Sensitivity Analysis in C-BAs

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Agenda

- Introduction
- Cost Benefits Analysis (C-BA)
- Sensitivity Analysis in C-BAs
- Presenting Sensitivity Analysis
- Sensitivity Analysis and the Tradespace
- Generalizations and Conclusion
- Questions

Introduction

Sensitivity analysis is the study of how uncertainty in the outputs is related to uncertainty in the inputs.

This is typically accomplished by finding (mathematically) how changes in the inputs change the results and then summarizing those findings.

Introduction

- Sensitivity Analysis is usually the most important and valuable part of the estimate
- Difficult to communicate the results
- Typically not effective as a result of too few alternatives in the C-BA
- Decision matrix C-BAs allow simple presentations of sensitivity analysis
- Can replace unnecessary analysis by shifting the focus to truly relevant topics

Cost Benefits Analysis Army

Broad variety of types of analysis including:

- Material Solutions
- Force Design Updates
- Support JCIDS documents
- Economic Analysis
- Portfolio Analysis
- Army C-BAs consider all the types of analysis discussed in CEBoK Module 13

(sources: U.S. Army Cost Benefit Analysis Guide at www.asafm.army.mil, CEBoK Module 13: Economic Analysis)

Cost Benefits Analysis General Idea

- Similar to Analysis of Alternatives
- Develops Alternatives
- Develop Cost Estimates (modified LCCE)
- Evaluate Benefits
- Compare Costs vs. Benefits
- Recommend Alternative
- Sensitivity Analysis / Risk Analysis

Cost Benefits Analysis
Comparing Costs and Benefits

A Popular Method is weighted benefits matrix (decision matrix):

- Written scoring criteria for how each benefits is scored
- Linear weighting of benefits
- Sum of those linearly weighted benefits
- Compare Costs vs. Benefits
- Recommend Alternative
  - May use a Cost-Benefits Index (CBI)

Cost Benefits Analysis
Comparing Costs and Benefits

The weighted benefits decision matrix:

- Is very natural to those with a knowledge of Linear Programming
- A variety of CBIs can be used
- Benefits can come from a standardized capabilities set, such as Joint Capabilities Areas

As a simple example consider 3 benefits: Range, Payload and Weight and 4 alternatives

Cost Benefits Analysis
Benefits Summary Example

	Wt	Alt 1 Raw	Alt 1 Wtd Score	Alt 2 Raw	Alt 2 Wtd score	Alt 3 Raw	Alt 3 Wtd Score	Alt 4 Raw	Alt 4 Wtd Score
Range	.5	1	**.5**	5	**2.5**	5	**2.5**	5	**2.5**
Payload	.25	3	**.75**	4	**1**	4	**1**	5	**1.25**
Weight	.25	2	**.5**	5	**1.25**	3	**.75**	5	**1.25**
Total			**1.75**		**4.75**		**4.25**		**5**

Higher is better

Cost Benefits Analysis Evaluation Example

	Alt 1	Alt 2	Alt 3	Alt 4
Cost	**300**	370	340	400
Benefits	1.75	4.75	4.25	**5**
CBI	171.4	**77.9**	80	80
Baselined CBI* (Alt 1 baseline)	NA	23.3	**16**	30.8

Higher benefits is better, lower CBI is better

* (cost increase over baseline)/(benefits increase over baseline)

Cost Benefits Analysis
Matrix Form

$$S = \begin{pmatrix} 1 & 5 & 5 & 5 \\ 3 & 4 & 4 & 5 \\ 2 & 5 & 3 & 5 \end{pmatrix} \qquad w = (\ .5 \quad .25 \quad .25)$$

b = w*S = (1.75 4.75 4.25 5)

c = (300, 370, 340, 400)

S (scores), w (weights), b (benefits), c (cost)

Sensitivity Analysis in C-BAs

C-BA sensitivity analysis differs from sensitivity analysis in other cost estimates because:

- The C-BA typically recommends an alternative rather than a project budget
- The C-BA deals with differences between alternatives
- As a result, the C-BA sensitivity analysis considers factors that change the recommendation instead of factors which change the cost estimate

Sensitivity Analysis in C-BAs

Consider sensitivity to:

- Weights
- Scores
- Cost Estimates
- Variations in user or Community of Practice surveys for weights and scores
- Combined effects (CBI: c/b = c/w*S is a "product" of three potential sensitivity inputs)
- Alternate indices

Sensitivity Analysis in C-BAs

Further Considerations:

- Is it possible to enumerate the “opinions”?
- Assumptions
- Are you recommending a solution or a cost?
- Correlation between the benefits (linear dependencies)
  - Principal Component Analysis
  - Discuss the significance of correlation
  - Seek additional alternatives

Sensitivity Analysis in C-BAs

Decision matrix C-BA format tends to:

- Facilitate communication of the analysis
- Focus sensitivity analysis on the weights and selection of benefits
  - Peer review of cost estimates
  - Well defined scoring criteria
  - Benefits are usually derived from Key Performance Parameters or Key System Attributes
  - Direct comparison of alternatives is more defensible (logical cost and benefits comparison between alternatives)

Sensitivity Analysis in C-BAs

Considerations when using Standard CBI (c/w*S):

- Analysts tend to propose alternatives that are cost-benefit efficient points
- A weight can be selected that makes any efficient point come out first
- As a result, the sensitivity analysis tends to focus on the weights
- Baselined CBI = (cost increase over baseline)/(benefits increase over baseline) is better

Presenting Sensitivity Analysis

Techniques:

- Describe conditions on weights that change the preferred alternative
- Mathematical measures for the simple benefits (w*S):

$(90/\theta) * w/\|w\| + ((\theta - 90)/\theta) * (S_i - S_b)/\|S_i - S_b\|$ is the "closest" weight to w so that the recommended alternative "b" and alternative "i" are tied.

$w \cdot (S_i - S_b)/\|w\| \|S_i - S_b\|$ is the cos(angle) between the w and that weighting

- Description of benefit trades that have significant cost impact (best)

Sensitivity Analysis Metrics
Angle Method

In our example, using a simple benefits criteria, Alt 4 was best:

	S	Si-Sb	w	w*(Si-Sb)	‖w‖	‖Si-Sb‖	cos(θ)
Alt 1	(1, 3, 2)	(-4, -2, -3)	(.5,.25,.25)	-3	0.612	5.385	-0.91
Alt 2	(5, 4, 5)	(0, -1, 0)	(.5,.25,.25)	-.25	0.612	1	-0.41
Alt 3	(5, 4, 3)	(0, -1 ,-2)	(.5,.25,.25)	-0.75	0.612	2.24	-0.55
Alt 4	(5, 5, 5)	(0, 0, 0)	(.5,.25,.25)	0	0.612	0	NA

Highest is best. -0.41 is Alternative 3. cos(**114**)=-0.41

Sensitivity Analysis Metrics
Angle Method

In the simple benefits example:

- Highest is best: -0.41. cos(114)= -0.41.

- The nearest weight that changes the best to alternative 2 is:

$$(90/\theta) * w/\|w\| + ((\theta - 90)/\theta) * (S_i - S_b)/\|S_i - S_b\|$$

$$(90/114) * (.5, .25, .25) /0.612 + (24/114)(0, -1, 0)/1$$

or (0.644, 0.112, 0.322)

- The original weights were (0.5, 0.25, 0.25). Since this significantly reduced the importance of benefit 2 (payload), we should discuss the validity of that change

Sensitivity Analysis Metrics Angle Method

Angle method for CBI (wS/c):

- For a simple cost benefits index, the equation for cos(angle) is

$$w \cdot (S_i/c_i - S_b/c_b) / \| w \| \, \| S_i/c_i - S_b/c_b \|$$

- $(90/\theta)*w/\|w\| + ((\theta-90)/\theta)*(Si/ci-Sb/cb)/\|Si/ci-Sb/cb\|$ is the closest weight that results in a tie score

Presenting Sensitivity Analysis

Techniques for communicating the results:

- Discuss the effects of varying one of the variables at a time
- Conditions that change the recommendation and the validity of those conditions
- Discussion of the tradespace about the selected alternative
  - Indicates and suggests the robustness of the recommendation
  - Allows for informed cost for performance trades

Sensitivity Analysis and the Tradespace

Description of the tradespace provides extremely high value for low effort:

- In practice, by the time you understand the C-BA problem, it is relatively easy to add alternatives that discriminate (single out benefits)
- If the original alternatives did not provide adequate sensitivity, refine the alternatives about the solution
- Provides cost per capability trade information

Sensitivity Analysis and the Tradespace

To refine the alternatives about the solution:

- Consider changes in
  - Quantities
  - Schedule
  - Risk management and Acquisition Strategies (see Correlation Matrices Revisited)
  - Performance
- Smaller changes than in the main analysis
- The more sensitive the main analysis was, the better the information to the tradespace

Sensitivity Analysis and the Tradespace

Some estimates of the tradespace surface:

- The plane orthogonal to the weight vector itself
- The vector orthogonal to the plane through the next best alternatives (requires the same number of alternatives as benefits)
- Use multivariate regression when there are more alternatives (consider weighted regression)

Sensitivity Analysis and the Tradespace

Caution:

- Know the purpose of the study
- Don't show too much tradespace analysis detail in a C-BA that didn't ask for it
  - -You will be asked to redo the main analysis using the new alternatives
  - - The results are misused and misapplied
- Use broad statements such as, "The variables that had the greatest impact on cost were the combined effects of weight reduction and schedule."

Sensitivity Analysis and the Tradespace Example

$$S = \begin{pmatrix} 1 & 5 & 5 & 5 \\ 3 & 4 & 4 & 5 \\ 2 & 5 & 3 & 5 \end{pmatrix}$$

If alternative 4 is recommended, a refinement of the alternatives could be obtained by reusing alternatives 2 and 3 and developing a new alternative to replace the base case:

$$R = \begin{pmatrix} 4 & 5 & 5 \\ 5 & 4 & 4 \\ 4 & 5 & 3 \end{pmatrix} \quad c = (\,350,\ 370,\ 340\,)$$

Sensitivity Analysis and the Tradespace Example

$$\text{inv}(R) = \begin{pmatrix} -.444 & .556 & 0 \\ .056 & -.444 & .5 \\ .5 & 0 & -0.5 \end{pmatrix}$$

$c = (350, 370, 340)$

$$\text{inv}(R) * c^{t} = \begin{pmatrix} 50 \\ 25 \\ 5 \end{pmatrix}$$

Sensitivity Analysis and the Tradespace Example

$$\text{inv}(M)*c^{t} = \begin{pmatrix} 50 \\ 25 \\ 5 \end{pmatrix}$$

- This indicates the "cost" of benefits 1, 2 and 3 near the preferred solution
- When combined with bounds and collinearity, this describes part of the tradespace
- Best if the recommendation is in the convex combination of the alternative scores
- When there are more alternatives, just use ordinary linear regression (alternatives are observations, benefits are variables)

Sensitivity Analysis and the Tradespace Example

$$\text{inv}(M)*c^{t} = \begin{pmatrix} 50 \\ 25 \\ 5 \end{pmatrix}$$

The sensitivity analysis statement that goes with this data is:

The results are insensitive to additional alternatives that trade benefits in the ratio of 50:25:5.

Also state the bounds within which these trades can be made.

Generalizations

- The examples with three weights generalize to more weights (dimensions)
- Power rule weighting $S_1^{W_1} S_2^{W_2} S_3^{W_3} \ldots$(use log)
- More alternatives is better and ordinary least squares is used for cost per benefits when good alternatives outnumber benefits
- The methodology is very useful for risk analysis and cost, performance and schedule tradespace analysis

Follow-up Topics

- Normalizing the benefit selection and weights
- Interdependence of the benefits (correlation)
- Cost as an independent variable (CAIV) tradespace analysis
- Including risk and schedule in the analysis

Conclusions

- Decision matrices in C-BAs facilitate sensitivity analysis
- Sensitivity analysis can easily include tradespace information
- Sensitivity analysis is customized to the purpose of the analysis
  - Are you recommending a solution or a cost?
- Use refined alternatives in order to facilitate good sensitivity analysis

Questions