Safety Last: Analysis of the Rayleigh Curve with Normalized Software Data

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Overview

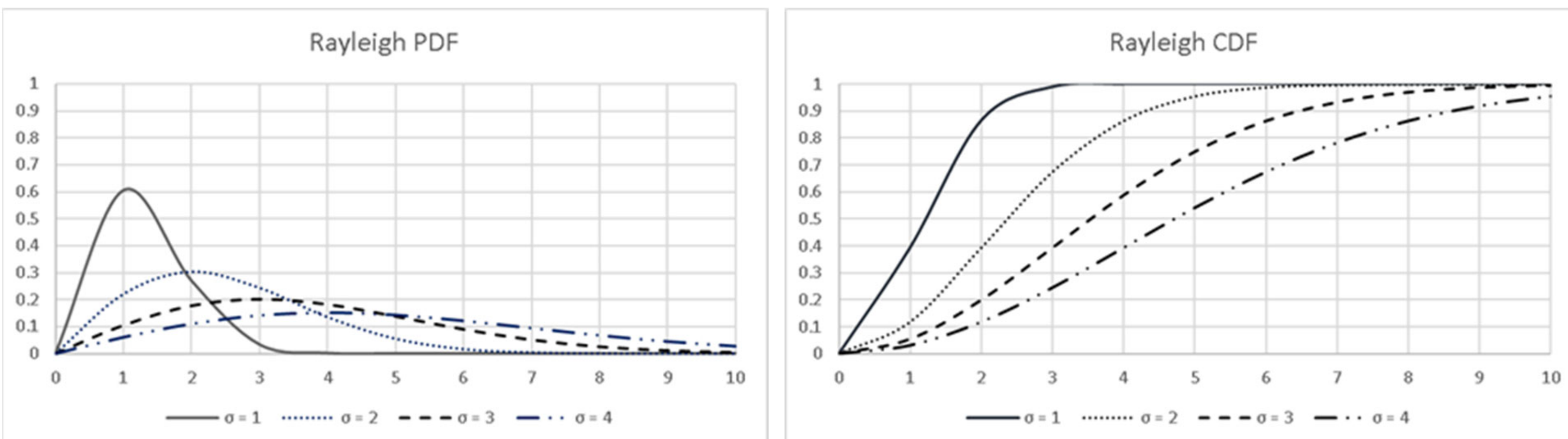
- Rayleigh Curve
- Normalizing the SRDR Database
- SRDR Using Rayleigh
- SRDR Quality Issues
- Rayleigh Peak Staffing Date Benchmarks
- Rayleigh as Weibull
- Boehm, Stutzke, and Strickland Rayleigh Curves
- Staffing Profiles Using Rayleigh
- Rayleigh and the New SRDR
- Future Work and Conclusions

Safety Last!

Software Estimation in DoD tends to focus more on effort and cost with little regard to time and phasing

Rayleigh Curve

- A continuous probability distribution named for John William Strutt, the 3rd Baron Rayleigh
- Defined by a positive shape parameter (σ):
  - $f(x;\sigma) = \frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)}, x \geq 0$ (PDF)
  - $F(x;\sigma) = 1 - e^{-x^2/(2\sigma^2)}$ (CDF)



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Rayleigh Curve - Staffing

- **Peter Norden first identified Rayleigh curves with staffing profiles – some staffing looks like Rayleigh**
- **Lawrence Putnam first applied Norden-Rayleigh curves to software staffing levels in his Software Lifecycle Model (SLIM)**
- **Norden-Rayleigh curves:**
  - $FTE(t) = \frac{Kt}{t_d{}^2} exp\left[\frac{-t^2}{[2t_d{}^2]}\right]$
    - FTE(t) = full-time equivalent personnel at time t
    - K = total project effort in man-months
    - t_d = point in time where peak staffing occurs
  - $E(t) = K\left(1 - exp\left[\frac{-t^2}{(2t_d{}^2)}\right]\right)$
    - E(t) = total effort expended from 0 to time t

Can Rayleigh curves be used with DoD Software Resources Data Reporting (SRDR) data to develop time-phasing benchmarks?

SRDR Database Ground Rules and Assumptions

- **Starting with the SRDR database of NOV 2018 (4084 records)**
  - Final SRDRs
  - "Good" Quality Tag
  - Populated Application Domain field
- **All data items should be of component or CSCI "size" in ESLOC:**
  - MDA Equivalent SLOC (ESLOC) = New + 50% (Modified) + 5% (Reuse) + 30% (AutoGen)
  - CSCI size is greater than 5K ESLOC, less than 200K ESLOC (same as Aerospace study)
- **All data items should have defined hours for Software Design, Code, and Test & Integration (DCTI)**
  - **Architecture/Design** hours are SW Design hours
  - **Code and Unit Test** hours are SW Code hours
  - **SW and System Integration, SW Qualification Testing** hours are SW Test and Integration hours
  - **Requirements Analysis** and **SW Developmental Test and Evaluation (DT&E)** hours are **not** part of DCTI hours
  - **Other** hours are distributed proportionally across all active phases

Total SRDR Records	4084
Final Records with Application Domains	569
CSCI-Sized Records	447
Records With Design, Code, Test Hrs	377

Normalization removes over 90% of the records from the dataset

SRDR Database Ground Rules and Assumptions – Rayleigh Specific

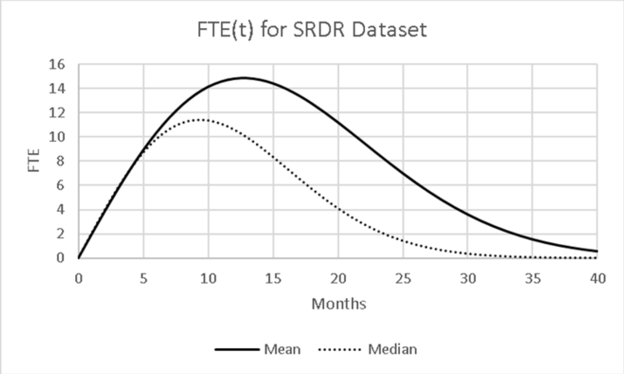
- **To use SRDR Data in Rayleigh analysis, we needed data with Rayleigh metrics populated**
  - Records need to have Peak Staff
  - Records need to have Development Months (Duration)
- **Duration calculated in months : Maximum Date (DCTI) – Minimum Date (DCTI)**

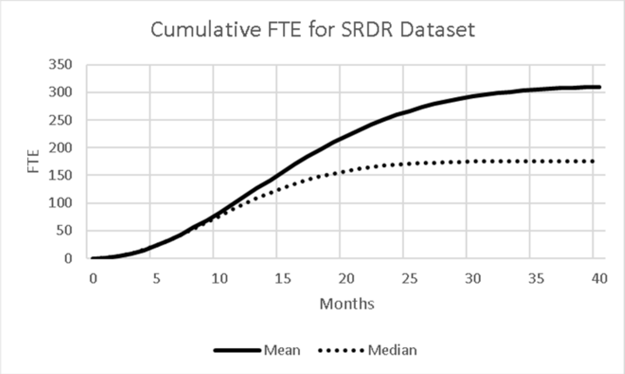
Normalized Dataset Records	377
Has Peak Staff	374
Has Development Months	373

- **For the remaining dataset records, solve for t_d**

$$t_d = \frac{K\,(0.6065)}{FTE_{MAX}}$$

FTE_{MAX} = Peak Staff





	K	Duration	td
Mean	311.76	38.8	12.73
Median	175.90	32	9.36

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SRDR Data Rayleigh Curves by Super-Domain

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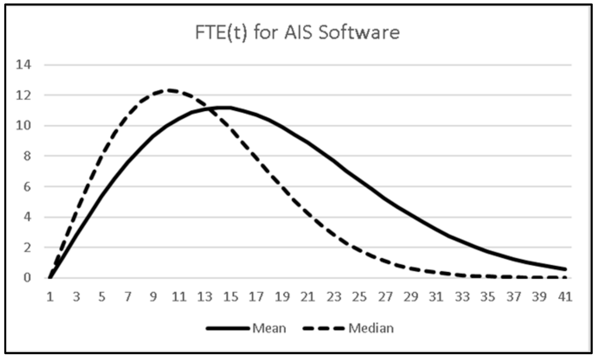
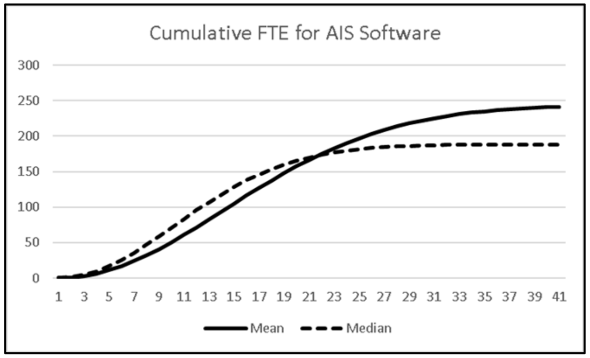
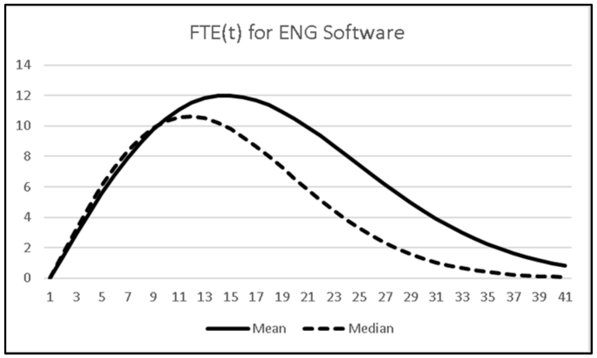
Automated Information Systems (AIS) Software – Mission Planning, Custom Automated Information Systems, Enterprise Information Systems, and Enterprise Service Systems

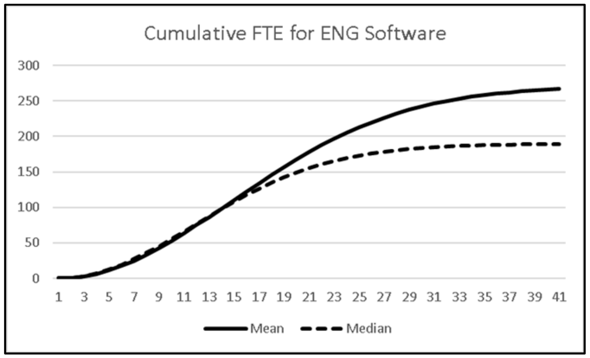




32 records	K	Duration	td
Mean	243.89	34.7	13.20
Median	188.25	24.5	9.25

Engineering Software – System Software, Process Control, Scientific/Simulation, Test/Measurement/Diagnostic Equipment





72 records	K	Duration	td
Mean	270.67	41.7	13.67
Median	189.20	30.5	10.79

SRDR Data Rayleigh Curves by Super-Domain (cont'd)

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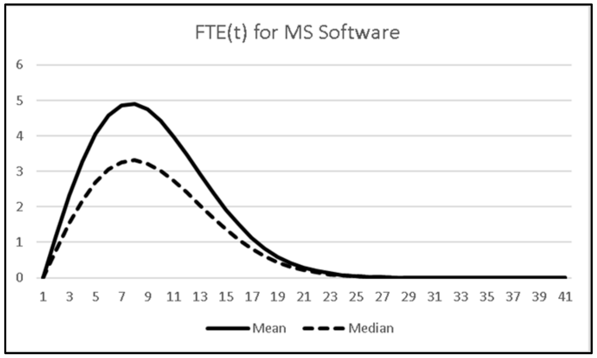
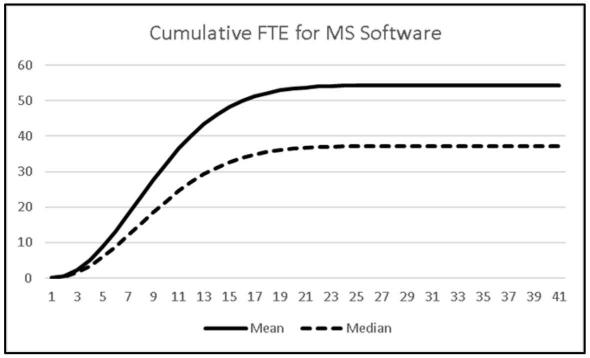
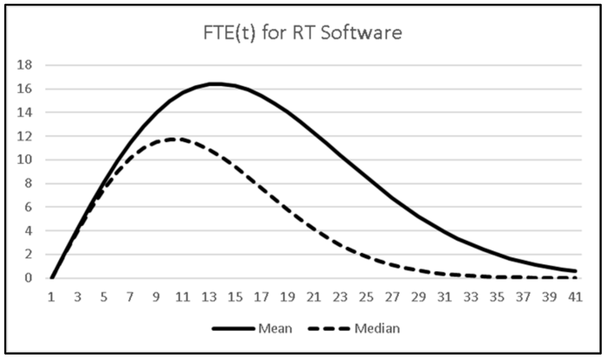
Support Software – Training and Software Tools

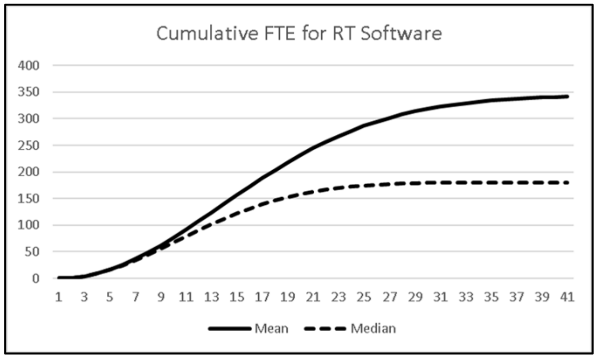




12 records	K	Duration	td
Mean	54.39	29.2	6.71
Median	37.27	27	6.82

Real-Time Software – Microcode/Firmware, Signal Processing, Vehicle Payload/Control, Command & Control, Communications





257 records	K	Duration	td
Mean	343.75	38.9	12.68
Median	180.64	35	9.32

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SRDR Quality Issues – Impossible Schedules

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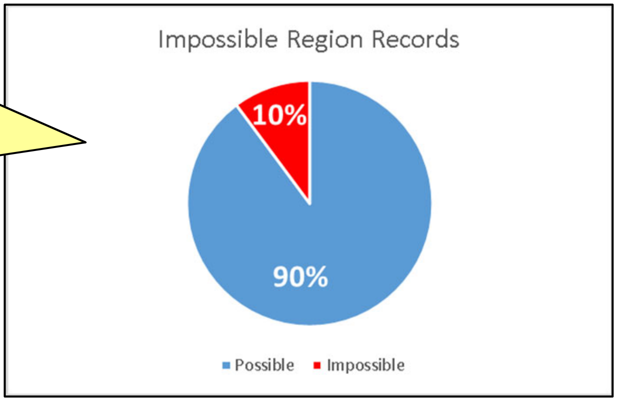
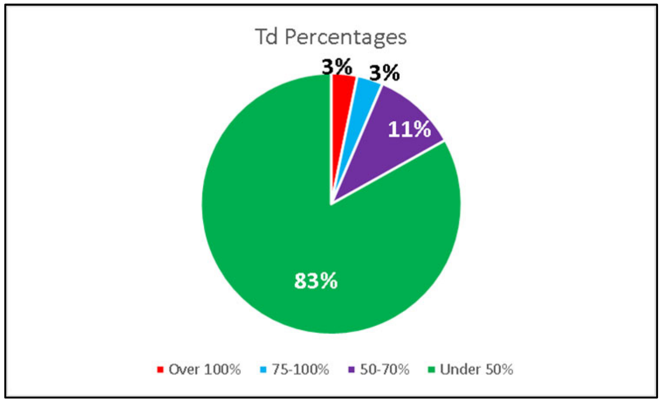
- **Proposed Benchmark – t_d percentage = t_d / Total Development Time**
- **Calculated t_d percentage for the 373 normalized records**
- **Some of the t_d percentage values were *above* 100% - peak staffing date occurs after delivery!**
- **Some of the t_d percentage values were above 75% - peak staffing occurs close to delivery**
- **Calculated Maximum Load for each record**
  - Maximum Load = Peak Staff * Total Development Time
  - Records with more Development Hours than Maximum Load are in an Impossible Region

- **Removal of 38 Impossible Region records**
- **New dataset is 335 records**





The SRDR Working Group addressed Impossible Schedule data and has added an indicator field in the SRDR database

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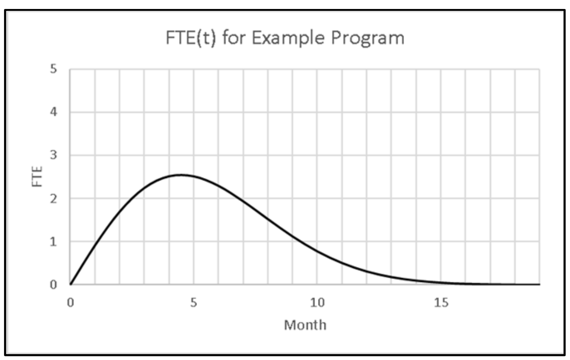
Rayleigh Peak Staffing Percentage Benchmarks

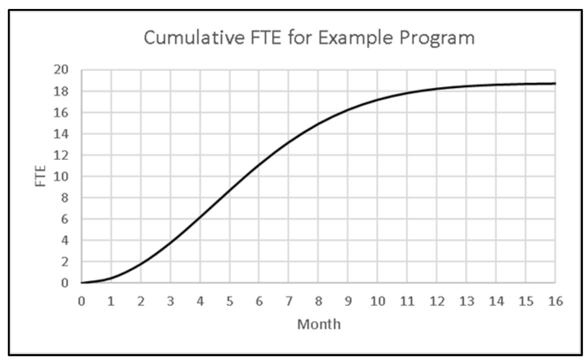
- **Mean and median t_d percentage values for the remaining 335 records calculated**

	Td Percentage		
	Records	Mean	Median
All	335	28%	28%
AIS	28	34%	35%
Engineering	65	31%	30%
Support	11	24%	18%
Real-Time	231	27%	26%

Outside of Support Software, median and mean t_d percentage values are very similar – either can be the "average"

- **Example: Generic software program with 3000 hours, planned development schedule of 16 months**
  - K = 3000 / 160 (average hrs per man-month) = 18.75 man months
  - t_d = 16 months * 28% = 4.48 months
  - FTE_{MAX} = (K * 0.6065) / t_d = 2.54 FTE





Rayleigh as Weibull

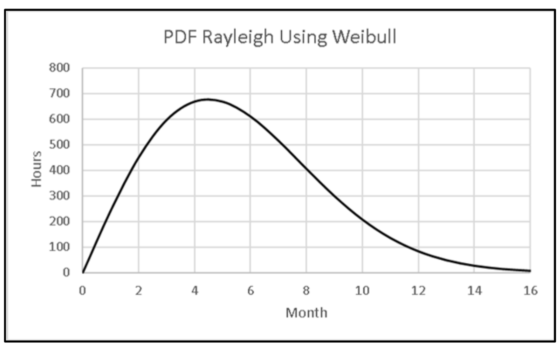
- **Rayleigh is actually a special case of the Weibull distribution**

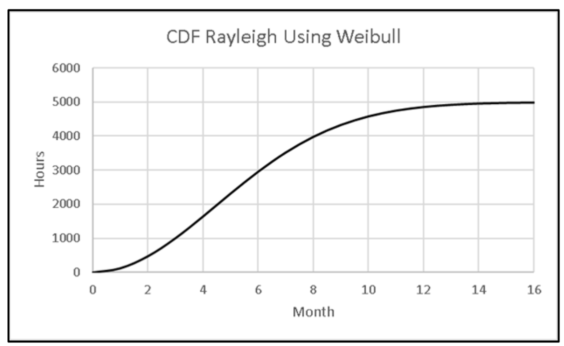
$$f(x;\lambda,k) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^{k}}$$

$$f(x;\ \lambda,k) = 1 - \ e^{-\left(\frac{x}{\lambda}\right)^{k}}$$

where k = 2 and the scale parameter (λ) = $\sqrt{2}\sigma$

- **Weibull is native to MS-Excel – easier to use than Rayleigh**
- **"=WEIBULL.DIST(x, alpha, beta, cumulative)"**
  - x = time t
  - alpha = 2
  - beta = SQRT(2) * t_d
  - cumulative = {FALSE (for PDF), TRUE (for CDF)}
- **Example: A generic software program estimated at 5000 hours with a duration of 16 months**
  - Use 28% for t_d percentage – t_d = √2*(0.28)TDEV = 6.336
  - PDF at time x: "=5000 * WEIBULL.DIST (x, 2, 6.336, FALSE)"





Boehm and Stutzke Rayleigh Normalizations

- Dr. Barry Boehm suggested that pure Rayleigh is inaccurate as no project starts with zero staff and Rayleigh starts at the origin
- Boehm suggested using only the portion of Rayleigh from $0.3t_d$ to $1.7t_d$

$$FTE(t) = K * \left(\frac{0.15 * TDEV + 0.7 * t}{0.25 * TDEV^2}\right) e^{\left(\frac{-(0.15 * TDEV + 0.7 * t)^2}{0.5 * TDEV^2}\right)}$$

- Dick Stutzke identified that this equation was not fully normalized for the new endpoints
- Stutzke normalized Boehm's formula by dividing results by 1.029

NOTE: Because of the defined endpoints of $0.3t_d$ and $1.7t_d$, t_d will always be 50% of the development duration using this normalization

Do *NOT* use the t_d benchmarks here

Strickland Weibull Rayleigh Normalizations

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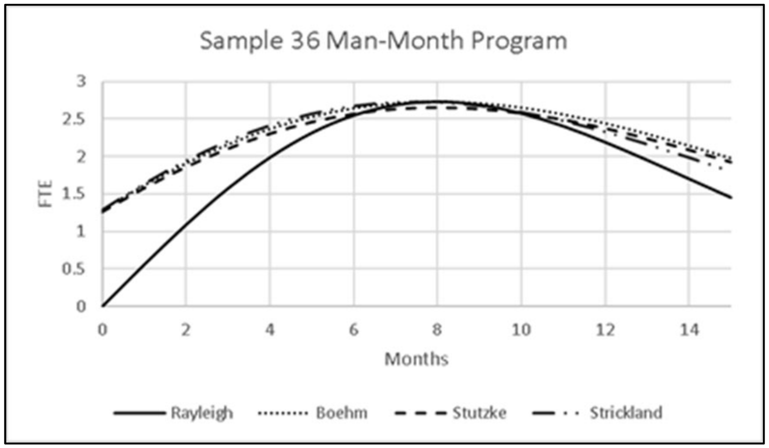
- Difference between the endpoints ($0.3t_d$ and $1.7t_d$) need to be spread over the development duration evenly
- Generic transformation for t:

$$Mult(t) = 0.3 + \frac{1.4t}{(TDEV - 1)}$$

- Expressed in Excel-like Weibull expression:

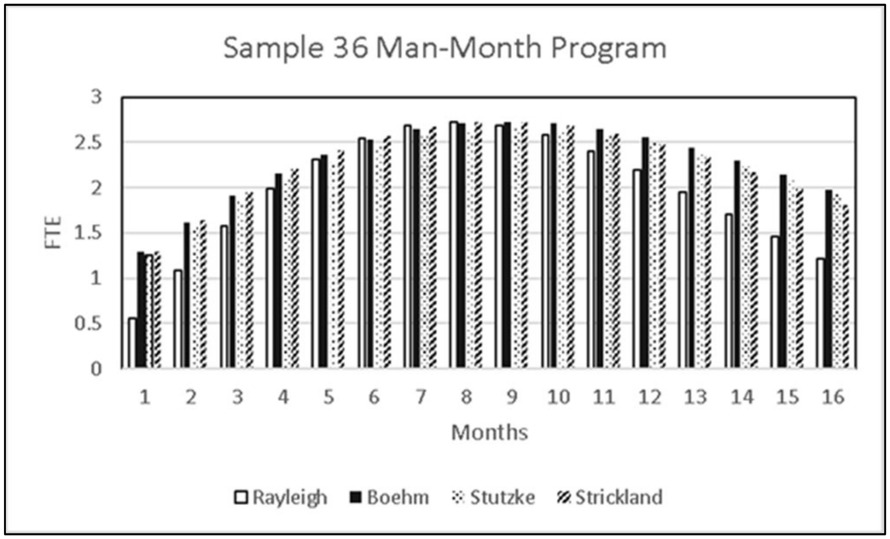
WEIBULL.DIST (Mult(t)*t_d, 2, SQRT(2)*t_d, FALSE)

- Example: 36 man-month program, 16 month duration



Strickland Weibull Rayleigh Normalizations (cont'd)

- **Analysts don't phase using continuous distributions, would rather have hours/staffing by month**
- **Rayleigh doesn't have to start in t=0, would start in t=1**
- **Boehm / Stutzke / Strickland normalizations start in t=0 but expressed as first month**
- **Example program as a discrete distribution:**



Sum of Discrete Distributions

Rayleigh	Boehm	Stutzke	Strickland
31.7	36.7	35.7	36.2

The Weibull distribution comes closest to the total in discrete calculations, but only when t_d = 50% duration

Staffing Profiles Using Rayleigh

- Without specific Earned Value data or effort/staffing linked in the SRDRs, we can't determine if the benchmark t_d percentage or Boehm normalization is a better representation
- Boehm – it makes sense that staffing peaks in the middle of SW development (Code and Unit Testing)
- Rayleigh – the shift in paradigm in SW development is for more effort and time to be spent in design and architecture
- Which should you use? Either, both are better than Uniform or Triangular
- MDA has developed an Excel worksheet that calculates Boehm, Stutzke, and Strickland monthly phasing given inputs – used by DAU in SW Estimation course

Hrs	MM	Duration	Months Y1	Total Cost ($M)
5000	32.9	16	12	

MM Distr.		1	2	3	4	5	6	7	8	9	10
	Boehm	1.2	1.5	1.7	2.0	2.2	2.3	2.4	2.5	2.5	2.5
	Stutzke	1.1	1.4	1.7	1.9	2.1	2.2	2.3	2.4	2.4	2.4
	Strickland	1.2	1.5	1.8	2.0	2.2	2.3	2.4	2.5	2.5	2.4

MM Per Yr		Yr1	Yr2	Yr3	Yr4	Yr5
	Boehm	25.5	8.1	0.0	0.0	0.0
	Stutzke	24.8	7.9	0.0	0.0	0.0
	Strickland	25.5	7.6	0.0	0.0	0.0

New SRDR Functionality

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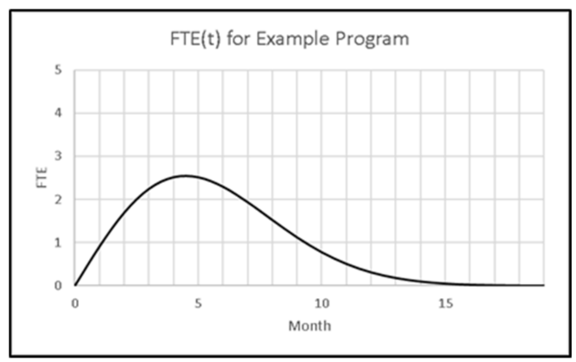
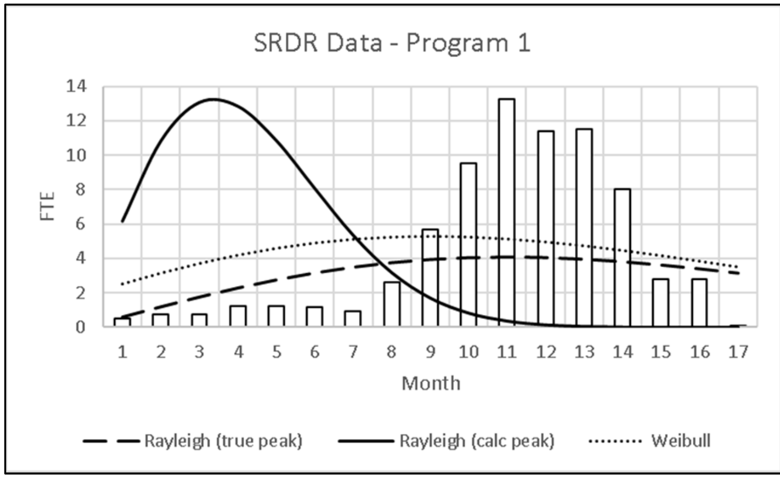
- In 2017, a new SRDR DID was developed by the SRDR Working Group to address necessary changes in data fidelity and stringency
- The new SW Development DID includes tasking for the contractor to report SW Development hours by activity, by month, by CSCI, by build
- Collections of data at this level can produce staffing profile curves for software and test Rayleigh Curve metrics

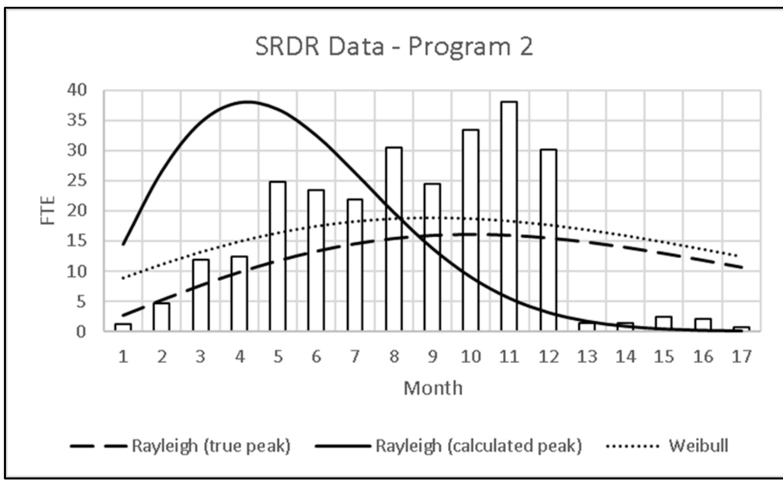
Prime Contractor **SECTION 3.4.1.1** Hours			M0 YYYYMMDD	M1 YYYYMMDD	M2 YYYYMMDD	M3 YYYYMMDD	M4 YYYYMMDD	M5 YYYYMMDD
WBS Element Code	**WBS Element Name**	**Activity ID**	**Activity Name**					
1.1.2.2.2	Software Release 1							
1.1.2.2.2.1	Software Release 1 CSCI 1							
1.1.2.2.2.2	Software Release 1 CSCI 1	X	Contractor-Defined Activity X					
1.1.2.2.2.3	Software Release 1 CSCI 1	Y	Contractor-Defined Activity Y					
1.1.2.2.2.4	Software Release 1 CSCI 1	Z	Contractor-Defined Activity Z					
1.1.2.2.2.2	Software Release 1 CSCI 2							
1.1.2.2.2.2	Software Release 1 CSCI 2	X	Contractor-Defined Activity X					
1.1.2.2.2.2	Software Release 1 CSCI 2	Y	Contractor-Defined Activity Y					
1.1.2.2.2.2	Software Release 1 CSCI 2	Z	Contractor-Defined Activity Z					
1.1.2.2.2.3	Software Release 1 CSCI n							
1.1.2.2.2.3	Software Release 1 CSCI n	X	Contractor-Defined Activity X					



SRDR Results



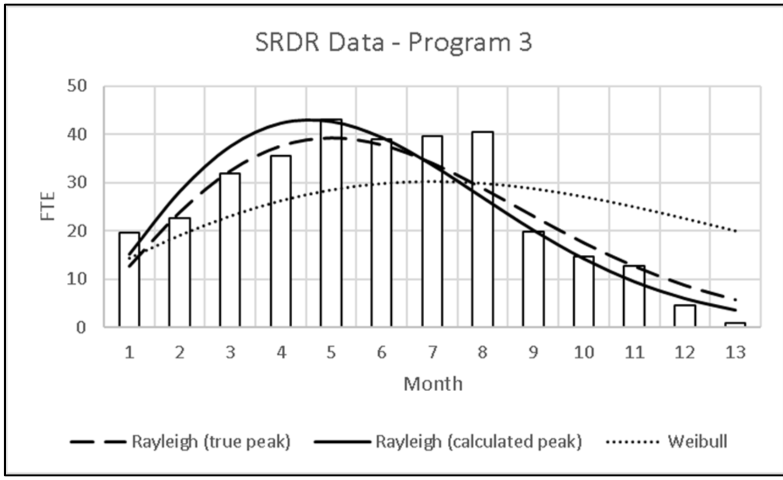
Duration	17
K (mm)	74.0
td	11
td%	65%
td (calc)	3.4



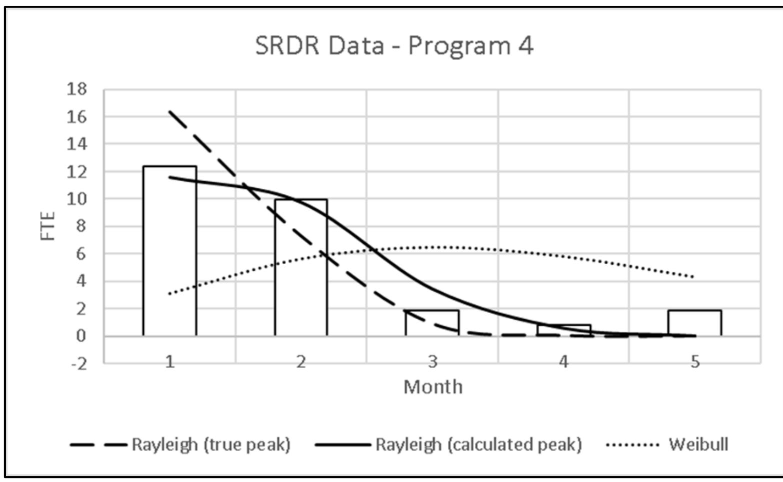
Duration	17
K(mm)	264.8
td	10
td%	59%
td (calc)	4.2

- Calculation of Rayleigh staffing curves using SRDR Final data
- Duration, K, and t_d (true peak) are known, t_d (calculated) is calculated given duration and K
- Program 1 – almost an inverse Rayleigh, peaked at 65% program completion
- Program 2 – closer behavior to Weibull at beginning, but a steep drop-off late

SRDR Results



Duration	13
K(mm)	324.5
td	5
td%	38%
td (calc)	4.6



Duration	5
K(mm)	26.9
td	1
td%	20%
td (calc)	1.3

- Program 3 – good match for Rayleigh, especially the calculated peak
- Program 4 – short duration, but good match for Rayleigh with a calculated peak
- Limited sample size, but Rayleigh is matching with a few programs; Weibull is not

Future Research and Conclusions

- Work with new SRDR submissions to identify staffing profiles for software development
- Utilize Rayleigh calculators to validate SRDR submissions

- The Rayleigh curve is an acceptable time-phasing distribution for software development
- Benchmarks using real, normalized data are available
- Using the Weibull distribution as a proxy for Rayleigh yields accurate results to a Boehm-normalized curve
- Tools and benchmarks are available to help cost estimators address time-phasing in software development

Questions