An Application of Data Mining Algorithms For Shipbuilding Cost Estimation

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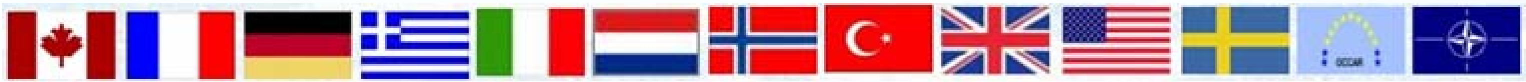
Background



NATO Research & Technology Organization (RTO) Systems Analysis and Studies (SAS) 076 Task Group:

- NATO Independent Cost Estimating and its Role in Capability Portfolio Analysis
- NATO SAS 076 Goal: Demonstrate practicality of NATO cost estimation guidelines
- Various systems (new and existing) analyzed.
    - Including:

The Acquisition Cost of the

Netherlands' Rotterdam class Landing Platform Dock Ships

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Background (cont.)

- The Netherlands’ Landing Platform Dock (LPD) ships:

Rotterdam L800
Commission in 1997

Johan de Witt L801
Commission in 2007

- Blind, ex post analysis:
  - The Netherlands withheld actual costs until after cost estimation exercise was completed.

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Outline

- Background
- Comprehensive data gathered for similar ships
- Two Data Mining methods applied:
  - M5 Model Tree (Parametric Approach)
  - Hierarchical Clustering (Costing by Analogy)
- Comparison: Actual vs. Estimated
- Conclusions

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Data

Ships “similar” to The Netherlands’ LPDs:

- Database of 57 ships in 16 classes from 6 nations
- 136 descriptive, technical, and cost attributes per ship

	Category	Number of Attributes
I	DESCRIPTION	6
II	CONSTRUCTION	8
III	DIMENSIONS	5
IV	PERFORMANCE	8
V	PROPULSION	9
VI	ELECTRICAL POWER GENERATION	3
VII	LIFT CAPACITY	35
VIII	FLIGHT DECK	19
IX	ARMAMENT	13
X	COUNTERMEASURES	5
XI	RADARS / TACAN / IFF / SONARS	13
XII	COMBAT DATA SYSTEMS	1
XIII	WEAPONS CONTROL SYSTEMS	1
XIV	OTHER CAPABILITIES	7
XV	COST DATA	3

Data (ships)

- Database of military or civilian auxiliary vessels of similar size / function to Rotterdam class ships:

Name	Number	Type	Rank	Commissioned	Country
Thomaston	LSD 28	LSD	1	1954	United States
Plymouth Rock	LSD 29	LSD	2	1954	United States
Fort Snelling	LSD 30	LSD	3	1955	United States
Point Defiance	LSD 31	LSD	4	1955	United States
Spiegel Grove	LSD 32	LSD	5	1956	United States
Alamo	LSD 33	LSD	6	1956	United States
Hermitage	LSD 34	LSD	7	1956	United States
Monticello	LSD 35	LSD	8	1957	United States
Anchorage	LSD 36	LSD	1	1969	United States
Portland	LSD 37	LSD	2	1970	United States
Pensacola	LSD 38	LSD	3	1971	United States
Mount Vernon	LSD 39	LSD	4	1972	United States
Fort Fisher	LSD 40	LSD	5	1972	United States
Whidbey Island	LSD 41	LSD	1	1985	United States
Germantown	LSD 42	LSD	2	1986	United States
Fort McHenry	LSD 43	LSD	3	1987	United States
Gunston Hall	LSD 44	LSD	4	1989	United States
Comstock	LSD 45	LSD	5	1990	United States
Tortuga	LSD 46	LSD	6	1990	United States
Rushmore	LSD 47	LSD	7	1991	United States
Ashland	LSD 48	LSD	8	1992	United States
Harpers Ferry	LSD 49	LSD	1	1995	United States
Carter Hall	LSD 50	LSD	2	1995	United States
Oak Hill	LSD 51	LSD	3	1996	United States
Pearl Harbour	LSD 52	LSD	4	1998	United States
Raleigh	LPD 1	LPD	1	1962	United States
Vancouver	LPD 2	LPD	2	1963	United States
La Salle	LPD 3	LPD	3	1964	United States
Austin	LPD 4	LPD	1	1965	United States
Ogden	LPD 5	LPD	2	1965	United States
Duluth	LPD 6	LPD	3	1965	United States
Cleveland	LPD 7	LPD	4	1967	United States
Dubuque	LPD 8	LPD	5	1967	United States
Denver	LPD 9	LPD	6	1968	United States
Juneau	LPD 10	LPD	7	1969	United States
Coronado	LPD 11	LPD	8	1970	United States
Shreveport	LPD 12	LPD	9	1970	United States
Nashville	LPD 13	LPD	10	1970	United States
Trenton	LPD 14	LPD	11	1971	United States
Ponce	LPD 15	LPD	12	1971	United States
Svalbard	W303	Icebreaker	1	2001	Norway
Carlskrona	M04	LPD	1	1982	Sweden
Atle	—	Icebreaker	1	1985	Sweden
Oden	—	Icebreaker	1	1989	Sweden
Protecteur	AOR 509	AOR	1	1969	Canada
Preserver	AOR 510	AOR	2	1970	Canada
Albion	L14	LPD	1	2003	United Kingdom
Bulwark	L15	LPD	2	2005	United Kingdom
Largs Bay	L3006	LSD	1	2006	United Kingdom
Lyme Bay	L3007	LSD	2	2007	United Kingdom
Mounts Bay	L3008	LSD	3	2006	United Kingdom
Cardigan Bay	L3009	LSD	4	2006	United Kingdom
Ocean	L12	LPH	1	1998	United Kingdom
Siroco	L9012	LSD	2	1998	France
Mistral	L9013	AAS	1	2006	France
Tonnerre	L9014	AAS	2	2007	France
Dixmude (BPC3)	L9015	AAS	3	2010	France

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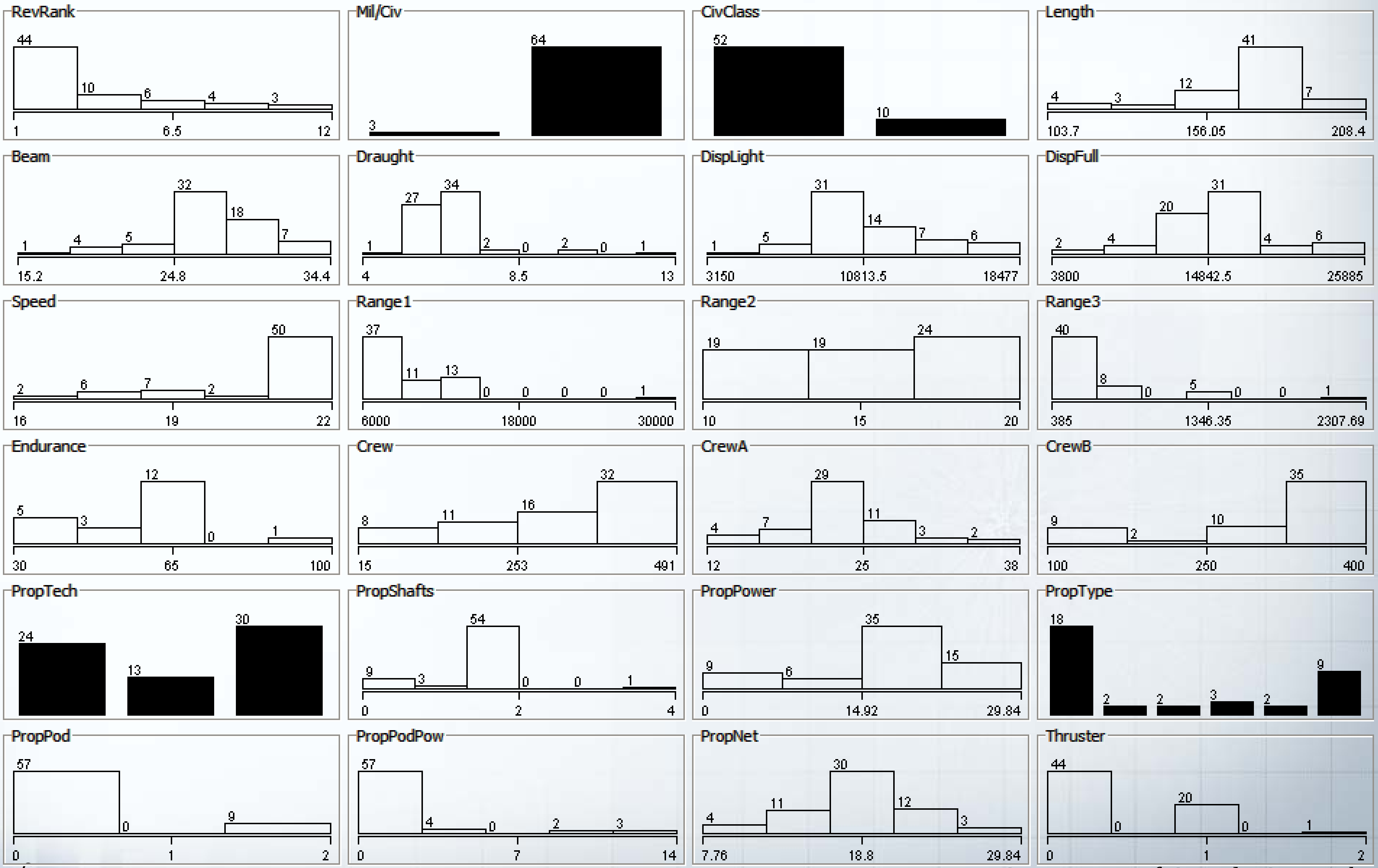
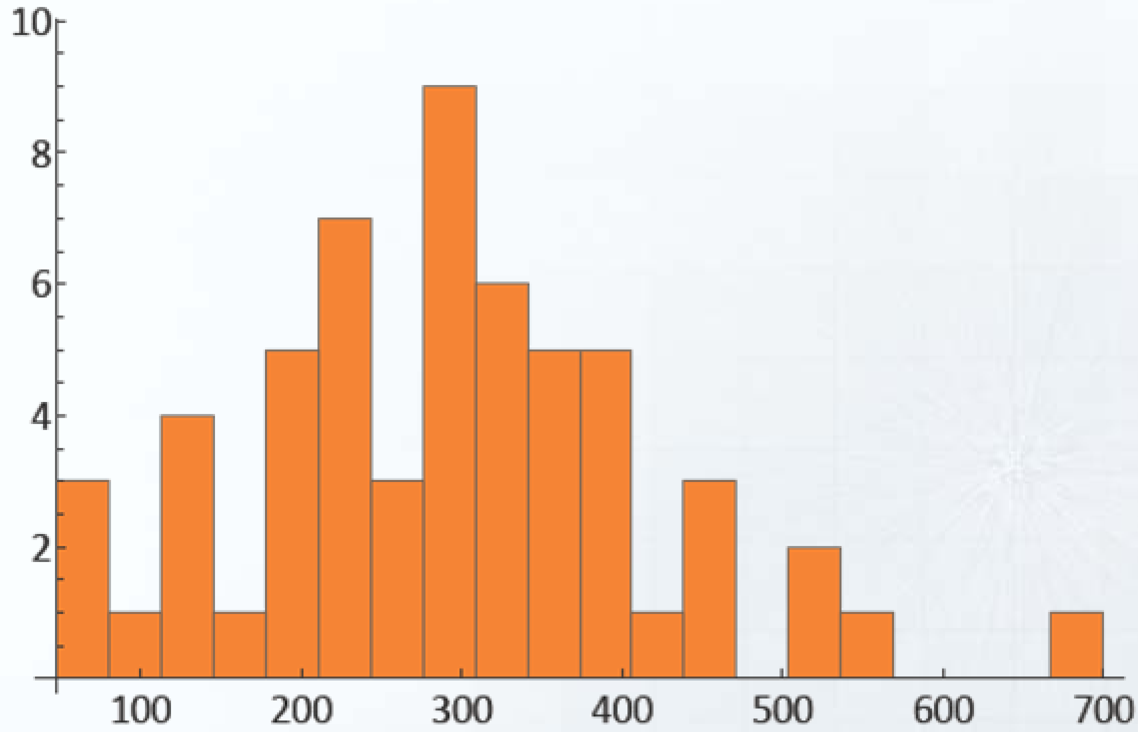
Data (sample of technical info)



Data (cost info)

- Ship costs normalized: fictitious notional common currency (NCC)
- Histogram of known costs for ships in database:



- Costs were log-transformed prior to analysis

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Cost Estimation Methods

I. Parametric Approach:

M5 Model Tree Algorithm

II. Costing by Analogy:

Hierarchical Clustering

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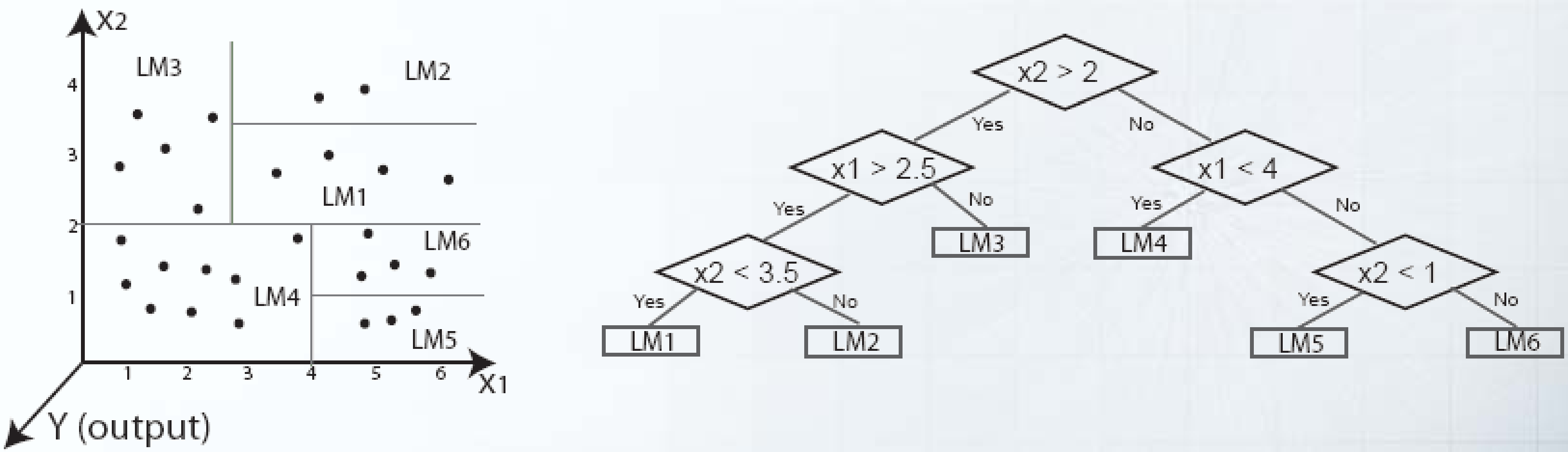
I. M5 Model Tree Approach

- Quinlan (1992) pioneered the M5 Model Tree Algorithm for numeric prediction
  - Combines decision trees and linear regression
  - Each tree node is a multivariate linear regression model
  - Only attributes used in decisions are used in regression



  - Small, easy to understand. Exploit local linearity.
  - Can excel with limited data. Handle numeric, notional, or missing data.

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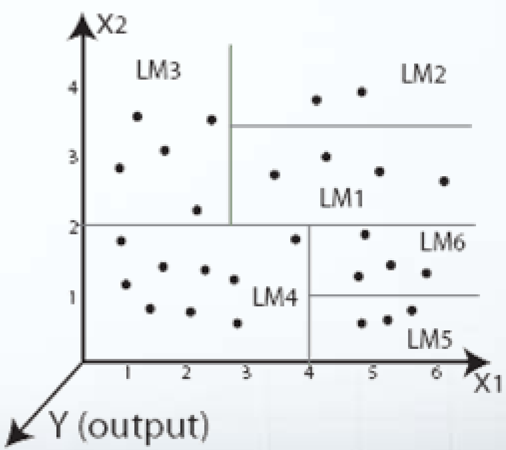
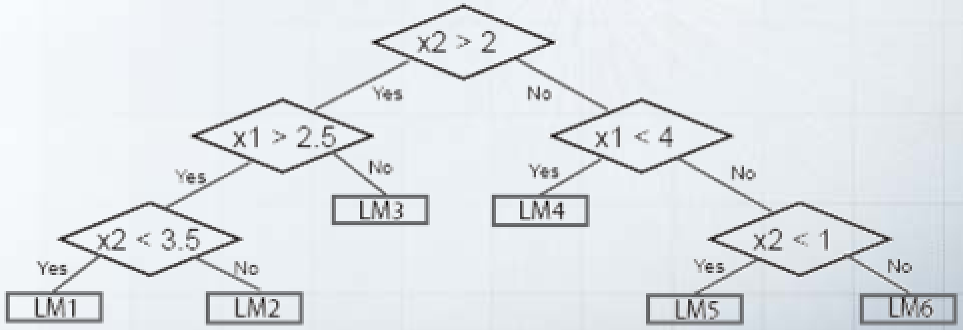
I. M5 Model Tree Approach (cont.)

- M5 Model Tree Algorithm

  Input: data set of ships (technical and cost data)

  1. Tree constructed recursively: choose attribute that best splits the data set in two (minimize estimation error)
  2. Construct multivariate linear regression models at each node
  3. Tree pruning (eliminate sub-trees if parent node estimates better)
  4. Smoothing process: make adjacent linear regression models smooth and continuous





I. M5 Model Tree Approach (cont.)

- Output:
  - net result is a tree type structure in which each leaf of the tree is a different regression model
  - Simple piece-wise linear (smoothed) models

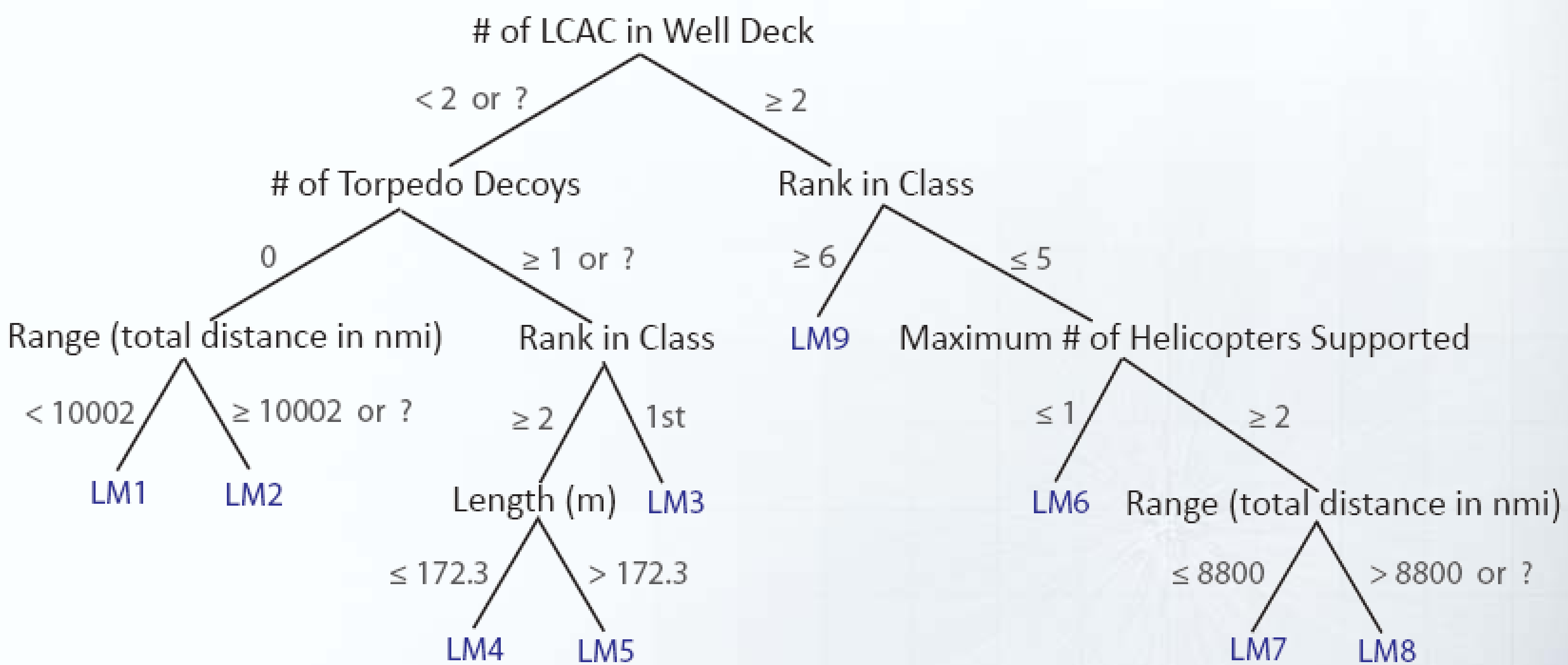
- Free, easy-to-use M5 Model Tree implementation:

WEKA: Waikato Environment for Knowledge Analysis

http://www.cs.waikato.ac.nz/~ml/weka/index.html

I. M5 Model Tree Approach (cont.)

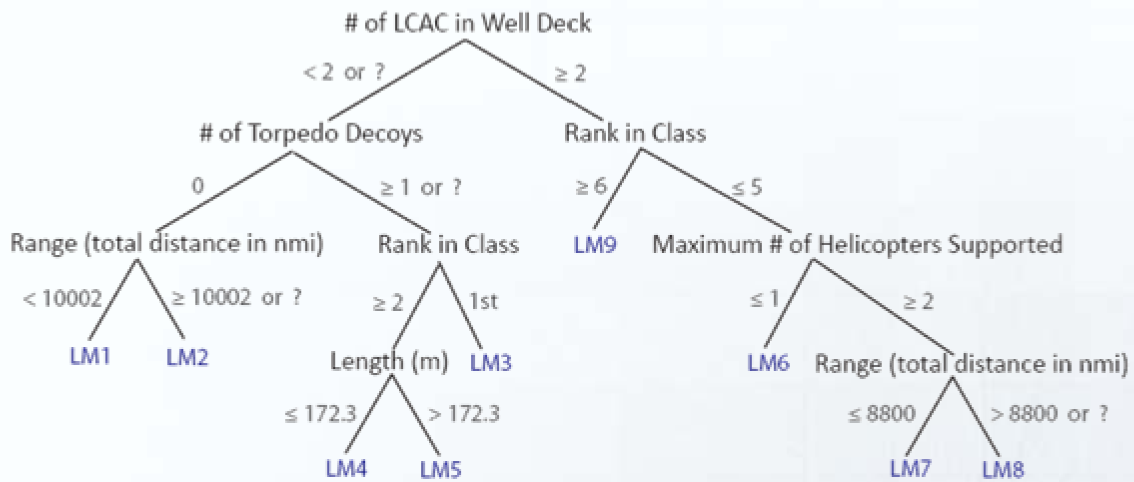
- Applied to our ship data set:



LCAC: air-cushioned landing craft

I. M5 Model Tree Approach (cont.)

- Applied to our ship data set:



LM1	LM2	LM3	LM4	LM5	LM6	LM7	LM8	LM9
Svalbard	Carlskrona	Thomaston	Plymouth Rock	Lyme Bay	Anchorage	Whidbey Island	Raleigh	Tortuga
Protecteur	Atle	Largs Bay	Fort Snelling	Mounts Bay	Portland	Germantown	Vancouver	Rushmore
Preserver	Oden	Ocean	Point Defiance	Cardigan Bay	Pensacola	Fort McHenry	La Salle	Ashland
			Spiegel Grove		Mount Vernon	Gunston Hall	Mistral	Denver
			Alamo		Fort Fisher	Comstock	Tonnerre	Juneau
			Hermitage		Harpers Ferry	Austin	Dixmude (BPC3)	Coronado
			Monticello		Carter Hall	Ogden		Shreveport
			Siroco		Oak Hill	Duluth		Nashville
					Pearl Harbour	Cleveland		Trenton
						Dubuque		Ponce
						Albion		
						Bulwark		

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I. M5 Model Tree Approach (cont.)

LM1

$$\text{Log(Cost)} = 7.4297 - 0.0112 \times \text{rank in class} + 0.0045 \times \text{length (m)} - 0.0002 \times \text{range (sailing time in hrs)} + 0.0445 \times \text{\# of LCAC in well deck} + 0.1104 \times \text{\# of torpedo decoys}$$

LM2

$$\text{Log(Cost)} = 7.4208 - 0.0112 \times \text{rank in class} + 0.0045 \times \text{length (m)} - 0.0002 \times \text{range (sailing time in hrs)} + 0.0445 \times \text{\# of LCAC in well deck} + 0.1104 \times \text{\# of torpedo decoys}$$

LM3

$$\text{Log(Cost)} = 7.6222 - 0.0167 \times \text{rank in class} + 0.0041 \times \text{length (m)} - 0.0002 \times \text{range (sailing time in hrs)} + 0.0445 \times \text{\# of LCAC in well deck} + 0.0659 \times \text{\# of torpedo decoys}$$

LM4

$$\text{Log(Cost)} = 7.7567 - 0.0172 \times \text{rank in class} + 0.0032 \times \text{length (m)} - 0.0002 \times \text{range (sailing time in hrs)} + 0.0445 \times \text{\# of LCAC in well deck} + 0.0659 \times \text{\# of torpedo decoys}$$

- Only attributes referenced in tree decisions appear in LMs
- Intuitive, except for negative coefficient of sailing time range: Data explains anomaly:

  Median sailing range is 444hrs. Only 6 of 57 ships have range > 770hrs. The cost of these 6 ships are relatively low. E.g., Sweden's Oden costs 53M NCC and has a range of >2200hrs

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I. M5 Model Tree Approach (cont.)

- Good idea to look at stats of all M5 model tree attributes:

Attribute	Minimum	Median	Mean	Maximum
Rank	1	3	3.68	12
Length	103.7	173.8	170.3	203.4
Range (sailing time)	385	444	616	2308
Range (total distance)	7500	10003	8000	30000
# LCAC	0	2	2	4
# torpedo decoys	0	0	2	8
# of helicopters supported	0	5	6	18

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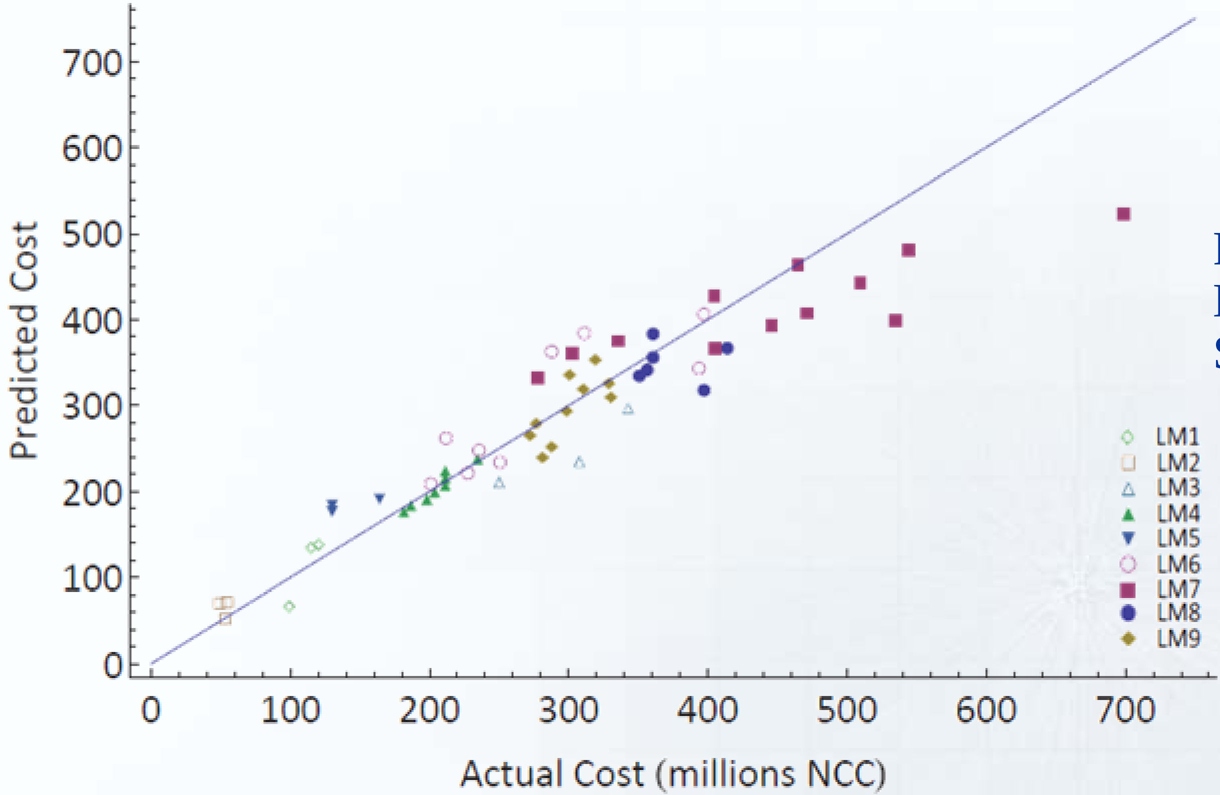
I. M5 Model Tree Approach (cont.)

- How well does the tree learn the known data?

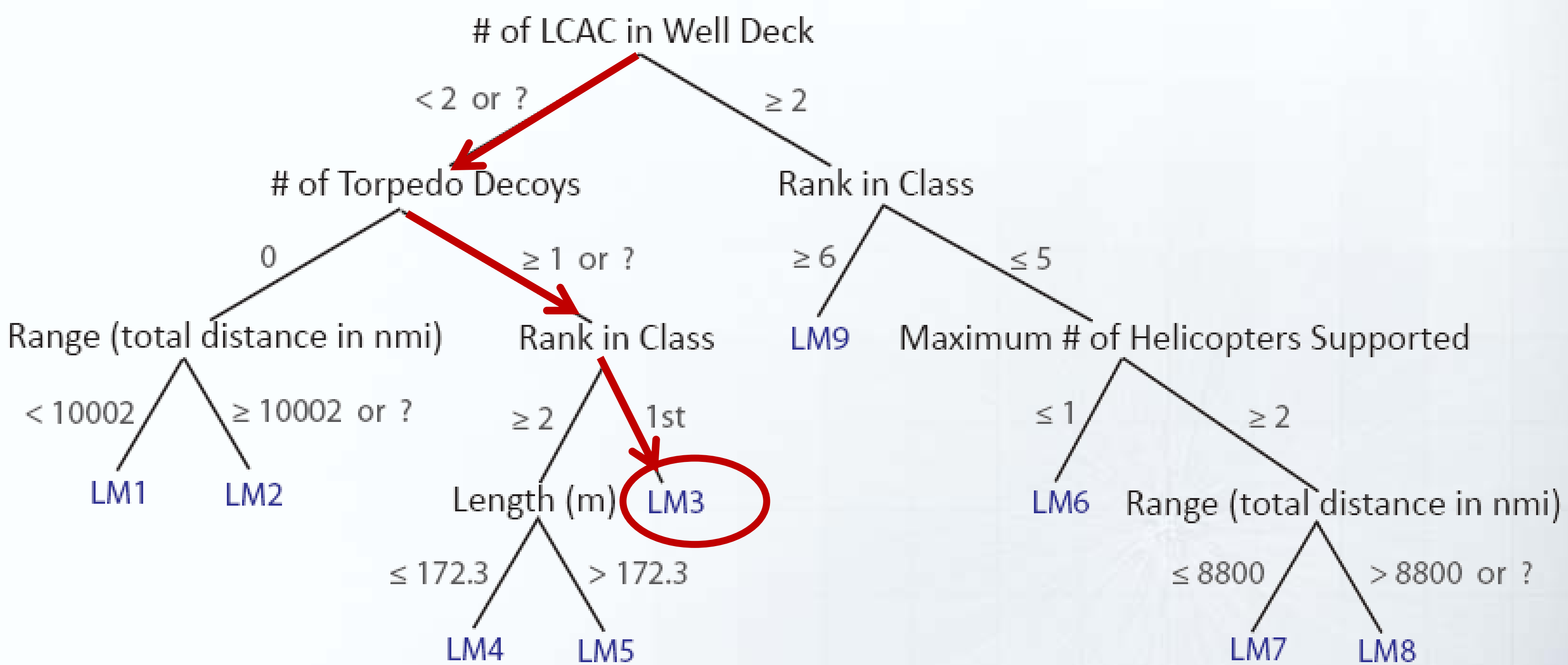


R^2= 0.92
Mean % error: 12%
Stnd. Dev.: 46.4M

	LM1	LM2	LM3	LM4	LM5	LM6	LM7	LM8	LM9
Mean % error:	22%	27%	17%	3%	33%	12%	14%	8%	6%
Standard deviation	24.3M	16.9M	53.0M	6.4M	45.6M	43.4M	78.0M	39.3M	24.3
# of instances:	3	3	3	8	3	9	12	6	10

I. M5 Model Tree Approach (cont.)

- Applied to Rotterdam and Johan de Witt ships:



Note: Royal Netherlands Navy considers the Rotterdam and Johan de Witt to be of separate classes (both rank = 1)

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I. M5 Model Tree Approach (cont.)

- Applied to Rotterdam and Johan de Witt ships:

LM3		Rotterdam	Johan de Witt
Log(Cost) =	7.6222		
	- 0.0167 × rank in class	1	1
	+ 0.0041 × length (m)	162.2m	175.35m
	- 0.0002 × range (sailing time in hrs)	500 hrs	833 hrs
	+ 0.0445 × # of LCAC in well deck	0	0
	+ 0.0659 × # of torpedo decoys	1	1

Cost estimates:

Rotterdam L800: 197.7M NCC

Johan de Witt L801: 212.3M NCC

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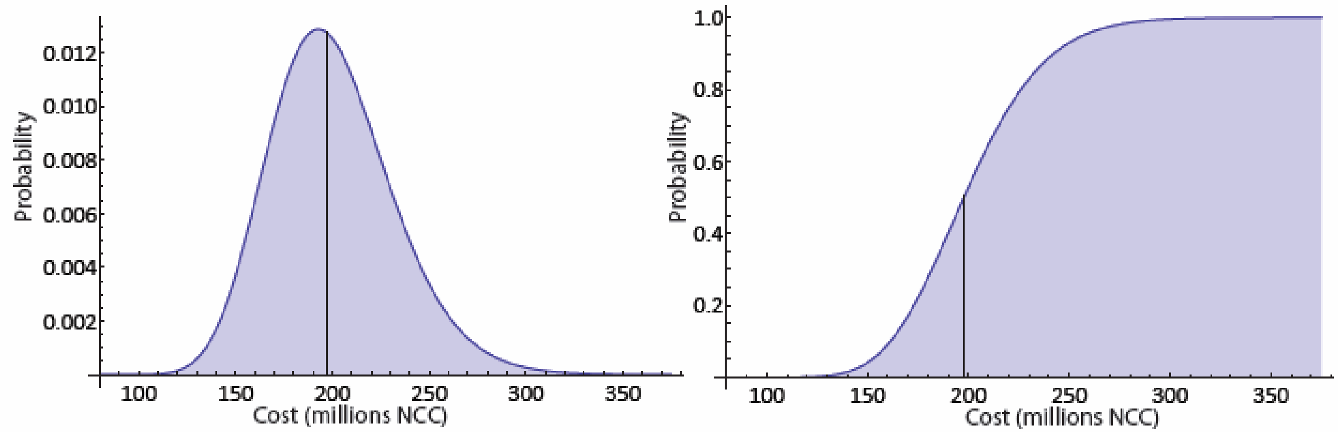
I. M5 Model Tree Approach (cont.)

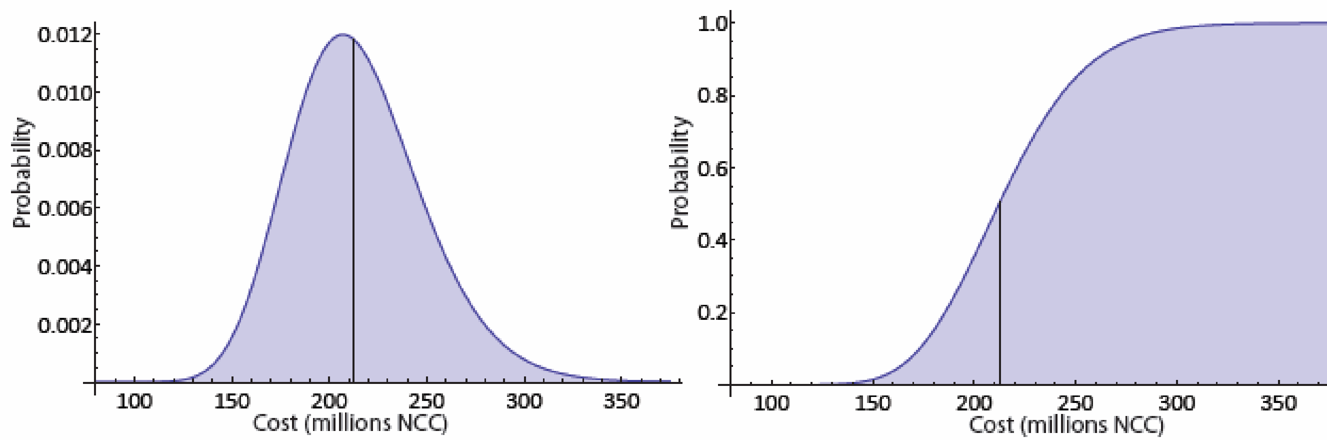
- Applied to Rotterdam and Johan de Witt ships:

Rotterdam:



(a) Probability density function (b) Cumulative distribution function

Johan de Witt:



(a) Probability density function (b) Cumulative distribution function

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Cost Estimation Methods

I. Parametric Approach:

M5 Model Tree Algorithm

II. Costing by Analogy:

Hierarchical Clustering

II. Hierarchical Clustering Approach

- Algorithmic way to determine which ships are most similar to the Rotterdam and Johan de Witt
- Nearest Neighbour Cluster Analysis idea:
  1. define a distance metric to measure similarity
  2. Compute average (weighted by distance) of all known ship costs to obtain an estimate "by analogy"

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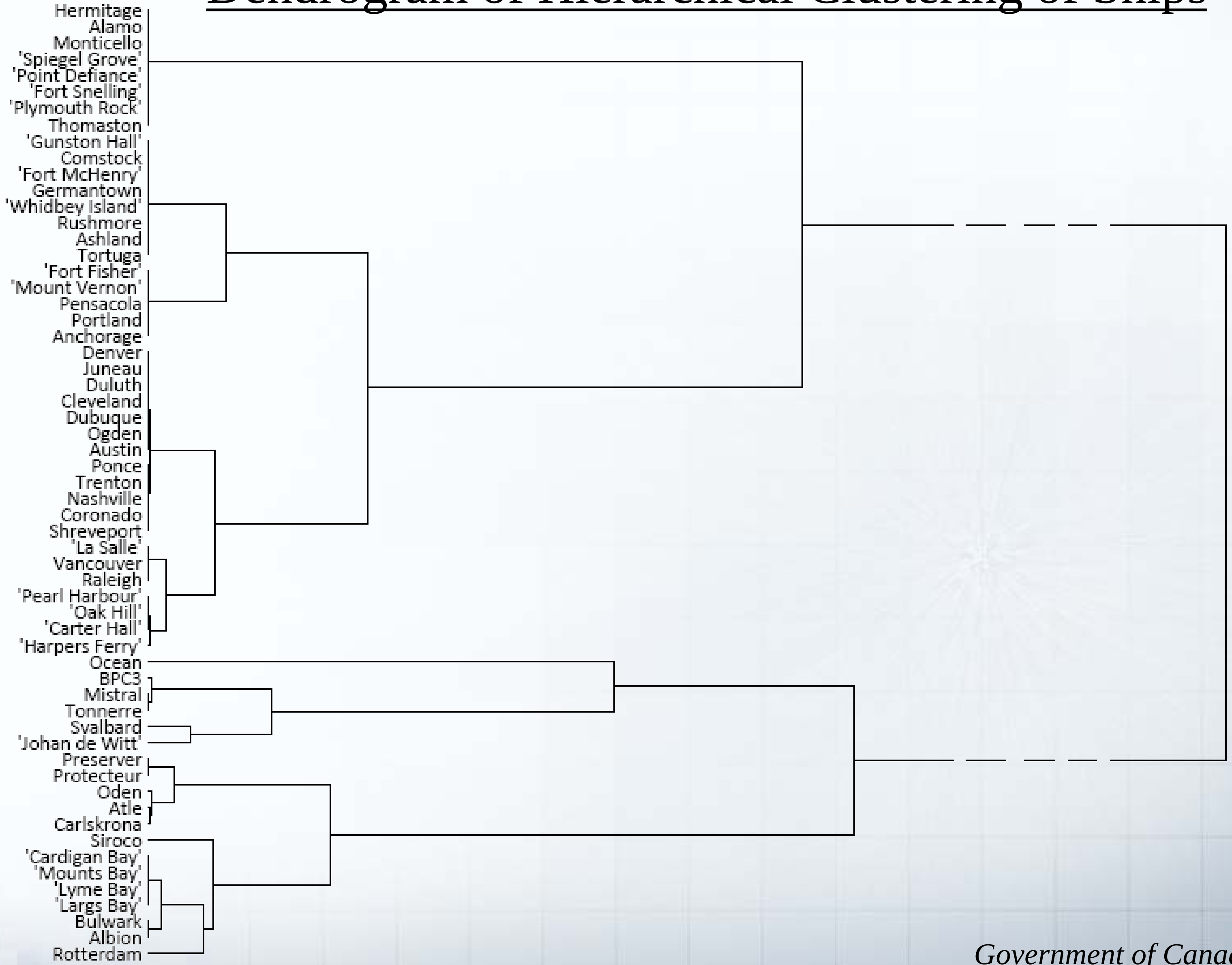
II. Hierarchical Clustering Approach (cont.)

Dendrogram of Hierarchical Clustering of Ships



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II. Hierarchical Clustering Approach (cont.)

- Cost estimate using distances:

$$d_{ijk} = \text{distance between ship } i \text{ and } j \text{ with respect to attribute } k$$
$$\in [0,1]$$

$$d_{ij} = \text{distance between ship } i \text{ and } j$$
$$= \sqrt{\sum_{k=1}^{M} d_{ijk}^2}$$

$$C_i = \text{known cost of ship } i$$

$$\tilde{C}_i = \sum_{j \neq i} \frac{C_i}{d_{ij}^2} \cdot \frac{1}{\sum_{j \neq i} \frac{1}{d_{ij}^2}} = \text{cost estimate of ship } i$$

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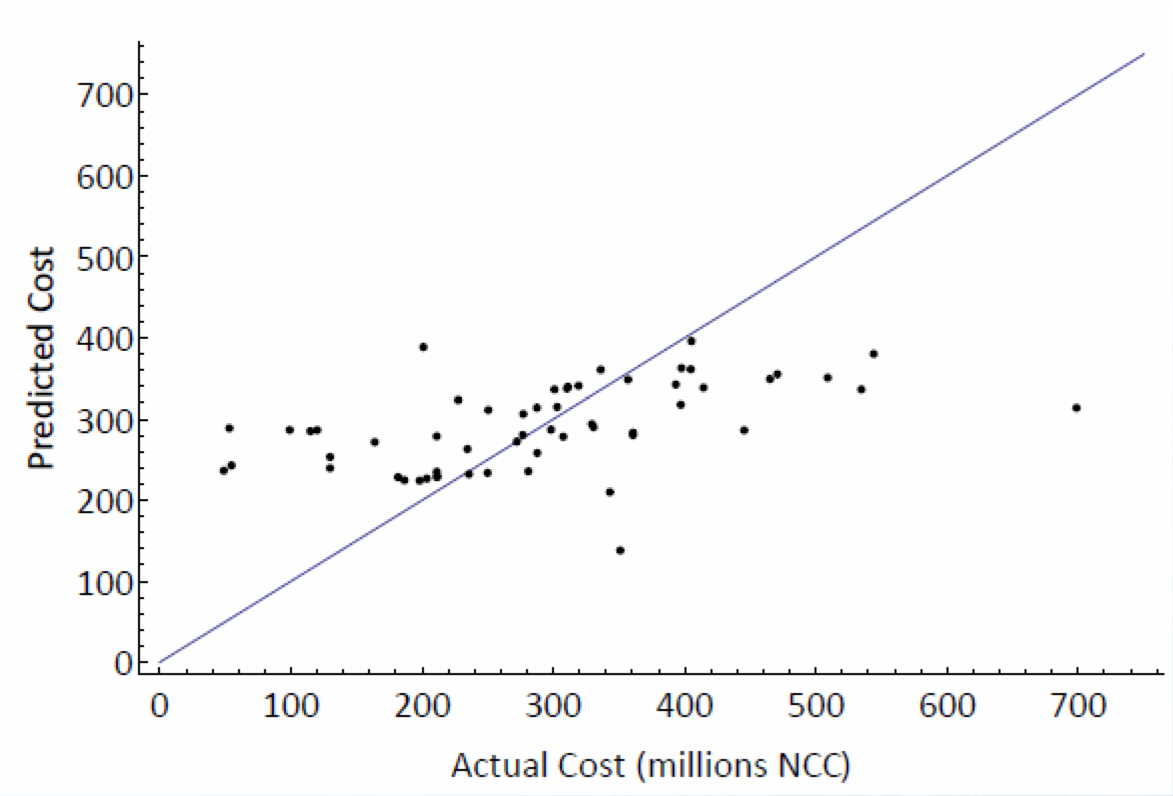
II. Hierarchical Clustering Approach (cont.)

- Cost estimate using distances:
- Not very smart: all attributes assumed to have equal importance



R^2= 0.23
Mean % error: 49%
Stnd. Dev.: 112M

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II. Hierarchical Clustering Approach (cont.)

- Cost estimate using **weighted-attribute** distances:
- Not all attributes are equal!

- Each attribute *k* given a weight w_k

$$\hat{d}_{ij} = \text{weighted distance between ship } i \text{ and } j$$

$$\hat{d}_{ij} = \sqrt{\sum_{k=1}^{M} (w_k d_{ijk})^2} \qquad \sum_{k=1}^{M} w_k = 1, \quad w_k \geq 0 \text{ for all } k$$

$$C_i = \text{known cost of ship } i$$

$$\hat{C}_i = \sum_{j \neq i} \frac{C_i}{\hat{d}_{ij}^2} \cdot \frac{1}{\sum_{j \neq i} \frac{1}{\hat{d}_{ij}^2}} = \text{(weighted) cost estimate of ship } i$$

$$\text{Minimize} \sum_{i=1}^{42} (C_i - \hat{C}_i)^2 \quad \text{(prediction error for known cases)}$$

→ Computationally intensive optimization (with all ~100 attributes)
(non-linear convex programming)

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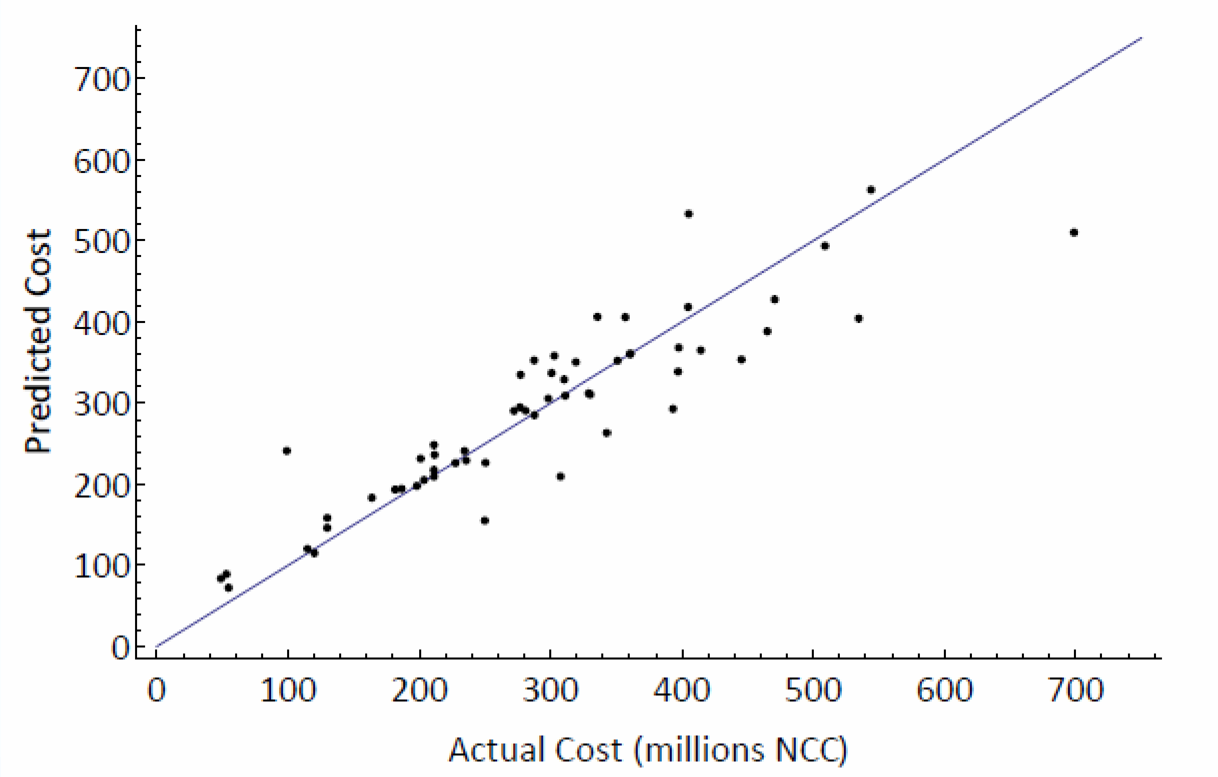
II. Hierarchical Clustering Approach (cont.)

- How well does the hierarchical clustering learn the known data?



R^2= 0.86
Mean % error: 16%
Stnd. Dev.: 55.9M

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II. Hierarchical Clustering Approach (cont.)

- Applied to Rotterdam and Johan de Witt ships:

Name	Distance	Name	Distance	Name	Distance
Rotterdam	0.000	Vancouver	0.312	Nashville	0.639
Largs Bay	0.034	La Salle	0.316	Trenton	0.646
Lyme Bay	0.034	Harpers Ferry	0.405	Ponce	0.650
Mounts Bay	0.035	Carter Hall	0.431	Whidbey Island	0.655
Cardigan Bay	0.035	Oak Hill	0.435	Germantown	0.659
Oden	0.039	Pearl Harbour	0.439	Fort McHenry	0.664
Carlskrona	0.044	Anchorage	0.546	Gunston Hall	0.668
Johan de Witt	0.046	Portland	0.550	Comstock	0.673
Atle	0.052	Pensacola	0.553	Tortuga	0.678
Albion	0.057	Mount Vernon	0.557	Rushmore	0.682
Bulwark	0.058	Fort Fisher	0.561	Ashland	0.687
Siroco	0.067	Austin	0.601	Thomaston	0.971
Svalbard	0.068	Ogden	0.606	Plymouth Rock	0.975
Protecteur	0.128	Duluth	0.610	Fort Snelling	0.979
Preserver	0.129	Cleveland	0.612	Point Defiance	0.983
Ocean	0.227	Dubuque	0.617	Spiegel Grove	0.987
Tonnerre	0.244	Denver	0.621	Alamo	0.992
Mistral	0.246	Juneau	0.626	Hermitage	0.996
BPC3	0.266	Coronado	0.630	Monticello	1.000
Raleigh	0.309	Shreveport	0.634		

<u>Cost estimates</u>

Rotterdam L800: 214.6M NCC

Johan de Witt L801: 243.9M NCC

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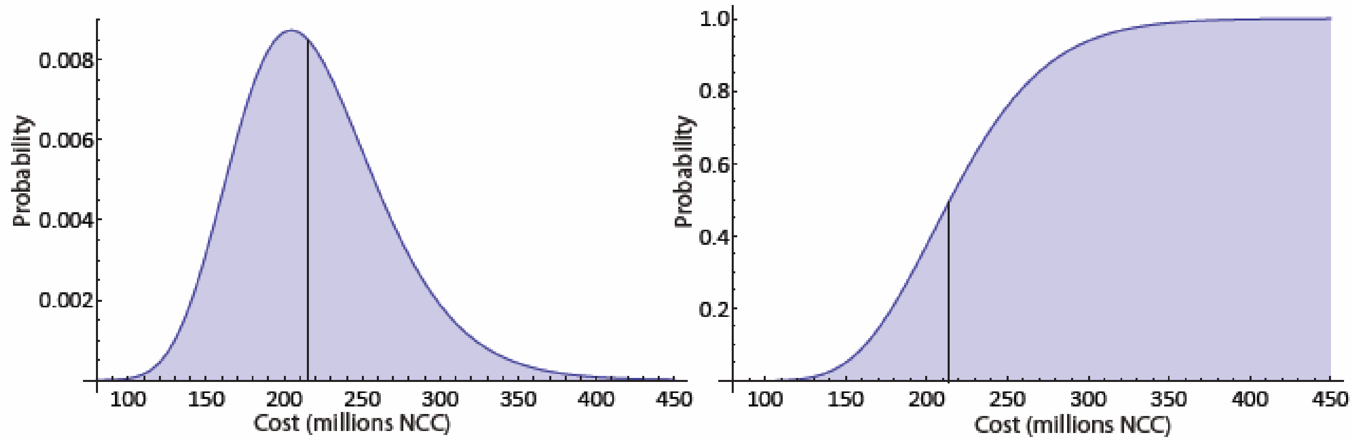
II. Hierarchical Clustering Approach (cont.)

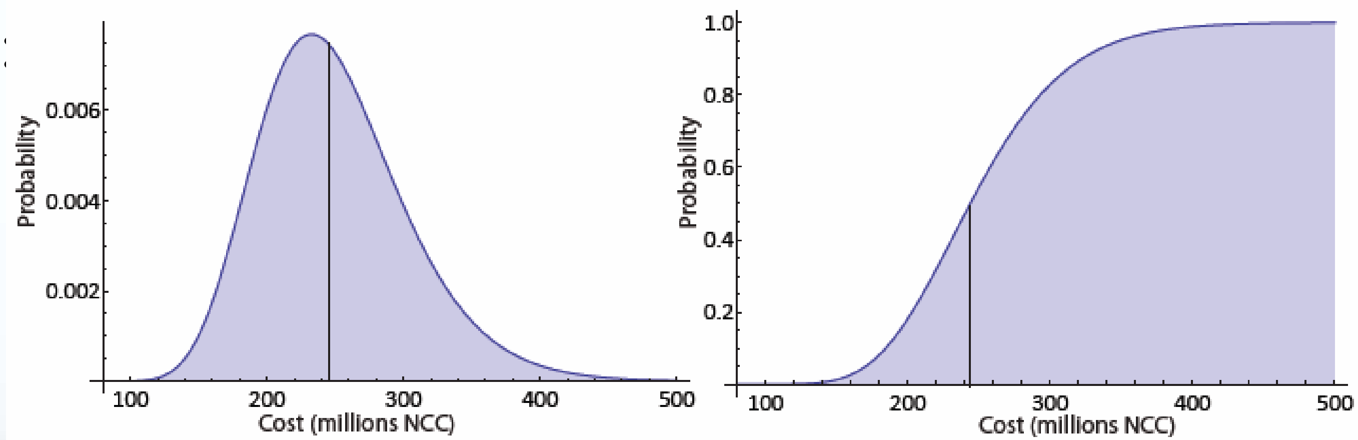
- Applied to Rotterdam and Johan de Witt ships:

Rotterdam:



(a) Probability density function (b) Cumulative distribution function

Johan de Witt:



(a) Probability density function (b) Cumulative distribution function

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Comparison to Actuals

- Once the cost estimates were documented, the Royal Netherlands Navy revealed the actual costs of the Rotterdam and Johan de Witt

- Estimate recap:

	M5 model tree	Hierarchical clustering	Actuals
HNLMS Rotterdam estimate	197.7M NCC	214.6M NCC	202.2M
HNLMS Johan de Witt estimate	221.3M NCC	243.9M NCC	253.7M
Coefficient of correlation	0.96	0.93	
Coefficient of determination	0.92	0.86	
Standard deviation	46.4M NCC	55.9M NCC	
Mean absolute % error	11%	16%	
Ability to learn known cases	✓	✓	
Optimized to predict unknown cases	✓	×	
Uses entire data set	✓	×	

- Cost figures in fictitious notional common currency

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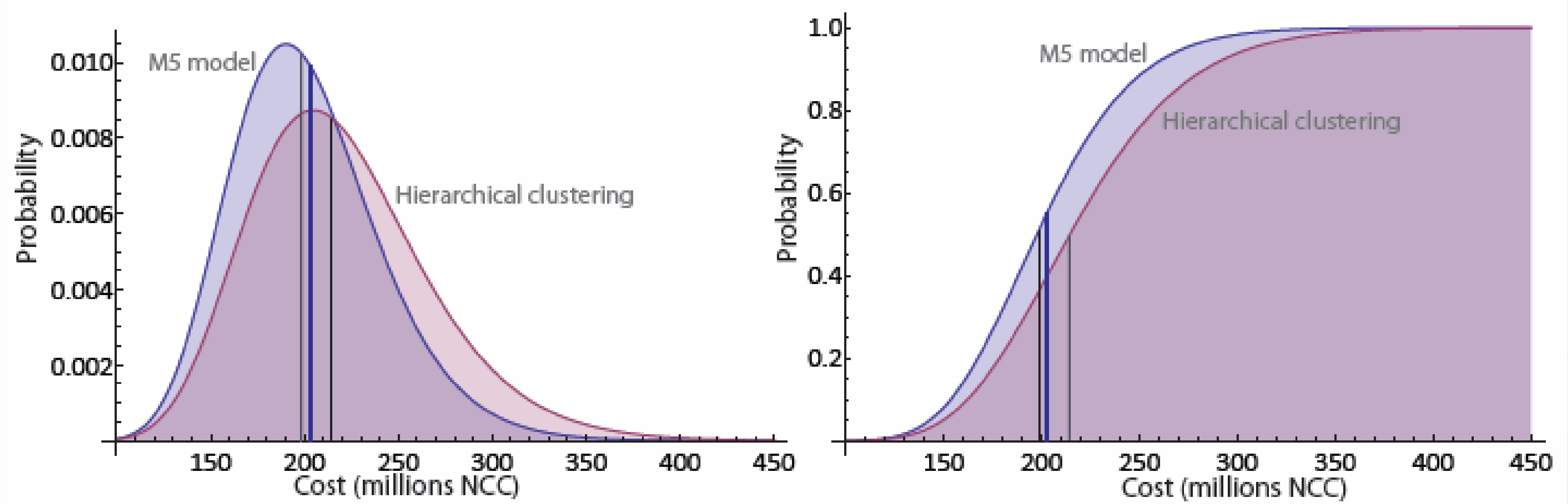
Comparison to Actuals (cont.)

- Rotterdam LPD estimates and actual:



(a) Probability density functions (b) Cumulative distribution functions

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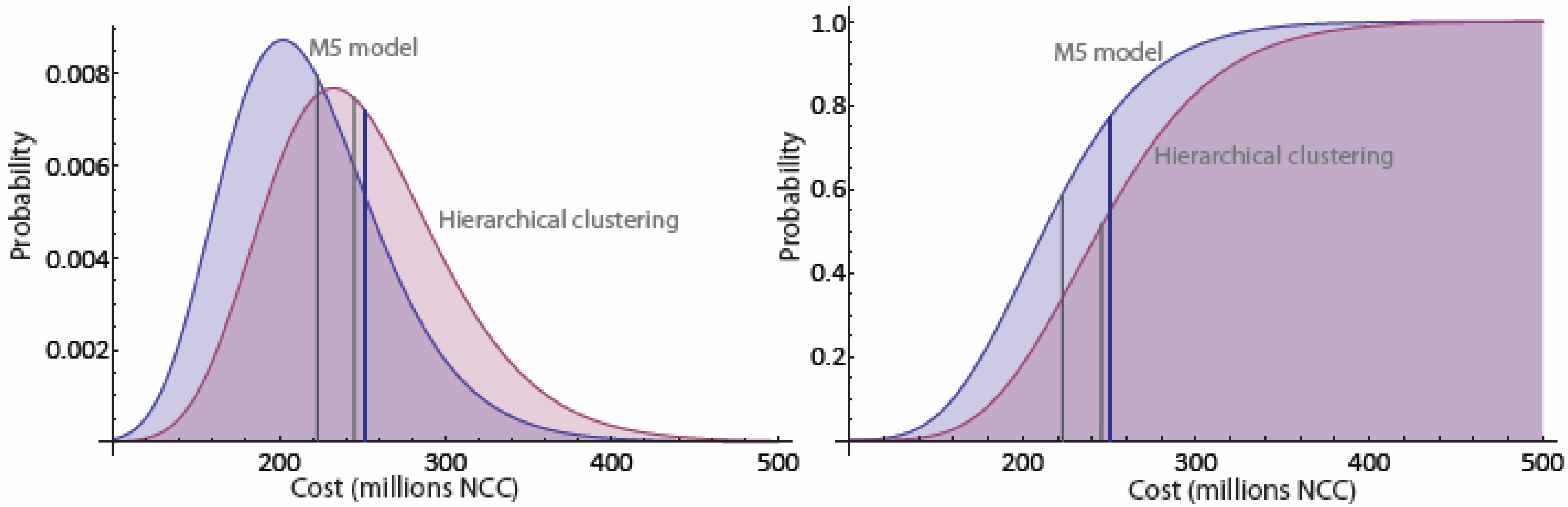
Comparison to Actuals (cont.)

- Johan de Witt LPD estimates and actual:



(a) Probability density functions (b) Cumulative distribution functions

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Estimates via Traditional Approaches

- **Simple Linear Regression on data set:**
  - Commonly use ship length — *$R^2 = 0.56$*

$$Log_{10}(cost) = 6.95 + 0.01 \times \text{ length of ship (in meters)},$$

  - Rotterdam estimate = 219.2M
  - Johan de Witt estimate = 289.7M

Actuals
202.2M
253.7M

- **Multiple Linear Regression**

$R^2 = 0.85$

$$\begin{aligned} Log_{10}(cost) = 5.7368 &- 0.0224 \times \text{ rank in class} \\ &+ 0.0121 \times \text{ length (in meters)} \\ &+ 0.0338 \times \text{ beam (in meters)} \\ &+ 0.1071 \times \text{ draught (in meters)} \\ &- 0.0001 \times \text{ full load displacement (in tonnes)} \\ &+ 0.0012 \times \text{ crew size} \\ &- 0.0876 \times \text{ number of propeller shafts} \\ &- 0.0239 \times \text{ number of guns of calibre } \geq 75, \end{aligned}$$

  - Rotterdam estimate = 158.9M
  - Johan de Witt estimate = 201.4M

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Conclusions

- Two novel approaches to cost estimation using known data mining algorithms
  - M5 Model Tree parametric approach
  - Hierarchical clustering analogy approach
- Proof of concept: blind, ex post analysis
- Incorporate multitude of cost driving factors, but remain top-down (suitable for planning and design phases)
- Should be considered by nations with lots of data (e.g., U.S. for estimating the LHA replacement)

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Comparison to Actuals (cont.)

- Recall discussion on non-intuitive sailing range coefficient in M5 model tree regression?

> ### I. M5 Model Tree Approach (cont.)
>
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>
> LM1
> $\text{Log(Cost)} = 7.4297 - 0.0112 \times \text{rank in class} + 0.0045 \times \text{length (m)} - 0.0002 \times \text{range (sailing time in hrs)} + 0.0445 \times \text{\# of LCAC in well deck} + 0.1104 \times \text{\# of torpedo decoys}$
>
> LM2
> $\text{Log(Cost)} = 7.4208 - 0.0112 \times \text{rank in class} + 0.0045 \times \text{length (m)} - 0.0002 \times \text{range (sailing time in hrs)} + 0.0445 \times \text{\# of LCAC in well deck} + 0.1104 \times \text{\# of torpedo decoys}$
>
> LM3
> $\text{Log(Cost)} = 7.6222 - 0.0167 \times \text{rank in class} + 0.0041 \times \text{length (m)} - 0.0002 \times \text{range (sailing time in hrs)} + 0.0445 \times \text{\# of LCAC in well deck} + 0.0659 \times \text{\# of torpedo decoys}$
>
> LM4
> $\text{Log(Cost)} = 7.7567 - 0.0172 \times \text{rank in class} + 0.0032 \times \text{length (m)} - 0.0002 \times \text{range (sailing time in hrs)} + 0.0445 \times \text{\# of LCAC in well deck} + 0.0659 \times \text{\# of torpedo decoys}$
>
> - Only attributes referenced in tree decisions appear in LMs
> - Intuitive, except for negative coefficient of sailing time range: Data explains anomaly:
>
>   Median sailing range is 444hrs. Only 6 of 57 ships have range > 770hrs. The cost of these 6 ships are relatively low. E.g., Sweden's Oden costs 53M NCC and has a range of >2200hrs

- Median is 444hrs.
- Rotterdam's range is **500** hrs (very close)
- Johan de Witt: **833** (outlier!) → neutralizing this attribute and re-applying M5 model tree yields revised estimate of **253.9M**

(Actual = 253.7M)

DEFENCE DÉFENSE

II. Hierarchical Clustering Approach (cont.)

- Principal Component Analysis of ship data base
  - Reduce dimensionality of data set
  - Solve optimization problem

Macro-Attribute	% of Data Variability Accounted for: Proportion	Cumulative
A1	17%	17%
A2	12%	29%
A3	11%	41%
A4	9%	49%
A5	8%	57%
A6	7%	64%
A7	6%	69%
A8	5%	74%
A9	4%	78%
A10	3%	81%
A11	3%	85%
A12	3%	88%
A13	3%	90%
A14	2%	92%
A15	2%	94%
A16	1%	95%

$$
\begin{aligned}
A2 = {} & 0.204 \times \text{length} \\
& + 0.196 \times \text{beam width} \\
& + 0.183 \times \text{vehicle space} \\
& + 0.18 \times \text{\# of expeditionary fighting vehicles} \\
& + 0.165 \times 1 \text{ if has a well deck, otherwise } 0 \\
& + 0.165 \times \text{width of the well deck} \\
& + 0.164 \times \text{length of the well deck} \\
& + 0.159 \times \text{\# of large personnel landing craft} \\
& + 0.156 \times \text{\# of Chinook helicopters supported} \\
& + 0.155 \times \text{full load displacement} \\
& + 0.153 \times \text{\# of combat data systems} \\
& + 0.150 \times \text{light load displacement} \\
& + 0.144 \times \text{well deck capacity} \\
& + 0.143 \times \text{\# of elevators} \\
& + 0.142 \times \text{vehicle fuel capacity} \\
& \text{etc.}
\end{aligned}
$$

DEFENCE R&D DÉFENSE

II. Hierarchical Clustering Approach (cont.)

- Principal Component Analysis of ship data base
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A4	9%	49%
A5	8%	57%
A6	7%	64%
A7	6%	69%
A8	5%	74%
A9	4%	78%
A10	3%	81%
A11	3%	85%
A12	3%	88%
A13	3%	90%
A14	2%	92%
A15	2%	94%
A16	1%	95%

Attribute	Weight
A1	0
A2	0.452
A3	0
A4	0
A5	0.334
A6	0
A7	0
A8	0
A9	0
A10	0.214