\right] = \$40{,}981 \text{ for year 4}$$
$$P = \$60{,}000\left[\frac{1}{(1.10)^5}\right] = \$37{,}255 \text{ for year 5} \qquad (10)$$
$$P = \$60{,}000\left[\frac{1}{(1.10)^6}\right] = \$33{,}868 \text{ for year 6}$$
$$P = \$60{,}000\left[\frac{1}{(1.10)^7}\right] = \$30{,}789 \text{ for year 7}$$

$$P = \$60{,}000\left[\frac{1}{(1.10)^8}\right] = \$26{,}124 \text{ for year 8}$$

which totals to a present value of $355,230 (ignoring rounding errors).

Table 2 adds additional rows to the previous example to demonstrate the selection between the same three alternatives using the equivalent annual amount method. The equivalent annual amount is shown in the table for each alternative just under the previously calculated present values. Machine C has the lowest annualized cost ($66,586) at our 10% rate of interest and would be chosen by this criterion as well. The $66,586 is calculated by multiplying the previously determined present value by the capital recovery factor for 8 years at 10% interest:

$$\begin{aligned} A &= \frac{P\left[i(1+i)^n\right]}{(1+i)^n - 1} \\ &= \$355{,}230\ (0.187444) \\ &= \$66{,}586 \end{aligned} \qquad (11)$$

As shown in the bottom portion of Table 2, 8-year cash flows of the annual amounts have the same present value as their respective original cash flows. For Machine C, for example, a uniform cash flow of $66,586 each year for 8 years has the same $355,230 present value as the original cash flow. Thus, either present value or equivalent annual amount capture the worth of the entire cash flow in a single value that can be compared directly to the present value or equivalent annual amount of another cash flow.

Table 2. Example of the Equivalent Annual Amount Method

Assumptions: End of period cash flows
Negative cash flows shown in parentheses
Discount rate used 10%

		Cases		
		A Existing Machine A	B Proposed Machine B	C Proposed Machine C
Purchase cost		$5,000	$30,000	$40,000
Design life		5	8	8
Remaining life (years)		3	8	8
Value if sold now		$3,000	NA	NA
Residual value at end of life		$500	$3,000	$4,000
Annual operating cost		$67,000	$63,000	$60,000
	Year	Total Cash Flows		
(Present value at end of year 0)	0		($27,000)	($37,000)
	1	($67,000)	($63,000)	($60,000)
	2	($67,000)	($63,000)	($60,000)
	3	($71,500)	($63,000)	($60,000)
	4	($67,000)	($63,000)	($60,000)
	5	($67,000)	($63,000)	($60,000)
	6	($67,000)	($63,000)	($60,000)
	7	($67,000)	($63,000)	($60,000)
	8	($66,500)	($60,000)	($56,000)
Undiscounted total		($540,000)	($528,000)	($513,000)
Present value		($360,588)	($361,701)	($355,230)
Equivalent annual amount		($67,590)	($67,799)	($66,586)
	Year	Equivalent Cash Flows		
`	0	($67,590)	($67,799)	($66,586)
	1	($67,590)	($67,799)	($66,586)
	2	($67,590)	($67,799)	($66,586)
	3	($67,590)	($67,799)	($66,586)
	4	($67,590)	($67,799)	($66,586)
	5	($67,590)	($67,799)	($66,586)
	6	($67,590)	($67,799)	($66,586)
	7	($67,590)	($67,799)	($66,586)
	8	($67,590)	($67,799)	($66,586)
Present value		($360,588)	($361,701)	($355,230)

SELECTION FROM MULTIPLE ALTERNATIVES

The discussion to this point has been limited to selecting the most cost-effective investment from only two or three alternatives. Extending the present value and equivalent annual amount methods to the problem of selecting from a large number of alternatives is straightforward - the alternative that presents the most positive (or least negative) present value or annual equivalent cost is the most economical choice. The IRR method can also be used to select from a large number of contending investments. There is, however, a potential pitfall when using IRR as the decision criterion when comparing multiple alternatives. It is not necessarily correct to calculate the IRR independently on each alternative cash flow and choose the option with the highest IRR. Instead the investment options are placed in ascending order (the alternative with the lowest investment cost is put first, followed by increasingly more costly alternatives, with the alternative having the highest investment cost put last). The incremental cash flow of the second compared to the first alternative is then subjected to the IRR method. If the resulting rate of return exceeds the minimum acceptable rate of return, the second alternative becomes the new defender. Otherwise, the first alternative is retained as the defender. The incremental cash flow of the third alternative compared to the defender is then developed, and the resulting marginal rate of return is compared to the minimum acceptable rate of return just as was done before. Either a new defender is found or the previous defender is retained, depending on whether the IRR is higher or lower respectively than the minimum acceptable IRR. This pair by pair incremental cash flow comparison approach is continued until all alternatives have been completed. The next highest investment is always selected as long as it meets or exceeds the minimum IRR. The justification for this approach is that an alternative is attractive if the incremental investment results in sufficient incremental returns to meet our minimum requirements. This is not necessarily the same as selecting the alternative that yields the overall highest IRR.

There can be alternatives that yield overall rates of return that are higher than the competing alternatives but that are unable to absorb our total available investment capital. Other alternatives that may cost more initially and yield a lower overall rate of return may actually be the wisest choice because they are able to put more of our capital to work at rates higher than our minimum rate of return. As we discuss later when we explore the proper choice of discount rates, the minimum acceptable rate of return is defined by the other opportunities that are available for investment. As long as a contemplated investment is able to give a rate of return that exceeds this opportunity it should be made.

The example problem is Table 3 demonstrates the foregoing premise. Six alternatives are compared; the first three are the same as in our previous problems using present value and equivalent annual amount. First of all, the total cash flows of these alternatives are used to calculate present value and equivalent annual amount. These two criteria had already been calculated for Machines A, B and C. Although previously we selected C based on its lowest present worth of costs equal to -$355,230, we can now see the new proposals for Machines D through F offer even lower cost alternatives. Because E’s present value of cost at -$351,426 is lower than either D or F, we favor Machine E as the most attractive of these options. The same selection is confirmed by the equivalent annual amounts tabulated just below the present values.

Table 3. Example of the Internal Rate of Return Method for Selecting from Multiple Alternatives

Assumptions: End of period cash flows
Negative cash flows shown in parentheses
Discount rate used 10%

		Cases					
		A Existing Machine A	B Proposed Machine B	C Proposed Machine C	D Proposed Machine D	E Proposed Machine E	F Proposed Machine F
Purchase cost		$5,000	$30,000	$40,000	$45,000	$50,000	$55,000
Design life		5	8	8	8	8	8
Remaining life (years)		3	8	8	8	8	8
Value if sold now		$3,000	NA	NA	NA	NA	NA
Residual value at end of life		$500	$3,000	$4,000	$4,500	$5,000	$5,500
Annual operating cost		$67,000	$63,000	$60,000	$58,500	$57,500	$56,800
	Year			Total Cash Flows			
(Present value at end of year 0)	0		($27,000)	($37,000)	($42,000)	($47,000)	($52,000)
	1	($67,000)	($63,000)	($60,000)	($58,500)	($57,500)	($56,800)
	2	($67,000)	($63,000)	($60,000)	($58,500)	($57,500)	($56,800)
	3	($71,500)	($63,000)	($60,000)	($58,500)	($57,500)	($56,800)
	4	($67,000)	($63,000)	($60,000)	($58,500)	($57,500)	($56,800)
	5	($67,000)	($63,000)	($60,000)	($58,500)	($57,500)	($56,800)
	6	($67,000)	($63,000)	($60,000)	($58,500)	($57,500)	($56,800)
	7	($67,000)	($63,000)	($60,000)	($58,500)	($57,500)	($56,800)
	8	($66,500)	($60,000)	($56,000)	($54,000)	($52,500)	($51,300)
Undiscounted total		($540,000)	($528,000)	($513,000)	($505,500)	($502,000)	($500,900)
Present value		($360,588)	($361,701)	($355,230)	($351,994)	($351,426)	($352,458)
Equivalent annual amount		($67,590)	($67,799)	($66,586)	($65,979)	($65,873)	($66,066)
	Year			Net Cash Flows			
(Present value at end of year 0)	0		($27,000)	($37,000)	($5,000)	($5,000)	($5,000)
	1		$4,000	$7,000	$1,500	$1,000	$700
	2		$4,000	$7,000	$1,500	$1,000	$700
	3		$8,500	$11,500	$1,500	$1,000	$700
	4		$4,000	$7,000	$1,500	$1,000	$700
	5		$4,000	$7,000	$1,500	$1,000	$700
	6		$4,000	$7,000	$1,500	$1,000	$700
	7		$4,000	$7,000	$1,500	$1,000	$700
	8		$6,500	$10,500	$2,000	$1,500	$1,200
Undiscounted total			$12,000	$27,000	$7,500	$3,500	$1,100
Net present value			($1,113)	$5,358	$3,236	$568	($1,032)
Equivalent annual amount			($209)	$1,004	$607	$107	($193)
Internal rate of return			8.87%	13.79%	25.56%	12.93%	4.36%

Now let us apply internal rate of return as the decision criterion in this problem. First, The internal rate of return calculation requires a cash flow with an investment (i.e., negative dollars) and a return (i.e., positive dollars). The total cash flows in Table 3 for our six alternatives are all negatives since they represent the expected costs of six different machines. Therefore, we must develop the net cash flows between these total cash flows because we are interested in knowing if the extra investment required by the increasingly more expensive options as we progress toward the right side of the table are worth the extra savings.

The net cash flow of our first proposal, Machine B, is calculated at the bottom of Table 3 in column B by subtracting the cash flow of Machine A from the cash flow of Machine B. Thus we obtain a net cash flow with an initial outlay of $27,000 followed by savings of $4000 for 2 years (the difference in operating cost), $8500 in year 3 (due to the replacement of Machine A), $4000 again for years 4 through 7, and finally $6500 in year 12 (due to the difference in residual value in the two machines). The net present value and net equivalent annual amount are displayed at the bottom of Table 3. Both criteria tell us Machine B is not a very good alternative when compared to Machine A. (Remember that we are now dealing with net cash flows in which it is to be hoped that the positive return exceeds the negative investment.) The internal rate of return of 8.87% is less than our specified minimum acceptable rate of return of 10%. Just as was the case when we examined the total cash flows with present value and equivalent annual amount, Machine B is to be rejected and Machine A continues as the defending alternative.

The net cash flow of the second proposal, Machine C, is calculated in column C by subtracting the cash flow of the defending alternative, Machine A, from the cash flow of Machine C. The net cash flow shows an investment of $37,000 followed by returns of $7000 for 2 years, $11,500 in year 3, and $7000 each year until the last year, when the marginal return is $10,500. For this alternative, both the net present value and net equivalent annual amount at 10% are positive, indicating that the option will give a return greater than the minimum required rate. Just how much more is indicated by the internal rate of return - exactly 13.79%. Thus Machine C becomes our new defender, supplanting the existing machine.

Column E gives the marginal investment and return profile for Machine D - another $5000 invested will get us an additional $1500 per year and $2000 in the final year. The net present value and net equivalent annual amount are positive at 10%, and the internal rate of return is 25.56%.

Column E gives us the marginal profile for Machine E - another $5000 invested will return $1000 for 7 years and $1500 in the final year. But is it worth it? The IRR is only 12.93% - not nearly as good as alternative D's 25.56%. Or is it? The answer is that Machine E does offer a better opportunity than D even though the rate of return is considerably less. How can this be? It is because the 12.93% is the marginal return only on the $5000 of extra investment, and since 10% represents our next best opportunity for putting our capital to work, if we do not choose to invest it in Machine E at 12.93%, then we will be able to earn only 10% elsewhere on our $5000. It is our choice - put the $5000 our at 12.93% or put it out at 10%. But if we succumb to this notion of picking E because its marginal return is greater than our minimum acceptable rate, will we not be loosing the opportunity of getting the 24.56% rate of return of Machine D? Not at all. Remember that E shows more savings per year than D (generally $1000 per year more). Alternative E offers us all the savings of D, plus some.

Finally, the net cash flow of Machine F in Table 3 results in all three criteria being undesirable, indicating that this alternative is not attractive. The additional investment of $5000 over the current defender, Machine E, yields $700 per year in returns and $1200 the final year, but the internal rate of return calculation indicates that this is only a 4.36% rate of return on the extra $5000. Our 10% opportunities offer us much more that that, and thus alternative F is rejected.

For those still troubled by the choice of alternative E and its 12.93% rate of return over D, look back at the top portion of Table 3 where the present value and equivalent annual amounts both indicated that Machine E is clearly the most cost-effective choice. As was said before, properly applied and interpreted all three-decision criteria give consistent results. Given the dangers of internal rate of return, would it not be better to stick with present value or equivalent annual amounts as more reliable decision criteria? Present value and equivalent annual amount will provide the correct choice every time with relatively low risk, but internal rate of return offers advantages at times. As was demonstrated in the example of Table 3, rate of return is an attractive criterion in choosing among multiple alternatives because it not only indicates the best alternative, but also perhaps gives the analyst and management a better intuitive feel for the degree of goodness through the comparison of the rate of return of the alternatives under consideration and the firm's other opportunities for rates of return. Present value and equivalent annual amount, on the other hand, convey less of a direct corollary meaning to the decision maker. Given the availability of computers in solving for each of the decision criteria that have been discussed, it is not infeasible to calculate each. An excellent treatment of the proper use of the various time value of money decision criteria is offered in reference [2].

THE RELATIONSHIP BETWEEN INTEREST AND INFLATION

A common mistake in discounted cash flow analysis is incorrectly accounting for the relationship between interest and inflation. Interest and inflation, although related in many ways, are totally different economic phenomena. Interest is a rent paid to the owner of money by the borrower of that money, or equivalently is a rate of return earned (or lost) on an investment. Thus interest, by having the power of making capital either grow or shrink over time, gives money a time value. Inflation also causes money to either increase or decrease in value over time, but this change in value is due to changes in the value of money as a standard unit of measure. The worth of a laborer's day may be put down in a cost estimate, but in periods of either inflation or deflation the value used will soon be out o date because laborer's are no longer being paid the same amount for a day's work. Inflation then, is not the same thing as interest. Inflation is instead a change in the measuring system - a change in the value of the dollar due to many reasons that have nothing to do with changes in value due to interest. (This disavowal of any connection between interest and inflation is only true in the microeconomic environment of engineering economics - the economics of a single firm for instance. In macroeconomics, interest and inflation are not so unrelated.)

Although interest and inflation are two different things, they are related to each other in fairly constant and predictable ways. Money that is earning a return (for example, money in an interest bearing account in a bank) is growing by an amount that can be calculated using compound interest equation. For example, a \$1000 principal amount invested at a 10% annual compounded interest rate grows in the following manner:

Year 1	Year 2	Year 3	Year 4
\$1000	$\$1000(1.1)^1$	$\$1000(1.1)^2$	$\$1000(1.1)^3$
\$1000	\$1100	\$1210	\$1331

However, if inflation is some positive rate (say 6%), then those earnings are also decreasing in value at a rate equal to the inflation rate. This can be calculated as follows:

Year 1	Year 2	Year 3	Year 4
\$1000	$\$1100/(1.06)^1$	$\$1200(1.06)^2$	$\$1331(1.06)^3$

\$1000	\$1038	\$1077	\$1118

This second calculation yields a cash flow that includes the effects of interest after correcting for inflation.
Since, as the previous example demonstrated, interest is calculated by multiplying the principle amount by the interest rate, and inflation is calculated by dividing by the inflation rate, the estimator can save a step by first adjusting the interest rate:

$$\text{adjusted interest} = \frac{1 + \text{interest rate}}{1 + \text{inflation rate}}$$

$$= \frac{1.10}{1.06}$$

$$= 1.038$$

and then the previous two-step operation becomes one step by using the adjusted interest rate:

Year 1	Year 2	Year 3	Year 4
\$1000	$\$1000(1.038)^1$	$\$1000(1.038)^2$	$\$1000(1.038)^3$
\$1000	\$1038	\$1077	\$1118

Economists call this interest rate after correcting for inflation the *real* rate of return or the *real* rate of interest. Over the long term, the rate of inflation and the rate of interest in the economy track each other and in fact the real rate of interest tends to remain fairly constant. Figure 6 shows this tracking trend in the inflation and interest rates in the United States from 1958 to 1986. As can be seen from the figure, when inflation is high, interest rates tend to rise a swell because the holders of capital in the economy resist lending their money out at rates that result in small real gains. As we discussed, the real interest rate in the market interest rate divided by the inflation rate. Performing this arithmetic on the data of the previous figure results in Figure 7. Until the 1980's real rates tended to remain under 4%. After 1980, inflation decreased dramatically while federal budget deficits kept the market rate of interest high, resulting in real rates of interest that were double the recent historical average.

There are a number of sources for indices of historical interest rates and inflation rates available for the cost analyst to consult. Historical interest and inflation data for the United States are available from the *Department of Commerce Statistical Abstract of the United States* [4], an annual publication available in most libraries or for sale by the Superintendent of Documents, U.S. Government Printing Office in Washington. Historical, as well as, forecasted indices are available from several econometric forecasting firms such as Data Resources, Inc., in Lexington, Massachusetts [5]. These sources publish a number of specialized indices for specific sectors of the economy and specific industries as well as the more general indices such as the GNP deflator and prime interest rate.

The choice of the proper interest rate to use in a discounted cash flow analysis is discussed at more length in the Section entitled "Choosing a Discount Rate." It is sufficient to say, for now, that the choice tends to be highly company specific depending on each company's cost of capital and other investment opportunities.

The proper choice of an historical inflation index is usually somewhat less company specific and more tied to the sector of the economy in which the company participates. The so-called GNP deflator of Figure 6 is a very general measure of inflation in the overall output of the economy. Familiar indices such as the Consumer Price Index and Producer Price index measure aggregate consumer and wholesale prices respectively. More applicable to the cost analyst are the detailed breakouts of these aggregate indices which re published by the previously mentioned sources. Lacking accurate company records on what historical inflation has been for their specific sector, the cost analyst would generally find such indices as those published by the government and the private econometric firms very useful.

Figure 6-7

Although historical inflation data are essential to the cost analyst for adjusting old cost data to more current price levels for use in an analysis, the value of including an allowance in cash flows for future general price inflation is open to argument. First of all, predicting future price increases is an inexact process at best. Such predictions are seldom reliable in any absolute sense, and frequently are inaccurate in even relative terms. Second, unless there is some overriding requirement for including general future price inflation in the cash flow estimate, it should be omitted, because the inclusion of inflation usually adds no useful information to an analysis of competing alternatives when all the alternatives will be affected equally by the general inflation adjustment. Such justifications for including general escalation include the case of the cash flow that is going to be used directly in a bid proposal or in some other budgetary manner. Budgetary inputs usually need to include inflation. However, if the cash flow analysis is to be used solely for the comparison of alternative investments such as equipment selection, there is probably no need to try to include the effects of future inflation unless - and this is a big unless – there is some component of the cash flow that is expected to inflate or deflate differentially with respect to the other components of the cash flows. Good examples of differentially inflating components might be labor or energy costs. For any such components where the cost analyst has evidence that differential inflation is likely, the anticipated escalation should be included in the cash flow. Note that including differential inflation does not change the price level of the analysis. The dollars of the cash flows are still in the base-year prices (usually termed "Constant 19XX Dollars") and should be clearly labeled as such in the documentation of the analysis.

The foregoing argument for performing non-budgetary cash flow analyses in constant rather than inflated dollars affects the choice of the discount rate to be used in the analysis. The rule is: Use a real discount rate for discounting cash flows that are in constant, non-inflated dollars and use a market discount rate (i.e., one that includes inflation) for discounting cash flows that have been inflated into future-year price levels.

There is some confusion in terminology in this area. The term "current" is used by economists to refer to data that are in then current dollars. For example, a table tracking the nation's GNP from 1950 through 1980 and labeled "current dollars" would mean that the 1950 data are in 1950 dollars, the 1951 data are in 1951 dollars, and so on.

If the table is in some constant-year dollars it would be labeled constant 19XX dollars. We follow this convention, using current to mean a cash flow that is in the price level of the year of the flow, and constant to mean a cash flow that excludes this allowance for general price inflation.

Either the constant-dollar approach or the current-dollar approach will give identical results in the typical application of selecting among alternatives. The following example demonstrates this fact as well as showing how differential inflation should be included in a cash flow analysis.

Let us consider an equipment selection problem involving any number of cash flows being compared. For one such cash flow, exactly the same present value will be obtained for both the constant-dollar and current-dollar approaches. Assume a purchase cost of $1000,000 followed by a labor operating cost of $20,000 for each year of a three-year operating life. There are no other expected costs. Let us further assume that the cost analyst, on checking with the personnel office, discover that owing to already negotiated labor contracts, operating cost for the equipment can be expected to increase at the general inflation rate plus 2% annually. The cost analyst consulted the company financial managers and determined that the appropriate market discount rate to use as the discount rate in the analysis is 12%. Consulting with economic seers, he determines that future general price inflation will be 8% annually.

Figure 8 shows a cash flow analysis of the given problem performed in current dollars (i.e., including general inflation) leading to the calculation of both present value and equivalent annual amount. In Figure 8 all dollars are in thousands. The cash flow profile is shown with the operations cost inflated by 10% annually, which includes both general inflation at 8% plus the expected escalation in operating cost of 2%. Thus the operating cost grows from $22 (thousand) in year 1 to $26.62 by year 3. Because the cash flow includes general inflation, a 12% discount rate that includes inflation is used to obtain a present value of $157.88. The application of a 12%, three-year capital recovery factor yields an equivalent annual amount of $65.73 in current dollars. As a check, the uniform cash flow of $65.73 is assembled at the bottom of Figure 8 and the present value calculated to insure that the same $157.88 is obtained.

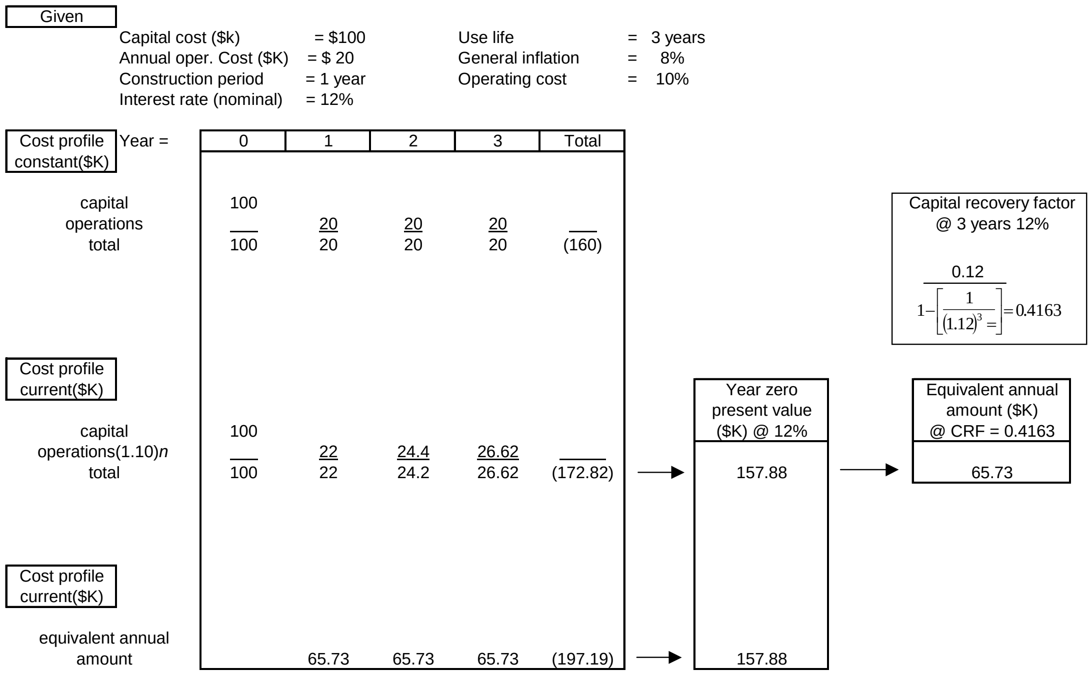


Figure 8. Calculating an equivalent annual amount with growing operating cost and inflation.

Figure 9 repeats the analysis but in constant-dollar terms. First, a real interest rate is calculated by dividing the 12% market rate by the expected rate of inflation of 85%. As shown, this results in a real discount rate of 3.7%. Also, a differential inflation rate is calculated for operations cost by dividing the 10% operations inflation rate by the general price inflation rate of 8%, resulting in a differential operations cost escalation rate of 1.85%. Now general price inflation can be ignored. The operations cost profile is inflated by the 1.85% rate, which results in the operations cost growing from $20.37 in year 1 to $21.13 in year 3. The cash flow is then discounted by the real discount rate to obtain the present value of $157.88 (which is the same present value obtained in Figure 8 working in inflated dollars). A real capital recovery factor of 0.3583 is calculated, as shown, and applied to the present value to obtain an equivalent annual amount of $56.57. As a check, the uniform cash flow of $56.57 is assembled at the bottom of Figure 9 and the present value calculated to insure that the same $157.88 is obtained.

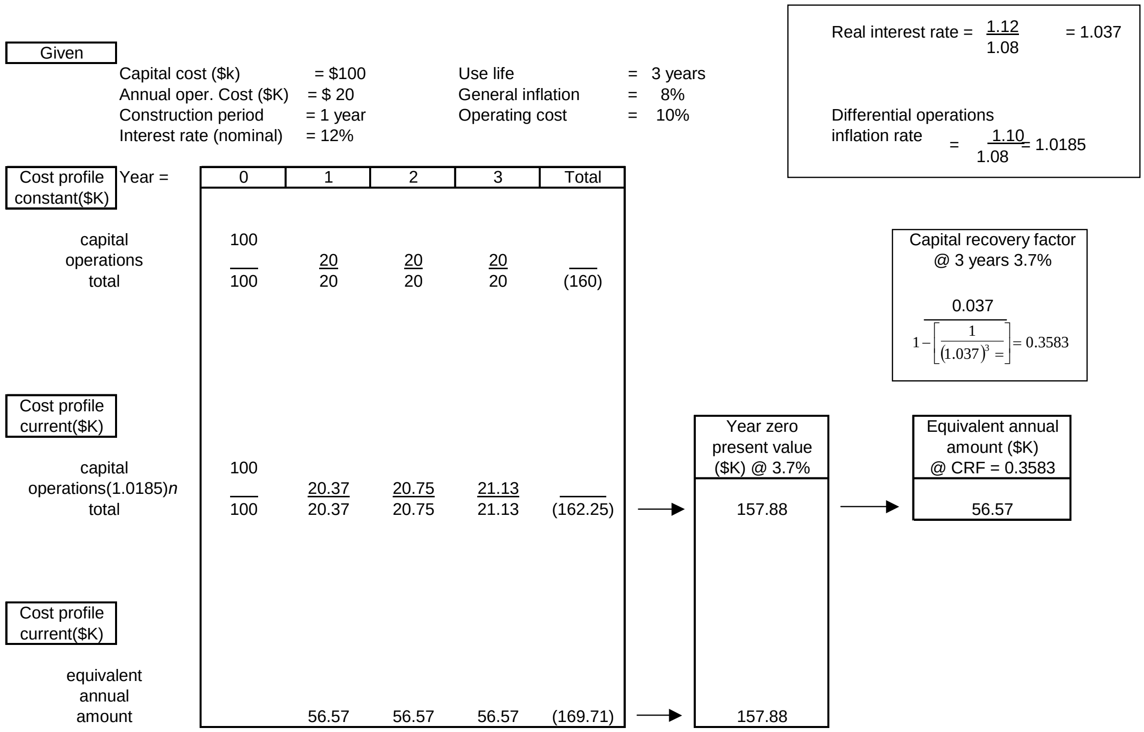


Figure 9. Calculating an equivalent annual amount with growing operating cost and constant dollars.

The equivalent annual amounts obtained by the two methods, \$65.73 and \$56.57, are also equivalent in terms of real value because uniform cash flows of these amounts discount back to equal present values. The only difference between the two values is the inclusion of an allowance for inflation in the \$65.73 value. In other words, the \$65.73 is in current dollars (over the year 0 to year 3 period), and the \$56.57 is in constant year 0 dollars. The next step in this equipment selection problem would be to calculate the present values and/or equivalent annual amounts of the other alternatives. Consistent use of either constant or current dollars would result in the same relative ranking between the alternatives.

This above example demonstrates that a cash flow analysis can be done either in inflated dollars using an interest rate that also includes inflation, or it can be performed using a real rate of interest and un-inflated (constant-year) dollars. As the example shows, the two approaches are identical in the net result. The key point is that the cash flows and the interest rate must be in the same terms; inflation must either be put in both or put in neither. This Golden Rule of Engineering Economics – do unto the discount rate as thou hast done unto the cash flow – is quite often violated. Usually the mistake is that constant-dollar cash flows are discounted with market discount rates, which in fact overstates the opportunity cost of capital to the firm and tends to cause attractive proposals to be rejected.

CHOOSING A DISCOUNT RATE

The interest rate chosen for discounting obviously has a great effect on which potential investment among alternatives will demonstrate the greatest worth. High discount rates will cause options with large initial investment costs and/or long payback periods to look comparatively less attractive than they would appear if the analysis had assumed a lower rate of interest. Lower discount rates will, conversely, make these same options appear more attractive. Improperly setting the discount rate can cause an organization either to forgo investment opportunities that should have been pursued or to commit to projects resources that could have been used to more beneficial effect.

In personal financial decisions the discount rate might be thought of as simply the going interest rate on loans at the local bank. Fir the individual trying to analyze, say, an electric versus gas-powered heating system for his house, and whose savings are tied up in not very liquid investments earning less than the rate at which the individual can borrow, then the loan interest rate is the correct discount rate. For business organizations, however, the cost of borrowed money is almost always too low to use as the discount rate in analysis of new investment opportunities. First, using the cost of borrowing for the discount rate theoretically can result in the choice of investment alternatives that yield no more return than can result in the choice of investment alternatives that yield no more return than the cost of the borrowed investment capital. A profit-making body that chooses ventures with no better possibilities than making just enough to pay the banker and bond holder has serious problems. (Although in fact regulated industries, such as utilities, theoretically might set their discounted rates not much above their weighted cost of capital, taking into account borrowing, bonds, and equity stock. Governments, too, are quite apt to set discount rates at very low levels for other reasons. This discussion, however, is primarily concerned with discount rates for more traditional business organizations.) Second, as mentioned before, almost all investment projects have some element of risk - setting the discount rate too near the borrowing rate might leave insufficient reserves for unpleasantries that sometimes occur. Therefore, generally speaking, a firm's cost of capital is too low to use as the minimum attractive rate of return.

When an organization's money is committed to an investment, the opportunity to use that money for gains in some alternate investment is forgone. The time value of money, then, is a result of forgone opportunities. This way of thinking about interest is know as the opportunity cost of capital concept. The following example may help to make the notion clearer. Assume that for the coming year a firm has a pool of investment funds - a capital budget, if you will - of $1 million. It might be that this particular business has a product line that is extremely profitable, and the demand for this product exceeds the company's present production capabilities. If an investment in additional production capability could reap a rate of return of 40% for instance, then any alternate investment should be required to at least meet this rate of return potential (all other considerations being equal). We can say, therefore, that the opportunity to make 40% establishes this rate as a minimum when considering alternate investments. At some point however, after half of our $1 million budget has been committed, we have enough production facilities to meet the market demand for our star product. What is the opportunity cost of capital for the next increment of our investment pool? It depends on the next best opportunity. It may be that our second most profitable product can get us a yield of 25%, and it too can accommodate some increase in production. Our minimum acceptable rate of return is now 25% because we would obviously not want to invest in any project offering less, as long as the 25% is available. Once the investment in this second product has been committed, and assuming there are still available funds in our budget, we find the next best opportunity in our list of potential projects, and this defines our new minimum acceptable rate of return. This process can be continued until the capital budget is exhausted.

It should be noted that the list of possible investments should not be limited necessarily to the stable of products in our hypothetical company. It could be that the capital budget is large enough that we would begin to spend money on projects that result in a rate of return less than what could be obtained by investing outside the company or by declaring the excess capital as dividends to the stockholders (who could in turn invest the money).

Ideally the rate of interest used in a discounted cash flow analysis should reflect the organization's minimum acceptable rate of return as defined by the other available potential investment opportunities. Proposed investments yielding returns higher than this minimum rate would therefore be accepted, and projects yielding less would be rejected.

SUMMARY

When the cost estimator is developing data that will be used to select alternative investment opportunities that have expenditures or receipts over time, the time value of money must be considered. Cash flows for each of the alternatives should be developed reflecting these expected disbursements and receipts. Each marginal level of investment should be individually compared to its marginal return. Any sunk costs or cash flow common to all alternatives should be excluded. The cash flows should represent investments of equal capability to the investing organization and have equal economic lifetimes. If it is expected that the alternatives will have relatively different income tax effects, then these effects should be included in the cash flows and the study performed as an after-tax analysis.

There are several decision criteria available for selecting between the alternative cash flows. The primary criteria are present value, equivalent annual amount, and internal rate of return.

The discounted cash flow analysis can be done either in constant dollars (ignoring future general price inflation) or in terms of cash flows inflated to their current price levels. Either method will result in the same preference between alternatives. Constant dollar analysis is generally preferred unless the cash flow data are to be used directly for budgetary planning. Care should be taken that the discount rate chosen for the analysis is real if constant dollar cash flows are involved or a market rate (including inflation) if the cash flows are inflated. The discount rate should reflect the organization's opportunity cost of capital as defined by other available investment opportunities.

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