# Statistical Foundations of Adaptive Cost-Estimating Relationships

**Stephen A. Book**
**MCR LLC**
**sbook@mcri.com**

**Melvin A. Broder**
**The Aerospace Corporation**
**melvin.a.broder@aero.org**

**Daniel I. Feldman**
**MCR LLC**
**dfeldman@mcri.com**

**SCEA-ISPA Joint Annual Conference & Training Workshop**
**Society of Cost Estimating and Analysis**
**International Society of Parametric Analysts**
**St Louis MO**
**2-4 June 2009**



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# Acknowledgments

**The authors are grateful to the members of MCR's quality-review team: Dr. Neal D. Hulkower, Dr. Jerry D. Hofmann, Raymond P. Covert, Nathan J. Menton, Timothy P. Anderson, and Jan B. Sterbutzel. The combined efforts of the team resulted in a significant improvement in the clarity of explanation of specific details of the conclusions drawn, as well as the elimination of various typos and other errors.**

# CER Error Sources

- **Inability of Any Cost-Estimating Relationship (CER) to Account for All Influences On Cost, No Matter How Many Cost Drivers it Contains – Too Bad, We Usually Can’t Do Anything About This**
- **Incorrectness of Algebraic CER Model to Which Cost Numbers in its Data Base are Statistically Fit – Tough Luck, Try Another Algebraic Form**
- **When CER is Applied to Estimate Cost at a Particular Cost-Driver Value $x$, Location of that Cost Driver Value with Respect to Cost-Driver Values Comprising Historical Cost Data Base – This Issue is What We Will Try to Resolve in This Briefing**
  - **If $x$ is Located Near Center of Range of Parameter Values, CER Will Provide Fairly Precise Estimate of Cost**
  - **If $x$ Is Located Far From Center of Range, CER-based Estimate Will Be Considerably Less Precise**

# Starting Point for the Study

- **Up to Now, Most Cost-Estimating Relationships (CERs) Have Been Based on Full Data Sets Consisting of All Cost and Technical Data Associated with a Particular Class of Products of Interest (e.g., Components or Subsystems of Satellites, Ground Systems, Aircraft, etc.)**
- **Exceptions Have Typically Involved Only …**
  - **The Analyst's Ability to Choose Particular Analogous Entities from a Full Data Set and Derive a CER from the Reduced Data Set, e.g. NAFCOM**
  - **Preliminary Removal of "Outliers" from the Full Data Set**
- **Last Year, the First Two Authors Described the Process of Deriving "Adaptive" CERs, Namely CERs that are "Tuned" to Specific Cost-Driver Values and Presented Several Examples of How to Carry Out that Process**

# Today's Presentation

- **The Goal of Such Specialization is to be Able to Apply CERs that Have Smaller Estimating Error and Narrower Prediction Bounds**
- **In Today's Presentation, We Will Explain the Mathematics behind the Analysis of Last Year's Adaptive CER Report, which will Point the Way Toward More Advanced Results**
  - **Adaptive CERs that are Applicable over the Entire Cost-Driver Range**
  - **Improvements in Our Understanding of Adaptive CER Standard Errors and Prediction Bounds**

# Contents

- **Ordinary Least-Squares (OLS) Regression CERs and their Quality Metrics**
- **Weighted Least-Squares (WLS) Regression CERs and their Quality Metrics**
- **Adaptive CERs via Quadratic-Distance Weighting**
- **The “Universal Adaptive CER”**
- **Prediction Bounds for OLS, WLS, and Adaptive CERs**
- **Conclusions**

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# Choice of Additive-Error Model

- **Normally, the Multiplicative-Error Regression Model is Preferred for CERs**
  - **Typically, Data-Base Cost Values Range over Large Intervals – Often by Two or Three Orders of Magnitude**
  - **So the Possible Output Values of the CER Will Range Over A Similar Interval**
  - **Therefore ±30% is More Meaningful as a Standard-Error Metric Than ±$30,000**
- **For What We are Considering in this Study, However, CER Output Will Range Over a Relatively Short (or even zero-length) Interval**
  - **The CER Will Be Usable for Only a Small Set of Cost-Driver Values (or maybe even only one cost-driver value)**
  - **Therefore, a Dollar-Valued Standard-Error Metric is Just as Meaningful as a Percentage Standard-Error Metric (and the mathematics is simpler)**
- **We Will Therefore Apply the Additive-Error Regression Model**

# Ordinary Least Squares

- **OLS "Best" Fits a Straight Line $y = a+bx$ to a Set of Data Points $(x_k,y_k)$ in Two-Dimensional Space**
  - $x_k$ **is the Value of the Cost Driver**
  - $y_k$ **is the Cost**
- **The OLS Criterion is That the Coefficients $a$ and $b$ are Selected so That the Sum of Squares of the Differences $d_k = y_k-(a+bx_k) = y_k-a-bx_k$ Between the Actual Costs and their Estimates is as Small as Possible**
- **The Mathematics Results in Numerical Values of $a$ and $b$ That Minimize the Quantity**

$$f(a,b) = \sum_{k=1}^{n} d_k^2 = \sum_{k=1}^{n} (y_k - a - bx_k)^2$$

# Coefficients of the OLS CER

$$b = \frac{n\sum_{k=1}^{n} x_k y_k - \left(\sum_{k=1}^{n} x_k\right)\left(\sum_{k=1}^{n} y_k\right)}{n\sum_{k=1}^{n} x_k^2 - \left(\sum_{k=1}^{n} x_k\right)^2}$$

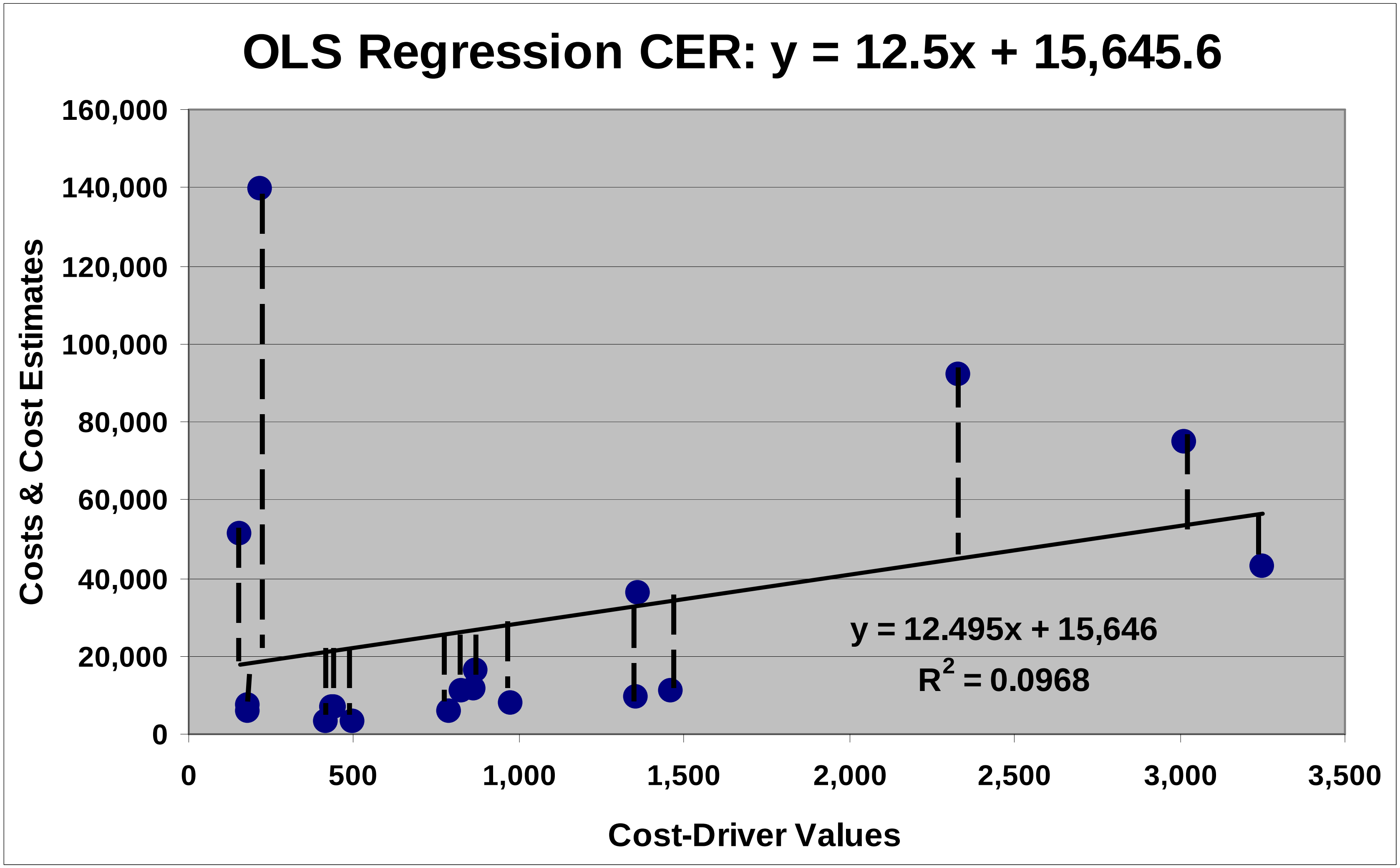
$$a = \frac{\sum_{k=1}^{n} y_k}{n} - b\,\frac{\sum_{k=1}^{n} x_k}{n}$$

# A Set of Actual Data

| Program | Cost-Driver Value x | Unit Cost y |
|---|---|---|
| A | 156.12 | 51,367.22 |
| B | 179.40 | 5,885.00 |
| C | 180.30 | 7,060.00 |
| D | 217.50 | 139,483.12 |
| E | 419.14 | 3,386.00 |
| F | 437.09 | 6,738.00 |
| G | 440.93 | 6,812.00 |
| H | 494.45 | 3,291.34 |
| I | 789.90 | 5,723.14 |
| J | 826.10 | 10,992.00 |
| K | 864.30 | 11,590.00 |
| L | 869.30 | 15,973.00 |
| M | 976.50 | 7,970.67 |
| N | 1,355.80 | 9,524.10 |
| O | 1,360.90 | 35,927.22 |
| P | 1,463.21 | 11,238.73 |
| Q | 2,332.10 | 92,059.97 |
| R | 3,017.73 | 74,649.00 |
| S | 3,253.00 | 42,915.23 |

**Note: This data set is a set of actual cost data; due to proprietary issues, however, the exact descriptions of the data points cannot be revealed.**

# OLS CER Based on Actual Data



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# OLS CER Quality Metrics

- **OLS Standard-Error of the Estimate**

$$SEE = \sqrt{\frac{\sum_{k=1}^{n}(y_k - a - bx_k)^2}{n-2}} = \sqrt{\frac{\sum_{k=1}^{n}y_k^2 - a\sum_{k=1}^{n}y_k - b\sum_{k=1}^{n}x_k y_k}{n-2}}$$

- **OLS Bias**

$$B = \frac{1}{n}\sum_{k=1}^{n}(a + bx_k - y_k) = 0.$$

- **OLS $R^2$**

$$R^2 = \frac{\left\{n\left(\sum_{k=1}^{n}x_k y_k\right) - \left(\sum_{k=1}^{n}x_k\right)\left(\sum_{k=1}^{n}y_k\right)\right\}^2}{\left\{n\left(\sum_{k=1}^{n}x_k^2\right) - \left(\sum_{k=1}^{n}x_k\right)^2\right\}\left\{n\left(\sum_{k=1}^{n}y_k^2\right) - \left(\sum_{k=1}^{n}y_k\right)^2\right\}}$$



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# OLS CER Quality Metrics Based on Set of Actual Data

- ***SEE = 34,336.83***
- ***B = 0***
- ***$R^2$ = = 0.0968 = 9.68%***

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# Weighted Least Squares

- **In "Weighted" Least Squares (WLS), the Problem is the Same, Except that the Points are not Considered of Equal Value**
- **Accompanying Each Data Point $(x_k,y_k)$ is a "Weight" $w_k$, so that the Data Set Consists of "Triples" $(x_k,y_k,w_k)$, Rather than Pairs $(x_k,y_k)$**
- **The WLS Criterion is that the Coefficients $a$ and $b$ are Selected so that the Weighted Sum of Squares of the Differences $(y_k-a-bx_k)$ is as Small as Possible**
- **The Mathematics Results in Numerical Values of $a$ and $b$ that Minimize the Quantity**

$$f(a,b) = \sum_{k=1}^{n} w_k d_k^2 = \sum_{k=1}^{n} w_k (y_k - a - bx_k)^2$$

- **Weights $w_k$ are Chosen as Follows:**
  - **Large When the Data Point is to Contribute Heavily to the CER**
  - **Small When the Data Point is to Contribute Only in a Minor Way, if at All, to the CER**

# The Weighted Least-Squares Solution

- **Applying some Calculus, we Can Derive Explicit Formulas for the Numerical Values of $a$ and $b$ that Minimize the Quantity** $f(a,b) = \sum_{k=1}^{n} w_k d_k^2 = \sum_{k=1}^{n} w_k (y_k - a - bx_k)^2$
- **The Resulting Expressions for $a$ and $b$ are as Follows:**

$$b = \frac{\left(\sum_{k=1}^{n} w_k\right)\left(\sum_{k=1}^{n} w_k x_k y_k\right) - \left(\sum_{k=1}^{n} w_k x_k\right)\left(\sum_{k=1}^{n} w_k y_k\right)}{\left(\sum_{k=1}^{n} w_k\right)\left(\sum_{k=1}^{n} w_k x_k^2\right) - \left(\sum_{k=1}^{n} w_k x_k\right)^2}$$

$$a = \frac{\sum_{k=1}^{n} w_k y_k}{\sum_{k=1}^{n} w_k} - b\,\frac{\sum_{k=1}^{n} w_k x_k}{\sum_{k=1}^{n} w_k}$$

**Reference: S.A. Book, "Deriving Cost-Estimating Relationships Using Weighted Least-Squares Regression," IAA/ISPA/AIAA Space Systems Cost Methodologies and Applications Symposium, San Diego CA, 10-11 May 1990. (Reference 1)**

# Weighting Data Points *A Priori*

- **Data Points Supporting CER Development May Not All be of Equal Value**
  - **Some May be Known With Greater Precision Than Others**
  - **Some May Be More Relevant to the Particular Estimating Task Than Others**
  - **Some May Be Very Far From the Cost-driver Region where Estimating is Most Commonly Done**
- **Should All Data Points Contribute Equally to the Computation of the CER?**

# The Actual Data Set
# Unweighted, i.e. All Weights = 1

| Program | Statistical Weight w | Cost-Driver Value x | Unit Cost C | Estimated Cost EST C |
|---|---|---|---|---|
| A | 1 | 156.12 | 51,367.22 | 17,596.30 |
| B | 1 | 179.40 | 5,885.00 | 17,887.18 |
| C | 1 | 180.30 | 7,060.00 | 17,898.42 |
| D | 1 | 217.50 | 139,483.12 | 18,363.23 |
| E | 1 | 419.14 | 3,386.00 | 20,882.67 |
| F | 1 | 437.09 | 6,738.00 | 21,106.95 |
| G | 1 | 440.93 | 6,812.00 | 21,154.93 |
| H | 1 | 494.45 | 3,291.34 | 21,823.65 |
| I | 1 | 789.90 | 5,723.14 | 25,515.22 |
| J | 1 | 826.10 | 10,992.00 | 25,967.53 |
| K | 1 | 864.30 | 11,590.00 | 26,444.83 |
| L | 1 | 869.30 | 15,973.00 | 26,507.30 |
| M | 1 | 976.50 | 7,970.67 | 27,846.74 |
| N | 1 | 1,355.80 | 9,524.10 | 32,586.00 |
| O | 1 | 1,360.90 | 35,927.22 | 32,649.72 |
| P | 1 | 1,463.21 | 11,238.73 | 33,928.06 |
| Q | 1 | 2,332.10 | 92,059.97 | 44,784.62 |
| R | 1 | 3,017.73 | 74,649.00 | 53,351.39 |
| S | 1 | 3,253.00 | 42,915.23 | 56,291.03 |
| Sums = | 19.00 | 19,633.77 | 542,585.74 | 542,585.74 |

| | | | | |
|---|---|---|---|---|
| a = | 15,645.619 | | Std Error = | 36,300.49 |
| b = | 12.495 | | Bias = | 0.00 |
| | | | $R^2$ = | 9.68% |



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# Quadratic-Distance Weighting

- **The "Q-Distance" Method Calls for Weighting Points by the Reciprocal of their Squared Distance Along the x-axis from a Cost-Driver Value of Interest**
- **Given that …**
  - $x_0$ **is the Value of the Cost Driver for the Product of Interest Being Investigated**
  - $x_k$ **is the** $x$ **Value of the** $k^{th}$ **Data Point**
  - $d_k$ **is the Distance from the** $x_0$ **Value to the Data Point** $x_k$
- **Then the Weight of the Data Point** $x_k$ **is the Reciprocal of the Quadratic (Squared) Distance, Namely** $w_k = d_k^{-2} = (x_k - x_0)^{-2}$

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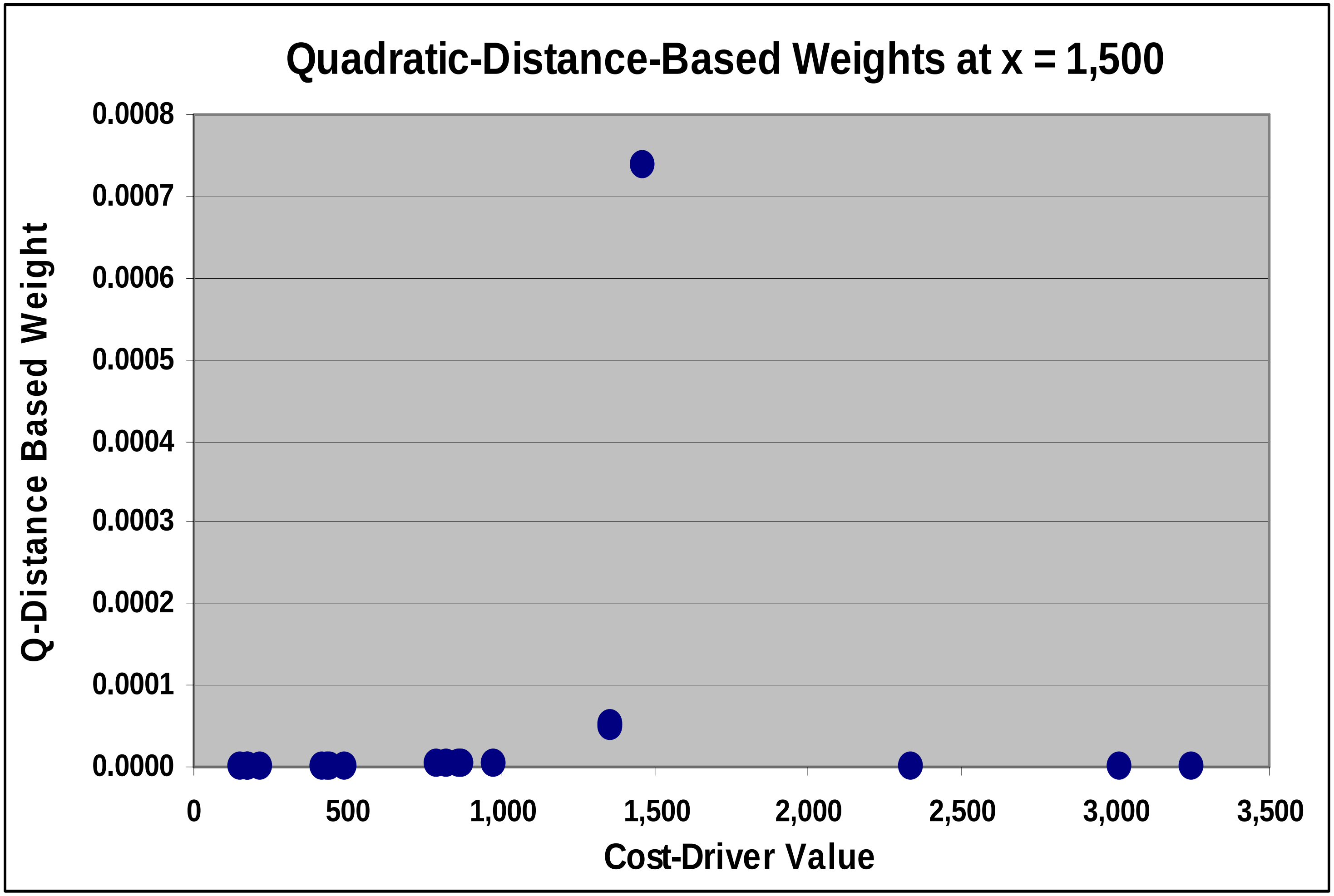
# Why $d_k^{-2}$?

- **There is an Infinite Number of Ways to Define the Weighting in Terms of a Cost Driver's Distance from $x_0$**
- **We Chose the Squared (Quadratic) Distance, because OLS Calculations Use the Squares of Residuals for Best Fit – this Forces the CER to Pass Through the Point $(\bar{x}, \bar{y})$, where $\bar{x}$ is the Mean of the Cost-Driver Values and $\bar{y}$ is the Mean of the Cost Values**
- **However, Other Weighting Schemes Can be Used**

# Quadratic-Distance-Based Weights at $x$ = 1,500



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# Weighting Points by their Q-Distance from $x$ = 1,500

- **Quadratic-Distance Weighting Anchored at $x$ = 1,500**

| Program | Statistical Distance d | Cost-Driver Value x | Unit Cost C | Estimated Cost EST C | Weights $w^* = 1/d^2$ |
|---|---|---|---|---|---|
| A | 1343.88 | 156.12 | 51,367.22 | 13,969.64 | 0.0000005537 |
| B | 1320.6 | 179.40 | 5,885.00 | 13,949.82 | 0.0000005734 |
| C | 1319.7 | 180.30 | 7,060.00 | 13,949.06 | 0.0000005742 |
| D | 1282.5 | 217.50 | 139,483.12 | 13,917.39 | 0.0000006080 |
| E | 1080.86 | 419.14 | 3,386.00 | 13,745.72 | 0.0000008560 |
| F | 1062.91 | 437.09 | 6,738.00 | 13,730.44 | 0.0000008851 |
| G | 1059.07 | 440.93 | 6,812.00 | 13,727.17 | 0.0000008916 |
| H | 1005.55 | 494.45 | 3,291.34 | 13,681.61 | 0.0000009890 |
| I | 710.1 | 789.90 | 5,723.14 | 13,430.08 | 0.0000019832 |
| J | 673.9 | 826.10 | 10,992.00 | 13,399.26 | 0.0000022020 |
| K | 635.7 | 864.30 | 11,590.00 | 13,366.74 | 0.0000024745 |
| L | 630.7 | 869.30 | 15,973.00 | 13,362.48 | 0.0000025139 |
| M | 523.5 | 976.50 | 7,970.67 | 13,271.22 | 0.0000036489 |
| N | 144.2 | 1,355.80 | 9,524.10 | 12,948.30 | 0.0000480916 |
| O | 139.1 | 1,360.90 | 35,927.22 | 12,943.96 | 0.0000516828 |
| P | 36.79 | 1,463.21 | 11,238.73 | 12,856.86 | 0.0007388230 |
| Q | 832.1 | 2,332.10 | 92,059.97 | 12,117.13 | 0.0000014443 |
| R | 1517.73 | 3,017.73 | 74,649.00 | 11,533.42 | 0.0000004341 |
| S | 1753 | 3,253.00 | 42,915.23 | 11,333.12 | 0.0000003254 |
| Sums = | 17,071.89 | 19,633.77 | 542,585.74 | 251,233.42 | 0.0008595547 |
| | | | | | $\Sigma w_k$ |
| a = | 14,102.56 | | | | |
| b = | -0.85 | | | | |



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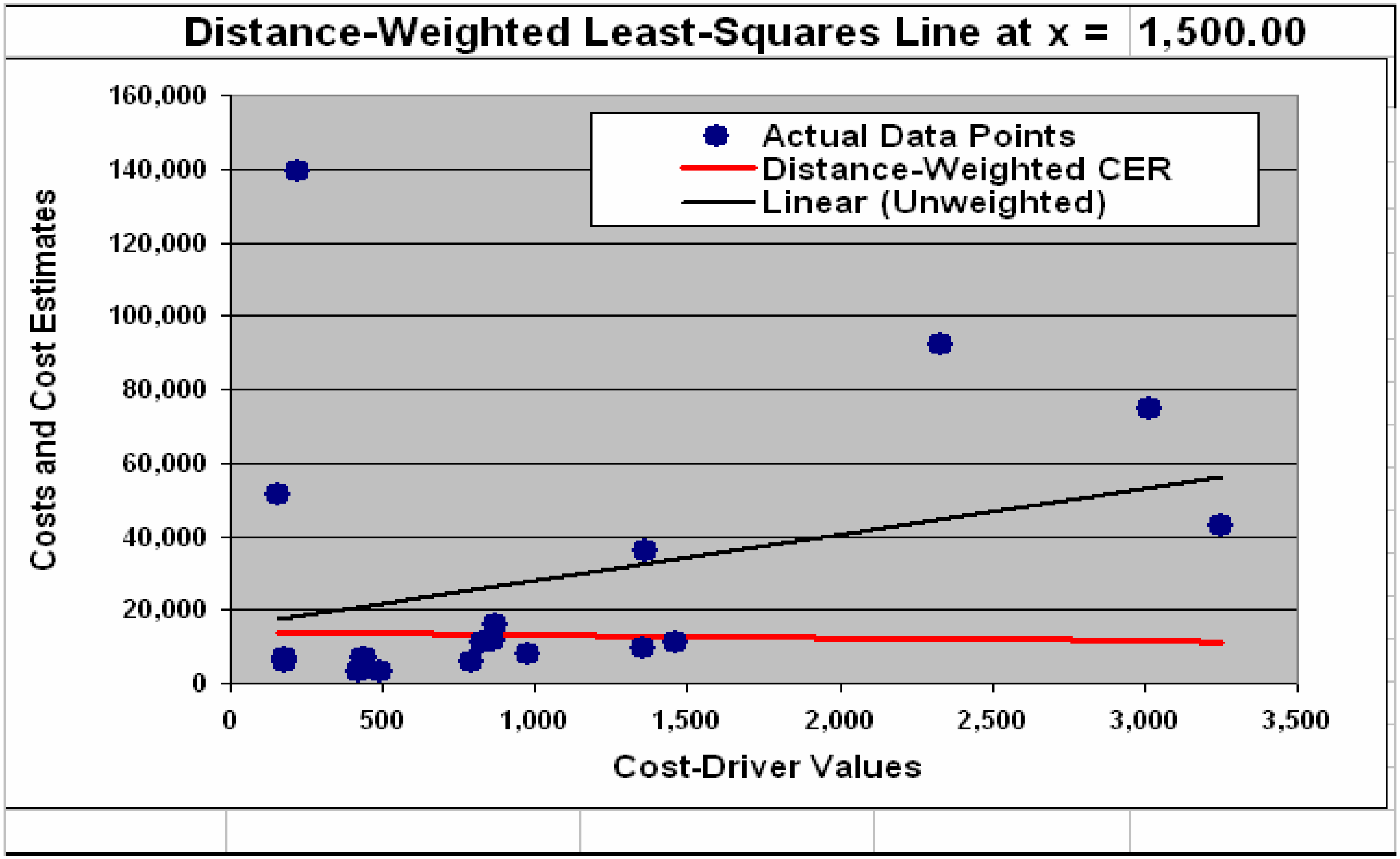
# Resulting Q-Distance CER for Real Data Set at $x$ = 1,500



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# "Normalizing" the Weights

- $\{w_1^*, w_2^*, \ldots, w_n^*,\}$ **is the Initial Set of Weights Assigned to the Data Points**
- **We Want to Find a Set of Weights** $\{w_1, w_2, \ldots, w_n,\}$ **that is Equivalent to the Initial Set in the Sense that the Relative Weight of the Data Points to Each Other is Preserved**
- **In Addition, We Want the "Normalized" Weights to Sum to *n*, i.e.** $\sum_{k=1}^{n} w_k = n$
- **The Definition of Normalized Weights that Works is**

$$w_j = n w_j^* \left( \sum_{k=1}^{n} w_j^* \right)^{-1}$$

- **Note, for Each Pair *i* and *j*, that** $w_i/w_j = w_i^*/w_j^*$**, so the Relative Weighting of the Data Points is Preserved**



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# Using the Normalized Weights

- From this Point on, We will be Using the Normalized Weights in Lieu of the Initial Weights
- The Resulting Expressions for $a$ and $b$ are as Follows:

$$b = \frac{n\left(\sum_{k=1}^{n} w_k x_k y_k\right) - \left(\sum_{k=1}^{n} w_k x_k\right)\left(\sum_{k=1}^{n} w_k y_k\right)}{n\left(\sum_{k=1}^{n} w_k x_k^2\right) - \left(\sum_{k=1}^{n} w_k x_k\right)^2}$$

$$a = \frac{\sum_{k=1}^{n} w_k y_k}{n} - b\,\frac{\sum_{k=1}^{n} w_k x_k}{n}$$

# Normalized Weighting of Points by their Q-Distance from $x$ = 1,500

- **Quadratic-Distance Weighting Anchored at $x$ = 1,500**

| Program | Statistical Distance d | Cost-Driver Value x | Unit Cost C | Estimated Cost EST C | Weights $w^* = 1/d^2$ | Normalized Weights w |
|---|---|---|---|---|---|---|
| A | 1343.88 | 156.12 | 51,367.22 | 13,969.64 | 0.0000005537 | 0.0122393710 |
| B | 1320.6 | 179.40 | 5,885.00 | 13,949.82 | 0.0000005734 | 0.0126746943 |
| C | 1319.7 | 180.30 | 7,060.00 | 13,949.06 | 0.0000005742 | 0.0126919878 |
| D | 1282.5 | 217.50 | 139,483.12 | 13,917.39 | 0.0000006080 | 0.0134389498 |
| E | 1080.86 | 419.14 | 3,386.00 | 13,745.72 | 0.0000008560 | 0.0189208738 |
| F | 1062.91 | 437.09 | 6,738.00 | 13,730.44 | 0.0000008851 | 0.0195653262 |
| G | 1059.07 | 440.93 | 6,812.00 | 13,727.17 | 0.0000008916 | 0.0197074643 |
| H | 1005.55 | 494.45 | 3,291.34 | 13,681.61 | 0.0000009890 | 0.0218611367 |
| I | 710.1 | 789.90 | 5,723.14 | 13,430.08 | 0.0000019832 | 0.0438370231 |
| J | 673.9 | 826.10 | 10,992.00 | 13,399.26 | 0.0000022020 | 0.0486731179 |
| K | 635.7 | 864.30 | 11,590.00 | 13,366.74 | 0.0000024745 | 0.0546985302 |
| L | 630.7 | 869.30 | 15,973.00 | 13,362.48 | 0.0000025139 | 0.0555692350 |
| M | 523.5 | 976.50 | 7,970.67 | 13,271.22 | 0.0000036489 | 0.0806578623 |
| N | 144.2 | 1,355.80 | 9,524.10 | 12,948.30 | 0.0000480916 | 1.0630398840 |
| O | 139.1 | 1,360.90 | 35,927.22 | 12,943.96 | 0.0000516828 | 1.1424200586 |
| P | 36.79 | 1,463.21 | 11,238.73 | 12,856.86 | 0.0007388230 | 16.3312905022 |
| Q | 832.1 | 2,332.10 | 92,059.97 | 12,117.13 | 0.0000014443 | 0.0319248625 |
| R | 1517.73 | 3,017.73 | 74,649.00 | 11,533.42 | 0.0000004341 | 0.0095960177 |
| S | 1753 | 3,253.00 | 42,915.23 | 11,333.12 | 0.0000003254 | 0.0071931025 |
| Sums = | 17,071.89 | 19,633.77 | 542,585.74 | 251,233.42 | 0.0008595547 | 19.0000000000 |
| | | | | | $\Sigma w_k$ | |
| a = | 14,102.56 | | [redacted] | | | |
| b = | -0.85 | | | | | |



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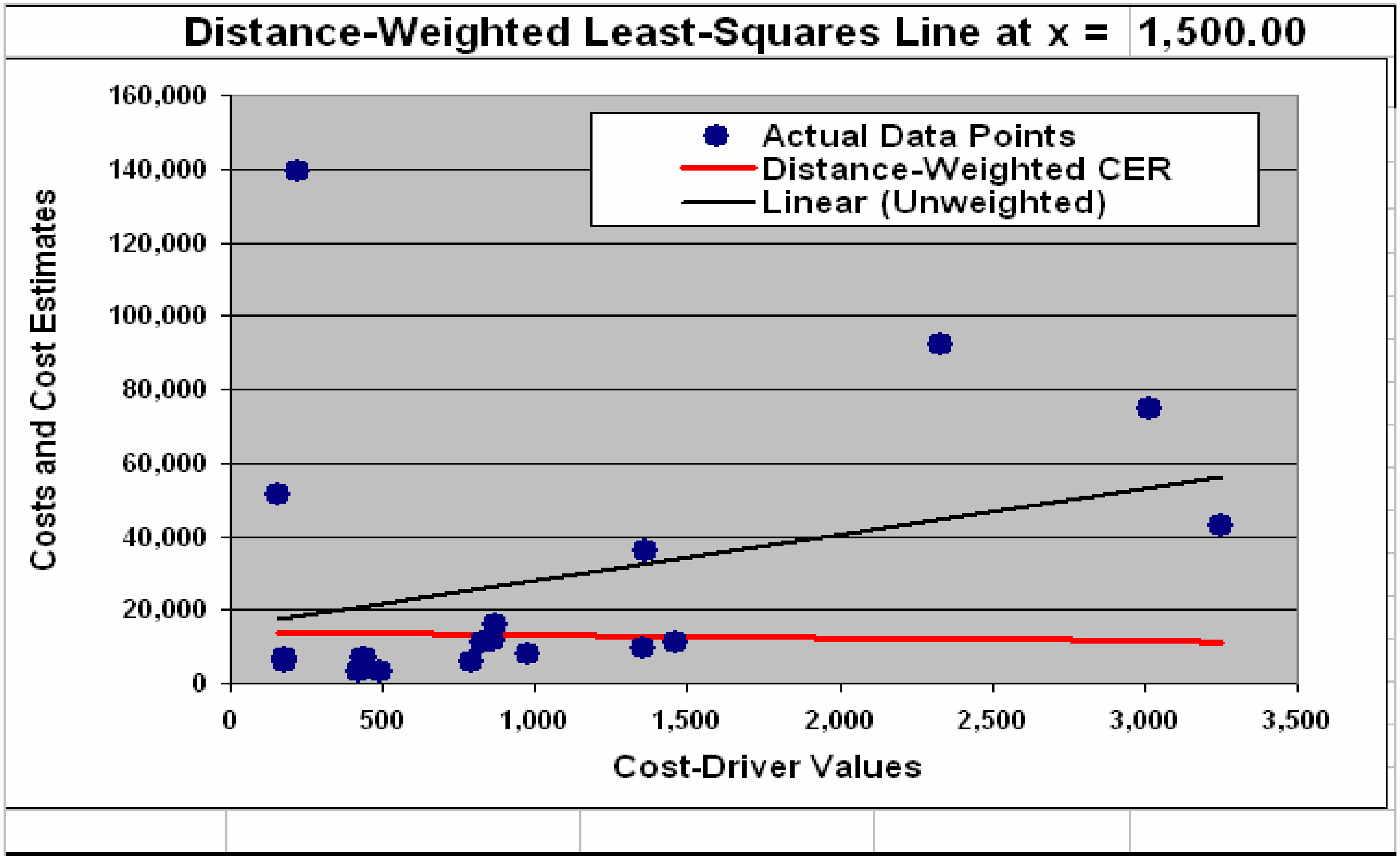
# Resulting Q-Distance CER Anchored at $x$ = 1,500



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# WLS CER Quality Metrics

- **WLS Standard-Error of the Estimate**

$$SEE_w = \sqrt{\frac{\sum_{k=1}^{n} w_k (y_k - a - bx_k)^2}{\sum_{k=1}^{n} w_k - 2}} = \sqrt{\frac{\sum_{k=1}^{n} w_k y_k^2 - a\sum_{k=1}^{n} w_k y_k - b\sum_{k=1}^{n} w_k x_k y_k}{n-2}}$$

- **WLS Bias (not necessarily 0)**

$$B_w = \frac{\left(\sum_{k=1}^{n} w_k (y_k - bx_k)\right) - \left(\sum_{k=1}^{n} (x_k - y_k)\right)}{n} = a - \frac{\left(\sum_{k=1}^{n} y_k\right) - b\left(\sum_{k=1}^{n} x_k\right)}{n}$$

- **WLS $R^2$**

$$R_w^2 = \frac{\left\{n\left(\sum_{k=1}^{n} w_k x_k y_k\right) - \left(\sum_{k=1}^{n} w_k x_k\right)\left(\sum_{k=1}^{n} w_k y_k\right)\right\}^2}{\left\{n\left(\sum_{k=1}^{n} w_k x_k^2\right) - \left(\sum_{k=1}^{n} w_k x_k\right)^2\right\}\left\{n\left(\sum_{k=1}^{n} w_k y_k^2\right) - \left(\sum_{k=1}^{n} w_k y_k\right)^2\right\}}$$

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# WLS CER Quality Metrics of Adaptive CER Based at 1,500

- ***SEE = 8,226.72***
- ***B = -15,334.33***
- $R^2 = 0.000218 = 0.0218\%$

# Contents

# The “Universal” Adaptive CER

- **The “Universal” Adaptive CER is Formed by Combining* the Various Individual Adaptive CERs of the Sort Derived Above Over the Range of Cost Drivers into One CER that Applies Over the Entire Range**
- **This Universal Adaptive CER is, as P. Foussier (Reference 3, Chart 5) Presciently Noted, “Highly Nonlinear”**
- **For the Data Set We Have Been Working with, We Can Consider the Cost-driver Range to Go from 50 to 3,500**
- **We Calculate a Quadratic-Distance Weighted CER and an Estimated Cost at Each Increment of 50 for Each of Those Cost-Driver Values**
- **Then We String All these Estimates Together and Interpolate between Successive Ones to Form the Universal Adaptive CER**

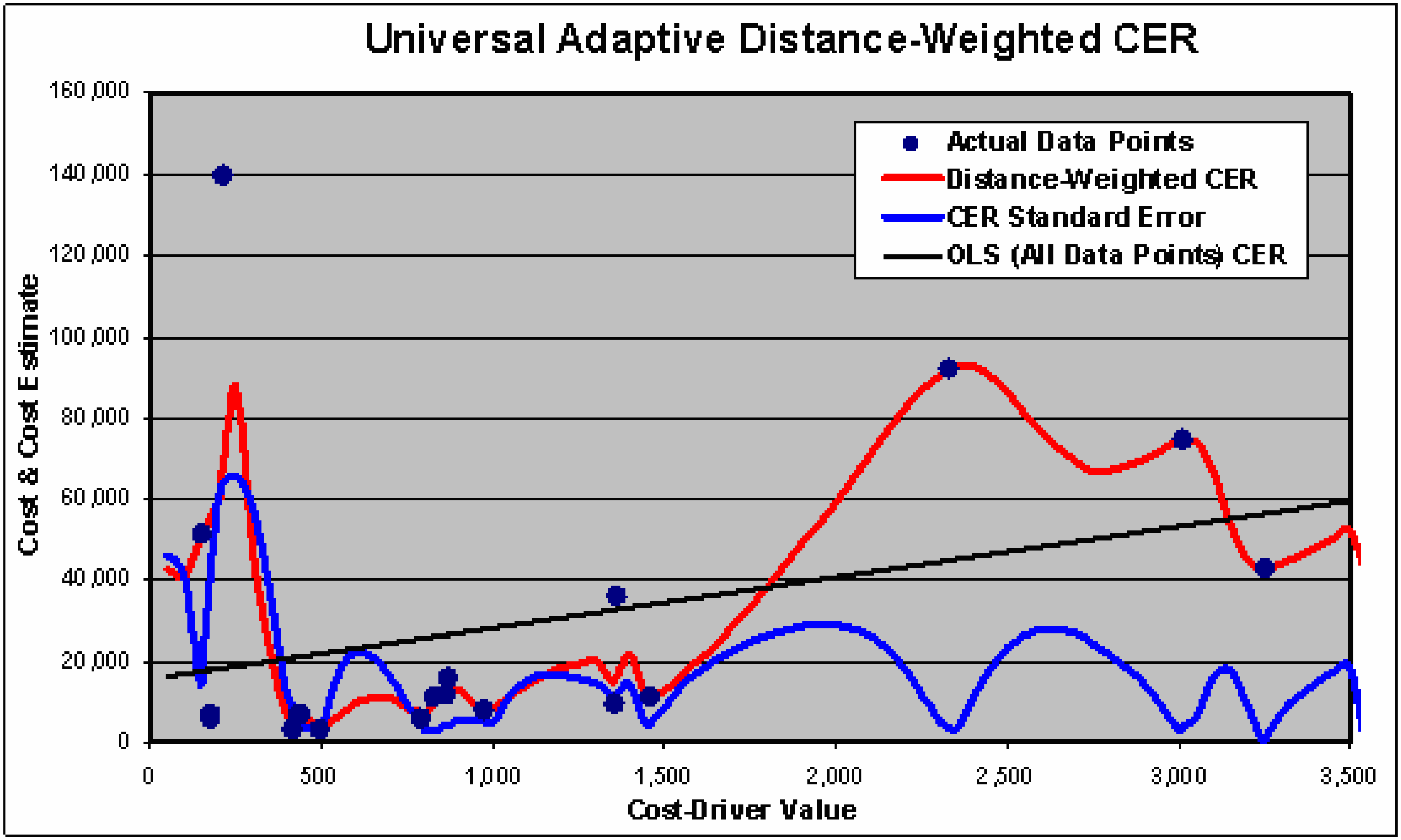
*** The idea of combining estimates at various points of the cost-driver range into one all-inclusive CER was suggested to us by Paul Wetzel of OpsConsulting LLC.**

# Universal Adaptive CER-Based Estimates at 50-Unit Increments

| Driver | EST Cost | Std Error |
|---|---|---|
| 50.00 | 42,739.31 | 46,098.71 |
| 100.00 | 40,817.29 | 41,490.92 |
| 150.00 | 49,546.82 | 15,013.91 |
| 156.12 | 50,880.53 | 20,862.57 |
| 179.40 | 55,953.88 | 43,110.41 |
| 180.30 | 56,150.02 | 43,970.50 |
| 200.00 | 60,443.18 | 62,797.07 |
| 217.50 | 69,749.17 | 63,712.78 |
| 250.00 | 87,031.73 | 65,413.39 |
| 300.00 | 46,425.71 | 57,676.55 |
| 350.00 | 22,733.56 | 36,873.63 |
| 400.00 | 7,006.95 | 11,986.04 |
| 419.14 | 6,760.42 | 9,109.80 |
| 437.09 | 6,529.22 | 6,412.39 |
| 440.93 | 6,479.76 | 5,835.34 |
| 450.00 | 6,362.94 | 4,472.36 |
| 494.45 | 3,589.46 | 3,084.58 |
| 500.00 | 3,243.16 | 2,911.31 |
| 550.00 | 6,829.12 | 17,776.83 |
| 600.00 | 9,959.40 | 22,010.11 |
| 650.00 | 11,310.17 | 21,033.96 |
| 700.00 | 10,929.01 | 16,492.92 |
| 750.00 | 8,652.67 | 9,456.12 |
| 789.90 | 7,175.24 | 4,565.75 |
| 800.00 | 6,801.25 | 3,327.84 |
| 826.10 | 9,756.59 | 3,386.63 |
| 850.00 | 12,462.82 | 3,440.47 |
| 864.30 | 12,666.50 | 4,059.71 |
| 869.30 | 12,737.72 | 4,276.23 |
| 900.00 | 13,174.99 | 5,605.64 |
| 950.00 | 9,208.15 | 5,651.88 |
| 976.50 | 8,832.68 | 5,342.38 |
| 1,000.00 | 8,499.71 | 5,067.91 |
| 1,050.00 | 11,462.16 | 11,841.54 |
| 1,100.00 | 14,296.49 | 15,323.02 |
| 1,150.00 | 16,537.15 | 16,912.27 |
| 1,200.00 | 18,230.99 | 17,020.52 |
| 1,250.00 | 19,495.31 | 16,029.95 |
| 1,300.00 | 20,310.23 | 14,631.94 |
| 1,350.00 | 14,974.31 | 11,522.07 |
| 1,355.80 | 15,774.27 | 11,821.74 |
| 1,360.90 | 16,477.67 | 12,085.24 |
| 1,400.00 | 21,870.45 | 14,105.41 |
| 1,450.00 | 11,840.86 | 4,214.92 |
| 1,463.21 | 12,101.01 | 5,274.84 |

| Driver | EST Cost | Std Error |
|---|---|---|
| 1,500.00 | 12,825.54 | 8,226.72 |
| 1,550.00 | 16,621.72 | 13,974.93 |
| 1,600.00 | 20,492.26 | 17,569.25 |
| 1,650.00 | 24,526.56 | 20,350.34 |
| 1,700.00 | 28,831.03 | 22,668.31 |
| 1,750.00 | 33,415.50 | 24,632.61 |
| 1,800.00 | 38,247.16 | 26,275.33 |
| 1,850.00 | 43,285.50 | 27,589.48 |
| 1,900.00 | 48,497.85 | 28,534.71 |
| 1,950.00 | 53,862.57 | 29,032.00 |
| 2,000.00 | 59,364.10 | 28,954.26 |
| 2,050.00 | 64,981.01 | 28,118.23 |
| 2,100.00 | 70,666.52 | 26,286.86 |
| 2,150.00 | 76,319.27 | 23,197.58 |
| 2,200.00 | 81,744.09 | 18,634.07 |
| 2,250.00 | 86,609.89 | 12,543.91 |
| 2,300.00 | 90,430.47 | 5,163.31 |
| 2,332.10 | 91,836.14 | 3,730.10 |
| 2,350.00 | 92,619.98 | 2,930.89 |
| 2,400.00 | 92,676.25 | 10,907.76 |
| 2,450.00 | 90,463.37 | 17,895.26 |
| 2,500.00 | 86,410.39 | 23,227.16 |
| 2,550.00 | 81,412.53 | 26,603.62 |
| 2,600.00 | 76,466.46 | 28,091.64 |
| 2,650.00 | 72,322.92 | 27,995.50 |
| 2,700.00 | 69,366.76 | 26,697.11 |
| 2,750.00 | 66,431.86 | 24,540.98 |
| 2,800.00 | 67,242.40 | 21,772.29 |
| 2,850.00 | 67,904.22 | 18,495.58 |
| 2,900.00 | 69,545.45 | 14,613.82 |
| 2,950.00 | 71,913.26 | 9,720.21 |
| 3,000.00 | 74,219.40 | 3,000.69 |
| 3,017.73 | 74,164.83 | 4,164.89 |
| 3,050.00 | 74,065.53 | 6,283.82 |
| 3,100.00 | 67,141.02 | 15,848.64 |
| 3,150.00 | 54,415.99 | 17,689.83 |
| 3,200.00 | 45,424.15 | 9,943.35 |
| 3,250.00 | 42,927.10 | 501.90 |
| 3,253.00 | 42,978.65 | 868.74 |
| 3,300.00 | 43,786.36 | 6,615.99 |
| 3,350.00 | 45,762.39 | 11,482.72 |
| 3,400.00 | 47,971.96 | 14,864.14 |
| 3,450.00 | 50,126.95 | 17,319.87 |
| 3,500.00 | 52,149.51 | 19,185.52 |

# Universal Adaptive CER Graphed with Standard Error and Actual Data



# Contents

- **Ordinary Least-Squares (OLS) Regression CERs and their Quality Metrics**
- **Weighted Least-Squares (WLS) Regression CERs and their Quality Metrics**
- **Adaptive CERs via Quadratic-Distance Weighting**
- **The "Universal Adaptive CER"**
- **Prediction Bounds for OLS, WLS, and Adaptive CERs**
- **Conclusions**

# Prediction Bounds on CER-Based Estimates

- **Estimating the Cost of Developing or Producing a New Subsystem or Component is Essentially Trying to Predict the Future, which Means that Any Such Estimate Contains Uncertainty**
- **Uncertainty in CER-Based Estimates Can be Expressed in Terms of Prediction Bounds, the Algebra of which is Completely Understood Only in the Case of CERs Derived by Classical OLS Linear Regression**
- **As a Result, an Explicit Formula Exists for "Prediction Intervals" that Bound Cost Estimates Based on CERs that Have Been Derived by Applying OLS to Historical Cost Data**

# The Formula for Prediction Bounds in the Case of OLS-Derived CERs

- **The Formula for the *(1-α)th* Percent Upper and Lower Prediction Bounds on the True Cost $y$, Based on the Estimate $ESTy$ from the CER is the Following:**

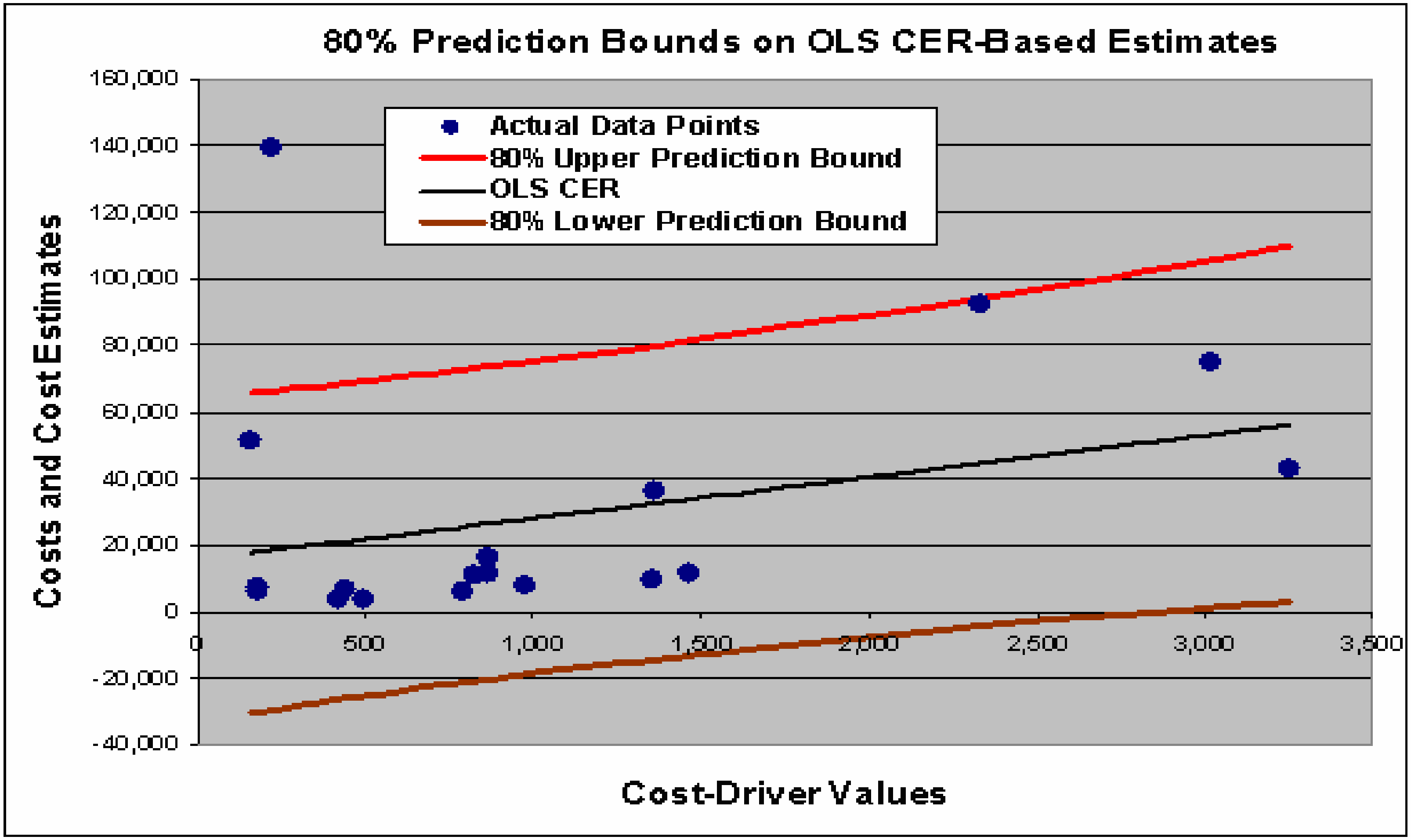
$$ESTy \pm t_{\alpha/2,n-2} * SEE\sqrt{1+\frac{1}{n}+\frac{(x-\bar{x})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}}$$

- **Here $t_{\alpha/2,n-2}$ is the *(1-α)th* Percentage Point of the *t* Distribution, $\bar{x}$ is the Mean of the Cost-Driver Values in the Data Base, $x$ is the Cost-Driver Value at which the Estimate is Being Made, and $SEE$ is the Standard Error of the CER**

# 80% Upper and Lower Prediction Bounds on the OLS CER

| Program | Cost-Driver Value x | Unit Cost y | 80% Upper Bound | OLS EST y | 80% Lower Bound |
|---|---|---|---|---|---|
| A | 156.12 | 51,367.22 | 65,673.53 | 17,596.30 | -30,480.93 |
| B | 179.40 | 5,885.00 | 65,907.23 | 17,887.18 | -30,132.88 |
| C | 180.30 | 7,060.00 | 65,916.29 | 17,898.42 | -30,119.45 |
| D | 217.50 | 139,483.12 | 66,292.88 | 18,363.23 | -29,566.43 |
| E | 419.14 | 3,386.00 | 68,400.42 | 20,882.67 | -26,635.08 |
| F | 437.09 | 6,738.00 | 68,593.51 | 21,106.95 | -26,379.62 |
| G | 440.93 | 6,812.00 | 68,634.94 | 21,154.93 | -26,325.09 |
| H | 494.45 | 3,291.34 | 69,216.65 | 21,823.65 | -25,569.35 |
| I | 789.90 | 5,723.14 | 72,574.56 | 25,515.22 | -21,544.12 |
| J | 826.10 | 10,992.00 | 73,003.23 | 25,967.53 | -21,068.17 |
| K | 864.30 | 11,590.00 | 73,459.69 | 26,444.83 | -20,570.03 |
| L | 869.30 | 15,973.00 | 73,519.75 | 26,507.30 | -20,505.14 |
| M | 976.50 | 7,970.67 | 74,824.83 | 27,846.74 | -19,131.35 |
| N | 1,355.80 | 9,524.10 | 79,710.04 | 32,586.00 | -14,538.05 |
| O | 1,360.90 | 35,927.22 | 79,778.56 | 32,649.72 | -14,479.12 |
| P | 1,463.21 | 11,238.73 | 81,168.85 | 33,928.06 | -13,312.74 |
| Q | 2,332.10 | 92,059.97 | 94,145.23 | 44,784.62 | -4,576.00 |
| R | 3,017.73 | 74,649.00 | 105,728.61 | 53,351.39 | 974.17 |
| S | 3,253.00 | 42,915.23 | 109,940.12 | 56,291.03 | 2,641.94 |

# 80% Prediction Bounds Graphed with OLS CER and Actual Data



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# The Formula for Prediction Bounds in the Case of WLS-Derived CERs

- **When the Weights are Normalized, the Expressions for the *(1-α)th* Percent Upper and Lower Prediction Bounds on the True Cost $y$ at the Cost-Driver Value $x_p$, Based On Estimates $ESTy$ from WLS-Based Adaptive CERs are the Following:**

$$ESTy \pm t_{\alpha/2,n-2} * SEE_w \sqrt{\frac{1}{w_p} + \frac{1}{n} + \frac{n(x_p - \bar{x})^2}{n\left(\sum_{k=1}^{n} w_k x_k^2\right) - \left(\sum_{k=1}^{k} w_k x_k\right)^2}}$$

- **One Way to Obtain a Usable Value, if Needed, for $w_p$ When $x_p$ is Not in the Data Base from which the Adaptive CERs are Derived is to Interpolate between the Weights of the Nearest Data-base Points**

# 80% Prediction Bounds on the Adaptive CER Based at $x$ = 800

| Program | Cost-Driver Value x | Unit Cost y | 80% Upper Bound | WLS EST y | 80% Lower Bound |
|---|---|---|---|---|---|
| A | 156.12 | 51,367.22 | 67,335.731428 | -5,734.14 | -78,804.008697 |
| B | 179.40 | 5,885.00 | 65,146.948025 | -5,280.91 | -75,708.771200 |
| C | 180.30 | 7,060.00 | 65,062.330513 | -5,263.39 | -75,589.110360 |
| D | 217.50 | 139,483.12 | 61,564.835765 | -4,539.16 | -70,643.158038 |
| E | 419.14 | 3,386.00 | 42,608.518817 | -613.53 | -43,835.578046 |
| F | 437.09 | 6,738.00 | 40,921.251654 | -264.07 | -41,449.391167 |
| G | 440.93 | 6,812.00 | 40,560.306422 | -189.31 | -40,938.927733 |
| H | 494.45 | 3,291.34 | 35,529.986321 | 852.64 | -33,824.697703 |
| I | 789.90 | 5,723.14 | 8,126.533982 | 6,604.62 | 5,082.700610 |
| J | 826.10 | 10,992.00 | 10,459.318778 | 7,309.38 | 4,159.436356 |
| K | 864.30 | 11,590.00 | 15,439.587849 | 8,053.07 | 666.561891 |
| L | 869.30 | 15,973.00 | 16,099.371097 | 8,150.42 | 201.463800 |
| M | 976.50 | 7,970.67 | 30,313.734118 | 10,237.44 | -9,838.849438 |
| N | 1,355.80 | 9,524.10 | 80,730.945765 | 17,621.85 | -45,487.245014 |
| O | 1,360.90 | 35,927.22 | 81,409.009710 | 17,721.14 | -45,966.730098 |
| P | 1,463.21 | 11,238.73 | 95,011.748000 | 19,712.96 | -55,585.820690 |
| Q | 2,332.10 | 92,059.97 | 210,542.762967 | 36,628.96 | -137,284.838305 |
| R | 3,017.73 | 74,649.00 | 301,708.981386 | 49,977.16 | -201,754.659776 |
| S | 3,253.00 | 42,915.23 | 332,992.265384 | 54,557.52 | -223,877.228359 |
| Sums | 19,633.77 | 542,585.74 | | 215,542.66 | |



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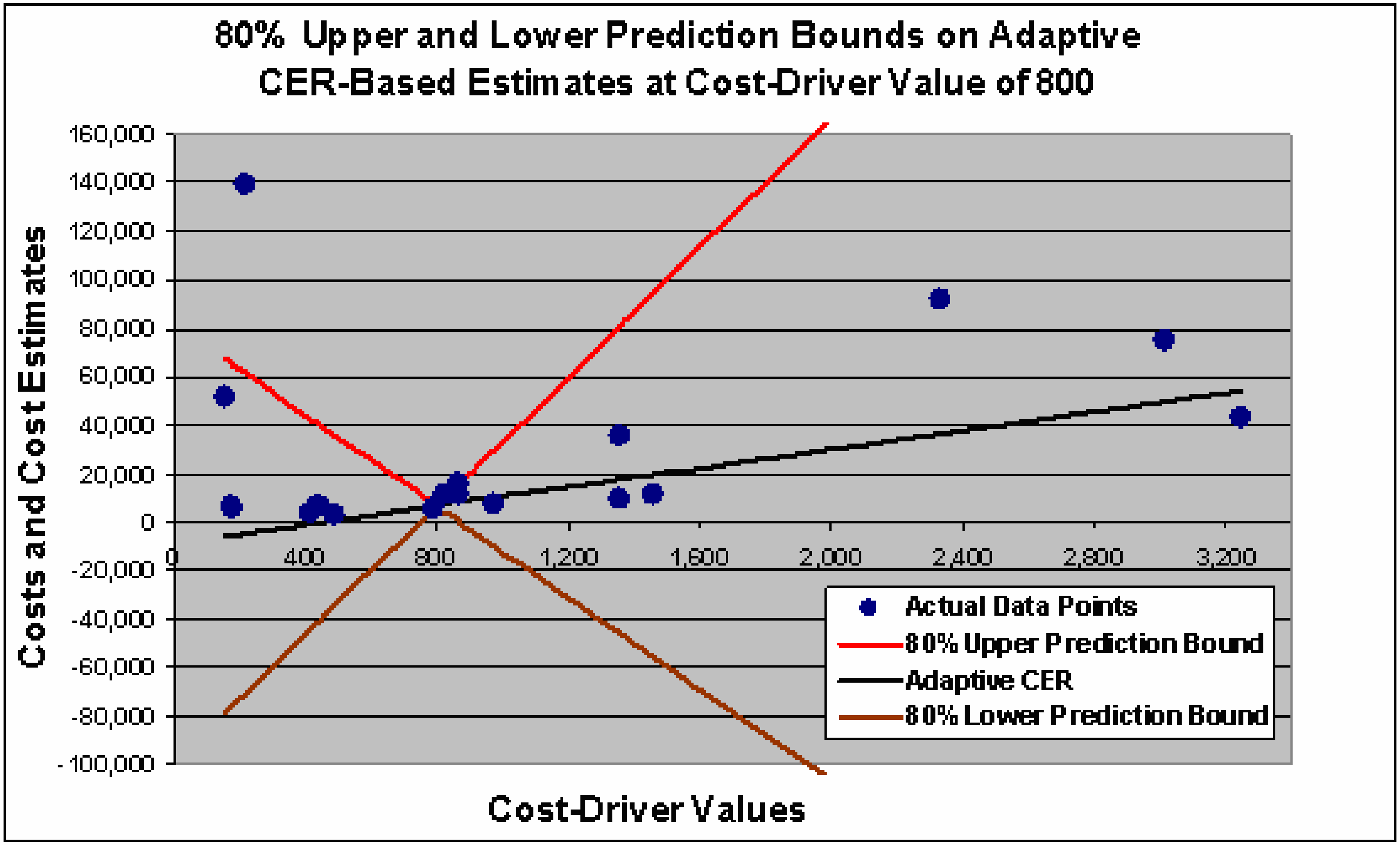
# 80% Prediction Bounds on the Adaptive CER Based at $x$ = 800



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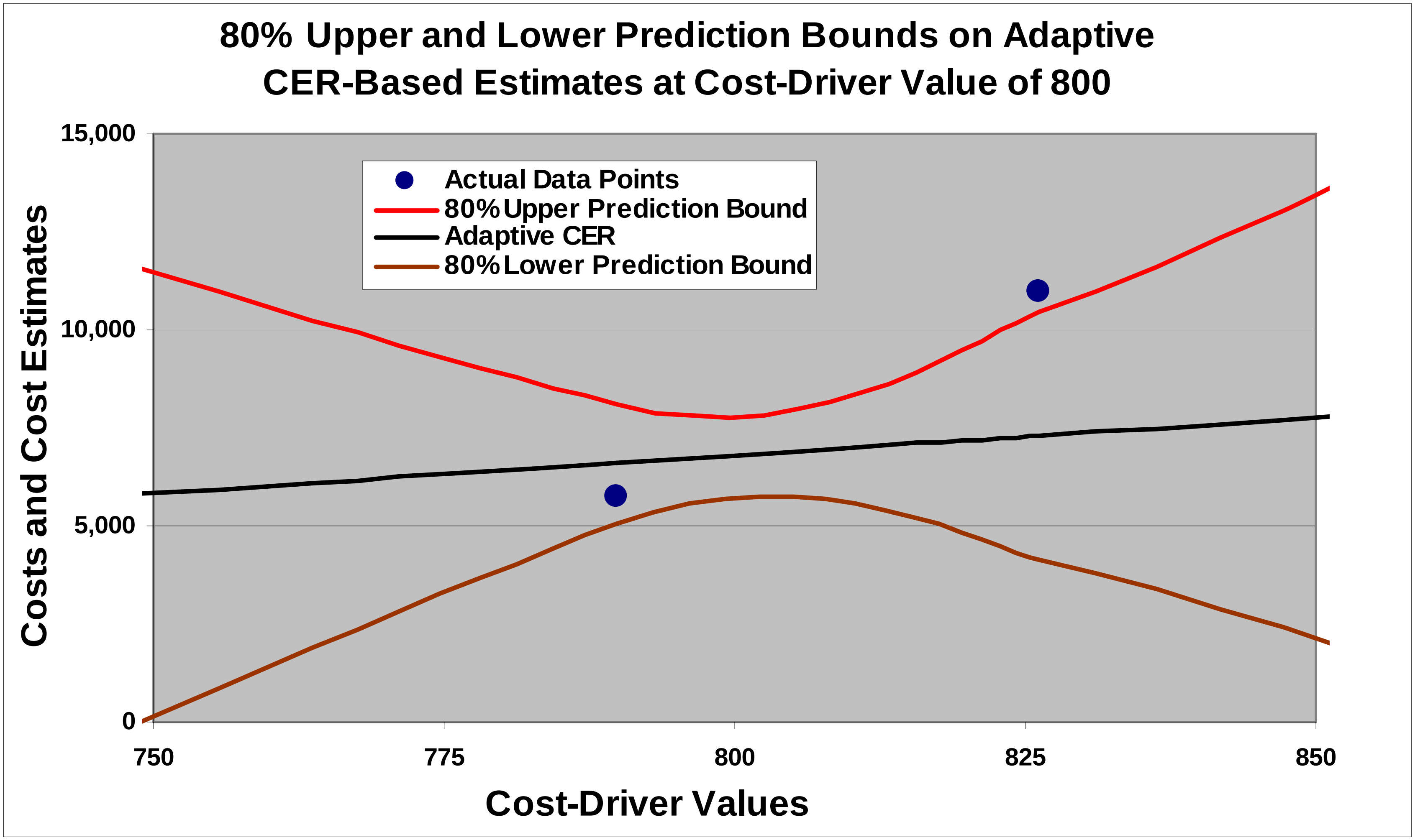
# Prediction Bounds Near Cost-Driver Value of 800



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# 80% Prediction Bounds on the Adaptive CER Based at $x$ = 300

| Program | Cost-Driver Value x | Unit Cost y | 80% Upper Bound | WLS EST y | 80% Lower Bound |
|---|---|---|---|---|---|
| A | 156.12 | 51,367.22 | 65,389.279544 | 61,698.97 | 58,008.663971 |
| B | 179.40 | 5,885.00 | 62,372.227016 | 59,227.74 | 56,083.244080 |
| C | 180.30 | 7,060.00 | 62,255.776784 | 59,132.20 | 56,008.619347 |
| D | 217.50 | 139,483.12 | 57,462.441876 | 55,183.32 | 52,904.189048 |
| E | 419.14 | 3,386.00 | 36,867.788626 | 33,778.67 | 30,689.557986 |
| F | 437.09 | 6,738.00 | 35,381.736102 | 31,873.23 | 28,364.726492 |
| G | 440.93 | 6,812.00 | 35,064.501531 | 31,465.60 | 27,866.707881 |
| H | 494.45 | 3,291.34 | 30,658.711130 | 25,784.31 | 20,909.907048 |
| I | 789.90 | 5,723.14 | 6,491.040727 | -5,578.52 | -17,648.087346 |
| J | 826.10 | 10,992.00 | 3,534.857637 | -9,421.25 | -22,377.363947 |
| K | 864.30 | 11,590.00 | 415.759782 | -13,476.29 | -27,368.336816 |
| L | 869.30 | 15,973.00 | 7.527753 | -14,007.05 | -28,021.632368 |
| M | 976.50 | 7,970.67 | -8,743.802865 | -25,386.63 | -42,029.453100 |
| N | 1,355.80 | 9,524.10 | -39,698.603983 | -65,650.37 | -91,602.134324 |
| O | 1,360.90 | 35,927.22 | -40,114.762116 | -66,191.75 | -92,268.734323 |
| P | 1,463.21 | 11,238.73 | -48,463.042557 | -77,052.24 | -105,641.431258 |
| Q | 2,332.10 | 92,059.97 | -119,355.526647 | -169,287.31 | -219,219.087245 |
| R | 3,017.73 | 74,649.00 | -175,292.373781 | -242,068.82 | -308,845.271266 |
| S | 3,253.00 | 42,915.23 | -194,486.501042 | -267,043.38 | -339,600.262830 |
| Sums | 19,633.77 | 542,585.74 | | -597,019.57 | |

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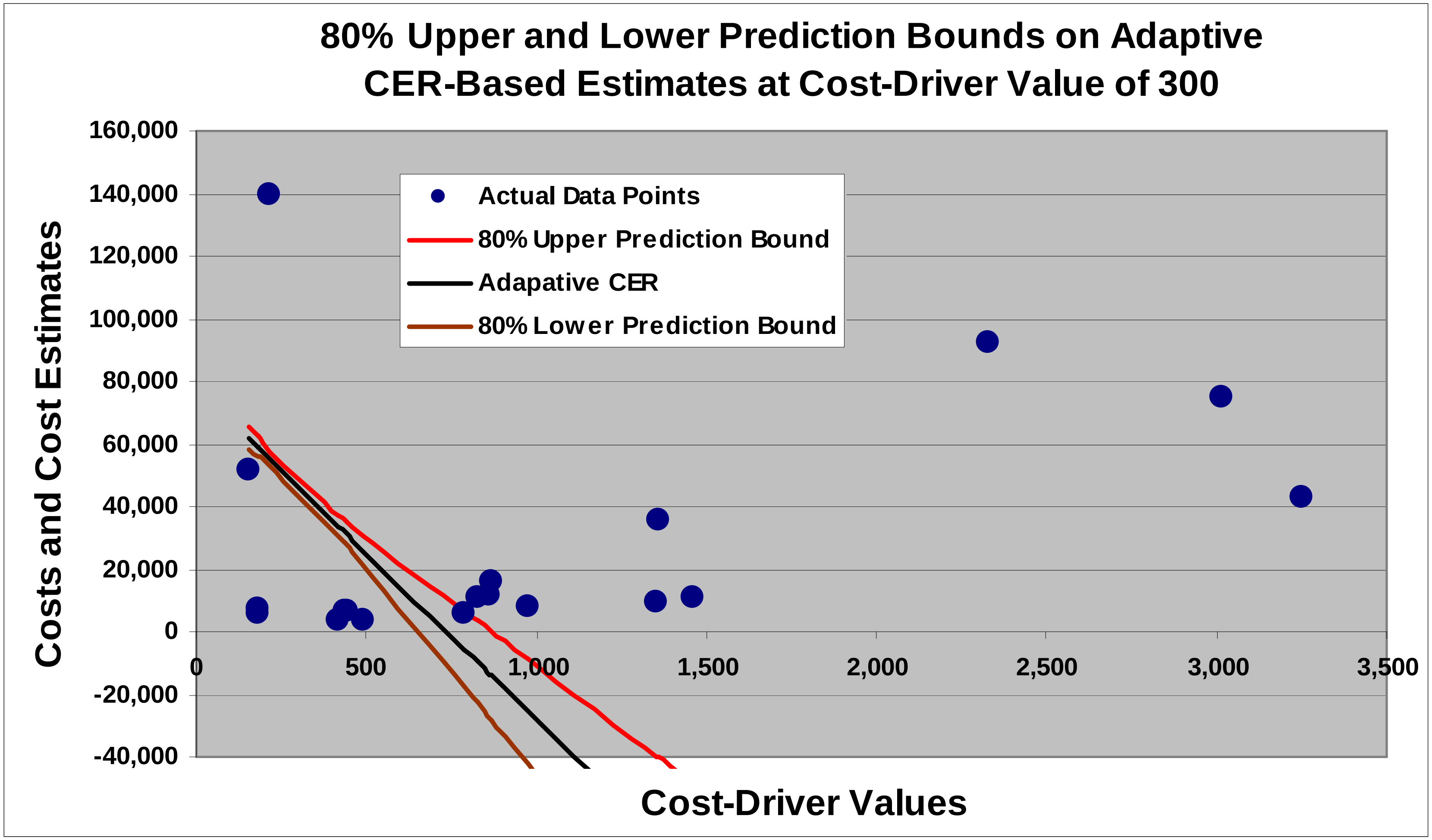
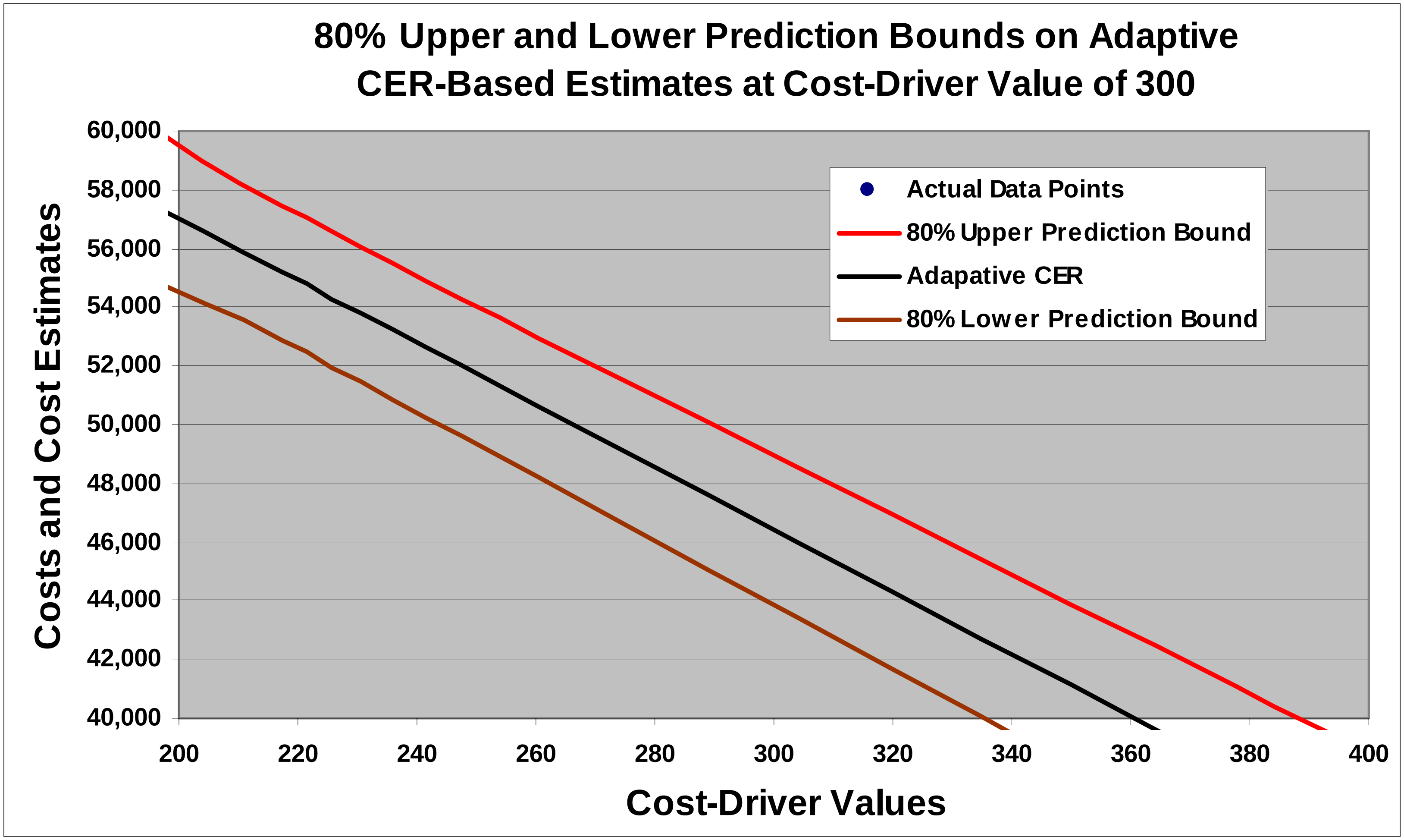
# 80% Prediction Bounds on the Adaptive CER Based at $x$ = 300



# Prediction Bounds Near Cost-Driver Value of 300



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# 80% Prediction Bounds on the Adaptive CER Based at $x$ = 3,000

| Program | Cost-Driver Value x | Unit Cost y | 80% Upper Bound | WLS EST y | 80% Lower Bound |
|---|---|---|---|---|---|
| A | 156.12 | 51,367.22 | 202,434.005312 | 34,104.71 | -134,224.591913 |
| B | 179.40 | 5,885.00 | 201,384.901034 | 34,433.09 | -132,518.729992 |
| C | 180.30 | 7,060.00 | 201,344.342887 | 34,445.78 | -132,452.781730 |
| D | 217.50 | 139,483.12 | 199,667.940092 | 34,970.51 | -129,726.920845 |
| E | 419.14 | 3,386.00 | 190,581.137616 | 37,814.77 | -114,951.604146 |
| F | 437.09 | 6,738.00 | 189,772.232090 | 38,067.96 | -113,636.306880 |
| G | 440.93 | 6,812.00 | 189,599.184936 | 38,122.13 | -113,354.928569 |
| H | 494.45 | 3,291.34 | 187,187.341936 | 38,877.06 | -109,433.220060 |
| I | 789.90 | 5,723.14 | 173,873.151720 | 43,044.57 | -87,784.019292 |
| J | 826.10 | 10,992.00 | 172,241.840172 | 43,555.19 | -85,131.460894 |
| K | 864.30 | 11,590.00 | 170,520.403443 | 44,094.02 | -82,332.354836 |
| L | 869.30 | 15,973.00 | 170,295.084698 | 44,164.55 | -81,965.979897 |
| M | 976.50 | 7,970.67 | 165,464.262738 | 45,676.67 | -74,110.913120 |
| N | 1,355.80 | 9,524.10 | 148,371.862469 | 51,026.94 | -46,317.989913 |
| O | 1,360.90 | 35,927.22 | 148,142.044389 | 51,098.87 | -45,944.294515 |
| P | 1,463.21 | 11,238.73 | 143,531.737673 | 52,542.02 | -38,447.695941 |
| Q | 2,332.10 | 92,059.97 | 104,382.272484 | 64,798.25 | 25,214.232669 |
| R | 3,017.73 | 74,649.00 | 75,911.693364 | 74,469.49 | 73,027.283557 |
| S | 3,253.00 | 42,915.23 | 92,744.870060 | 77,788.12 | 62,831.365052 |
| Sums | 19,633.77 | 542,585.74 | | 883,094.70 | |



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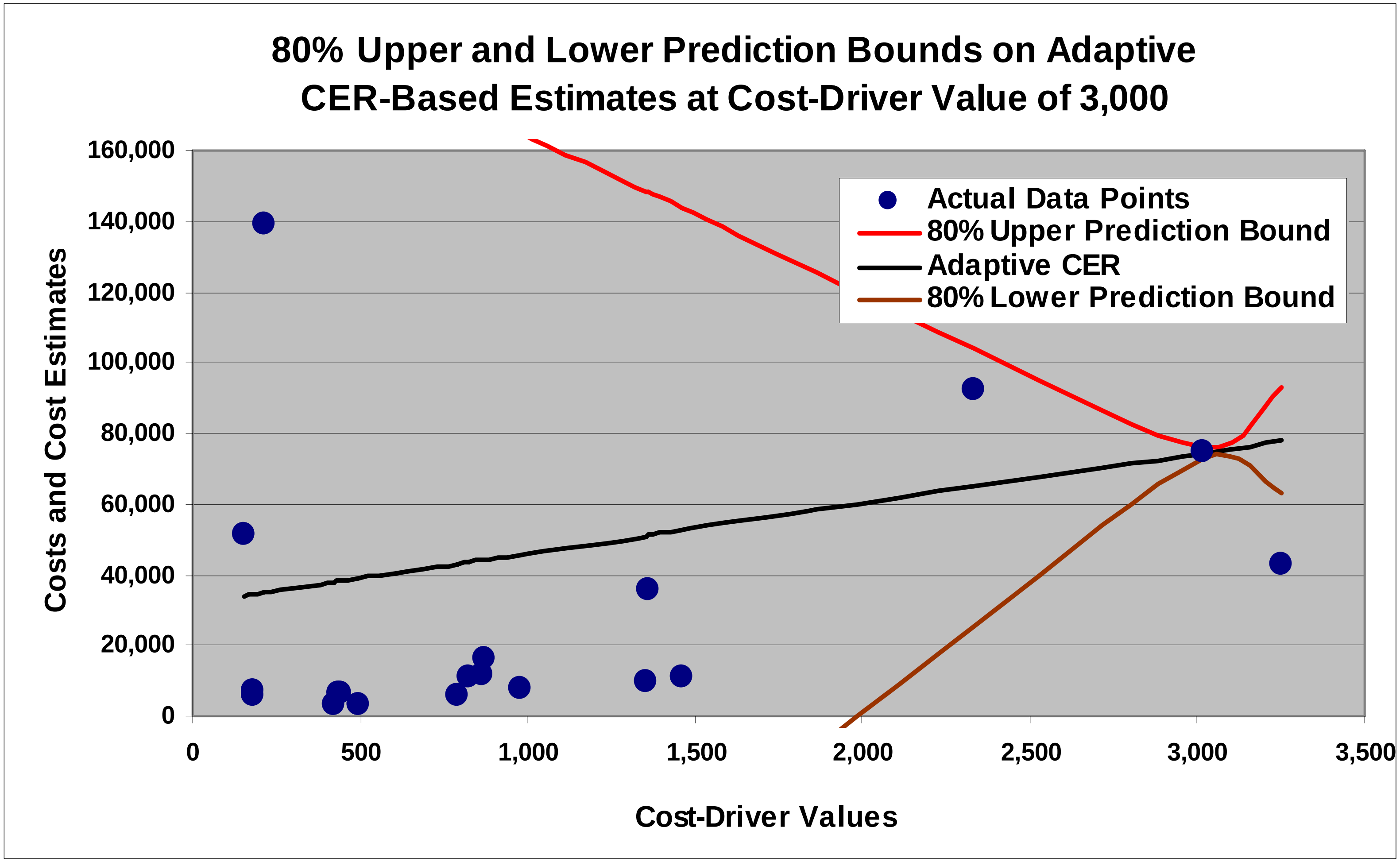
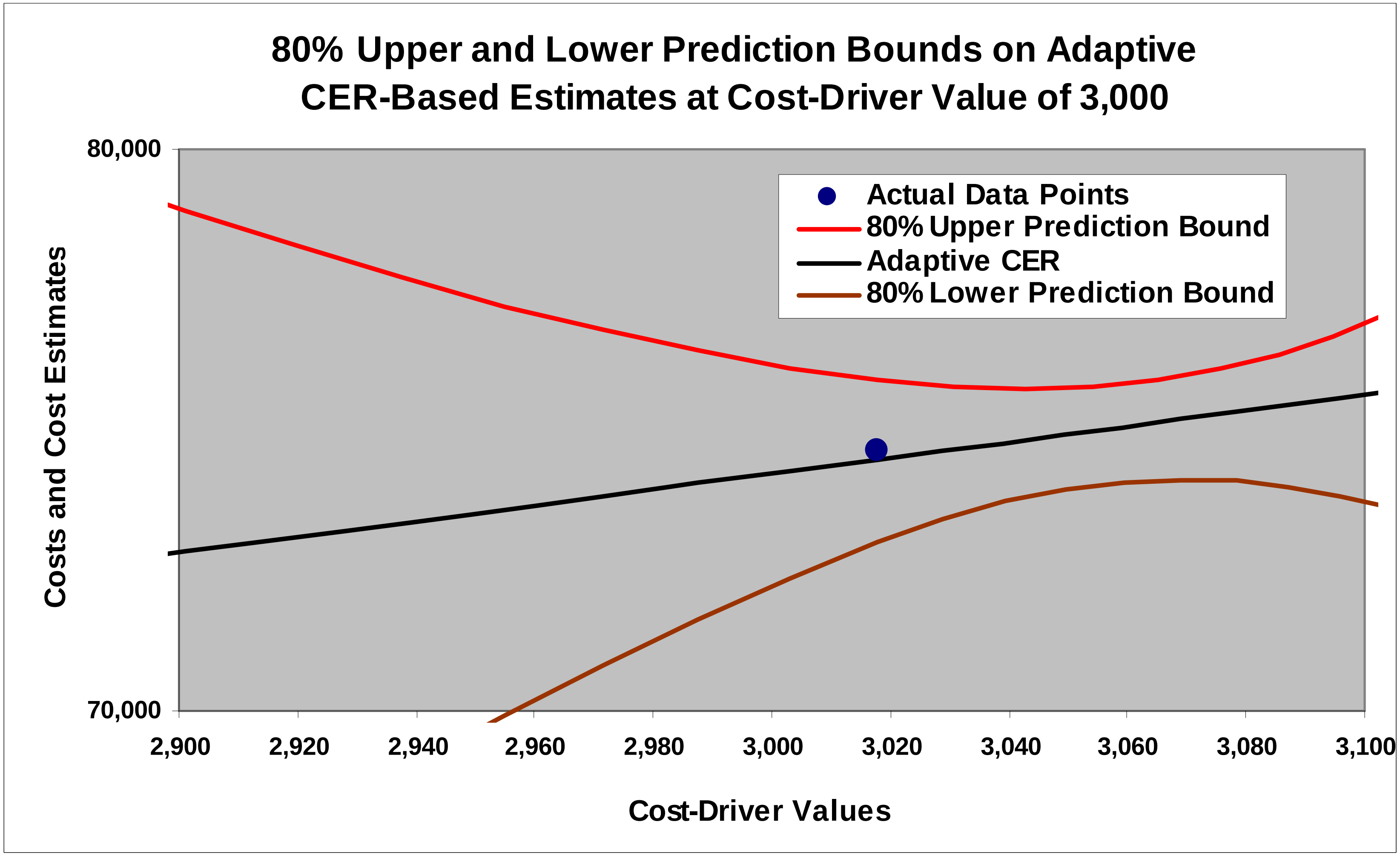
# 80% Prediction Bounds on the Adaptive CER Based at $x$ = 3,000



# Prediction Bounds Near Cost-Driver Value of 3,000

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# 80% Prediction Bounds on the Universal Adaptive CER

| Driver | Cost | 80% Upper Bound | EST Cost | 80% Lower Bound |
|---|---|---|---|---|
| 50.00 | | 62,922.60536 | 42,739.31 | 22,556.01954 |
| 100.00 | | 58,907.24807 | 40,817.29 | 22,727.33210 |
| 150.00 | | 56,054.74733 | 49,546.82 | 43,038.89123 |
| 156.12 | 51,367.22 | 59,905.78998 | 50,880.53 | 41,855.27867 |
| 179.40 | 5,885.00 | 74,603.67051 | 55,953.88 | 37,304.09301 |
| 180.30 | 7,060.00 | 75,171.88754 | 56,150.02 | 37,128.14511 |
| 200.00 | | 87,612.53844 | 60,443.18 | 33,273.82964 |
| 217.50 | 139,483.12 | 97,311.65891 | 69,749.17 | 42,186.69003 |
| 250.00 | | 115,347.90219 | 87,031.73 | 58,715.55405 |
| 300.00 | | 71,377.71021 | 46,425.71 | 21,473.71561 |
| 350.00 | | 38,704.87919 | 22,733.56 | 6,762.24433 |
| 400.00 | | 12,204.28249 | 7,006.95 | 1,809.62688 |
| 419.14 | 3,386.00 | 10,701.37240 | 6,760.42 | 2,819.47622 |
| 437.09 | 6,738.00 | 9,303.25537 | 6,529.22 | 3,755.18780 |
| 440.93 | 6,812.00 | 9,004.15958 | 6,479.76 | 3,955.36231 |
| 450.00 | | 8,300.59270 | 6,362.94 | 4,425.27919 |
| 494.45 | 3,291.34 | 4,923.86478 | 3,589.46 | 2,255.05196 |
| 500.00 | | 4,503.97498 | 3,243.16 | 1,982.35231 |
| 550.00 | | 14,529.66385 | 6,829.12 | -871.42873 |
| 600.00 | | 19,484.26578 | 9,959.40 | 434.52824 |
| 650.00 | | 20,409.64947 | 11,310.17 | 2,210.70010 |
| 700.00 | | 18,067.87906 | 10,929.01 | 3,790.13759 |
| 750.00 | | 12,749.77204 | 8,652.67 | 4,555.56975 |
| 789.90 | 5,723.14 | 9,150.40455 | 7,175.24 | 5,200.06839 |
| 800.00 | | 8,241.00254 | 6,801.25 | 5,361.49607 |
| 826.10 | 10,992.00 | 11,221.66628 | 9,756.59 | 8,291.51518 |
| 850.00 | | 13,951.60979 | 12,462.82 | 10,974.03604 |
| 864.30 | 11,590.00 | 14,422.75320 | 12,666.50 | 10,910.25030 |
| 869.30 | 15,973.00 | 14,587.63569 | 12,737.72 | 10,887.80057 |
| 900.00 | | 15,604.93947 | 13,174.99 | 10,745.03389 |
| 950.00 | | 11,653.20568 | 9,208.15 | 6,763.08930 |
| 976.50 | 7,970.67 | 11,143.81693 | 8,832.68 | 6,521.53760 |
| 1,000.00 | | 10,696.45067 | 8,499.71 | 6,302.97553 |
| 1,050.00 | | 16,599.85083 | 11,462.16 | 6,324.47866 |
| 1,100.00 | | 20,939.42063 | 14,296.49 | 7,653.56932 |
| 1,150.00 | | 23,860.71099 | 16,537.15 | 9,213.58728 |
| 1,200.00 | | 25,595.54200 | 18,230.99 | 10,866.44026 |
| 1,250.00 | | 26,430.13239 | 19,495.31 | 12,560.49388 |
| 1,300.00 | | 26,643.54266 | 20,310.23 | 13,976.92318 |
| 1,350.00 | | 19,965.36192 | 14,974.31 | 9,983.26614 |
| 1,355.80 | 9,524.10 | 20,888.41315 | 15,774.27 | 10,660.11771 |
| 1,360.90 | 35,927.22 | 21,705.81036 | 16,477.67 | 11,249.53159 |
| 1,400.00 | | 27,979.59574 | 21,870.45 | 15,761.29785 |
| 1,450.00 | | 13,664.60151 | 11,840.86 | 10,017.11120 |
| 1,463.21 | 11,238.73 | 14,382.93075 | 12,101.01 | 9,819.08646 |

| Driver | Cost | 80% Upper Bound | EST Cost | 80% Lower Bound |
|---|---|---|---|---|
| 1,500.00 | | 16,394.47396 | 12,825.54 | 9,256.59722 |
| 1,550.00 | | 22,698.80390 | 16,621.72 | 10,544.64424 |
| 1,600.00 | | 28,144.34523 | 20,492.26 | 12,840.17489 |
| 1,650.00 | | 33,397.70393 | 24,526.56 | 15,655.41028 |
| 1,700.00 | | 38,715.42463 | 28,831.03 | 18,946.63797 |
| 1,750.00 | | 44,154.10037 | 33,415.50 | 22,676.89870 |
| 1,800.00 | | 49,694.94147 | 38,247.16 | 26,799.37082 |
| 1,850.00 | | 55,295.14895 | 43,285.50 | 31,275.84370 |
| 1,900.00 | | 60,905.74386 | 48,497.85 | 36,089.94751 |
| 1,950.00 | | 66,472.33508 | 53,862.57 | 41,252.81236 |
| 2,000.00 | | 71,925.95418 | 59,364.10 | 46,802.23932 |
| 2,050.00 | | 77,167.60488 | 64,981.01 | 52,794.40890 |
| 2,100.00 | | 82,049.60587 | 70,666.52 | 59,283.42427 |
| 2,150.00 | | 86,358.35570 | 76,319.27 | 66,280.18315 |
| 2,200.00 | | 89,805.61668 | 81,744.09 | 73,682.56956 |
| 2,250.00 | | 92,036.69169 | 86,609.89 | 81,183.08854 |
| 2,300.00 | | 92,664.98297 | 90,430.47 | 88,195.96100 |
| 2,332.10 | 92,059.97 | 93,449.79687 | 91,836.14 | 90,222.47807 |
| 2,350.00 | | 93,889.12293 | 92,619.98 | 91,350.84125 |
| 2,400.00 | | 97,402.84769 | 92,676.25 | 87,949.65291 |
| 2,450.00 | | 98,222.52317 | 90,463.37 | 82,704.22441 |
| 2,500.00 | | 96,484.73846 | 86,410.39 | 76,336.03984 |
| 2,550.00 | | 92,951.10708 | 81,412.53 | 69,873.94518 |
| 2,600.00 | | 88,646.33546 | 76,466.46 | 64,286.59020 |
| 2,650.00 | | 84,454.68294 | 72,322.92 | 60,191.16611 |
| 2,700.00 | | 80,929.09901 | 69,366.76 | 57,804.41474 |
| 2,750.00 | | 77,054.82704 | 66,431.86 | 55,808.89003 |
| 2,800.00 | | 76,663.27434 | 67,242.40 | 57,821.52197 |
| 2,850.00 | | 75,905.67799 | 67,904.22 | 59,902.76737 |
| 2,900.00 | | 75,867.66447 | 69,545.45 | 63,223.22554 |
| 2,950.00 | | 76,119.31586 | 71,913.26 | 67,707.21079 |
| 3,000.00 | | 75,518.29497 | 74,219.40 | 72,920.49668 |
| 3,017.73 | 74,649.00 | 75,966.58830 | 74,164.83 | 72,363.08019 |
| 3,050.00 | | 76,786.35756 | 74,065.53 | 71,344.69813 |
| 3,100.00 | | 74,002.10190 | 67,141.02 | 60,279.92945 |
| 3,150.00 | | 62,069.86209 | 54,415.99 | 46,762.11593 |
| 3,200.00 | | 49,725.94543 | 45,424.15 | 41,122.36282 |
| 3,250.00 | | 43,144.36743 | 42,927.10 | 42,709.82647 |
| 3,253.00 | 42,915.23 | 43,354.47526 | 42,978.65 | 42,602.82964 |
| 3,300.00 | | 46,653.41208 | 43,786.36 | 40,919.29842 |
| 3,350.00 | | 50,744.79550 | 45,762.39 | 40,779.98882 |
| 3,400.00 | | 54,430.68793 | 47,971.96 | 41,513.24151 |
| 3,450.00 | | 57,664.17277 | 50,126.95 | 42,589.72220 |
| 3,500.00 | | 60,512.13570 | 52,149.51 | 43,786.89143 |

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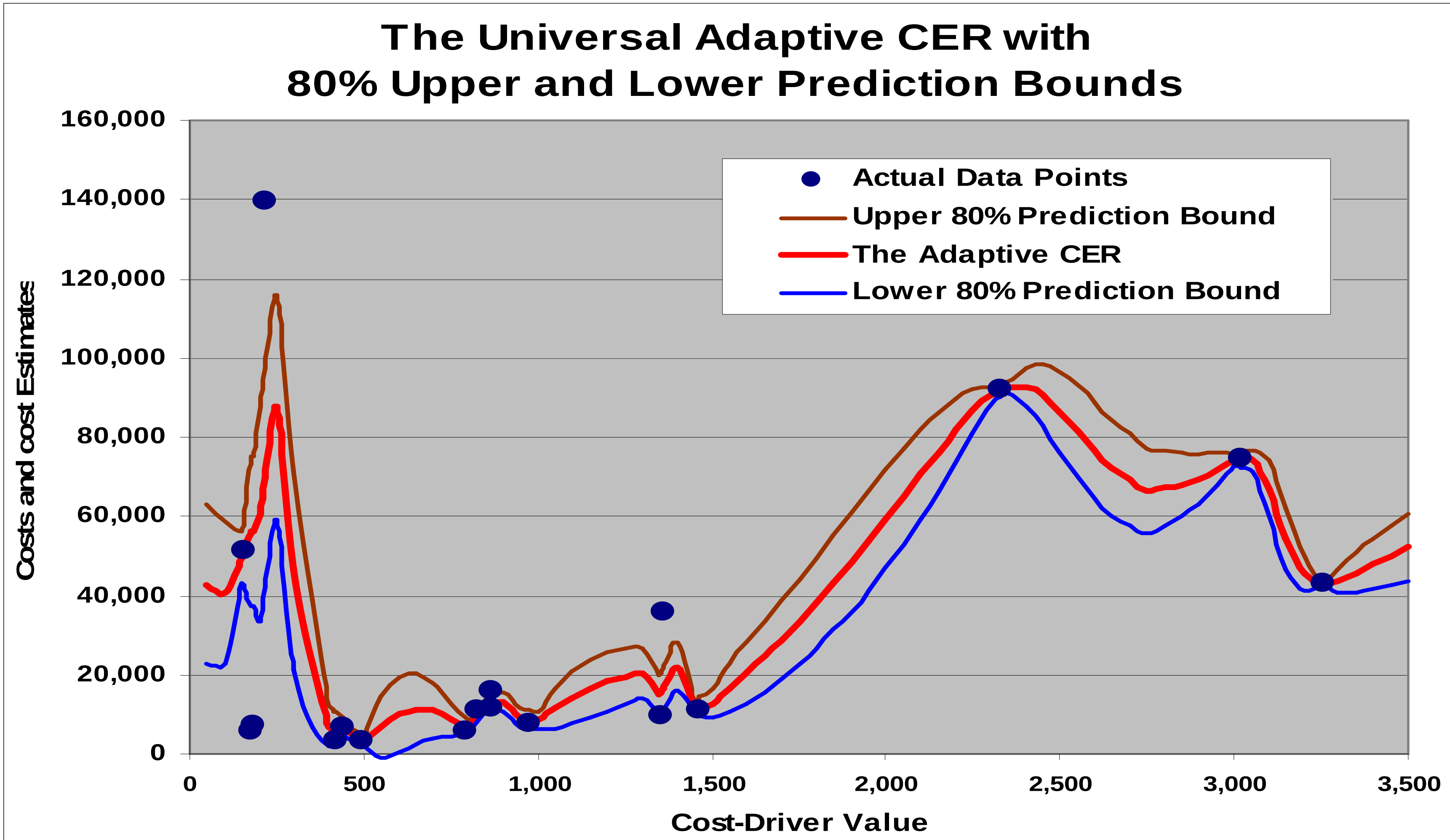
# 80% Prediction Bounds on the Universal Adaptive CER



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# Contents

# Concluding Remarks

- **CERs are the Mainstay of Parametric Cost Estimating – Their Major Drawback is the Uncertainty of Applicability in Any Particular Estimating Situation**
- **Deriving Adaptive CERs Requires More Work Than Deriving Full Data Set CERs, but it Offers the Possibility of More Credible and Precise Estimates**
  - **Adaptive CERs Can be "Tuned" to Specific Cost-Driver Ranges, or**
  - **They Can be Merged to Form a Universal Adaptive CER that is Valid over the Entire Cost-Driver Range**
- **In This Report, We Have Provided the Mathematical Foundations of Calculating and Applying Adaptive CERs**

# References

1. **Book, S.A., "Deriving Cost-Estimating Relationships Using Weighted Least-Squares Regression," IAA/ISPA/AIAA Space System Cost Methodologies and Applications Symposium, San Diego CA, 10-11 May 1990, 19 pages.**
2. **Book, S.A.and Broder, M.A., "Adaptive Cost-Estimating Relationships," Space Systems Cost Analysis Group (SSCAG) Meeting in Noordwijk, Holland, 15-16 May 2008, and ISPA/SCEA Joint Annual Conference & Training Workshop, Industry Hills CA, 24-27 June 2008, 80 Charts.**
3. **Foussier, P., "Space System Study," Space Systems Cost Analysis Group (SSCAG), Montreal QB, 16 May 2007, 41 Charts.**