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## Don't Get Caught In the Learning Curve Vacuum

EXPLORING THE IMPACT OF PRODUCT COMMONALITY ON COST ESTIMATES

## PATRICK N. MCCARTHY

Manufacturing and production labor hours are frequently estimated using learning curve analysis. It is common practice to develop a projection of what the hours per unit will be for each unit of a production run given a learning curve method (Cum Average or Unit), learning curve slope and a theoretical first unit cost/hours. Labor hours are commonly estimated for a single end item and do not consider the similarity of the item in the estimate with other items that are being produced concurrently, have been produced recently or will be produced in the near future. By neglecting other end items in an integrated production environment, learning and subsequent gains in efficiency that may be taking place among the same personnel for similar items is often ignored or assessed independently. This similarity between end items is referred to as commonality. In this paper, we will address approaches for defining and quantifying commonality between end items. Alternative methods of accounting for commonality in cost estimates will also be reviewed by demonstrating various learning curve adjustments that can be made and how they impact a cost estimate. Applying these adjustments will also introduce us to another critical, yet often overlooked aspect of learning curve theory production steady state.
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## IN TRODUCTION

When performing direct labor hour estimates, the objective is to develop a mathematical projection of what will occur in the future. In particular, the goal is to develop a model that will project the number of hours per unit with consideration for efficiencies gained as work is completed. This modeling is achieved through the application of learning curve theory.

Learning curves have long been used in manufacturing and production environments to estimate the increased efficiency, and subsequent reduction in costs, as production quantities increase. In 1936, T.P. Wright introduced learning curve theory through his work in the aircraft industry by observing that in a manufacturing environment that remains relatively static in terms of key variables, there will be a constant rate of reduction in direct labor costs each time the quantities double. The result of Wright's research is referred to as the Cumulative Average Theory. An alternative learning curve theory, the Unit Curve Theory, was developed by J.R. Crawford.

While the Wright and Crawford approaches both capture the improvement in cost over time, there are a number of variables that can affect the amount of learning that will take place and, subsequently, the amount of cost improvement that can be anticipated. These variables include production rate, attrition, labor union agreements, production breaks and business base impacts. While substantial research has been conducted on learning curve modifications that can account for these variables, the characteristic that is often overlooked is the impact of an integrated production environment's business base on the product being estimated. Specifically, the commonality among both materials and processes that are being utilized for products being produced in parallel or series on the same production line often goes unaccounted for.

## EXAMPLES OF OVERLOOKED COMMONALITY

Although there are several different scenarios that exist where commonality among end items can be overlooked when estimating labor costs, this paper will focus on two scenarios particularly relevant to the defense industry:

1. Weapon Systems with Multiple Platforms or Variants Being Produced Concurrently
2. New Weapon System Production Integrated with a Legacy System

In the defense industry, these scenarios occur frequently. As with most products, a weapon system's usefulness and effectiveness will decline over time. In some cases, a weapon system that was designed, developed and produced within the last 10-15 years may no longer be relevant due to the evolving nature of mission requirements. A replacement weapon system may need to be developed and fielded to meet the new requirements. To avoid a gap in system availability, there will often be an overlap in the production ramp down of the legacy system and production ramp up of the new system. This overlap period can often last for several years. In addition, the new system may have several different variant packages that are built from a single platform to provide the warfighter with a family of systems that can provide a multitude of functions. Both scenarios, occurring either independently or simultaneously within the same production environment, can have a significant impact on the hours required to produce a weapon system.

## KEY QUESTIONS TO BE ASKED (AND ANSWERED) IN ADDRESSING COMMONALITY

Before we can attempt to account for commonality in direct labor estimates using learning curves, two key questions need to be addressed:

1) How will commonality be defined and subsequently measured for the production environment and the end items in question?
2) How will learning curve(s) for each of the end items under consideration be adjusted to account for the commonality as defined in the first question and where on the learning curve(s) will the adjustment take place?

In the next two sections, we will explore considerations that will provide critical insight into how we address these questions.

## COMMONALITY DEFINED

We will begin our investigation into more objective approaches to commonality by looking at two distinctly different methods of defining commonality and integrating it into cost estimates - product commonality and process commonality. We will take a closer look at what differentiates these two approaches and also offer some pros and cons for each type concerning their application to learning curves in integrated production systems.

## Product Commonality

Product commonality considers the standardization of components, materials and subassemblies used to produce an end item.

## 1) PRODUCT COMMONALITY PROS

This type of commonality can range in complexity in terms of how it is calculated. Ahmed, Wazed and Yusoff (1) provide several different approaches for measuring product commonality that range in complexity. In the defense industry, the kinds and quantities for all components included in a Bill of Material (BOM) are usually established prior to production and can be easily analyzed and compared to those of other end items. In its most simplistic form, product commonality can be calculated as shown in Table 1 for a hypothetical weapon system consisting of six variants:

Table 1

|  | Variant 1 | Variant 2 | Variant 3 | Variant 4 | Variant 5 | Variant 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Total Parts in BOM | 980 | 1,020 | 1,035 | 1,100 | 1,134 | 1,190 |
| Parts Common to All Variants | 420 | 420 | 420 | 420 | 420 | 420 |
| \% of Common Parts in BOM | $42.9 \%$ | $41.2 \%$ | $40.6 \%$ | $38.2 \%$ | $37.0 \%$ | $35.3 \%$ |

a. The most significant issue with using product commonality is that a high number of low-priced, smaller items (e.g. nuts, bolts, washers, clamps, pins) can potentially constitute a large percentage of a weapon system's BOM. It is advantageous for a manufacturer to design their products with these types of materials in common in order to benefit from bulk ordering and the economies of scale; however, these parts can have limited impact on what makes a particular variant common or unique from an assembly perspective and utilizing them to develop a commonality index or percentage between multiple end items can be misleading.
b. The second issue with product commonality is that even after developing a commonality percentage or index for each variant, finding a suitable method for applying these percentages to learning curves can be challenging. We will discuss this challenge in this paper in one of our examples.

## Process Commonality

Process commonality considers the standardization of the machines, tools and production processes used to produce an end item.

1) PROCESS COMMONALITY PROS

Identifying the commonality of direct labor assembly and manufacturing processes will give clear understanding of how much common and unique work content is required to assemble each end item.

The work content referred to above is typically defined in the form of standard hours or Budgeted Work Standards (BWS). Various definitions exist on what exactly a time standard should represent. One definition of the work standard is that it should be considered a performance or efficiency ceiling for a particular task. For example, let's say a task is assigned a standard hour content of 7.62 hours per unit. According to this definition, 7.62 hours is the minimum anticipated amount of time we should expect this task to be performed in at any point of the production lifecycle.

Another definition of work standards is one that defines the time to complete a particular task when a worker is performing at $100 \%$ efficiency. This definition can vary among organizations. Some experts contend that 100\% efficiency is impossible to achieve. The argument is based on the concept that if actual hours recorded indicate a task's observed efficiency is $100 \%$ or greater, than the most likely cause is that the standard is too high and needs to be adjusted.

Regardless of the definition used, utilizing standard hour content to identify task-centric labor requirements is highly beneficial to identifying process commonality and as we will observe later in this paper, provides an opportune integration of the commonality assessment into learning curve models.

## 2) PROCESS COMMONALITY CONS

In order to accurately capture this type of commonality, work measurement must be performed for each variant in order to determine the amount of work content associated with discrete tasks and elements of the production process. The work measurement could be in the form of time studies, Predetermined Motion Time Systems (PMTS) or work sampling. Ideally this work measurement will take place simultaneously or after the development of work instructions so that work content can be measured for specific sub-elements or tasks. While this type of assessment is what ultimately provide the work standards addressed above, it can be time consuming and often requires highly-skilled professionals from various disciplines including Industrial Engineering, Manufacturing Engineering and Production Management to provide such analyses.

Based on the pros and cons discussed above, it is highly recommended to utilize process commonality whenever possible. While the work measurement methods associated with the process commonality analysis can be laborious, there is tremendous benefit to having detailed, accurate assessments of the work content that is required for each task associated with an assembly process. However, before we can begin to address the benefits of utilizing process commonality, we must first address how the commonality will be integrated into the learning curve models.

## ADJUSTING THE LEARNING CURVES TO ACCOUNT FOR COMMONALITY

As discussed earlier, the goal of using a learning curve model is to capture what is occurring, or has occurred, in a production environment and mathematically depict how the environment will behave in terms of efficiency gains due to learning. In order to apply commonality impacts to multiple products, it will be necessary to consider the estimated labor hours required for the different products while they are at similar points of the their production lifecycles.

## Production Steady State

One common phenomenon that occurs in production environments is when the system reaches a steady state and direct labor hours required remain constant. As Kar points out (2), the labor hour "decrease cannot continue indefinitely and eventually saturation would be expected to take place". As with commonality, it is important to define what a steady state in a production environment means, why it happens and what its impact will be on the cost estimate.

While no one definition exists for production steady state, it is generally safe to describe a process in steady state as one where direct labor hours required to produce an end item have plateaued over a sustained period of time and there is low variability in direct labor hours from unit-to-unit.
In order to understand why this plateau is reached, we must look in a broader context of what occurs in a production environment rather than simply assume the workforce has reached its maximum efficiency. There are often several factors that can cause the steady state to be reached. Before we discuss some of these factors, let's introduce this concept using an example.

## Steady State Example

Assume 750 units of a new weapon system are to be produced. Based on past data for new production launches, an $85.0 \%$ slope is assumed using the unit learning curve method to estimate the direct labor hours. The first unit, or T1, has been produced using 1,241 direct labor hours. The plant where production will occur has noticed that their improvement typically ceases around the $125^{\text {th }}$ unit. In Figure 1, we provide a look at two learning curves using unit curve theory - one that depicts continuous learning for all 750 units and one where learning stops at the $125^{\text {th }}$ unit.

Figure 1 - Production Steady State


What could cause the plateauing of the learning curve? As shown above in the shaded area, the learning curve model projection would lead us to believe that substantial learning could still take place and costs could continue to decrease after the $125^{\text {th }}$ unit. While it is usually assumed that several variables are held static during the production process being estimated (e.g. tooling, machines, design) there are other uncontrollable variables that can impact efficiency gains. These variables include promotions, attrition, business base fluctuations as well as other variables within the production dynamic that can impact personnel and learning. We depict these in Figure 2 below to counter the efficiency gains projected by the learning curve model:

Figure 2


As the top half of Figure 2 shows, while the learning curve model may predict that learning should continue on indefinitely, the variables depicted in the lower half of Figure 2 are often unpredictable and can contribute toward the production environment ending up in a state of equilibrium.

## APPROACH/METHODOLOGY

Now that the two key questions and aspects of the model have been defined, we will discuss how to combine commonality and production steady state into a single estimating application. First, we will assume the use of unit curve theory and introduce the variables that will be used:
$\mathrm{V}_{\mathrm{n}}=$ Weapon System Variant $n$
LCS $_{n}=$ Learning Curve Slope for Variant $n$
LCS $_{c}=$ Common Learning Curve Slope
$X_{\text {ss }}=$ Unit $X$ at Which Production Steady State Occurs
$Y_{\text {ssc }}=$ Hours of Steady State Work Content Common to all Variants $V_{1}$ through $V_{n}$ for $n=1,2 \ldots n$ (For Process Commonality)
$\mathrm{Y}_{\mathrm{SSCV} n \ldots \mathrm{n}}=$ Hours of Steady State Work Content Common to any subset of Variants $\mathrm{V}_{1}$ through $V_{n}$ for $\mathrm{n}=1,2 \ldots n$ (For Process Commonality)
$Y^{\prime}{ }_{s s n}=$ Total Hours of Steady State Work Content Unique to Individual Variants $V_{1}$ through $V_{n}$ for $n=1,2 \ldots n$

## Common Curve Development

The first step in accounting for commonality is to define the learning curve(s) that will account for work content common to all variants or sub-sets of variants. As defined above, $Y_{\text {ssc }}$ represents the work content common to all variants at unit $X_{s s}$. The work content $\left(Y_{n}\right)$ can be developed for each unit $X_{n}$ on the common learning curve as:

$$
\begin{gathered}
\mathrm{Y}_{\mathrm{SSXn}}=\frac{Y_{S S C}}{X_{S S}{ }^{\log \left(\mathrm{LCS}_{C}\right) / \log (2)}} \times X_{n}^{\log \left(\operatorname{LCS}_{C}\right) / \log (2)}, \text { for } \mathrm{X}_{\mathrm{n}}<\mathrm{X}_{\mathrm{SS}} \\
\mathrm{Y}_{\mathrm{SSXn}}=\mathrm{Y}_{\mathrm{SSC}}, \text { for } \mathrm{X}_{\mathrm{n}} \geq \mathrm{X}_{\mathrm{SS}}
\end{gathered}
$$

In addition to the curve that represents work content among all variants, we may be interested in developing commonality curves for subsets of variants when there are three or more variants under consideration. For example, there may be some commonality that exists in two out of three variants, three out of four variants or even nine out of ten variants. The math for developing the learning curve will be the same as above, however, we will define the common steady state hours for this subset as:
$\mathrm{Y}_{\text {SSCVn...n }}$, For $\mathrm{V}_{\mathrm{n} . . . \mathrm{n}}=$ Any combination of variants $\mathrm{V}_{1} \ldots \mathrm{~V}_{\mathrm{n}}$ greater than one and less than $n$

For example, for a weapon system with four variants, possible alternatives of commonality subsets are $Y_{\text {SSCV1,2 }}, Y_{\text {SSCV1,3 }}$,


## Variant Unique Curve Development

The next step in accounting for commonality is to extract all common work content from individual variant curves and generate curves for variant unique work content. For each individual Variant $n$, we start by generating the amount of steady state work content $\mathrm{Y}_{\mathrm{SS} 1-\mathrm{n}}$ at unit $\mathrm{X}_{\mathrm{ss}}$. This can either be arrived at by using a known first unit value ( T 1 ) for a particular variant coupled with the variant slope $\mathrm{LCS}_{n}$ or the steady state value may be arrived at simply by using the standard hour content for that variant if it is assumed the steady state work content is one in the same. We then remove the work content common to all variants ( $\mathrm{Y}_{\mathrm{ssc}}$ ) as well as the commonality accounted for in any subsets that include the variant in question:

$$
\mathrm{Y}^{\prime}{ }_{\text {SS1-n }}=\mathrm{Y}_{\text {SS1-n }}-\mathrm{Y}_{\text {SSC }}-\left(\sum_{S S S V x, \ldots, n}\right)
$$

The work content $\left(Y^{\prime}{ }^{\prime}{ }_{n}\right)$ can be developed for each unit $X_{n}$ on the unique learning curve for a specific variant as:

$$
\begin{gathered}
Y^{\prime} X_{n}=\frac{Y_{S S C}}{X_{S S}{ }^{\log \left(L C S S_{n}\right) / \log (2)}} \times X_{n}{ }^{\log \left(\operatorname{LCS}_{n}\right) / \log (2)} \text {, for } X_{n}<X_{s S} \\
Y^{\prime} X_{n}=Y^{\prime}{ }_{S S n}, \ldots, n, \text { for } X_{n} \geq X_{s S}
\end{gathered}
$$

## Compilation of Hours

Now that all common work content has been accounted for and the work content that makes each variant unique has been isolated, we must determine how to compile all of the hours in order to complete the direct labor cost estimate.

The first thing that needs to be done is to reference the anticipated production schedule in order to see how many units are anticipated for each variant. The reason this is necessary is so we know how many units should be accounted for in each learning curve. For instance, the curve common to all variants should account for all units in the estimate. Any common work content curves made up of subsets of variants should account for the total combined number of units for all variants in the subset.

If we are only interested in the total number of hours, we can simply use the sum of all hours estimated in all curves developed. If we are interested in calculating hours for specific variants, the analysis takes us back to the schedule. The reason for this is so that we can determine which variants' hours will be impacted by the learning impact on the common curve(s) depending on which variants are being produced earlier in the production schedule. We will address this technique as well as attempt to gain a better understanding of how these methodologies impact the cost estimate below in Example 1.

## EXAMPLE 1

In our first example, we examine a ground vehicle consisting of three distinct variants. Historical data exists that recommends a learning curve slope of $85.0 \%$ and that the steady state for new programs in this production environment are typically reached around the $150^{\text {th }}$ unit. Work instructions have been developed for these variants and three prototypes units have been completed for each variant. Using the work instructions, a hybrid approach of high-level time studies and PMTS, budgeted work standards have been developed for all three variants. The integrated production schedule for the weapon system is as follows in Table 2:

Table 2

| Month | Variant 1 | Variant 2 | Variant 3 | Monthly Total | Cum Total |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 6 | 3 | 2 | 11 | 11 |
| 2 | 12 | 5 | 3 | 20 | 31 |
| 3 | 15 | 6 | 4 | 25 | 56 |
| 4 | 21 | 14 | 11 | 46 | 102 |
| 5 | 21 | 15 | 11 | 47 | 149 |
| 6 | 21 | 15 | 12 | 48 | 197 |
| 7 | 21 | 15 | 12 | 48 | 245 |
| 8 | 21 | 15 | 12 | 48 | 293 |
| 9 | 21 | 15 | 12 | 48 | 341 |
| 10 | 21 | 15 | 12 | 48 | 389 |
| 11 | 21 | 15 | 12 | 48 | 437 |
| 12 | 21 | 15 | 12 | 48 | 485 |
| 13 | 21 | 15 | 12 | 48 | 533 |
| 14 | 21 | 15 | 12 | 48 | 581 |
| 15 | 21 | 15 | 12 | 48 | 629 |
| 16 | 21 | 14 | 11 | 46 | 675 |
| 17 | 21 | 14 | 10 | 45 | 720 |
| 18 | 8 | 4 | 3 | 15 | 735 |
| Total | 335 | 225 | 175 | 735 | 735 |

The work measurement analysis resulted in the following steady state hours of common work content among variants:
Table 3

|  | V1 | V2 | V3 |
| :--- | :---: | :---: | :---: |
| Steady State Hours | 216.6 | 223.9 | 250.6 |
| Hours Common to 1,2,3 | 135.0 | 135.0 | 135.0 |
| Hours Common to 1,2 | 35.0 | 35.0 | 0.0 |
| Hours Common to 1,3 | 29.0 | 0.0 | 29.0 |
| Hours Common to 2,3 | 0.0 | 31.0 | 31.0 |
| Variant Unique Hours | 17.6 | 22.9 | 55.6 |

We can now begin to apply the commonality application discussed earlier. Let us begin by introducing the variables:
$\mathrm{V}_{1}=$ Personnel Carrier Variant
$\mathrm{V}_{2}=$ Combat Variant
$\mathrm{V}_{3}=$ Ambulance Variant
$\mathrm{LCS}_{1}=\mathrm{LCS}_{2}=\mathrm{LCS}_{3}=\mathrm{LCS}_{\mathrm{C}}=88.0 \%$
$X_{\mathrm{ss}}=150$
$\mathrm{Y}_{\mathrm{ssc}}=135 \mathrm{Hrs}$.
$Y_{\mathrm{sscv} 1,2}=35 \mathrm{Hrs}$.
$Y_{\mathrm{sscv} 1,3}=29 \mathrm{Hrs}$.
$Y_{\mathrm{sscv} 2,3}=31 \mathrm{Hrs}$.

We will start by generating learning curves for each of the variants without commonality using unit curve theory and the aforementioned parameters as seen below in Figure 3:

Figure 3


The learning curves for all three variants enter into the steady state at unit 150 and the total estimated hours are $196,972.7$. The next step is to extract the common work content from learning curves above and generate the common curves.

For $\mathrm{Y}_{\mathrm{SSC}}=135 \mathrm{Hrs} ., \mathrm{LCS}_{\mathrm{C}}=88.0 \%$ and $\mathrm{X}_{\mathrm{SS}}=150$, we generate a curve for 735 units:
Figure 4


For $\mathrm{Y}_{\mathrm{SSCV} 1,2}=35 \mathrm{Hrs} ., \mathrm{Y}_{\mathrm{SSCV} 1,3}=29 \mathrm{Hrs} ., \mathrm{Y}_{\mathrm{SSCV} 2,3}=31 \mathrm{Hrs} .$, we generate curves for 560,510 and 400 units respectively using $L_{C S}=88.0 \%$ and $X_{s s}=150$ :

Figure 5


The last step is to generate our learning curves for work content that is unique to each variant. We start by calculating the steady state hour requirements for each of the three variants, which we will refer to as $Y^{\prime}{ }_{s s 1}, Y^{\prime}{ }_{s s 2}, Y^{\prime}{ }_{s s 3}$ :
$Y^{\prime}{ }_{S S 1}=Y_{S S 1}-Y_{S S C}-Y_{S S C V 1,2}-Y_{S S C V 1,3}=216.6-135.0-35.0-29.0=17.6 \mathrm{Hrs}$
$Y^{\prime}{ }_{S S 2}=Y_{S S 2}-Y_{S S C}-Y_{S S C V 1,2}-Y_{S S C V 2,3}=223.9-135.0-35.0-31.0=22.9 \mathrm{Hrs}$
$Y^{\prime}{ }_{\mathrm{SS} 3}=Y_{\mathrm{SS} 3}-Y_{\mathrm{SSC}}-Y_{\mathrm{SSCV} 1,3}-Y_{\mathrm{SSCV} 2,3}=250.6-135.0-29.0-31.0=55.6 \mathrm{Hrs}$

Based on these values and the slopes discussed above, the resulting learning curves using unit curve theory are presented in Figure 6:

Figure 6


It is worth noting that while the same slopes were used for these variant unique curves that were used for the original curves for Variants 1, 2 and 3, it is at this point in the process that the slopes should be revisited and validated based on the work scope remaining. For instance, after extracting all common work content for Variant 1, the remaining 17.6 hours per unit of steady state work content may consist of mostly the machining and fabrication of components. This type of work will typically have a higher slope percentage than general assembly labor. This scenario could also be true if the work consisted of highly specialized assembly processes, welding or paint techniques. Scenarios such as these should be reviewed on a case-by-case basis with slope modifications being made as necessary.

Now that all of the commonality has been accounted for and the learning curves have been generated, we can complete our direct labor estimate. If we are interested in just the total hours of all variants, the hours for each of the seven curves generated can be added together for a total of 181,093.3 hours. However, if we are interested in estimating direct labor hours requirements by variant, we must consider the production schedule. The first step is to consider how many units worth of learning will be realized on each curve on a monthly basis. For example, the curve that accounts for commonality across all three variants will experience eleven units worth of learning in the first month (Variant 1-6 Units, Variant 2 - 3 Units, Variant 3 - 2 Units). Given that $3,359.3$ total hours are expected to be required for these units, we assume an HPU of 305.4 for that curve in month one. Using this methodology, we create HPUs for each curve for each month in Table 4:

## Table 4

| Month | Common 1,2,3 <br> Curve HPU | Common 1,2 <br> Curve HPU | Common 1,3 <br> Curve HPU | Common 2,3 <br> Curve HPU | Unique <br> Variant 1 <br> Curve HPU | Unique <br> Variant 2 <br> Curve HPU | Unique <br> Variant 3 <br> Curve HPU |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 305.4 | 82.3 | 69.7 | 80.9 | 44.4 | 64.9 | 166.5 |
| 2 | 215.3 | 58.2 | 49.6 | 59.7 | 31.8 | 49.2 | 130.8 |
| 3 | 180.7 | 48.8 | 41.5 | 50.8 | 26.6 | 42.0 | 112.6 |
| 4 | 157.3 | 42.7 | 36.3 | 43.6 | 23.5 | 36.4 | 96.0 |
| 5 | 140.9 | 38.5 | 32.7 | 38.3 | 21.4 | 32.1 | 84.0 |
| 6 | 135.0 | 35.8 | 30.4 | 35.2 | 20.0 | 29.6 | 77.0 |
| 7 | 135.0 | 35.0 | 29.1 | 33.0 | 19.0 | 27.8 | 72.1 |
| 8 | 135.0 | 35.0 | 29.0 | 31.4 | 18.2 | 26.5 | 68.5 |
| 9 | 135.0 | 35.0 | 29.0 | 31.0 | 17.6 | 25.5 | 65.7 |
| 10 | 135.0 | 35.0 | 29.0 | 31.0 | 17.6 | 24.6 | 63.4 |
| 11 | 135.0 | 35.0 | 29.0 | 31.0 | 17.6 | 23.9 | 61.5 |
| 12 | 135.0 | 35.0 | 29.0 | 31.0 | 17.6 | 23.3 | 59.8 |
| 13 | 135.0 | 35.0 | 29.0 | 31.0 | 17.6 | 22.9 | 58.4 |
| 14 | 135.0 | 35.0 | 29.0 | 31.0 | 17.6 | 22.9 | 57.1 |
| 15 | 135.0 | 35.0 | 29.0 | 31.0 | 17.6 | 22.9 | 56.0 |
| 16 | 135.0 | 35.0 | 29.0 | 31.0 | 17.6 | 22.9 | 55.6 |
| 17 | 135.0 | 35.0 | 29.0 | 31.0 | 17.6 | 22.9 | 55.6 |
| 18 | 135.0 | 35.0 | 29.0 | 31.0 | 17.6 | 22.9 | 55.6 |

The last step is to apply this average HPU for each month and curve to the number of units planned for each of the three variants by month. For example, we can expect Variant 1 to require 4,646 total hours in the sixth month ( 21 units $x$ $(135.0+35.8+30.4+20.0)$ ). In Table 5, we show the complete table of hours required by variant.

## Table 5

|  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Month | Variant 1 | Variant 2 | Variant 3 | Total |
| 1 | $3,010.3$ | $1,600.4$ | $1,244.8$ | $6,682.6$ |
| 2 | $4,258.7$ | $1,912.2$ | $1,366.3$ | $8,166.8$ |
| 3 | $4,464.0$ | $1,933.9$ | $1,542.4$ | $8,505.4$ |
| 4 | $5,456.2$ | $3,919.9$ | $3,665.3$ | $13,587.2$ |
| 5 | $4,902.6$ | $3,746.8$ | $3,254.6$ | $12,450.8$ |
| 6 | $4,646.2$ | $3,533.4$ | $3,331.2$ | $12,082.8$ |
| 7 | $4,579.8$ | $3,462.5$ | $3,230.7$ | $11,883.0$ |
| 8 | $4,561.7$ | $3,419.3$ | $3,167.7$ | $11,801.4$ |
| 9 | $4,549.5$ | $3,397.1$ | $3,128.7$ | $11,773.0$ |
| 10 | $4,547.6$ | $3,384.3$ | $3,101.1$ | $11,777.5$ |
| 11 | $4,547.6$ | $3,373.4$ | $3,078.0$ | $11,791.0$ |
| 12 | $4,547.6$ | $3,364.1$ | $3,058.2$ | $11,809.5$ |
| 13 | $4,547.6$ | $3,359.0$ | $3,040.8$ | $11,835.3$ |
| 14 | $4,547.6$ | $3,359.0$ | $3,025.5$ | $11,868.7$ |
| 15 | $4,547.6$ | $3,359.0$ | $3,011.9$ | $11,904.0$ |
| 16 | $4,547.6$ | $3,135.1$ | $2,756.4$ | $11,472.1$ |
| 17 | $4,547.6$ | $3,135.1$ | $2,505.8$ | $11,268.5$ |
| 18 | $1,732.4$ | 895.7 | 751.7 | $4,477.0$ |
| Total | $78,541.8$ | $54,290.3$ | $48,261.2$ | $181,828.3$ |
|  |  |  |  |  |

As you can see the total hours required for the program is still 181,828.3. As a reference, let's compare these totals with those from the original curves where a steady state was assumed, but commonality was not considered. We will also consider what the totals would have been had we recognized neither commonality nor steady state.

Table 6

| Scenario | Total Hours |
| :---: | :---: |
| No Commonality, No Steady State | $191,931.29$ |
| Steady-State, No Commonality | $196,972.68$ |
| Commonality and Steady-State | $181,093.34$ |

Compared to the initial curves without commonality or a steady state, the commonality results in a $5.65 \%$ reduction in total hours. The commonality curve results in an $8.06 \%$ reduction in total hours when compared to the curve that only accounts for a steady state.

Before moving on to the next example, we will also address the challenge discussed earlier regarding product commonality. Suppose time studies and detailed work measurement were not an option in determining process commonality and we had to rely on the following material data to identify product commonality:

Table 7

|  | Variant 1 | Variant 2 | Variant 3 |
| :--- | :---: | :---: | :---: |
| Total Parts in BOM | 1,100 | 1,190 | 1,275 |
| Parts Common to All Variants | 700 | 700 | 700 |
| \% of Common Parts in BOM | $63.6 \%$ | $58.8 \%$ | $54.9 \%$ |
| Parts Common to Variants 1,2 | 120 | 120 | 0 |
| \% of Common Parts in BOM | $10.9 \%$ | $10.1 \%$ | $0.0 \%$ |
| Parts Common to Variants 1,3 | 80 | 0 | 80 |
| \% of Common Parts in BOM | $7.3 \%$ | $0.0 \%$ | $6.3 \%$ |
| Parts Common to Variants 2,3 | 0 | 92 | 92 |
| \% of Common Parts in BOM | $0.0 \%$ | $7.7 \%$ | $7.2 \%$ |
| Unique Variant Parts | 200 | 278 | 403 |
| \% of Common Parts in BOM | $18.2 \%$ | $23.4 \%$ | $31.6 \%$ |

We can see that product commonality tells us that Variants 1, 2 and 3 will have $18.2 \%, 24.3 \%$ and $31.6 \%$ of unique work content respectively. The rest of the work content falls into a commonality classification of some sort. What is not clear is how to extract this commonality from learning independent variant curves and create curves that account for the commonality. For instance, we knew from work measurement that the steady state hours for the curve representing the learning associated with commonality across all three variants was 135.0 hours per unit and could be subtracted from each of the original curves for all three variants. When considering materials, it is difficult to perform this same exercise since each the 700 parts that are common across all three variants represent a different percentage of each variant's BOM. This is true for each subset of commonality as well. Between the uncertain extraction process coupled with the potential for materials being misleading in terms of work content, product commonality should be avoided if at all possible.

## EXAMPLE 2

This example looks at a legacy program that is still in full-rate production, but will soon be phased out by a replacement program that will eventually produce 400 units. It is anticipated that the legacy program's full-rate production will overlap with the low-rate initial production of the replacement system. The legacy program has been at steady state for several years and consistently requires 1,000 hours per unit. Analysis of historical data reveals that this program reached steady state at the $200^{\text {th }}$ unit. Learning typically occurs at an $82.5 \%$ rate for this production facility. Work measurement studies have revealed that the replacement system will require 1,200 hours per unit of which 300 hours is common with the legacy system.

As with the first example, we begin by developing the learning curve for the replacement system (Note - a curve for the legacy system is unnecessary, as we know that system is already in steady state):

Figure 7

| Replacement System Hours |
| :---: |
| 6000 |
|  |
|  |  |
|  |  |
|  |
|  |

Extracting the 300 hours from the curve to account for commonality and applying the same approach from Example 1 results in the following curve:

Figure 8


Finally, we can view our final hour requirements for the replacement system, broken out by common and unique work content:

## Figure 9



Accounting for commonality results in a reduction of 22,073.7 hours (3.9\%) as compared to the base estimate for the replacement system.

## MODEL LMITATIONS AND OTHER CONSIDERATIONS

While this paper has attempted to heighten awareness of the impact of commonality on learning curve estimates, as with many other cost estimating tools and methodologies, there is not always going to be one correct answer or approach to performing the estimate. In this section, we detail some of the challenges and limitations of the model we have presented.

## Where to Draw the Line on Commonality Analysis

The first challenge with incorporating commonality into cost estimates is knowing how much commonality to try and account for. For example, if we are looking at only two end items, that answer is simple - we are interested in the commonality between only those two end items. What about if we are looking at three end items? Are we interested in the commonality that is present only across all three end items or are we also interested in the commonality across each combination of two of the three variants? What if we are looking at a system with ten variants? Before trying to answer that question, let's look at how quickly the number of combinations grows where commonality exists as a function of the number of variants. In Table 8, we present a table that identifies all possible combinations where commonality could be considered by calculating the number of subsets of size $s$ that can be selected from a set of $n$ end items using the equation:

$$
\frac{n!}{(s!*(n-s)!)}
$$

## Table 8

## Possible Combinations For Various End Item Quantities

|  |  | Number of End Items ( $n$ ) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $n>10$ |
|  | 2 | 1 | 3 | 6 | 10 | 15 | 21 | 28 | 36 | 45 | $\frac{n!}{(2!*(n-2)!)}$ |
|  | 3 | - | 1 | 4 | 10 | 20 | 35 | 56 | 84 | 120 | $\frac{n!}{(3!*(n-3)!)}$ |
|  | 4 | - | - | 1 | 5 | 15 | 35 | 70 | 126 | 210 | $\frac{n!}{(4!*(n-4)!)}$ |
|  | 5 | - | - | - | 1 | 6 | 21 | 56 | 126 | 252 | $\frac{n!}{(5!*(n-5)!)}$ |
|  | 6 | - | - | - | - | 1 | 7 | 28 | 84 | 210 | $\frac{n!}{(6!*(n-6)!)}$ |
|  | 7 | - | - | - | - | - | 1 | 8 | 36 | 120 | $\frac{n!}{(7!*(n-7)!)}$ |
|  | 8 | - | - | - | - | - | - | 1 | 9 | 45 | $\frac{n!}{(8!*(n-8)!)}$ |
|  | 9 | - | - | - | - | - | - | - | 1 | 10 | $\frac{n!}{(9!*(n-9)!)}$ |
|  | 10 | - | - | - | - | - | - | - | - | 1 | $\frac{n!}{(10!*(n-10)!)}$ |
|  | $s>10$ | - | - | - | - | - | - | - | - | - | $\frac{n!}{(s!*(n-s)!)}$ |

As you can see, the number of possible combinations grows dramatically as the number of end items and potential subsets increases. If we were looking to account for commonality in an integrated production environment with ten end items, it is doubtful we would truly be interested in several hundred subsets of commonality as well as developing the all of the subsequent learning curves. As a general rule of thumb, it is probably best to keep the commonality assessment manageable in terms of development, maintenance and application.

Another approach to keep in mind is to identify opportunities to reduce the end items into more manageable buckets. For instance, maybe each of the ten end items from the table above can be easily classified into two or three broader categories or platforms. Doing so would enable the estimate to still account for commonality, but in a much more manageable amount of time.

## Impact of Schedule and Rate

Another key thing to consider is the impact that production schedule will have on the model. For instance, Example 1 dealt with an integrated production environment where all variants under consideration were being produced every month and in reasonable sized quantities at that. What if that was not the case. What if we were considering a system where a variant had a production break of 6 months? Or 12 months? In scenarios such as these, additional considerations must be made towards how the common and individual curves are developed. For instance, if a particular variant had a reasonably long break in production, the curve accounting for that variant's work content may need to have a loss of learning application modeled into it such as the method proposed by Anderlohr (3) or something similar.

## CONCLUSIONS

Throughout this paper, we have explored two very common occurrences in integrated production environments commonality across end items and production steady state - but that are often overlooked when modeling learning curves. The methodology proposed for identifying commonality utilizes a very common production practice in work measurement. The identification of production steady state is less prevalent in industry. As described in this paper, the identification of a steady state will rely not just on any one discipline. In order to accurately gauge when steady state was reached for previous programs and what was going on directly and indirectly with a production process, subject matter expertise should be sought from several areas including, but not limited to, Production Management, Industrial Engineering, Human Resources, Quality, Design Engineering and others. The identification of a specific technique or techniques for identifying production steady state certainly warrants further research. In addition, identifying relevant techniques for applying product commonality or other alternatives when work measurement is not feasible also deserve substantial consideration for future research.

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## BIOGRAPHY

Pat McCarthy, an Associate with Booz Allen Hamilton, has over 14 years of experience in Federal and private industry performing cost analyses and Industrial Engineering studies. He has previously worked as a Pricing Analyst and Team Chief with the Army Contracting Command and as a Senior Industrial Engineer and Team Leader with General Dynamics Land Systems. Pat is CCEA certified and holds B.S. and M.S. degrees in Industrial Engineering from Purdue University.

