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The Percentile Problem: *How Much Is Enough?*

Society of Cost Estimating and Analysis (SCEA) Conference

June 2008

Kevin Cincotta

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Acknowledgments

- The author wishes to recognize and thank the following individuals and institutions for their contributions:
  - Timothy Anderson, Aerospace Corporation
  - Dr. Stephen Book, MCR LLC
  - Dr. David Lee, LMI

Un débauché de profession est rarement un homme pitoyable.

-De Sade, *Les infortunes de la vertu*

Agenda

- Background
- Implications
- Adding Correlation
- Proposed Solution
- The Unfortunate Reality
- Conclusions & Recommendations

Background

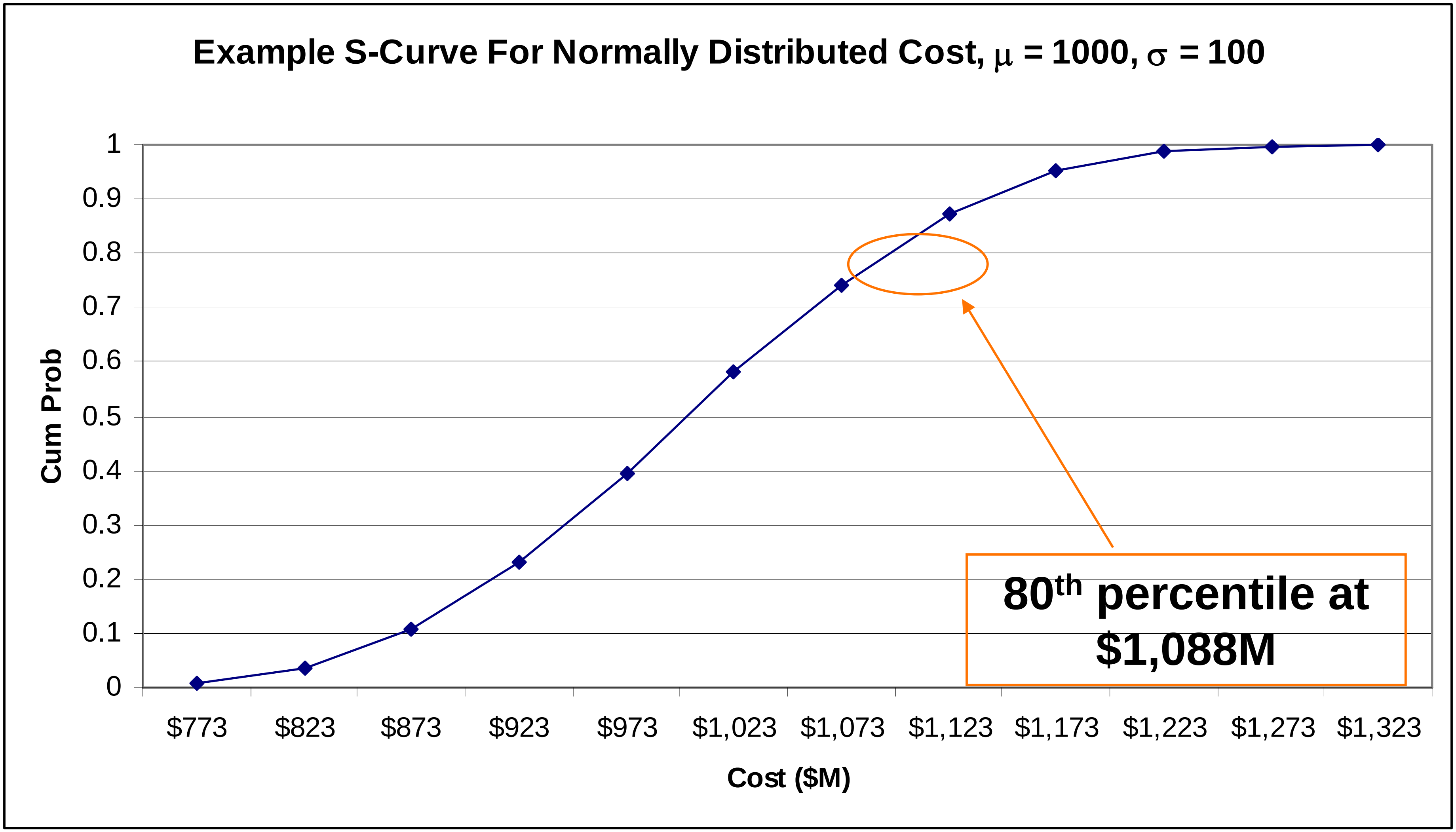
- In recent years, agency-level guidance has instructed cost estimators to provide decision-makers with a **range of possible costs**, rather than a single point estimate
- This range is often expressed as a cumulative distribution function (cdf), or “S-Curve”
- Using the S-Curve, decision-makers select the percentile at which to budget (e.g. 80th)

Example Guidance

"The space acquisition system is strongly biased to produce unrealistically low cost estimates throughout the acquisition process. These estimates lead to unrealistic budgets and unexecutable programs. We recommend, among other things, that the government budget space acquisition programs to a most probable (80/20) cost…"

-From the *Report of the Defense Science Board/Air Force Scientific Advisory Board Joint Task Force on Acquisition Of National Security Space Programs*, May 2003.

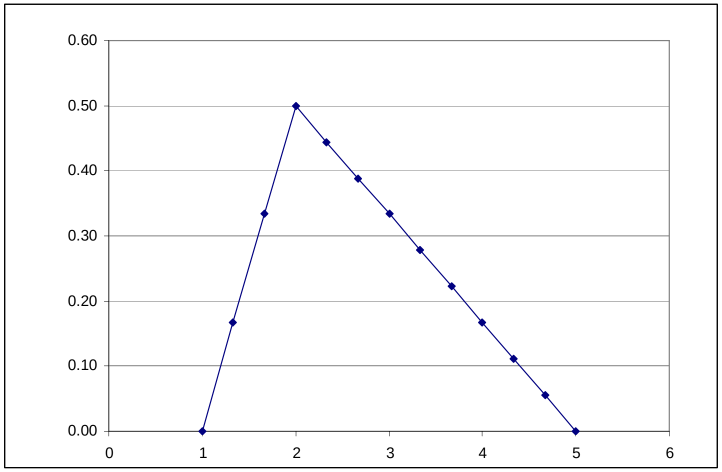
What Does It Mean?



Risk dollars = $1,088M - $1,000M = $88M

But We Often Buy Down More Risk...

ELEMENT	LB	ML	UB	Mean	StDev	Distribution
1	1	2	5	2.67	0.85	Triangular
2	1	2	5	2.67	0.85	Triangular
3	1	2	5	2.67	0.85	Triangular
4	1	2	5	2.67	0.85	Triangular
5	1	2	5	2.67	0.85	Triangular
	5	**10**	**25**	**13.33**	**1.90**	**Normal**



Cost Measure	Percentile	Value	Diff from PE	Diff from 50th
Sum of Point Est	0.04	10.00	--	--
Mean	0.50	13.33	3.33	--
80th	0.80	14.93	4.93	1.60

Risk dollars = $14.93M – $10M = $4.93M (not $1.6M)

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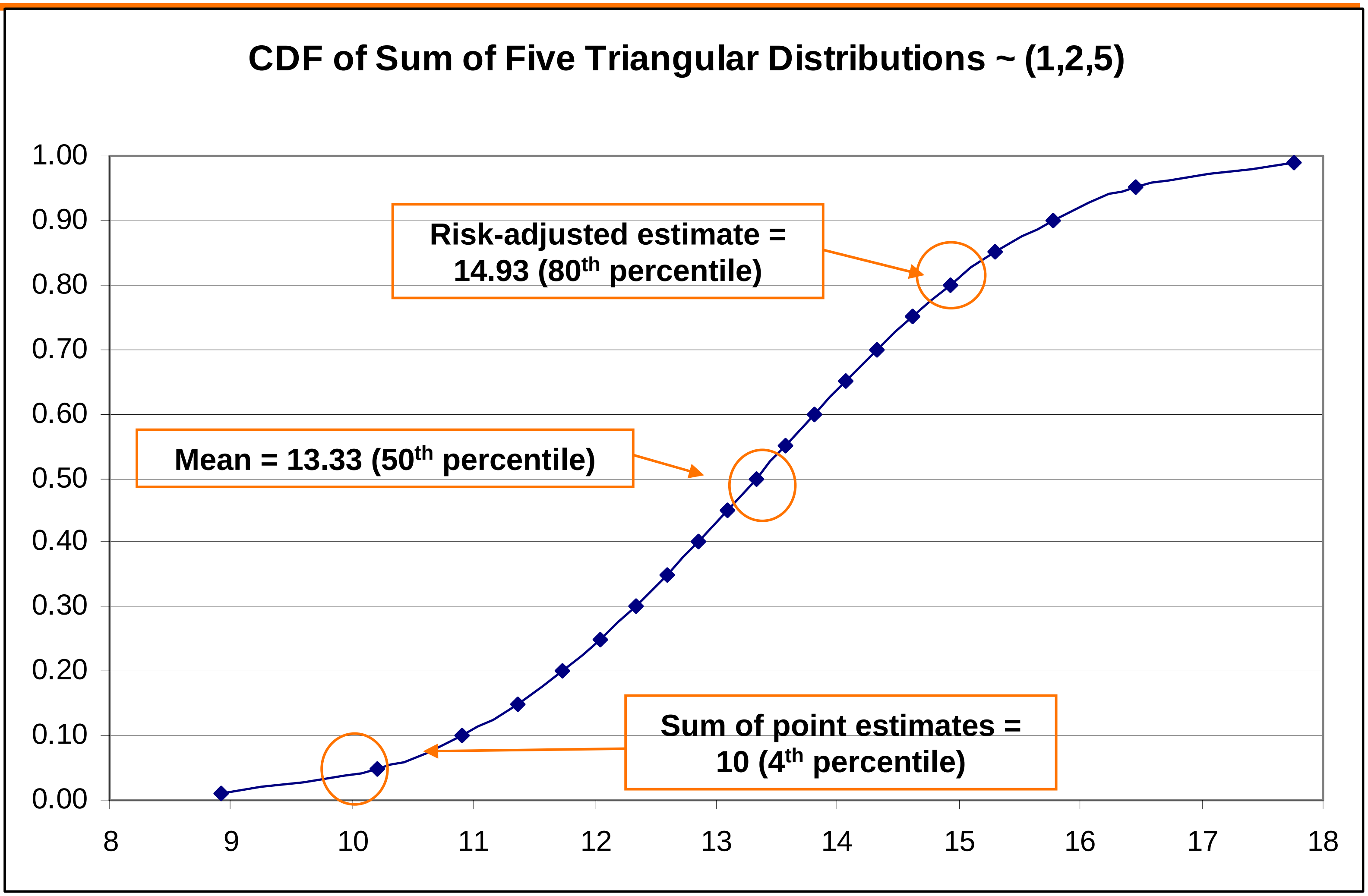
What Does It *Really* Mean?



Risk dollars = 14.93 – 10 = 4.93 (49.3% above PE)

Implications

- Suppose that, across an entire portfolio of programs, we follow this guidance and budget at the 80th percentile.
- Assuming our programs are independent, at what percentile are we budgeting for the entire portfolio?

We Are Budgeting at the 98th Percentile!

Program	LB	ML	UB	μ	σ	Estimate	Percentile
1	1	2	5	2.67	0.85	3.45	0.80
2	1	2	5	2.67	0.85	3.45	0.80
3	1	2	5	2.67	0.85	3.45	0.80
4	1	2	5	2.67	0.85	3.45	0.80
5	1	2	5	2.67	0.85	3.45	0.80
TOTAL	**5**	**10**	**25**	**13.33**	**1.90**	**17.25**	**0.98**

The sum of the 80th percentiles for each program lies at roughly the 98th percentile of the portfolio.

Result Generalizes Across Distributions, Sample Sizes, and Parameters

DISTRIBUTION	N	ELEMENTS	PORTFOLIO PERCENTILE
Triangular	5	Identical a,b,c	98.04%
Normal	10	Various μ, σ	99.45%
Lognormal	10	Various μ, σ	97.76%

The individual programs need not be independently identically distributed (iid). As long as they are independent, the sum of 80th percentiles is near the portfolio-level 98th percentile

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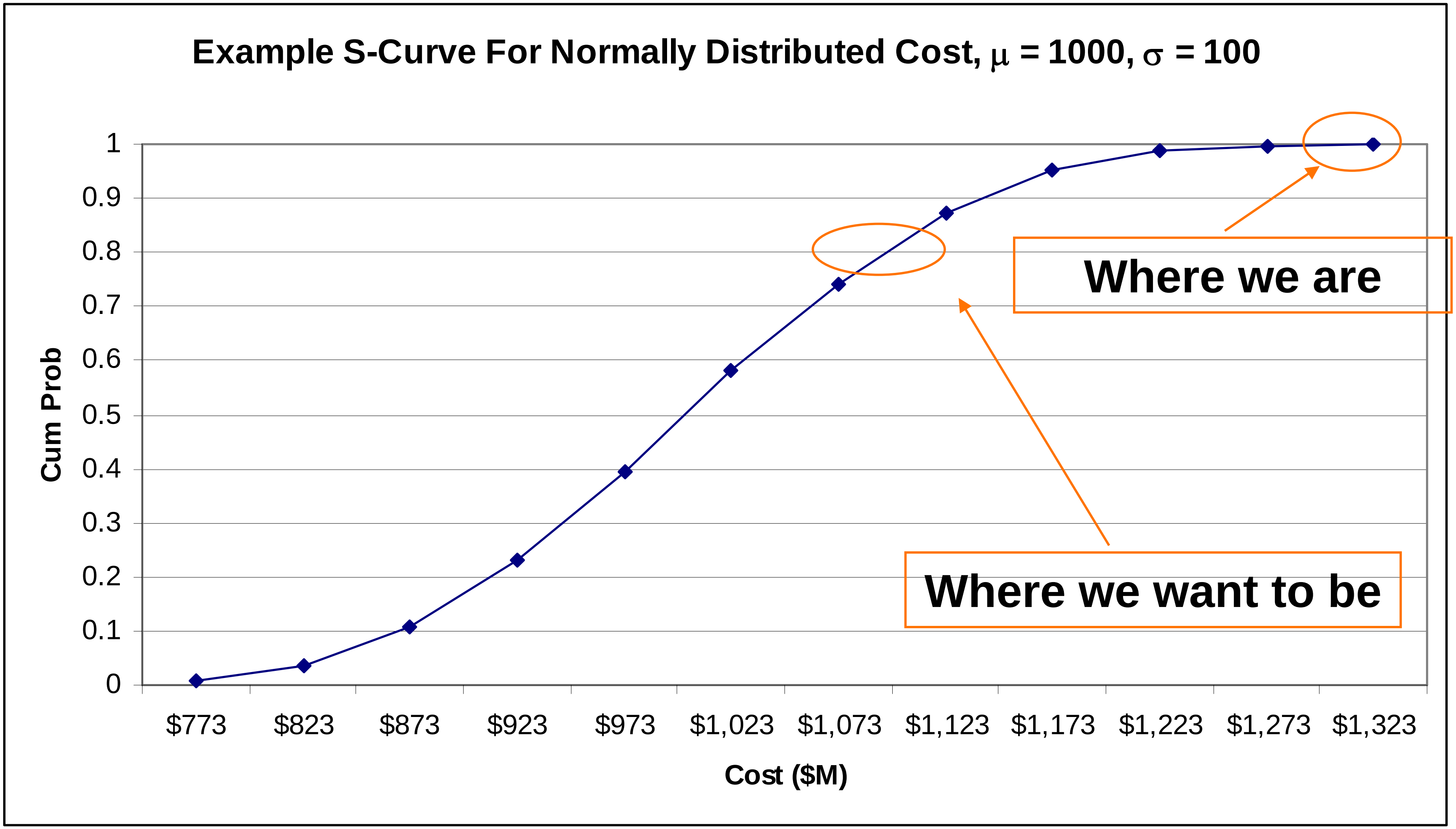
What are the Consequences?



Extra Risk dollars = $1,212M - $1,088M = $124M (12.4%)

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Perfect Positive Correlation: No Problem!

- When program costs (within a portfolio) are perfectly positively correlated, the sum of 80th percentiles is, in fact, the 80th percentile
- If X and Y are perfectly correlated, and $X \sim N(\mu,\sigma^2)$ then $Y=aX$

$$Var(Y) = Var(aX) = a^2 Var(X)$$

$$StDev(Y) = a\sigma$$

- Thus, both the mean and standard deviation of Y are exactly proportionate to the mean and variance of X
  - For example, suppose that you are estimating SE/PM cost (Y) as 10% per year of recurring production cost (X)
  - Then X and Y are perfectly correlated
  - Y is at its 80th percentile whenever X is at its 80th percentile

Perfect Negative Correlation: No Need for Risk Analysis

- Now suppose that two programs' costs are perfectly negatively correlated
  - Example: if a certain adversary's missile test succeeds, we must spend \$100M on Program A, and \$100M less on Program B. If the adversary's missile test fails, we do the opposite.
  - Then cost overruns and underruns exactly offset, at the portfolio level
- As long as the two programs are roughly the same size, we are "perfectly hedged," and we don't need complex risk analysis:

$$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$$

$$\rho = -1 = Cov(X,Y)/\sigma_x\sigma_y$$

$$\text{So } Cov(X,Y) = -\sigma_x\sigma_y$$

$$Var\ (X+Y) = Var(X) + Var(Y) - 2\sigma_x\sigma_y$$

$$= 0 \text{ when } Var(X) = Var(Y)$$

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A More Likely Scenario: (Somewhat) Positive Correlation

- Suppose we have the same sample data as before, with the programs' costs are somewhat correlated
- As our prior examples showed, "the more independent they are, the worse it is."
- So milder correlations mitigate, but do not eliminate, the problem of mistakenly over-budgeting:

ρ	Sum of 80th*
0.00	99.5%
0.20	93.2%
0.40	88.2%
0.60	84.6%
0.80	82.0%
1.00	80.0%

* Assumes 10 normally distributed program costs, with variances similar to that of a recent space program

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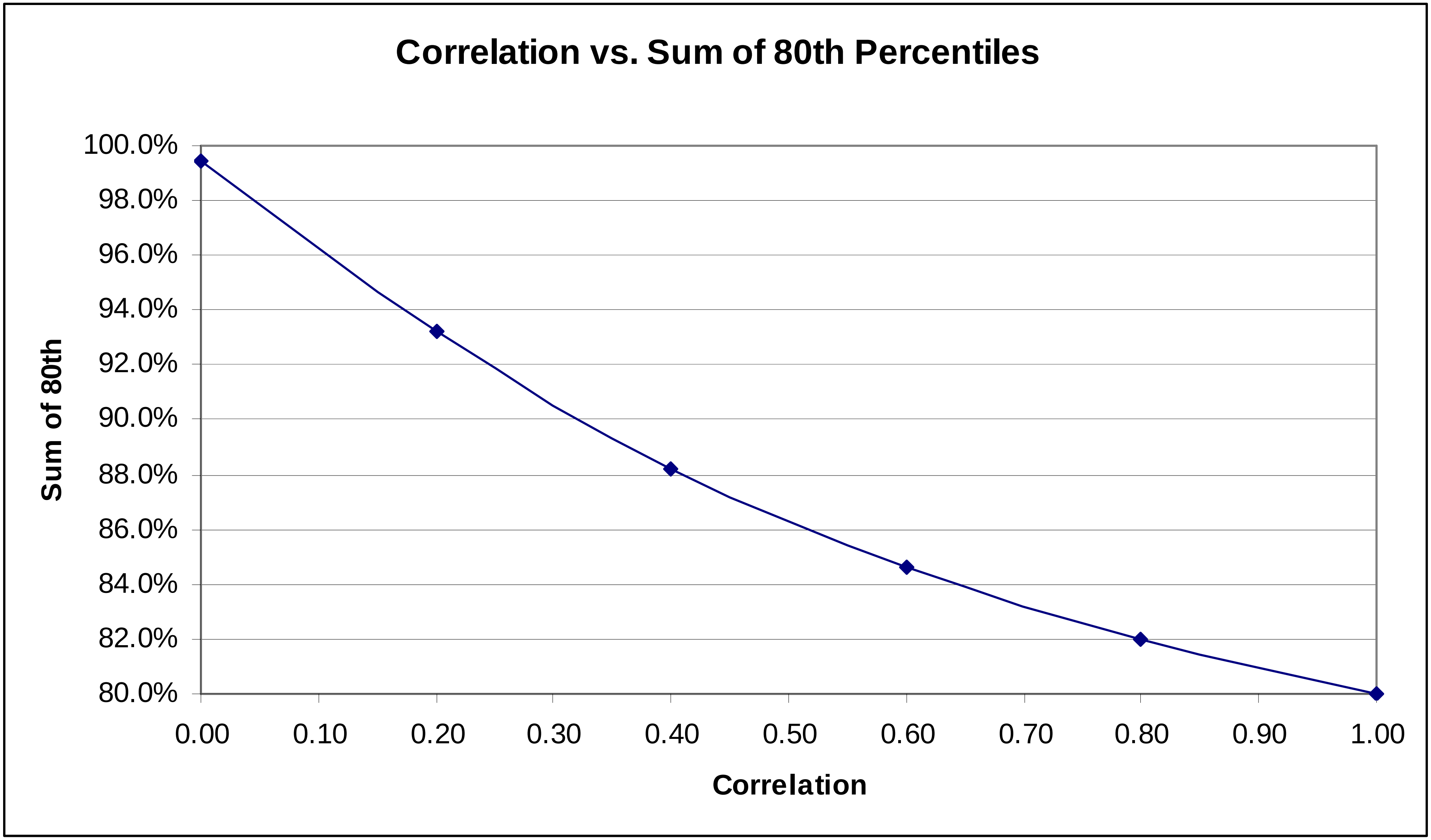
The Price of Independence*



What About Changing The Portfolio Size?

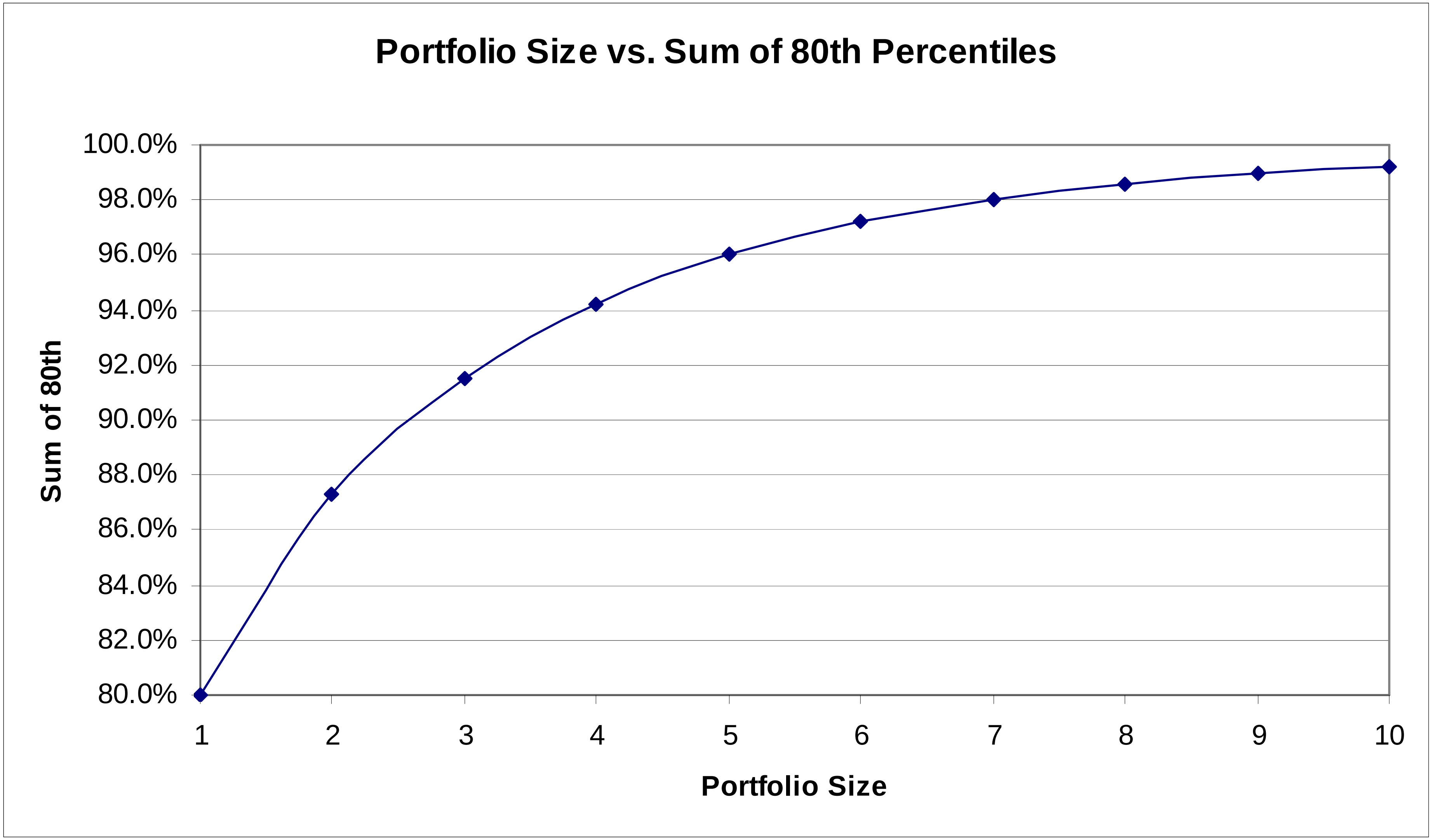
- Suppose that we hold the correlation constant ($\rho = 0.4$) and allow the number of programs (n) to vary
- Clearly, each program compounds the problem, pushing the percentile of the total ever-higher
- As with independence, large sample sizes make the problem worse:

n	Sum of 80th*
1	80.0%
2	87.3%
3	91.5%
4	94.2%
5	96.0%
6	97.2%
7	98.0%
8	98.6%
9	99.0%
10	99.3%

* Assumes 10 normally distributed program costs, with variances similar to that of a recent space program

The Price of Large Portfolios*

* Assumes ρ = 0.4



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Proposed Solution

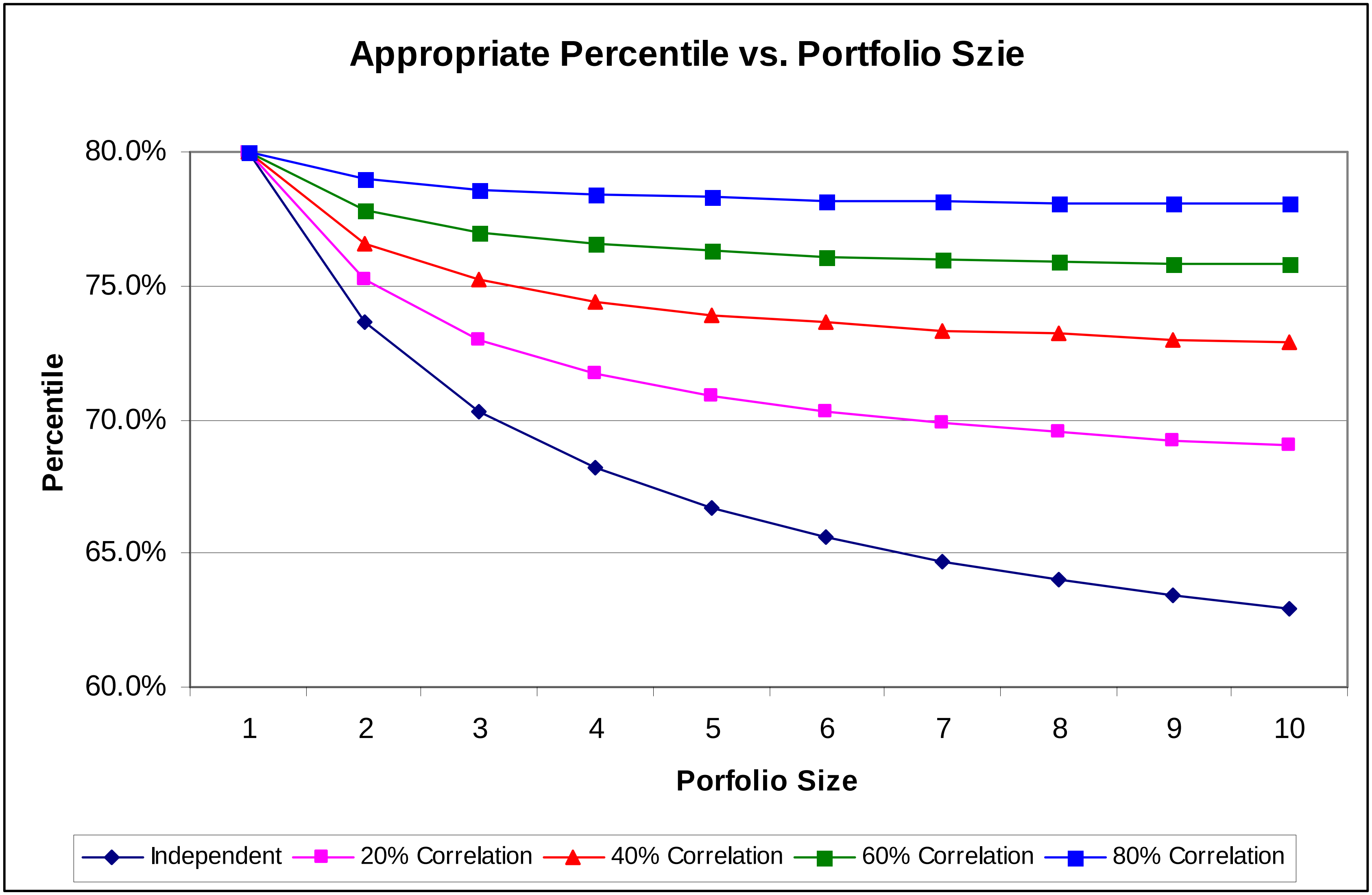
- Budget at a percentile, such that the sum of each of those percentiles equals the 80th percentile for the total portfolio
- This varies by:
  - Assumed distribution of each program
  - Portfolio size
  - Correlation among elements within programs
- However, the solution is **reliably between the 61st and 68th percentiles when the programs are independent**
  - Higher for correlated programs; lower for large portfolio sizes
- Tim Anderson (2004, 2006) provides the algebra behind the solution, but it is easy to program in Excel using *Goal Seek* or *Solver*—or in any other major risk or statistical package

Proposed Solution (Example)

Program	LB	ML	UB	μ	σ	Estimate	Percentile
1	1	2	5	2.67	0.85	2.99	0.66
2	1	2	5	2.67	0.85	2.99	0.66
3	1	2	5	2.67	0.85	2.99	0.66
4	1	2	5	2.67	0.85	2.99	0.66
5	1	2	5	2.67	0.85	2.99	0.66
TOTAL	**5**	**10**	**25**	**13.33**	**1.90**	**14.94**	**0.80**

In this example, the 66th percentile of each i.i.d triangular distribution (1,2,5) corresponds to the 80th percentile of the portfolio

Proposed Solution (General Case)



We're Done, Right?

- Anderson concludes (citation in *References*):

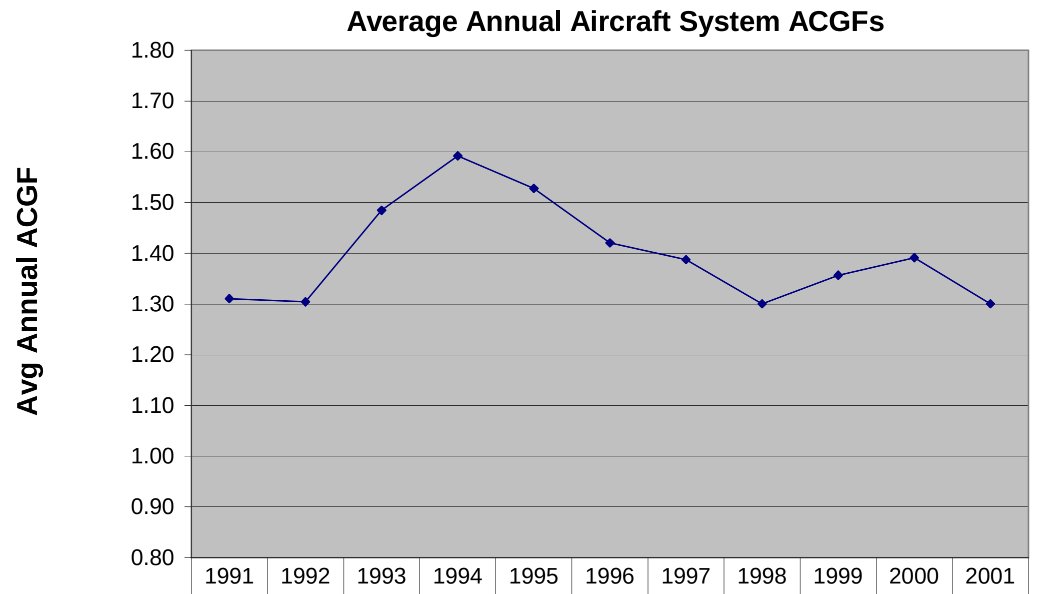
  it is inefficient to budget each program at its 80th percentile. *Too much money gets tied up. Moreover, given a limited budget, the decision-maker would likely have no choice but to cut programs that would probably do just fine if budgeted at a lower percentile. After all, by definition, each program has an 80% chance of coming in at or below its 80th percentile.*

- This assumes that there is no systemic downward-bias in program-level cost estimating. But the implications of this assumption are counterfactual:
  - Cost overruns and underruns would be equally likely
  - The average cost growth factor (CGF) would be consistently at or near 1.00

The Unfortunate Reality

- The unfortunate reality is that programs are several times more likely to overrun than underrun, and the average *annual* CGF for aircraft programs **has been reliably estimated at 1.30***

Does it make sense to advise budgeting at a lower percentile in this environment?



Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Avg Annual CGFs	1.31	1.30	1.48	1.59	1.53	1.42	1.39	1.30	1.36	1.39	1.30

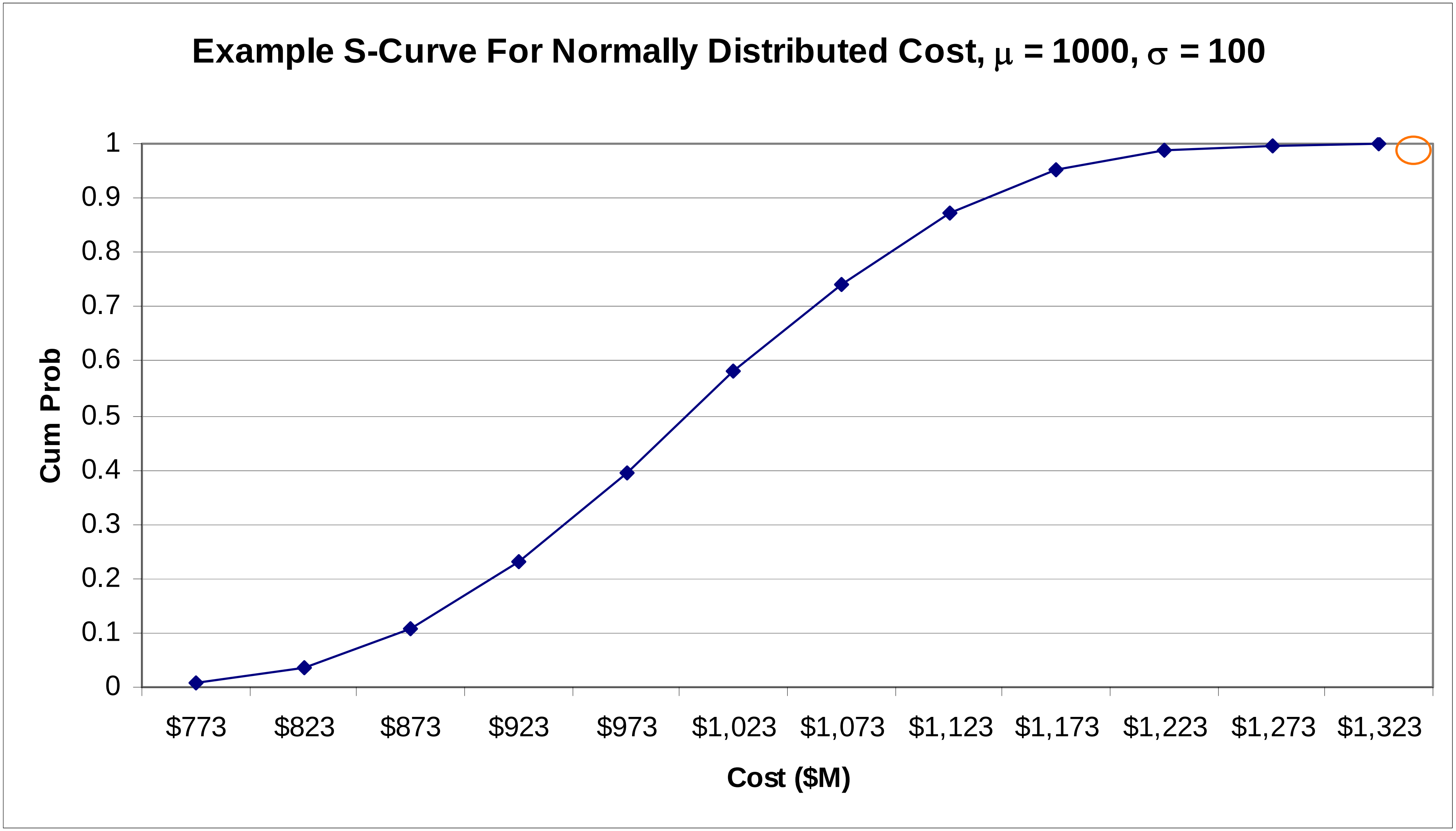
*Coleman et al (2004). Full citation in *References*

Where Does That Leave Us?

- Suppose that our point estimate (X~N(1000,100)) is 30% downward-biased
- Then the true point estimate is $1000M * 1.3 = $1300M
- Holding variance constant, the true 80th percentile is NORMINV(1300,100,.8) = $1384M
- This lies at the 99.99th percentile of our original N(1000,100) distribution, so that **98th percentile budgeting is not enough**
- These are conservative assumptions, because we typically underestimate the variance (not just the mean), and this ignores the fact that the 30% factor is an **annual** one

Unfortunate Reality: We can't afford *not* to budget at the 98th percentile

Even with Conservative Assumptions, The "True" 80th Percentile is Off The Chart...



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Conclusions & Recommendations

- Budgeting each program at the 80th percentile generally does not give an 80th percentile portfolio-level cost
- Instead, the cost is typically at a much higher percentile of the estimate (e.g. 98th)
- In an environment where cost overruns and underruns were equally likely, it would not make sense to budget this way
- But overruns are much more common than underruns, so in fact, we cannot afford *not* to budget this way
- Recommendations:
  - Short term: Continue to budget each program at 80th percentile
  - Long term: Improve CERs, so that estimated 80th percentile corresponds to actual 80th percentile; then implement Anderson recommendations

References

- Abate, Capt. Christopher; Coleman, Richard; Grenier, Maj. Michael; and Reynolds, Daniel. *An Analysis of Aircraft & Missile Systems Cost Growth and Implementation of Acquisition Reform Initiatives Using a Hybrid Adjusted Cost Growth Model*. Air Force Institute of Technology (AFIT) and TASC. Presented at SCEA National Conference, Los Angeles, CA (2004)
- Anderson, Timothy P. *The Trouble with Budgeting at the 80th Percentile*. Aerospace Corporation.
  - Presented at the Military Operations Research Society (MORS) Symposium, Monterey, CA (2004)
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- *Defense Science Board/Air Force Scientific Advisory Board Joint Task Force on Acquisition Of National Security Space Programs* (2003)

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