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Avoiding Pitfalls When Applying Learning to Your Estimate

Selected Topics: Sums of Learning Curves, Fixed Costs, and the End of Learning

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Agenda

- Introduction
- Prerequisites
- Selected Topics:
  - Sums of Learning Curves
  - Learning Curves with Fixed Cost
  - End of Learning
- Conclusion

Introduction

- This presentation will attempt to expand on the fundamental concept of the Learning Curve Equation to consider a few interesting scenarios

Prerequisites

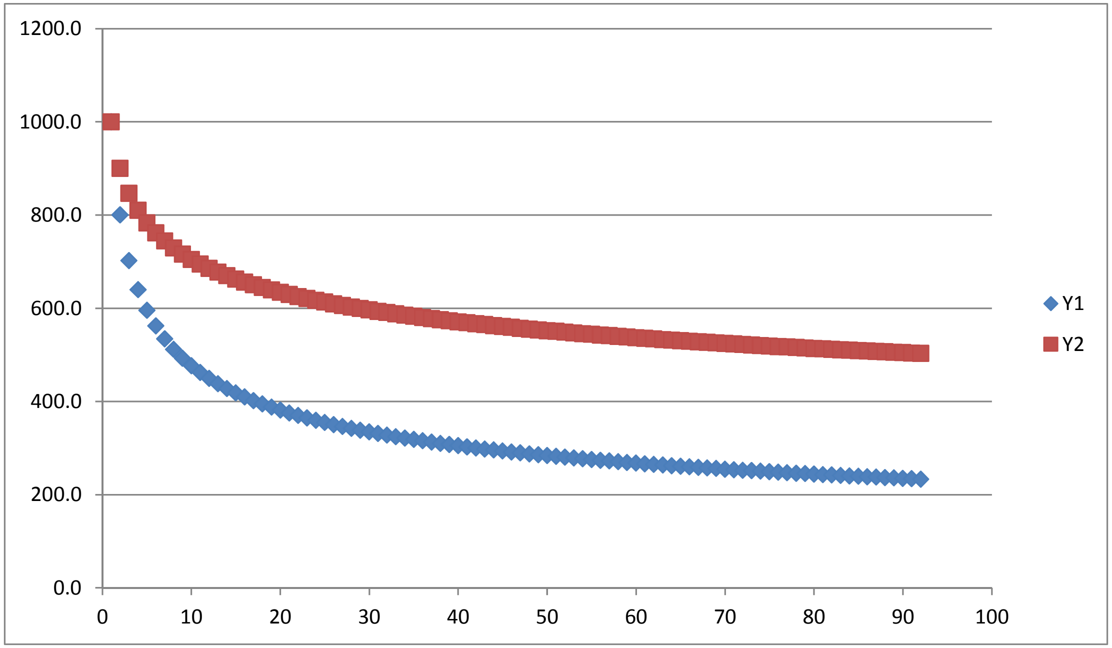
- Throughout this paper, Unit Theory will be used
- The Learning Curve Equation
  - $Y(x) = Ax^b$
  - Y: Cost
  - A: First Unit Cost (also called T1)
  - X: Unit Number
  - B: exponent such that 2^b is the Learning Curve Slope(LCS)
- When Quantity doubles, Cost decreases by a fixed percentage (LCS)
  - $\frac{Y(2x)}{Y(x)} = \frac{A(2x)^b}{A(x)^b} = \frac{(2x)^b}{(x)^b} = \frac{2^b x^b}{x^b} = 2^b$

Sums of Learning Curves

- Hypothetical example
  - Lets say you want to model learning that occurs for the cost of Widgets
  - Each Widget consists of several components, each with its own learning rate
  - Some of these components may exhibit no learning

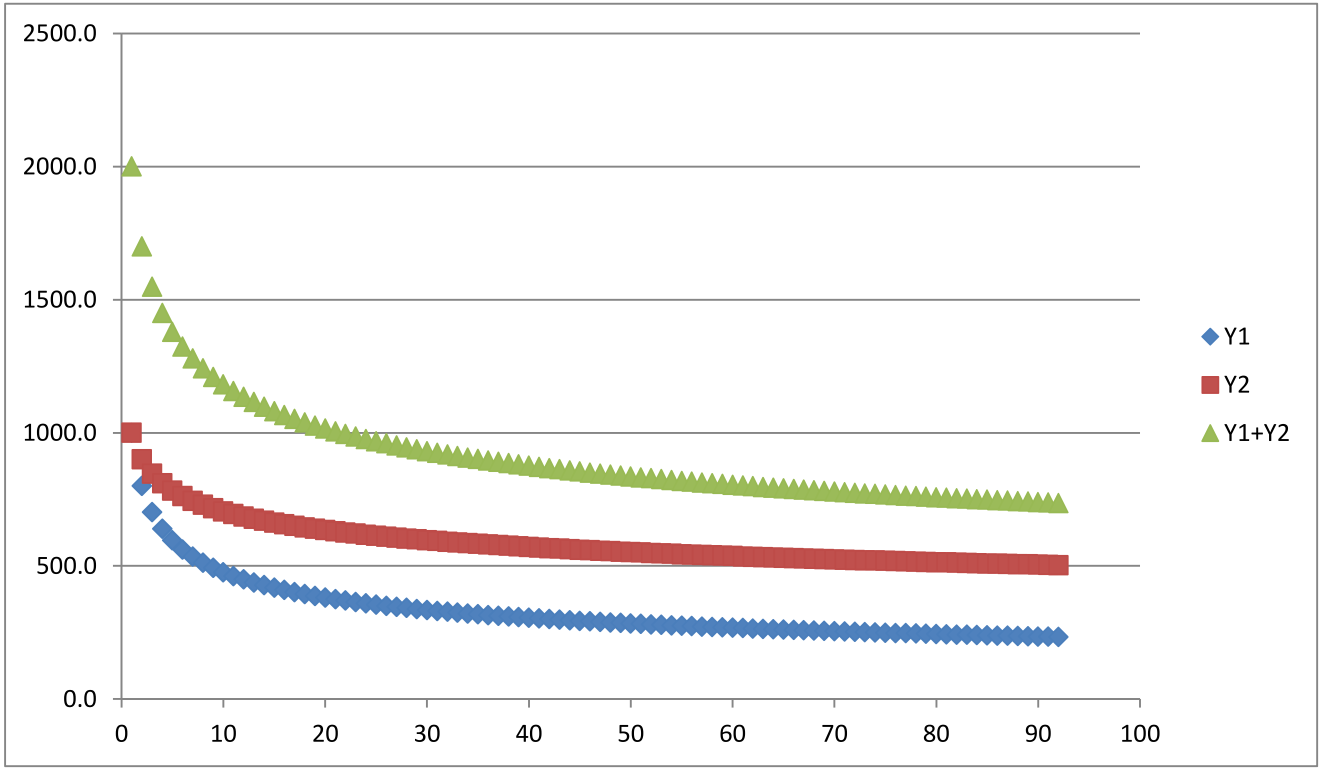
Sums of Learning Curves

- Let's start with a simple example:
  - Two Learning Curves with the same T1 cost but different Learning Curve Slopes
  - $Y1 = Ax^{b1}$
  - $Y2 = Ax^{b2}$



Sums of Learning Curves

- What is the nature of Z=Y1+Y2?
  - Looks like a learning curve, right?



Sums of Learning Curves

- What happens when we try to derive the slope of Z?

$$\frac{Z(2x)}{Z(x)} = \frac{A(2x)^{b1} + A(2x)^{b2}}{A(x)^{b1} + A(x)^{b2}}$$

$$= \frac{(2x)^{b1} + (2x)^{b2}}{x^{b1} + x^{b2}}$$

$$= 2^{b1}\frac{x^{b1}}{x^{b1} + x^{b2}} + 2^{b2}\frac{x^{b2}}{x^{b1} + x^{b2}}$$

- This quantity, which represents the "slope" of Z, actually depends on x
- Can be thought of as a weighted average of the slopes of Y1 and Y2

Sums of Learning Curves

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- For x=1, "slope" of Z is:

$$\frac{(2)^{b1}+(2)^{b2}}{(1)^{b1}+(1)^{b2}}=\frac{(2)^{b1}+(2)^{b2}}{2}$$

- This is the average of the slopes of Y1 and Y2
- What Happens as x increases without bound?
- Remember, $-1 \le b2 \le b1 \le 0$

$$\lim_{x\to\infty} 2^{b1}\frac{x^{b1}}{x^{b1}+x^{b2}} + 2^{b2}\frac{x^{b2}}{x^{b1}+x^{b2}} = 2^{b1}$$

- So the slope of Z tends towards the greater of the slopes of Y1 and Y2

Sums of Learning Curves

- Let's Generalize to any number of curves
- Also, Curves with different First Unit Costs

$$Z = \sum_i A_i x^{b_i}$$

- What happens when we try to derive the slope of Z?

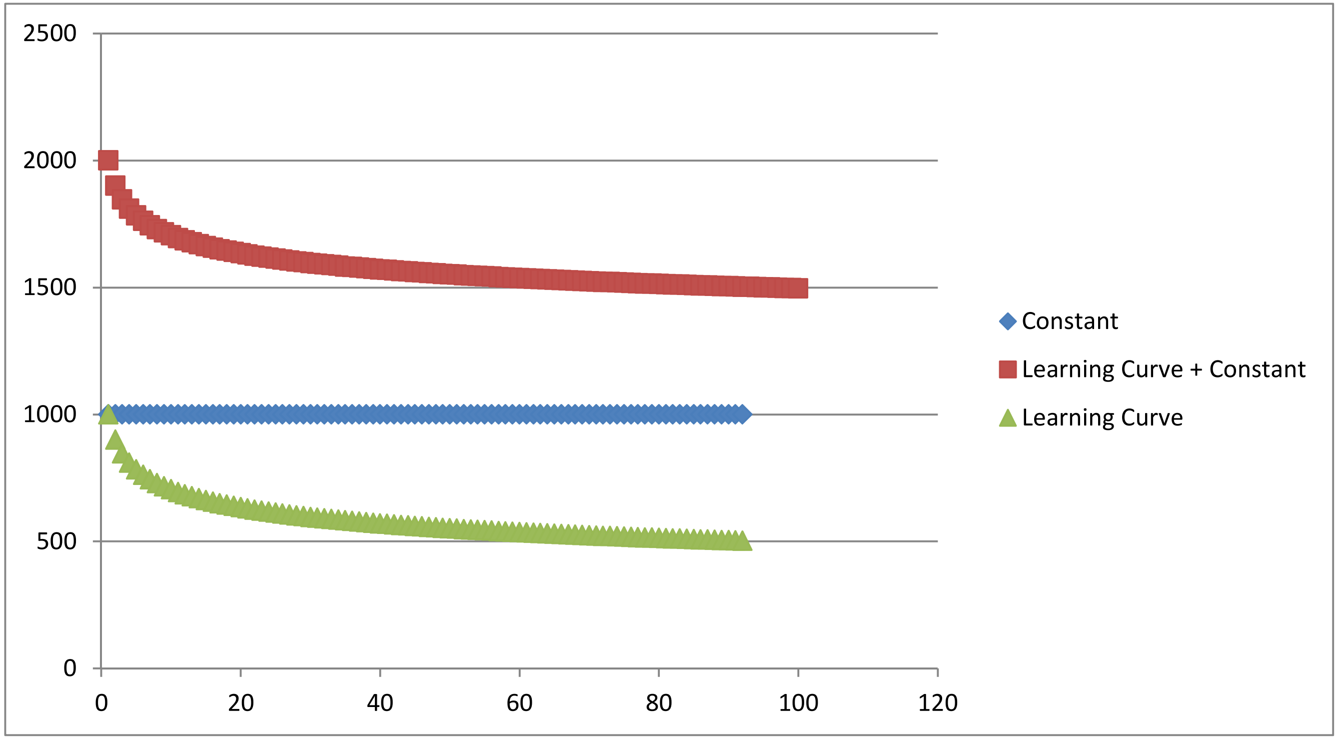
$$\frac{Z(2x)}{Z(x)} = \frac{\sum_i A_i (2x)^{b_i}}{\sum_j A_j x^{b_j}}$$

$$\sum_i 2^{b_i} \frac{A_i x^{b_i}}{\sum_i A_j x^{b_j}}$$

- Once again, this can be thought of as a weighted average of the slopes of each component curve of Z

Learning Curve with Fixed Cost

- Suppose there are fixed (i.e. non-learnable) costs included in the cost of an item
- Instead of $Y = Ax^b$, you have $Y = C + Ax^b$



Learning Curve with Fixed Cost

- $Y = C + Ax^b$
  - This can be thought of as a specific case of the sum of 2 learning curves, where one curve has slope of 100%
- Using what we know about Sums of Learning Curves, we know that
  - The "slope" of Y is non-constant
  - The "slope" at x=1 is $\frac{100+2^b}{2}$

The End of Learning

- When does learning end?
- Mathematically, we know that

$$\lim_{x \to \infty} Ax^b = 0 \; when \; b < 0$$

- But costs will never really reach zero
  - Nothing is free!
- So should we be worried?
  - Not Really

The End of Learning

- Lets Examine what happens as we move out to the right
  - From Units x to x+1, cost decrease is $\frac{A(x+1)^b}{Ax^b} = (\frac{x+1}{x})^b$
  - Say the Learning Curve Slope is 80% and b=-0.322
  - From 100 to101, the change in cost is $(\frac{101}{100})^{-0.322} = 0.996802$
  - From 1000 to1001, the change in cost is $(\frac{1001}{1000})^{-0.322} = 0.999678$
  - From 10000 to10001, the change in cost is 0.999968
  - From 100000 to100001, the change in cost is 0.999997
- Very quickly, the change from unit to unit gets very small

Conclusion

- When modeling learning, we should consider whether these issues
  - Are there multiple components with different learning rates included in the cost?
  - Are there fixed costs included?
  - How far out on the curve am I going?
- Possible Extensions
  - Real World Examples of sums and fixed costs learning problems
  - Ways to model non-constant learning rates