Case Study: A Parametric Model for the Cost per Flight Hour

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Acronyms

AAP	Allied Administrative Publication
AFMC	Air Force Materiel Command (US Air Force)
AIC	Akaike Information Criterion
ALCCP	Allied Life Cycle Cost Publication
CALS	Continuous Acquisition and Lifecycle Support
CAPE	Cost Assessment and Program Evaluation (US)
CER	Cost Estimating Relationship
CI	Confidence Interval
COTS	Commercial-Off-The-Shelf
CTOL	Conventional Takeoff and Landing
CPFH	Cost per Flight Hour
CRUA	Cost Risk and Uncertainty Analysis
DAU	Defense Acquisition University (US)
DoD	Department of Defense (US)
DoDCAS	DoD Cost Analysis Symposium
FAA	Federal Aviation Administration
HAF	Hellenic Air Force
ISPA	International Society of Parametric Analysts
JSF	Joint Strike Fighter
LCC	Life Cycle Cost
LCM	Life Cycle Management
MEDEVAC	Medical Evacuation
MTOW	Maximum Takeoff Weight
MUPE	Minimum Unbiased Percentage Error
NASA	National Aeronautics and Space Administration (US)
NATO	North Atlantic Treaty Organization
OLS	Ordinary Least Squares
O&S	Operating and Support
OSD	Office of the Secretary of Defense (US)
PI	Prediction Interval
RDT&E	Research, Development, Test, and Evaluation
RMS	Reliability-Maintainability-Supportability
ROM	Rough Order of Magnitude
SAR	Search and Rescue
SCEA	Society of Cost Estimating and Analysis
SFC	Specific Fuel Consumption
VAMOSC	Visibility and Management of Operating and Support Costs (US Navy)
ZMPE	Zero Bias Minimum Percent Error

Introduction

The Hellenic Air Force (HAF)'s mission[1] is to organize, staff, mobilize, and train its personnel, in order to develop an air power capable of dissuasion, intensive and prolonged air operations, obtaining and retaining air superiority, securing the air defense of the country, and providing air protection and support to ground and maritime operations. During peacetime, HAF also conducts public service operations supporting many aspects of public interest, such as fire-fighting, search and rescue (SAR), air transports and medical evacuations (MEDEVAC).

Equipment

Aircraft

Fighters

- F-16C/D Blk30, 50 Fighting Falcon
- F-16C/D Blk52+ Fighting Falcon
- F-16C/D Blk52+adv Fighting Falcon
- Mirage 2000E/BGM
- Mirage 2000-5
- F-4E Phantom II
- RF-4E Phantom

Support

- C-130H/B Hercules
- C-27J Spartan
- EMB-145H AEW&C
- EMB-135
- Gulfstream V

Fire-Fighting

- CL-215
- CL-415
- PZL

Trainers

- T-41D
- T-6A Texan II
- T-2E Buckeye

Hellicopters

- AS-332C1 Super Puma
- A-109E Power
- B-212
- AB-205

Table 1: The Hellenic Air Force (HAF) fleet.[2]

The diversity in HAF's mission profiles is portrayed in the different aircraft types. In order to fulfil a particular mission, an aircraft should meet analogous technical and performance specifications. Do the aircraft physical and performance characteristics affect its *Operating and Support* (O&S)[3] cost? If yes, how? During the procurement process there is an emphasis in affordability and cost management issues, therefore the answers to the aforementioned questions are critical for the comparison and evaluation of new ("unknown") systems.

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[1] Hellenic Air Force official site, https://www.haf.gr/en/mission

[2] https://www.haf.gr/en/equipment

[3] OSD/CAPE Operating and Support Cost-Estimating Guide (2014), Chapter 6.

Despite the lack of actual data from the *Utilization* and *Support life cycle stages*[4] where the largest portion of the *Life Cycle Cost* (LCC)[5] is incurred, an analyst must carry out a timely and reliable O&S cost estimate. At this critical time point, the capability of conducting a parametric estimate is an asset.

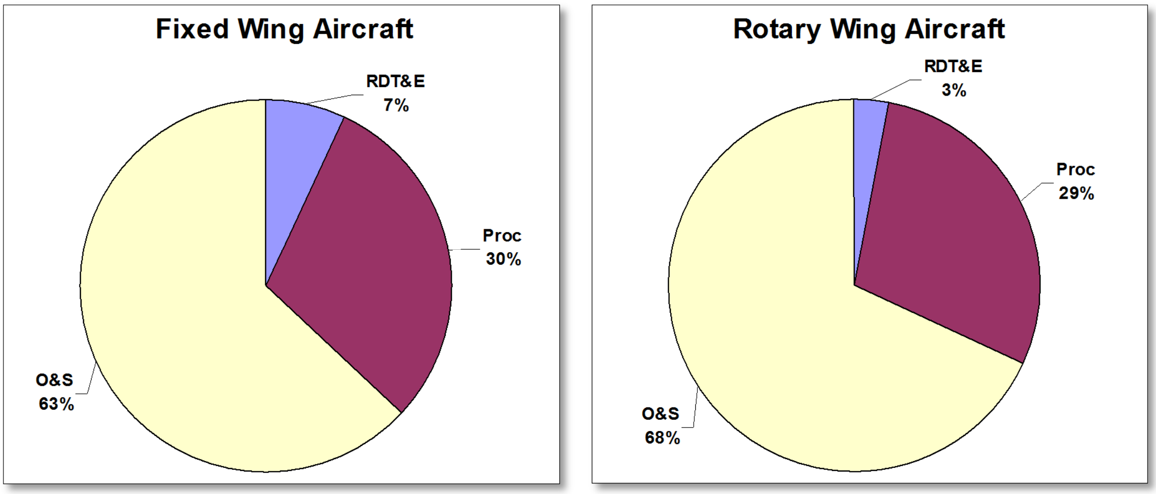

Figure 1: Typical allocation of aircraft life cycle cost.[6]

The parametric estimating technique

The parametric or "top-down" technique is a relatively fast and inexpensive estimating tool. Properly applied, it may provide reliable predictions and, most important, timely estimates. According to ISPA/SCEA Parametric Handbook:[7]

> "*Parametric estimating is a technique that develops cost estimates based upon the examination and validation of the relationships which exist between a project's technical, programmatic, and cost characteristics as well as the resources consumed during its development, manufacture, maintenance, and/or modification. Parametric models can be classified as simple or complex. Simple models are cost estimating relationships (CERs) consisting of one cost driver. Complex models, on the other hand, are models consisting of multiple CERs, or algorithms, to derive cost estimates.*"

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[4] AAP-48 NATO System Life Cycle Stages and Processes (2013)

[5] ALCCP-1 NATO Guidance on Life Cycle Costs (2008)

[6] OSD/CAPE Operating and Support Cost-Estimating Guide (2014), Chapter 2, fig. 2-2

[7] ISPA/SCEA Parametric Handbook, 4th Edition (2008)

The parametric technique is applicable during the early stages of a system's the life cycle, amidst analogy and engineering estimating techniques:

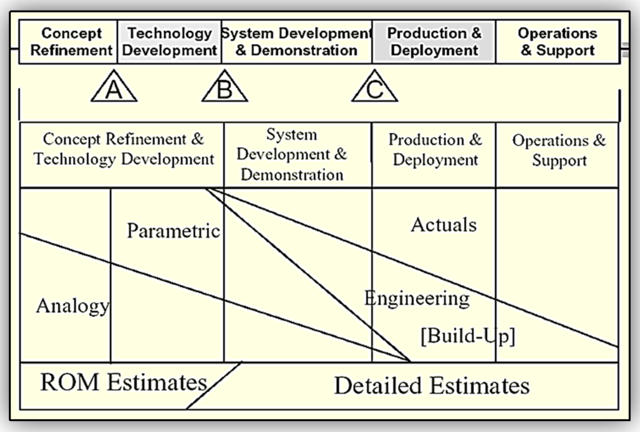

Figure 2: Typical application of estimating techniques through a system's life cycle stages.[8]

The parametric technique uses regression analysis, a statistical process for estimating the relationships among variables. Regression analysis helps an analyst to understand how the typical value of the dependent variable (response or criterion variable) changes when any one of the independent variables (predictors or explanatory variables) is varied, while the other independent variables are held fixed.

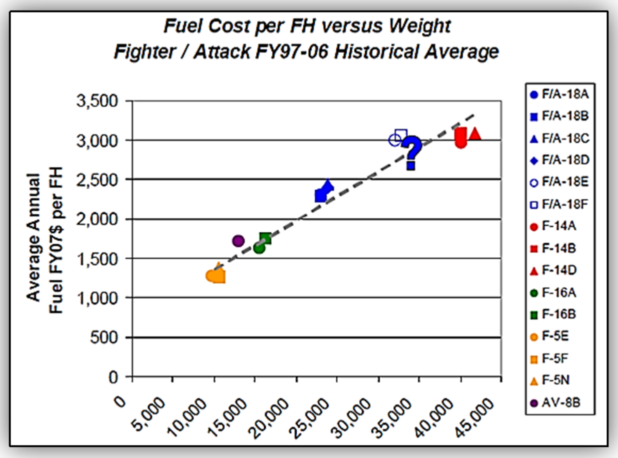

Figure 3: The development of a simple parametric model: Application of regression analysis to identify a CER between the fuel CPFH and the weight of fighter aircraft.[9]

[8] DAU Integrated Defense Acquisition, Technology, and Logistics LCM Framework chart, v5.2 (2008).

Pros and cons of the parametric technique

The implementation of the parametric technique is a blended process and the interpretation of the results has to be done with extreme caution. An analyst should always consider the following pros and cons about the parametric technique:

Pros:

- It does not require actual and detailed cost information about a new system. Compared to the engineering or "bottom-up" cost estimating technique it requires less data, duration, and resources.
- It may reveal strong CERs between cost and Reliability-Maintainability-Supportability (RMS) metrics,[10] helping to optimize maintenance and logistic procedures.
- A parametric model can be easily adjusted when the main cost drivers change. The CERs may be easily updated and sensitivity analysis may be applied.
- It is a sound statistical process and can be objectively validated.
- The uncertainty of the estimate can be quantified, allowing cost risk analysis.
- There are many available commercial-off-the-shelf (COTS) parametric tools. Additionally, general-purpose statistical packages support the parametric technique.

Cons:

- It is a rigorous statistical technique (uses regression analysis).
- CERs are often considered "black boxes," especially if they derive from COTS tools with unknown data libraries, and/or if the CER mathematical expression can't be logically explained.
- Appropriate data adjustments might be required before the analysis, depending on the selected regression method (OLS, OLS-Log space, MUPE, ZMPE). Also, standard error adjustments for sample size and relevance might be required.[11]
- CERs must be frequently updated to ensure validity.
- The validity of the prediction interval (PI) heavily depends on the residuals diagnostics.
- The decision makers may feel "itchy" to base their final decision on a parametric estimate (probably won't be statisticians).

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[9] M. Carey, DoDCAS 2010, Naval Center for Cost Analysis, "Navy VAMOSC."

[10] TO 00-20-2, Maintenance Data Documentation, (Change 2 - 2007), Appendix L: "*Air Force Standard Algorithms.*"

[11] USAF Cost Risk and Uncertainty Analysis Handbook (2007), par. 2.2.2.1 and 2.2.2.3.

- Wide-ranging prediction intervals may render the estimate useless. Why not use the *rule of thumb* instead?

Building a parametric model for the Hellenic Air Force

This case study investigates the relationship between historical CPFH[12] data and specific aircraft characteristics. The objective is to identify a strong CER that will be used to estimate the hypothetical CPFH for "unknown" aircraft.

Constraints & requirements	Results
Use the sample of 22 aircraft operated by the Hellenic Air Force.	**OK**. The sample is taken from Table 1.
Use the appropriate cost information.	**OK**. Current CPFH data used, excluding the *indirect support* cost category.
Use cost drivers (independent variables) that are easily accessible and quantifiable.	**OK**. The cost drivers are physical and performance characteristics.
The model must be as less complex as possible and include no more than two cost drivers.	**OK**. The selected model includes two independent variables.
The model should be statistically significant at the 5% level.	**OK**. p-value = $3 \cdot 10^{-8}$ (Table 4)
The model should capture at least 75% of the CPFH variance.	**OK**. $R^2_{adj} = 0.82$ (Table 4)
The model's prediction intervals must be valid.	**OK**. The residuals pass all tests (Table 5). There are many outliers though (Figure 6).
The model's mathematical expression should make sense.	**OK**. The model suggests that the aircraft weight and the engine specific fuel consumption correlate positively with the CPFH.

Table 2: A generic view of the constraints / requirements and the parametric model performance.

[12] The CPFH includes the following 6 main cost categories, according to the O&S cost element structure: *Unit-level manpower*, *unit operations*, *maintenance*, *sustaining support*, *continuing system improvements*, and *indirect support*. Since the purpose of the parametric model is the comparison of alternatives, the *indirect support* cost category is excluded from the analysis.

Variable	Simple CER's regression line	Variable adjustment
dependent: CPFH		log-transformation
independent: Length	hyperbolic	log-transformation
independent: Empty weight	hyperbolic	log-transformation
independent: MTOW	hyperbolic	log-transformation
independent: SFC (max)	hyperbolic	log-transformation
independent: Speed (max)	hyperbolic	log-transformation
independent: Ceiling	exponential	$\times 10^{-4}$

Table 3: The variables used for the analysis. The log-transformations support the implementation of linear CERs.

The examination of the independent variables may reveal multicollinearity issues. Two or more independent variables may be highly correlated, for example Log(empty weight) and Log(MTOW), meaning that one can be linearly estimated from the others with a substantial degree of accuracy. A parametric model should not include strongly correlated independent variables, because its predictive ability degrades. The variables correlation matrix offers an overview of the existing correlations:

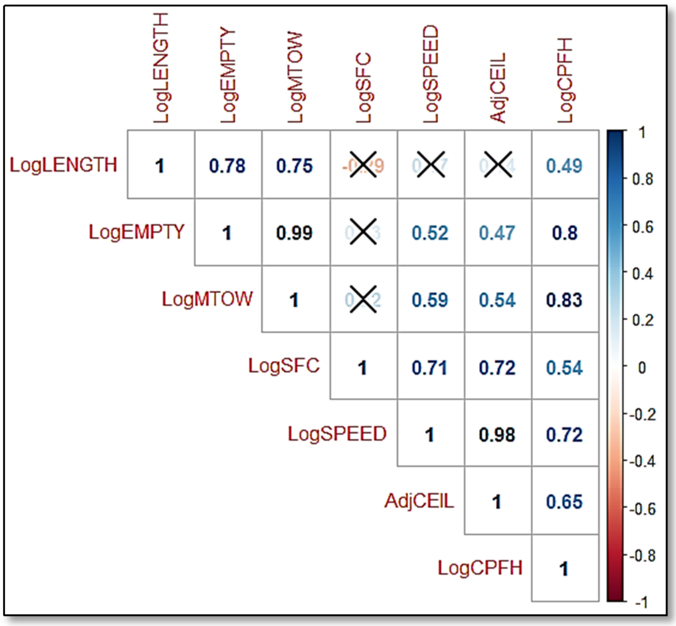

Figure 4: The variables correlation matrix. The symbol "X" indicates the insignificant correlations at the 5% sig. level. Multicollinearity is evident among several independent variables.

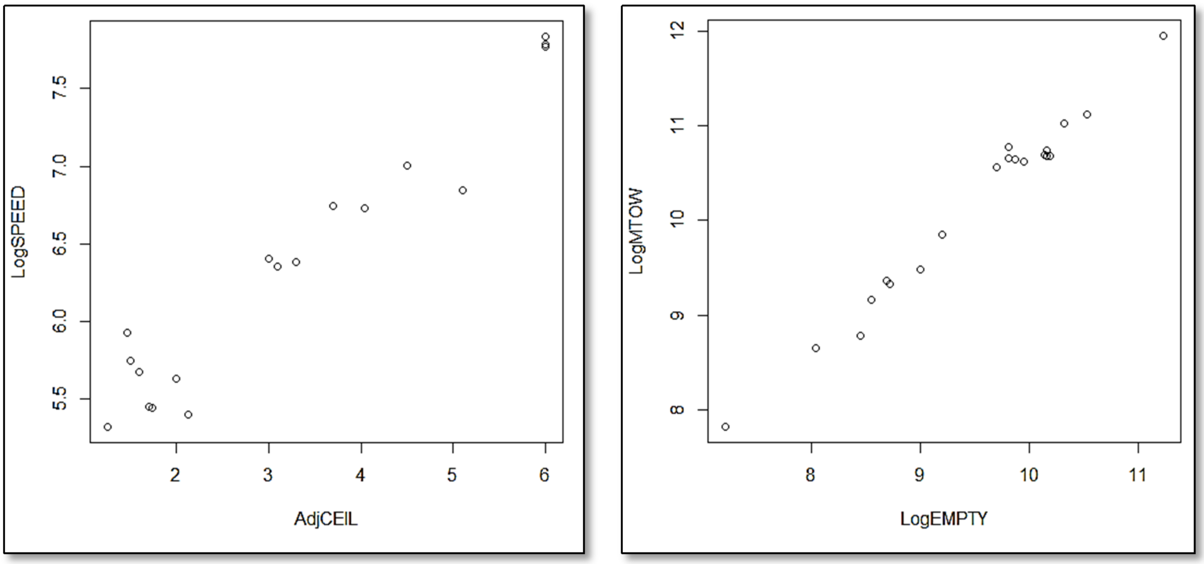

Figure 5: Examples of multicollinearity.

Selection of the optimal CER

The highest correlation coefficient between Log(CPFH) and the independent variables is $r = 0.83$. Therefore, Log(MTOW) would be the best choice for building a simple linear CER. Unluckily, this model doesn't comply at least with one of the requirements in Table 2, which is: $R^2_{adj} \geq 0.75$ (indeed, $r^2 = 0.83^2 = 0.69 < 0.75$).

The next step is to investigate all CERs of the form: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2$. The implementation of stepwise regression along with the Akaike Information Criterion (AIC), as the measure of the CERs relative quality, derives the following model:

$$\text{Log(CPFH)} = \beta_0 + \beta_1 \text{Log(empty weight)} + \beta_2 \text{Log(SFC)},$$

where β_0, β_1, β_2 are known coefficients.

Notably, the two selected independent variables do not correlate significantly (Figure 4), so there is no multicollinearity in the selected model. Also, the interaction of the two independent variables is not significant hence the term X_1X_2 is omitted from the right hand of the equation. Although the model explains a remarkable 82.15% of the Log(CPFH) variance, it does not demonstrate analogous predictive ability on the training set.[13] Indeed, 7 of the 22 actual costs fall outside the 95% prediction interval (notice the existence of outliers in Figure 6).

[13] The dataset that generated the model.

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Call:
lm(formula = LogCPFH ~ LogEMPTY + LogSFC)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.42125 -0.08515 -0.02154  0.09199  0.50650 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)    ........    ....     6.570 2.74e-06 ***
LogEMPTY       ........    ....     7.984 1.73e-07 ***
LogSFC         ........    ....     4.827 0.000117 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2553 on 19 degrees of freedom
Multiple R-squared:  0.8385,           Adjusted R-squared:  0.8215 
F-statistic: 49.31 on 2 and 19 DF,      p-value: 3.009e-08

Correlation of Coefficients:
         (Intercept) LogEMPTY
LogEMPTY -0.99               
LogSFC    0.17       -0.13   
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Table 4: The optimal model's properties.

Residuals diagnostics

The construction of valid prediction or confidence intervals relies on the assumptions that the residuals are normal, have constant variance and no autocorrelations. Remarkably, the residuals of the selected model pass all tests:

Test	Null hypothesis	*p*-value	Reject the null hypothesis at the 5% sig. level?
Shapiro-Wilk normality test	normality	0.161	NO
Breusch-Pagan test for heteroscedasticity	constant variance	0.332	NO
Durbin-Watson test for autocorrelation	no autocorrelations	0.342	NO

Table 5: The residuals diagnostics.

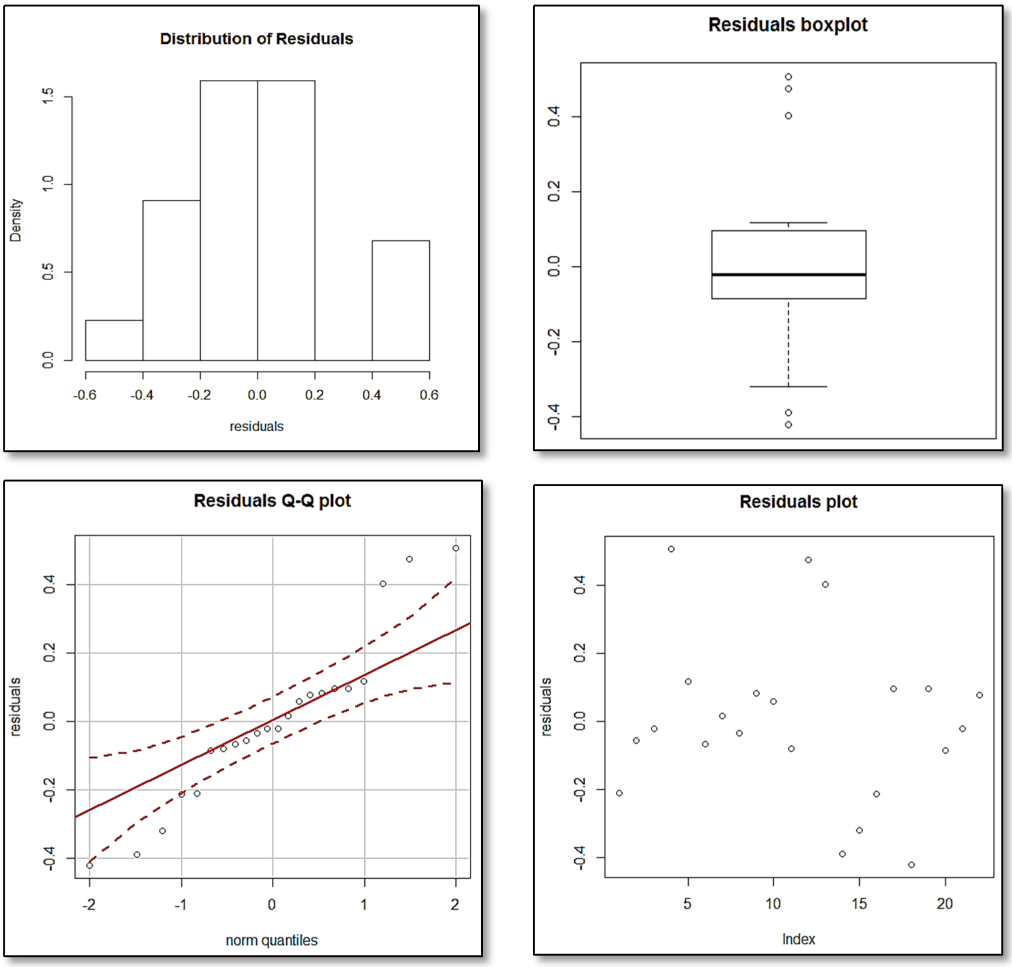

Figure 6: Typical residuals graphs. Although the residuals pass the tests, the existence of many outliers should keep the analyst alerted on the validity of the model's prediction intervals.

Making predictions for an "unknown" system

The Lockheed Martin F-35 Lightning II is a family of fifth generation, single-seat, single engine, stealth multirole fighters undergoing final development and testing by the US. The F-35 program, also known as the Joint Strike Fighter (JSF), is the most expensive weapon system in history with a projected service life up to 2070. The JSF is designed and built by an aerospace industry team lead by Lockheed Martin. Besides the US, many NATO members & close US allies participate in the funding of the JSF development. Several additional countries have ordered, or are considering ordering, the F-35.

Figure 7: The Lockheed Martin F-35A CTOL variant[14] and its power plant Pratt & Whitney F135-PW-100 afterburning turbofan engine.[15]

Supposing that the Hellenic Air Force considers the procurement of a new fighter aircraft, a rough O&S cost estimate of the alternatives, including the JSF, will be required. According to the parametric model, the F-35A empty weight (= 29,098 lb) and the F135-PW-100 specific fuel consumption (≈ 1.95 lb/lbf·h) must feed the right hand side of the model, in order to get an estimate for the cost per flight hour:

$$\text{Log(CPFH)} = \beta_0 + \beta_1 \text{ Log}(29{,}098) + \beta_2 \text{ Log}(1.95).$$

The CPFH distribution properties are estimated through two different approaches:

a. Theoretical approach. The mean ($\mu = 8.9434$) and standard deviation ($\sigma = 0.1066$) of the dependent variable are estimated explicitly, according to the regression analysis theory. Log(CPFH) is assumed to be normally distributed; therefore, CPFH follows a lognormal distribution with parameters μ and σ. Any CPFH percentile or prediction interval is then estimated according to the identified lognormal distribution.

b. Monte-Carlo simulation. According to the coefficient correlation matrix (Table 4), an algorithm generates pseudorandom values for 3 student-t distributed variables (with 19 degrees of freedom) that correspond to the model's coefficients β_0, β_1, and β_2. These 3 random values feed the right hand side of the above equation to compute a value for the CPFH. After this process has been repeated a million times, the mean ($\mu = 8.9435$) and standard deviation ($\sigma = 0.1254$) of the Log(CPFH) are estimated using Monte-Carlo simulation. Finally, the

[14] Image source: https://en.wikipedia.org/wiki/Lockheed_Martin_F-35_Lightning_II#/media/File:F-35A_three-view.PNG

[15] Image source: http://www.pw.utc.com/Content/F135_Engine/img/b-2-4_f135-ctol-cutaway-high.jpg

CPFH is fitted by a lognormal distribution with parameters μ and σ. Any CPFH percentile or prediction interval may be estimated according to either the fitted lognormal distribution properties, or the simulation output.

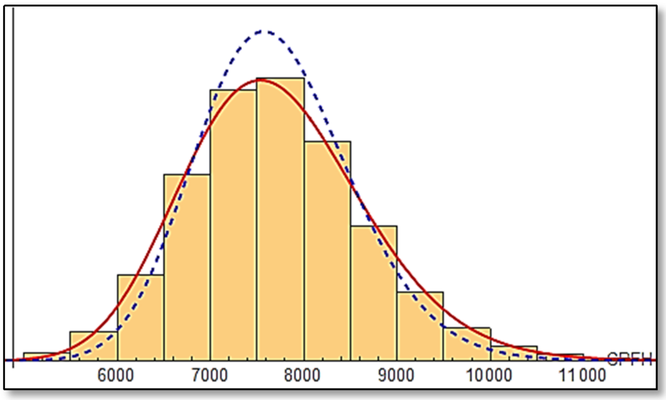

Figure 8: A lognormal distribution with $\mu = 8.9434$ and $\sigma = 0.1066$ (blue dashed line) denotes the theoretical CPFH estimate. A lognormal distribution with $\mu = 8.9435$ and $\sigma = 0.1254$ (red line) fits the simulation-generated CPFH (histogram).

Property	Theoretical output	Simulation output
Log(CPFH) mean	8.9434	8.9435
Log(CPFH) standard deviation	0.1066	0.1254
CPFH mean	7,701 €	7,719 €
CPFH median	7,658 €	7,658 €
CPFH mode	7,571 €	7,539 €
CPFH standard deviation	823 €	973 €
CPFH 80th percentile	8,376 €	8,481 €
CPFH 95% prediction interval	6,214 to 9,436 €	5,975 to 9,822 €
Prob(CPFH > 10,000 €)	0.61%	1.83%
Cost risk (80th percentile - mode)	805 €	942 €

Table 6: The parametric model's predictions on the F-35A cost per flight hour (excluding *indirect support* cost), assuming it had been operated by HAF. The theoretical regression model underestimates the uncertainty of the estimate.[16]

[16] USAF Cost Risk and Uncertainty Analysis Handbook (2007), par. 2.2.2.3.

Epilogue

The parametric estimating technique may provide timely cost estimates for "unknown" systems, through the utilization of cost estimating relationships deriving from historical datasets. The reliability of parametric estimates depends on many factors which an analyst must be aware of. This case study offers an overview on the development of a parametric model that estimates the cost per flight hour for "unknown" aircraft. The cost derives as a function of the aircraft empty weight and the engine's specific fuel consumption. As an example, the F-35A cost per flight hour is estimated under the hypothetical scenario that it is operated by the Hellenic Air Force.

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