# A Refresher on Engineering Economics 

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... Tom and me found the money that the robbers hid in the cave, and it made us rich. We got six thousand dollars apiece - all gold. It was an awful sight of money when it was piled up. Well, judge Thatcher, he took it and put it out at interest, and it fetched us a dollar a day apiece, all year round - more than a body could tell what to do with.

Mark Twain in Adventures of Huckleberry Finn

## Topics

$>$ This session covers the most common analytical techniques used in discounted cash flow studies to account for the time value of money when choosing between alternatives.
$>$ The underlying logic of the concepts is explained and the techniques of present value, equivalent annual amount, and internal rate of return are presented.
$>$ Special coverage is included for cases involving the choice between multiple alternatives, the relationship between interest and inflation, and the choice of the proper discount rate.

## The Time Value of Money Concept and When It Applies

$>$ All money has time value.

- Borrowed money has the time value equal to the interest payments made on the loan, whereas invested money has a time value equal to the returns or income that accrue from the investment.
- Money that is held as cash or in noninterest bearing accounts has time value because the money is forgoing either profits or interest that could have been earned if it had been invested in some other way.
$>$ Whenever an organization is contemplating an investment that involves more than one option, the time value of money will potentially affect the decision.
- This is true regardless of the source of the funding.



## The Time Value of Money Concept and When It Applies

$>$ It is not necessary that borrowed funds be involved, or is it necessary that the funds are visibly forgoing interest because they re being taken out of some interest bearing account.
$>$ Thus it is advisable for the cost analyst to consider the time value of money whenever the costs of two or more alternatives are being compared that have disbursements and/or receipts distributed over time.

- Relying solely on payback period or return on investment (ROI) criteria, which, as normally calculated, do not recognize the time value of money, is inadvisable.



## Cash Flows

> The techniques for analyzing the time value of money are generally referred to as discounted cash flow analysis techniques
$>$ A cash flow is the expected life cycle costs and revenues (or savings) of a contemplated investment
> Machine A is an existing machine with 3 years left in its design life of 5 years, an annual operating cost of $\$ 67,000$, and a residual value (say, in this case, the scrap value after deducting the cost of dismantling the machine) of $\$ 500$, and a value of $\$ 3000$ if sold now.
> Machine $B$ is a proposed replacement costing $\$ 30,000$, with an annual operating cost of $\$ 60,000$, and a residual value of $\$ 3000$ at the end of its operational life of 8 years.

- Because Machine A has 3 years of service left and Machine B would last 8 years, the assumption is made that Machine A would be replaced with an identical machine for $\$ 5000$ at the end of the 3 years which would give 5 more years of service, thus equalizing the service of the two machines at 8 years each.


Figure 1 Cash flow diagrams

## Costs Only? Costs and Revenues?

> It is useful to recognize that discounted cash flow analysis applies to several different cash flow situations the cost analyst might encounter.
> The problem may involve only cash outflows such as an equipment selection, which is comparing alternatives that have identical output capability (like our preceding example).
$>$ Because the revenue is the same for all options, only the life cycle costs need by considered in the time value of money analysis.
$>$ Sometimes the problem might be an investment decision analysis between options that have not only different initial purchase and operating costs but also have different expected revenues as well.

$>$ In such cases the cash flows carried into the time value of money analysis will need to include these revenue differences.

## Cost Only Versus Cost and Revenues (or Savings)

$>$ The first situation (comparing the cost of equipment options with identical revenues) can be represented by an allnegative cash flow for each of the options under consideration.
> The discounted cash flow analysis would be concerned with determining which of the options has the least negative cost after the time value of money is taken into account.
$>$ For cases in which the cash flows of each alternative include different revenue effects, the identical comparative techniques are applied, but the analyst would be interested in discovering which alternative resulted in the most positive result after taking into account the time value of money (presuming, of course, that the revenues or other cash inflows of the investment exceed the costs).



## Net Cash Flows

$>$ It is equivalent to work with net cash flows.
$>$ The net cash flow between two alternatives is simply the difference between the two cash flows.
$>$ Net cash flows will typically show a differential initial investment (negative cash flows) followed by later differential returns (positive cash flows).
$>$ The net cash flow of our example comparison of two machines is bar charted.
> Any net positive value would
 indicate that the savings of Machine $B$ justify the extra cost of Machine B.

## STRUCTURING THE DISCOUNTED CASH FLOW ANALYSIS

> Use of the discounted cash flow analysis method requires that there be at least two alternatives--frequently there will be several.
> Often one of the alternatives in an investment analysis is to do nothing; instead, simply continue with the present system.
$>$ Other options may involve minor modifications to the present method to make it more efficient.
$>$ There may be alternatives that require major modifications to the present system and other options that involve replacement of the existing method with totally new methods.
$>$ Discounted cash flow analysis is a tool
 for selecting the best from among those cash flows defined.

## Multiple Levels of Investment

$>$ Some of the options may have multiple levels of investment with incremental costs bringing incremental levels of performance.
$>$ To the extent practical, such options should be broken down such that each marginal investment can be evaluated against its marginal return.
> There are sometimes alternatives that have attractive cash flows when compared to the competing alternatives but that actually contain sub-elements that, if separately analyzed based on the sub-element's incremental investment versus incremental return, might be unattractive.
> The elimination of such sub-elements will enhance the overall alternative's performance.


## Exclusion of Common Cash Flows and Sunk Costs

$>$ It is only necessary to quantify and consider the cash flow differences between alternatives.
> Cash flows common to all options should be excluded.
> Likewise, sunk costs are of no consequence.
$>$ The fact that one option (usually the existing method) has had a large previous sum of money invested in it should not bias the analysis.
$>$ What is important is to identify, from the current moment onward, which of the alternatives is most economical.


## Equal Capabilities Between Alternatives

> The cash flow analysis should be structured so that all alternatives are compared on an equal basis.
> For example, it is usually the case that newer alternatives being considered have a higher performance than the old method.
> If this higher performance manifests itself as lower operating costs or higher revenues, then it will be taken into account when the cash flows are estimated for alternative methods.
> Sometimes however, synthetic adjustments must be made to cash flows to equalize the capability between alternatives.
> One alternative may have certain capabilities that other alternatives do not possess.
> In such cases, the scope of the study can be expanded to include the services provided by the most capable alternative.
> At some point, adjustments to the cash flows of the systems being compared reach the point where any remaining differences must be treated in a non-quantitative manner.


## Equal Economic Lifetimes

> Alternatives with unequal economic lifetimes represent another common situation in discounted cash flow analyses because it is often the case that a new method will have a longer useful life than the old method.
$>$ This can be compensated for by assuming that the short-lived options are replaced at the end of their lifetimes with an identical replacement.
> Thus an analysis comparing an alternative with a 5 -year life to one with a 10-year life would require a 10-year cash flow and the inclusion of the replacement cost at the end of year 5 in the cash flow for the 5-year option.
$>$ However, it still may be difficult to construct cash flows that end simultaneously for all options.
> In such cases a residual value approach can be used wherein the remaining value is included as a positive cash flow for any
 options that are not at the end of their lifetimes when the cash flow analysis terminates.

## Income Tax Considerations

> If the alternatives under consideration do not all have the same tax impact on the firm, then the cash flow analysis should be structured as an after-tax study by quantifying the tax differences in the cash flows in the appropriate time periods.
> Although it would involve a major accounting study to precisely quantify the different tax consequences among alternatives, it will often suffice to consider the differences in taxes due to the net deduction on capitalized (depreciable) assets and the net deduction due to operating expenses.
> For example, consider a comparison between option A, an existing manufacturing operation with an annual operating cost of $\$ 10,000$, and option B, a 10 -year lifetime capital equipment improvement that would cost $\$ 16,000$ but reduce the cost of operations by $\$ 3000$ annually.
> The existing operation can be assumed to result in a yearly operating expense tax deduction of $\$ 10,000$ which, after taxes, at a rate of $40 \%$, would yield $\$ 4000$ of tax savings.
> Option B would result in a depreciation deduction of $\$ 1600$ per year (assuming straightline depreciation), which would convert to a tax savings of $\$ 640$, and an operating expense deduction of $\$ 7000$, which would be worth $\$ 2800$, for a total annual tax savings of $\$ 3440$.
> For studies in which the alternatives involve different expected revenues, then the cash flows must include these revenues and the estimated tax on the revenues.


## Disregard Payment Schedules Due to Financing Arrangements

- A point of confusion in cash flow analysis similar to the erroneous inclusion of depreciation in the cash flow is that of formulating cash flows that correspond to the payment schedule on the loan for an investment, as opposed to constructing cash flows that correspond to the cash flow obligation.
$>$ Whether an investment that is being analyzed is to be financed by borrowed money or paid for either totally or in part by cash is not relevant to the timing of money in the cash flow analysis.
$>$ The fact that the organization is financing an investment and will be making principal and interest payments over a period of time will be implicitly accounted for by the time value of money techniques to be introduced.
$>$ The cash flow should therefore include the total purchase cost in the period or periods in which the purchase is
 expected to be made.


## Uncertainties and Risk

$>$ Differences in perceived risk (technical and economic risks) can be handled in several ways.
$>$ The most straightforward way to account for risks is to add contingencies to the cost estimates of the alternatives.
$>$ Another method used to analyze risk is sensitivity analyses.
$>$ A third common approach to the problem of risk is to include an allowance for risk in the discounted rate used.

- Raise the minimum acceptable rate of return.
> Two more statistical approaches to risk analysis are Monte Carlo simulation and decision tree analyses.
- The simulation approach requires a cash flow model that will accept probability distributions for each variable in the analysis instead of single deterministic values.
- Decision tree analysis is an approach used to analyze the uncertainties in investment analysis by laying out (usually in a tree-oriented structure) the various alternatives available to the decision maker.
- The probabilities of the events along the paths are estimated, and statistical methods are used to calculate the overall economic expectations of the investment.


## Decision Criteria

> Once cash flows have been developed for each alternative, there are several time value of money decision criteria available to the cost analyst to apply to the problem of choosing between cash flows.
$>$ These include

- Present value comparisons (sometimes called net present value analysis or present worth analysis),
- Equivalent annual amount comparisons (also called uniform annual amount and other similar names),
- Internal rate of return (sometimes called discounted rate of return, interest rate of return, and other names).
- These techniques, properly applied, are essentially equivalent and give
 consistent results.


## Present Value

$>$ The most fundamental of these criteria, and probably the most commonly used, is the present value.
$>$ The present value of a series of future cash flows is the value that it would be necessary to invest at the present time at a specified interest rate to be able just to make the future cash disbursements and receipts of the cash flow and completely exhaust the investment balance.
$>$ The present value of $\$ 1000$ one year from the present time at $10 \%$ interest is $\$ 909.09$.

- That is, if one had \$909.09 earning $10 \%$ for year, then a $\$ 1000$ payment could be made leaving a zero balance.
- The present value of $\$ 1000$ ten years from now at $10 \%$ interest is $\$ 385.54$ ( $\$ 1000$ divided by 1.10 to the power of 10 ).
- Both values, $\$ 385.54$ now and $\$ 1000$ ten years
 from now are exactly equivalent at 10\% interest.


## Present Value (cont'd)

$>$ The present value of a series of cash flows is calculated by summing the present values of each of the individual present values of the cash flow.
> For example, the present value of the following cash flow:

| Year 0 | Year 1 | Year 2 |
| :--- | :--- | :--- |
| $-\$ 1000$ | $\$ 800$ | $\$ 800$ |

can be calculated as:

$$
\begin{aligned}
\text { PV } & =\left[-1000 /(1.10)^{0}\right]+\left[800 /(1.10)^{1}\right]+\left[800 /(1.10)^{2}\right] \\
& =-1000+727.27+661.16 \\
& =388.43
\end{aligned}
$$

$>$ In this example, $\$ 388.43$ is said to be the discounted present value of the given cash flow.
$>$ The word discount relates to the fact that dollars in the future are not worth as much as dollars now, and the value of future dollars must be discounted both as a function of the interest rate and as a function of how far they are into the future.

## Present Value (cont'd)

$>$ Mathematically a cash flow is discounted to its present value by calculating the present worth of each of its periodic amounts at the time selected as the present.
$>$ It does not matter what instant in time is selected as "the present" as long as each cash flow being compared is discounted to the same "present."

- The present may be defined as the year 1914.
- Or it can be defined just as well as the year corresponding to the first cash flow, which is the normal convention in discounted cash flow analyses.


## Present Value (cont'd)

> It could also be defined as some "future present" such as the year 2121 or, say, the year of the last cash flow in the analysis.
$>$ In this case the calculations would utilize negative exponents.
$>$ Let us say we want to repeat our discounting example but define the present to be the end of year 2.
$>$ The resulting present value would be:

|  | Year 0 |
| ---: | :--- |
| PV | Year 1 |
| $=\left[-1000 /(1.1)^{-2}\right]+\left[800 /(1.10)^{-1}\right]+\left[800 /(1.10)^{-0}\right]$ |  |
| $=$ | $-1000(1.10)^{2}+800(1.10)^{1}$ |
| $=$ | $+800(1210$ |
| $=$ | +880.00 |

> This $\$ 470.00$ value is the present value of our cash flow because we have temporarily defined the end of year 2 to be the present.
$>$ It should be apparent from the second line of this calculation that we are performing an operation that is equivalent to what we would do if asked to calculate the future value of the cash flow at the end of year two.
$>$ That is, $\$ 470.00$ is also the future value of the cash flow, or the worth of the cash flow at $10 \%$ interest at the end of year 2.
> The same result for future value could be obtained by calculating the future value of the $\$ 388.43$ that we initially calculated as the year zero present value.
$>$ The future value of $\$ 388.43$ two years hence is

$$
F V=\$ 388.43(1.10)^{2}=\$ 470.00
$$

## Present Value (cont'd)

$>$ Thus, the present value at the end of year zero of $\$ 388.43$ is equivalent to the future value of $\$ 470.00$ at the end of year two, both of which are equivalent to the original 3-year cash flow of -\$1000, \$800, \$800.
$>$ In fact, there are an infinite number of other equivalent values because there are an infinite number of periods that we could define as present or future.
$>$ Thus present value and future value are equivalent concepts, both of which collapse a time series of dollar amounts into a single dollar amount.
> This amount represents the worth of the entire cash flow it replaces taking into account the time value of money.

## Discounting

$>$ Therefore, the general equation for present value is

$$
P=F \frac{1}{(1+i)^{n}}
$$

where $P$ is present value, $F$ is the future cash flow amount, $i$ is the discount rate decimal equivalent (e.g., 0.10 for 10\%), and $n$ is the number of periods separating the present and the future time periods.
$>$ Most textbooks on engineering economics contain tables of the function for discount factors at various interest rates for any number of years.
$>$ Today, most discounted cash flow analyses are performed on computers, which simplifies the mathematics of discounting cash flows.
$>$ Many of the software packages that are popular among cost estimators (such as spreadsheets) have time value of money functions such as present value.

## Future Value

$>$ Some cost analysts prefer to use future value as a decision criterion.
$>$ This is equivalent to defining some future time period as the present and is perfectly valid.
$>$ Instead of calculating the value of a cash flow at a period of time close to the outset of the activity, the analyst calculates the value of all the cash flows closer to the end period of the cash flows.
$>$ The equation is just the reciprocal of present value:

$$
F=P(1+i)^{n}
$$

## Equivalent Annual Amount

> Another technique used as a criterion for selection in discounted cash flow analysis is the equivalent annual amount.
$>$ In this approach, the cash flows being compared are all converted to a constant annual amount over a specified time period that has the same present value as the original cash flow.
> In other words, the original cash flow, which may have periodic amounts varying in magnitude from period to period, is converted to a uniform cash flow (one with the same dollar magnitude in each period) that has a present value equivalent to the original cash flow.
$>$ Mathematically, the procedure is composed of two steps:

- The first calculates the present value of the series of cash flows just as in the present value technique.
- The second step uses what is generally called a capital recovery factor to calculate what constant amount of money spread over $n$ periods at $i$ interest rate would have the original present value.
- Therefore, to calculate the annual equivalent over $n$ periods at $i$ interest of a cash flow with a present value of $P$, use:

$$
\frac{P\left[i(1+i)^{n}\right]}{(1+i)^{n}-1}
$$

where $A$ is the equivalent annual amount, and the parenthetical expression being multiplied by $P$ is the capital recovery factor.

## Equivalent Annual Amount (cont'd)

$>$ For example, a cash flow that has been found to have a present value of $\$ 10,000$ can also be represented over 5 years at $10 \%$ interest with an equivalent annual amount of \$2637.97.
$>$ The capital recovery factor equation is the same equation that lenders use to calculate the payment on a loan;

- $\$ 2637.97$ is the same value one would be quoted by a banker as the annual payment on a $\$ 10,000.00$ loan for 5 years (while for monthly payments $n$ would be entered as the number of months over which the loan was to be financed and $i$ would be entered as one-twelfth of the annual interest rate).
- Once two or more cash flows have been thus annualized, the preferred choice is the one with the highest positive present value or lowest negative present value.


## Equivalent Annual Amount (cont'd)

$>$ One advantage of the equivalent annual amount approach is that in applications where the analysis is choosing the most cost effective production method for a product, the equivalent annual amount can be calculated over a period of time corresponding to the product's revenue life cycle, and the results of the analysis can be presented as a unit cost for the product.
$>$ For example, if the production is 1000 units per year, the cost per unit in the preceding example could be quoted at $\$ 2.64$ per unit.
$>$ Said another way, to realize a $10 \%$ rate of return, the products must be sold for \$2.64 apiece.
$>$ For the next alternate production method, the cash flow could also be converted to a cost per unit and compared to the $\$ 2.64$ value.
$>$ A lower cost would cause the corresponding method to be selected whereas a higher cost per unit would cause its rejection.

## Equivalent Annual Amount (cont'd)

$>$ A second advantage to the equivalent annual amount technique is that in many cost analyses the recurring annual cost (of a production method for example) is known.
$>$ Since this cost is already "annualized" there is no need to perform any other time value of money calculations on these amounts.
$>$ All that is required is to annualize any costs that are not on an annual basis (nonrecurring capital cost for example) using the appropriate capital recovery factor and to add the result to the know annual cost.
$>$ Let us say that a proposed alternative method to produce our product involves the purchase of a $\$ 5000$ machine and a $\$ 1000$ annual operating cost made up of materials, labor, and all other recurring production costs.

- Assuming a 5-year life for our machine, the same capital recovery factor as used before would be applied to get a $\$ 1318.99$ equivalent annual amount for the machine cost ( $0.2638 \times \$ 5000$ ).
$>$ This could be added directly to the other known annual costs of $\$ 1000$ to get the total equivalent annual amount of $\$ 2318.99$, which works out to $\$ 2.32$ per unit.
$>$ Since this is less than the $\$ 2.64$ cost per unit of the former method, the new method is preferred.


## Assumption of an Infinite Horizon

> Although most cash flow analyses do set a limit on the length of the life cycle that is considered, limiting the economic horizon to one or more multiples of the service lives actually understates the value of the preferred alternative.
$>$ The justification for this statement is that if two or more alternatives for future investment are compared in a discounted cash flow analysis, and one alternative is chosen because it demonstrates economic benefits over the other alternative is chosen because it demonstrates economic benefits over the other alternatives, then this benefit stream will likely extend indefinitely into the future.
> Because at the end of the useful like of the chosen alternative, the alternative will either be replaced with yet another alternative that is at least as, and probably more, cost-effective.
> Thus the benefits of the chosen alternative will continue forever at a level as great as that shown for it in the original analysis.

## Infinite Horizon (cont'd)

$>$ This concept, also called perpetual worth or capitalized costs, can be used in the comparison of alternatives by first calculating the equivalent annual amount of each alternative and then dividing the result by the interest rate.
$>$ For out preceding cash flow, with an equivalent annual amount of \$2637.97 over 5 years at an interest rate of $8 \%$, the present value of an infinite horizon annual amount would be $\$ 2637.97$ divided by 0.08 or $\$ 32,975$.
$>$ That is, the present value of a series of $\$ 2637.97$ annual cash flows stretching into the future forever is $\$ 32,975$.
$>$ Although this idea overwhelmed Huck Finn, a moment's reflection will illuminate the principle because $\$ 32,975$ put into an $8 \%$ bearing investment now would yield an annual interest income of $\$ 2637.97$ forever without ever touching the principal.
$>$ Incidentally, Huck's and Tom Sawyer's $\$ 6000$ put out at interest by Judge Thatcher earning a dollar a day works out to be about 6\% interest, apparently the going rate in the time of the novel (the mid-1800s).

## Infinite Horizon (cont’d)

> For discounted cash flow analyses of relatively long life cycles (say 30 years or longer) and/or relatively high interest rates (say 15\% or more), the assumption of infinite horizon may not yield present value quantities significantly higher than those that would have been obtained without infinite horizon.
$>$ This is because discount factors decrease over time and do so more rapidly at higher discount rates.
$>$ For all practical purposes, discount factors approach zero after three decades or so for interest rates above $15 \%$.
$>$ Any cash flow extending beyond this (including one going to infinity) is essentially zeroed out by the discounting process


Figure 4.4. Discount factors decrease over time.

## Internal Rate of Return

$>$ Present values, future values, and equivalent annual amounts are all really just extensions of the same basic concept.
$>$ Properly applied, they all give consistent and reliable results when choosing among alternatives.
$>$ A fourth technique often used as a decision criterion in discounted cash flow analyses is the internal rate of return (IRR).
$>$ Assuming that a new method is being compared to an existing method, in a non-discounted analysis one would say that an incremental investment of, say, \$1000 that results in returns of \$200 per year is an 20\% rate of return.
$>$ This simple return on investment (ROI) calculation fails to take into account that each dollar in the $\$ 200$ returns flowing in each year are not worth, in the time value of money sense, the same as the dollars in the $\mathbf{\$ 1 0 0 0}$ invested at time zero.
$>$ The internal rate of return is a discounted rate of return that does correct for the differing time values of the dollars in the cash flows.

## Internal Rate of Return (cont'd)

$>$ The internal rate of return for a given cash flow is defined as the discount rate that results in a present value of zero.
$>$ Thus the IRR method finds the interest rate that equalizes the present value of the investment and the present value of the returns.
$>$ For our ROI cash flow example, the IRR is $\mathbf{1 5 . 0 9 8 \%}$.

- This can be verified by discounting the cash flow-a negative $\$ 1000$ followed by 10 positive $\$ 200$ amounts-by $15.098 \%$.
- A net present value of zero will be found.
$>$ Since the simple ROI result of $20 \%$ does not take into account the time value of money, it is to be expected in this case that IRR should be less than ROI.


## Internal Rate of Return (cont'd)

$>$ Once IRR is known for a cash flow, it is compared to the minimum acceptable rate of return, and the investment is either accepted or rejected accordingly.
$>$ Considerations involved in specifying the minimum acceptable rate of return are discussed later "Choosing A Discount Rate."
$>$ Presuming that the minimum acceptable rate of return is known, the use of IRR as an investment decision criterion has a certain appeal because of this straightforward manner in which the investment decision is made-as long as the contemplated investment meets the organization's minimum rate of return requirement, then it is considered acceptable.

## Internal Rate of Return (cont'd)

$>$ A potential problem with the use of the IRR as an investment decision criterion can occur in certain circumstances.
$>$ Because the calculation of IRR involves finding a solution to a complex function, certain cash flows that are not well behaved may cause either multiple solutions or no solutions.
$>$ A cash flow that begins negative and then turns positive will generally cause no problems in the IRR calculation.
$>$ Cash flows with multiple sign changes can, however, result in multiple mathematically valid solutions, a disturbing outcome in investment analysis.
$>$ n such an event, the cost estimator can rely on the criteria of present value and/or equivalent annual amount.

## THE RELATIONSHIP BETWEEN INTEREST AND INFLATION

$>$ A common mistake in discounted cash flow analysis is incorrectly accounting for the relationship between interest and inflation.
$>$ Interest and inflation, although related in many ways, are totally different economic phenomena.
$>$ Interest is a rent paid to the owner of money by the borrower of that money, or equivalently is a rate of return earned (or lost) on an investment.
$>$ Thus interest, by having the power of making capital either grow or shrink over time, gives money a time value.
$>$ Inflation also causes money to either increase or decrease in value over time, but this change in value is due to changes in the value of money as a standard unit of measure.
$>$ Inflation then, is not the same thing as interest.
$>$ Inflation is instead a change in the measuring system-a change in the value of the dollar due to many reasons that have nothing to do with changes in value due to interest.

## THE RELATIONSHIP BETWEEN INTEREST AND INFLATION

$>$ Although interest and inflation are two different things, they are related to each other in fairly constant and predictable ways.
$>$ Money that is earning a return (for example, money in an interest bearing account in a bank) is growing by an amount that can be calculated using compound interest equation.
$>$ For example, a $\mathbf{\$ 1 0 0 0}$ principal amount invested at a $10 \%$ annual compounded interest rate grows in the following manner:

| Year 1 | Year 2 | Year 3 | Year 4 |
| :--- | :--- | :--- | :--- |
| $\$ 1000$ | $\$ 1000(1.1)^{1}$ | $\$ 1000(1.1)^{2}$ | $\$ 1000(1.1)^{3}$ |
| $\$ 1000$ | $\$ 1100$ | $\$ 1210$ | $\$ 1331$ |

$>$ However, if inflation is some positive rate (say 6\%), then those earnings are also decreasing in value at a rate equal to the inflation rate. This can be calculated as follows:

| Year 1 | Year 2 | Year 3 | Year 4 |
| :--- | :--- | :--- | :--- |
| $\$ 1000$ | $\$ 1100 /(1.06)^{1}$ | $\$ 1200(1.06)^{2}$ | $\$ 1331(1.06)^{3}$ |
| $\$ 1000$ | $\$ 1038$ | $\$ 1077$ | $\$ 1118$ |

## THE RELATIONSHIP BETWEEN INTEREST AND INFLATION

$>$ Since, as the previous example demonstrated, interest is calculated by multiplying the principle amount by the interest rate, and inflation is calculated by dividing by the inflation rate, the estimator can save a step by first adjusting the interest rate:

| adjusted interest | $=\frac{1+\text { interest rate }}{1+\text { inflation rate }}$ |
| ---: | :--- |
|  | $=\frac{1.10}{1.06}$ |
|  | $=1.038$ |

$>$ And then the previous two-step operation becomes one step by using the adjusted interest rate:

| Year 1 | Year 2 | Year 3 | Year 4 |
| :--- | :--- | :--- | :--- |
| $\$ 1000$ | $\$ 1000(1.038)^{1}$ | $\$ 1000(1.038)^{2}$ | $\$ 1000(1.038)^{3}$ |
| $\$ 1000$ | $\$ 1038$ | $\$ 1077$ | $\$ 1118$ |

## THE RELATIONSHIP BETWEEN INTEREST AND INFLATION

$>$ Economists call this interest rate after correcting for inflation the real rate of return or the real rate of interest.
$>$ Over the long term, the rate of inflation and the rate of interest in the economy track each other and in fact the real rate of interest tends to remain fairly constant.
$>$ When inflation is high, interest rates tend to rise as well because the holders of capital in the economy resist lending their money out at rates that result in small real gains.
$>$ As we discussed, the real interest rate in the market interest rate divided by the inflation rate.

## Including Inflation in the Analysis

$>$ Although historical inflation data are essential to the cost analyst for adjusting old cost data to more current price levels for use in an analysis, the value of including an allowance in cash flows for future general price inflation is open to argument.
$>$ First of all, predicting future price increases is an inexact process at best. Such predictions are seldom reliable in any absolute sense, and frequently are inaccurate in even relative terms.
$>$ Second, unless there is some overriding requirement for including general future price inflation in the cash flow estimate, it should be omitted, because the inclusion of inflation usually adds no useful information to an analysis of competing alternatives when all the alternatives will be affected equally by the general inflation adjustment.
$>$ Such justifications for including general escalation include the case of the cash flow that is going to be used directly in a bid proposal or in some other budgetary manner.
$>$ Budgetary inputs usually need to include inflation.
$>$ However, if the cash flow analysis is to be used solely for the comparison of alternative investments such as equipment selection, there is probably no need to try to include the effects of future inflation there is some component of the cash flow that is expected to inflate or deflate differentially with respect to the other components of the cash flows.

- Good examples of differentially inflating components might be labor or energy costs.
- For any such components where the cost analyst has evidence that differential inflation is likely, the anticipated escalation should be included in the cash flow.
- Note that including differential inflation does not change the price level of the analysis.


## Inflation (cont'd)

$>$ The foregoing argument for performing non-budgetary cash flow analyses in constant rather than inflated dollars affects the choice of the discount rate to be used in the analysis.
$>$ The rule is: Use a real discount rate for discounting cash flows that are in constant, non-inflated dollars and use a market discount rate (i.e., one that includes inflation) for discounting cash flows that have been inflated into future-year price levels.
$>$ The Golden Rule of Engineering Economics - do unto the discount rate as thou hast done unto the cash flow - is quite often violated.

- Usually the mistake is that constant-dollar cash flows are discounted with market discount rates, which in fact overstates the opportunity cost of capital to the firm and tends to cause attractive proposals to be rejected.
- There is some confusion in terminology in this area. The term "current" is used by economists to refer to data that are in then current dollars.
- For example, a table tracking the nation's GNP from 1950 through 1980 and labeled "current dollars" would mean that the 1950 data are in 1950 dollars, the 1951 data are in 1951 dollars, and so on.
- If the table is in some constant-year dollars it would be labeled constant 19XX dollars.


## CHOOSING A DISCOUNT RATE

$>$ The interest rate chosen for discounting obviously has a great effect on which potential investment among alternatives will demonstrate the greatest worth.
$>$ High discount rates will cause options with large initial investment costs and/or long payback periods to look comparatively less attractive than they would appear if the analysis had assumed a lower rate of interest.
$>$ Lower discount rates will, conversely, make these same options appear more attractive. Improperly setting the discount rate can cause an organization either to forgo investment opportunities that should have been pursued or to commit to projects resources that could have been used to more beneficial effect.
$>$ In personal financial decisions the discount rate might be thought of as simply the going interest rate on loans at the local bank.
$>$ For business organizations, however, the cost of borrowed money is almost always too low to use as the discount rate in analysis of new investment opportunities.
$>$ A profit-making body that chooses ventures with no better possibilities than making just enough to pay the banker and bond holder has serious problems.
$>$ Second, almost all investment projects have some element of risk

- Setting the discount rate too near the borrowing rate might leave insufficient reserves for contigencies that sometimes occur.
- Therefore, generally speaking, a firm's cost of capital is too low to use as the minimum attractive rate of return.


## CHOOSING A DISCOUNT RATE (cont’d)

> When an organization's money is committed to an investment, the opportunity to use that money for gains in some alternate investment is forgone.
> The time value of money, then, is a result of forgone opportunities.
$>$ This way of thinking about interest is known as the opportunity cost of capital concept.

- Assume that for the coming year a firm has a pool of investment funds-a capital budget, of \$1 million.
> It might be that this particular business has a product line that is extremely profitable, and the demand for this product exceeds the company's present production capabilities.
> If an investment in additional production capability could reap a rate of return of $40 \%$ for instance, then any alternate investment should be required to at least meet this rate of return potential (all other considerations being equal).
$>$ We can say, therefore, that the opportunity to make 40\% establishes this rate as a minimum when considering alternate investments.
- At some point however, after half of our $\$ 1$ million budget has been committed, we have enough production facilities to meet the market demand for our star product.
> It may be that our second most profitable product can get us a yield of $25 \%$, and it too can accommodate some increase in production.
> Our minimum acceptable rate of return is now $25 \%$ because we would obviously not want to invest in any project offering less, as long as the $25 \%$ is available.
$>$ This process can be continued until the capital budget is exhausted.


## Summary

$>$ When the cost estimator is developing data that will be used to select alternative investment opportunities that have expenditures or receipts over time, the time value of money must be considered.
$>$ Cash flows for each of the alternatives should be developed reflecting these expected disbursements and receipts.
$>$ Each marginal level of investment should be individually compared to its marginal return. Any sunk costs or cash flow common to all alternatives should be excluded.
$>$ The cash flows should represent investments of equal capability to the investing organization and have equal economic lifetimes.
$>$ If it is expected that the alternatives will have relatively different income tax effects, then these effects should be included in the cash flows and the study performed as an after-tax analysis.
$>$ There are several decision criteria available for selecting between the alternative cash flows. The primary criteria are present value, equivalent annual amount, and internal rate of return.
> The discounted cash flow analysis can be done either in constant dollars (ignoring future general price inflation) or in terms of cash flows inflated to their current price levels.

- Constant dollar analysis is generally preferred unless the cash flow data are to be used directly for budgetary planning.
- Care should be taken that the discount rate chosen for the analysis is real if constant dollar cash flows are involved or a market rate (including inflation) if the cash flows are inflated.
$>$ The discount rate should reflect the organization's opportunity cost of capital.

