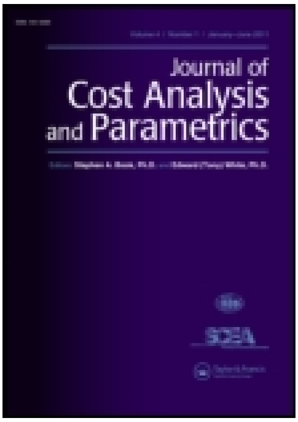

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Brian Gillespie [a] & Darrell Hamilton [b]

[a] Missile Defense Agency, Colorado Springs, Colorado, USA

[b] Defense Acquisition University, Huntsville, Alabama, USA

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A Closed-Form Solution for the Production-Break Retrograde Method

BRIAN GILLESPIE[1] and DARRELL HAMILTON [2]

[1]Missile Defense Agency, Colorado Springs, Colorado
[2]Defense Acquisition University, Huntsville, Alabama

This article explores and discusses concepts surrounding the multi-step retrograde analysis process for learning curve production breaks that was popularized by George Anderlohr, in his 1969 Industrial Engineering article "What Production Breaks Cost." Mr. Anderlohr based much of his analysis using the cumulative average curve method, but the basic principles have been widely accepted and used to calculate the equivalent calculation using the unit theory learning curves. Because Mr. Anderlohr's method is considered the standard for such calculations and because the method is relatively simple to perform, not much has been written to either simplify the process or to explain what appear to be anomalies in his methodology and other designated official sources such as that published by the Government Accountability Office (GAO). The article will briefly explore and answer the more vexing of the anomaly issues and then introduce a single closed-form equation to bypass the multi-step method which can save the cost analyst time and minimizes opportunities for trivial mathematical errors.

Introduction

George Anderlohr (1969), in an *Industrial Engineering* article, "What Production Breaks Cost," proposed a methodology for calculating the impact of a break in production of multiple units of a manufactured item. Anderlohr defined a production break as "the time lapse between the completion of a contract for the manufacture of certain units of equipment and the commencement of a follow-on order for identical units." In general, a production break occurs whenever a production line stops active manufacturing for a period of time long enough for the manufacturing knowledge to begin to diminish. Production breaks result in an increase in the cost of manufacturing, and it is important for cost estimators to be able to easily use the quantitative tools laid out by Anderlohr, in particular his retrograde method, to determine their impact.

The Anderlohr method, considered a "best practice" by the Government Accountability Office (GAO, 2009), examines the various factors that contribute to loss of learning and applies them to calculate a lost learning factor (LLF). Specific details of the work stoppage are immaterial to the calculation but could be due to an extended labor dispute, delays and changes in contract order quantities, or long delays in modernization efforts for a production line. Calculating the LLF is described in detail in Anderlohr (1969) and will not be repeated in this discussion. The LLF can then be employed, via retrograde analysis, to determine the starting point on the learning curve for future production.

Address correspondence to Darrell Hamilton, Defense Acquisition University, Huntsville, AL 35806. E-mail: Darrell.hamilton@dau.mil; or Brian Gillespie, Missile Defense Agency, Colorado Spring, CO 80912. E-mail: Brian.gillespie@mda.mil

When taught in most instructional settings, the retrograde method is taught using a five-step method that will work for either the Cum Average or the Unit Learning Curve Theories. Anderlohr demonstrated the application of the retrograde method by using the cumulative average learning curve theory. However, unlike the Cum Average Theory, each step in the unit learning curve theory is a simple closed-form solution. This article takes advantage of that formula simplicity to combine the five-step process into a single closed-form calculation that bypasses the multi-step retrograde analysis process. Use of the single equation in the unit learning curve theory case will save the cost analyst time and minimize opportunities for trivial mathematical errors on the part of the cost analyst.

Complexities of Cum Average Theory

While the steps in applying the retrograde method to the Cum Average Theory are the same as the Unit Theory, the math gets more complicated. In the Cum Average Theory, the calculations are applied to the item-value curve, not directly to the cumulative average curve. As can be seen in the formulas depicted in Figure 1, the item-value curve of $Y_X = Y_1(X^{b+1} - (X-1)^{b+1})$ is a complex curve that does not lend itself to a simple formula-based solution when solving for the item (X) when the item-value (Y_X) is known. As will be shown in more detail later in Step 4, this is the most mathematically complex step in the retrograde process. It can easily be solved using Goal Seeker or Solver in Excel™, but it is not a simple exercise to derive a closed-form solution.

Applying the retrograde method to Unit Theory-based learning curves is significantly easier. In the unit learning curve theory, the item-value curve is the learning curve of $Y_X = AX^b$. Solving for X when Y_X is known is a simple rearrangement of the formula to $X = (Y_X/Y_1)^{1/b}$. Therefore, it is possible to describe the whole retrograde method in Unit Theory using a series of simple mathematical formulas. The complexity in Cumulative Average Theory of solving for X when Y_X is known may be the main reason that most instructional examples on calculating the retrograde analysis have avoided using the cumulative average learning curve theory.

Comments on Sensitivity and Mathematical Rigor

It should be noted that calculation of the LLF as well as the entire retrograde calculation is an approximation technique that is useful for calculating the ballpark impact of a work stoppage. The LLF, while not discussed in detail in this article, consists of a series of estimates

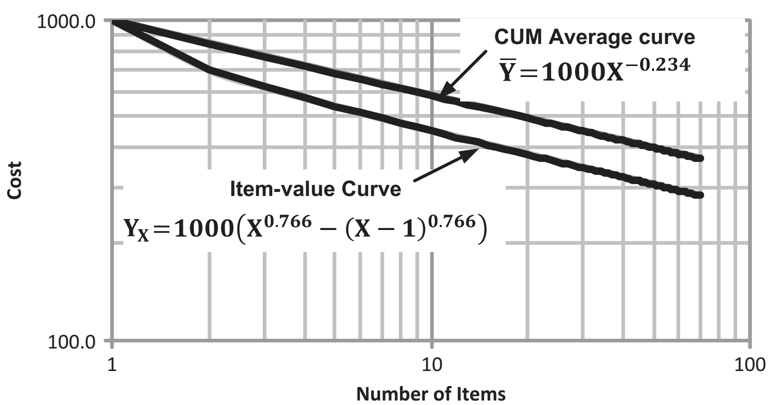

FIGURE 1 Anderlohr Cumulative Average Curve vs. Item-Value Curve.

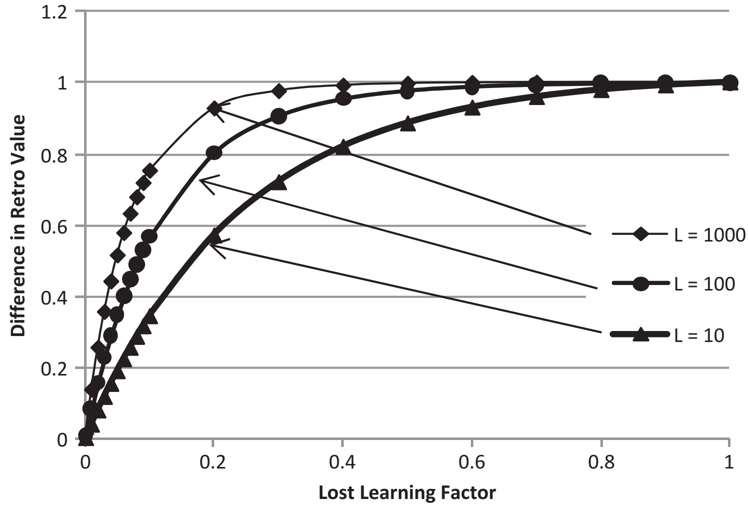

FIGURE 2 Differences in solutions provided by Anderlohr and GAO methodologies.

based on expert judgments. Each of the five or more estimates are only rough estimates which, in the aggregate, point to an order-of-magnitude value that is believable, but not necessarily accurate enough to claim more than a significant digit or two. This, coupled with the uncertainty usually surrounding the accuracy of the learning curve slope, leaves us with an estimate that is not as mathematically rigorous as the decimal values would often suggest. Efforts to use the calculated values beyond the nearest whole number are neither prudent nor justifiable in the long term. As such, for this article, we will retain significant digits for the calculations, but always round the final answer to the nearest whole number.

Another source of difference can arise from noting that the GAO method differs slightly from the original method proposed by Anderlohr. It was pointed out by one of this journal's anonymous referees that Anderlohr based his calculations on the last item to be produced, while GAO bases their calculations on the next unit to be produced. If we analyze the results in detail, the two methods produce results that differ by up to one unit as shown in Figure 2. While one unit would normally be considered significant in most large-cost, small-volume cost calculations, it should be noted that the methods used to calculate the LLF are based on approximations of hard-to-measure input parameters. Reasonable uncertainty bounds of at least one percentage point per estimation will produce differences greater than the difference in results produced by the two methods. The difference between the methods is actually dwarfed by any reasonable error term that could be applied to the calculation of the LLF. Therefore, for the purposes of this article, we will ignore the slight differences in the two methods. For simplicity purposes, we will use the GAO method of referencing our calculations from the next item to be produced.

Traditional Anderlohr Retrograde Method

In the Anderlohr method, the impact of a production break is determined through a series of estimates that allows an analyst to calculate a percentage of learning lost, known as the LLF. The LLF is then used to calculate an equivalent number of production items where the knowledge gained to more efficiently produce those units was "lost." The cost of the next item to be produced is based on the cost of a theoretical, previously produced item that had the same calculated, equivalent amount of learned knowledge. In effect, the Learning Curve

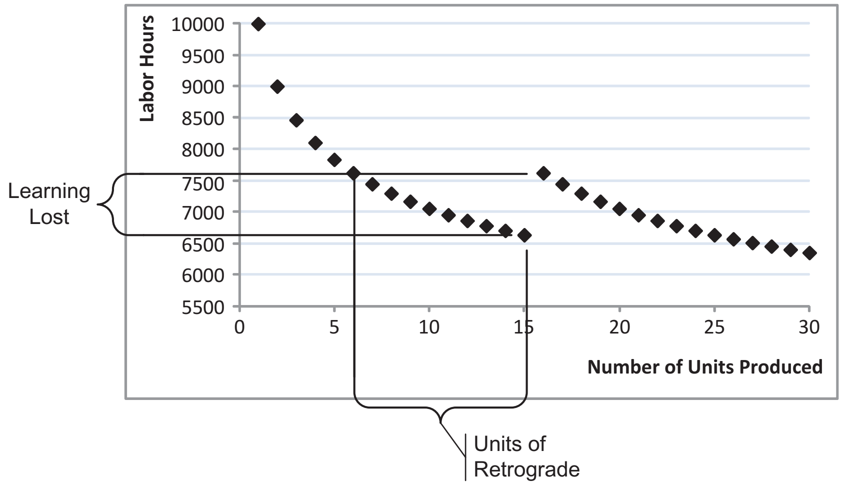

FIGURE 3 Diagram of lost learning.

is reset to a higher cost to restart production at the next unit in the production sequence as if all units that were "lost" had never been produced.

For the purpose of this discussion, we will take as an example the production of a fictitious component, the X5. The first X5 component costs 10,000 labor hours to build and 15 units of the X5 have so far been produced. Production followed a 90% learning curve trend before a labor dispute resulted in a six-month production break. The break resulted in an estimated 29% lost learning factor due to management changes and assembly crew losses. Figure 3 illustrates the X5 example. This factor will first be applied by a method described in Anderlohr's paper that has become known as the Retrograde Method.

The typical instructional steps are as follows:

Step 1. *Find the amount of learning achieved to date.*

In order to find the learning achieved (LA), one subtracts the cost of the last unit produced (Y_{F-1}) from the cost of the first unit (Y_1). For notation purposes, the number of the last unit produced before the production break ($F - 1$) is always one unit less than the first unit after the production break (F):

$$LA = Y_1 - Y_{F-1}. \tag{1}$$

Since 15 units were produced before the break, Equation (1) becomes:

$$LA = Y_1 - Y_{15}.$$

Additionally,

$$b = \frac{\ln(slope)}{\ln(2)} = \frac{\ln(0.90)}{\ln(2)} = -0.1520,$$

$$Y_{15} = Y_1 \times X^b = 10{,}000 \times 15^{-0.1520} = 6{,}625.7.$$

Therefore,

$$LA = 10{,}000 - 6{,}625.7 = 3{,}374.3.$$

Step 2. *Estimate the amount of learning lost.*

The amount of learning lost (LL), in this case measured in hours, is calculated by multiplying the learning achieved (LA) by the lost learning factor (LLF):

$$LL = LA \times LLF, \tag{2}$$

$$LL = LA \times LLF = 3{,}374.3 \times 0.29 = 978.6.$$

Step 3. *Estimate the cost of the first unit after the break.*

The Anderlohr Retrograde Method asserts that the cost of the first unit after a production break is equal to the combined cost of the learning lost (LL) and the cost of the first unit after the break in production as if no break in production had occurred.

The new cost associated with the first unit after the production break (F) is indicated by an accent mark in order to distinguish it from the value it would have if there were no break in production:

$$Y'_F = Y_F + LL. \tag{3}$$

Since the first unit post-production break is the 16th unit, Equation (3) becomes:

$$Y'_{16} = Y_{16} + LL,$$

where Y_{16} is calculated the same way as Y_{15}:

$$Y_{16} = Y_1 \times X^b = 10{,}000 \times 16^{-0.1520} = 6{,}561.0.$$

Therefore,

$$Y'_{16} = Y_{16} + LL = 6{,}561.0 + 978.6 = 7{,}539.6.$$

Step 4. *Find the unit whose cost is equal to* Y'_F.

Rearranging the basic learning curve formula, $Y_X = Y_1 \times X^b$, it is possible to directly calculate the unit X whose value most closely matches the cost of the first new unit.

$$X = \left(\frac{Y_X}{Y_1}\right)^{1/b}.$$

For this example,

$$X = \left(\frac{7539.6}{10{,}000}\right)^{1/-0.1520} = 6.4 \sim 6.$$

Step 5. *Find the number of units of retrograde.*

The number of units of retrograde is illustrated in Figure 3 and is defined as the number of units (m) in the learning curve lost due to the production break. It can be calculated by subtracting the X value from the number of the next unit to be produced calculated in Step 4:

$$m = F - X. \tag{4}$$

In this example:

$$m = 16 - 6 = 10.$$

Unit Theory Closed-Form Method

It can be observed that with the application of a bit of algebra, many of the values in the above five-step process cancel out, leaving a simple one-step formulation for the same process. This derivation makes use of Equations (1), (2), (3), and (4) and will result in a final equation dependent only on the unit number of the first unit after the production break F, the lost learning factor LLF, and the learning curve slope b.

The derivation starts with the recognition that the cost of the first unit after the production break is equal to the cost of the same unit as if no break had occurred, plus the lost learning, i.e., Equation (3).

Using Equation (4), the cost of the first new unit after the production break is:

$$Y'_F = Y_1 \times (F - m)^b .$$

Plugging this back into Equation (3) along with the value for Y_F gives:

$$Y_1 \times (F - m)^b = Y_1 \times F^b + LL. \tag{5}$$

What remains is to substitute a value for the *Learning Lost LL*. Equation (2) gives the value of LL to be:

$$LL = LA \times LLF, \tag{2}$$

where the Learning Achieved LA is given by Equation (1):

$$LA = Y_1 - Y_{L,} \tag{1}$$

$$LA = Y_1 - Y_1 \times (F - 1)^b,$$

$$LA = Y_1 \times \left(1 - (F - 1)^b\right).$$

Giving an equivalent structure for Equation (2) as:

$$LL = LLF \times Y_1 \times \left(1 - (F - 1)^b\right). \tag{6}$$

Substituting Equation (6) for LL into Equation (5) gives:

$$Y_1 \times (F - m)^b = Y_1 \times F^b + LLF \times Y_1 \times \left(1 - (F - 1)^b\right). \tag{7}$$

By dividing both sides of the equation by Y_1 and solving Equation (7) for the units of retrograde (m) gives:

$$m = F - \left(F^b + LLF \times \left(1 - (F-1)^b\right)\right)^{1/b}. \tag{8}$$

Equation (8) represents the closed-form solution to the Anderlohr Retrograde Method. It is interesting from a mathematical perspective to observe that knowing the cost of any unit (including the first unit) is not necessary in order to find the number of units needed for the retrograde calculation.

Repeating the same example using the fictitious component X5 gives:

$$m = F - \left(F^b + LLF \times \left(1 - (F-1)^b\right)\right)^{1/b},$$

$$m = 16 - \left(16^{-0.1520} + 0.29 \times \left(1 - (16-1)^{-0.1520}\right)\right)^{1/(-0.1520)},$$

$$m = 16 - (0.6561 + 0.29 \times (1 - 0.6626))^{-6.5789},$$

$$m = 16 - (0.6561 + 0.0978)^{-6.5789},$$

$$m = 16 - 6.4 \cong 10.$$

Conclusion

This article has introduced a single closed-form equation, derived from the Anderlohr Retrograde Method, which has as its output the number of units of production retrograde in the learning curve due to a production break. This method provides the exact same answer in a single calculation step as the Anderlohr Retrograde Method provides in five. The usefulness of this method as opposed to the traditional method will most likely be based on the application and tools being used.

Acknowledgment

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About the Authors

Dr. Brian Gillespie, Ph.D., is a Cost Estimator within the Missile Defense Agency Cost Estimating and Analysis Directorate (DOC) supporting the Space Tracking and Surveillance System (STSS). In his life Dr. Gillespie has delivered papers, competed in bowling leagues, taught college calculus, coached introductory wushu, climbed mountains, and performed on a flute before an audience. Dr. Gillespie received his Doctor of Philosophy in Engineering Physics from the University of Virginia for his work researching the bond order potentials of group IV semiconductors. Dr. Gillespie was born and raised in northern Virginia. He, his wife and daughter reside in Colorado Springs, CO.

Darrell Hamilton is a Professor of Business Cost Estimating at the Defense Acquisition University (DAU), teaching most cost estimating courses offered by DAU. He also is currently the course manager for BCF215, "Operating and Support Cost Analysis" and is the primary author and course manager for a new course called "Cost-Benefit Analysis for Army Programs." Prior to rejoining the government at the Missile Defense Agency in 2008 as a Senior Cost Analyst, he held various positions in industry such as Strategic Director of Project Management and Quality (Software Systems) for LapCorp, Managing Director of Marketing Automation at Trans World Airlines (TWA) and Project Manager of Operations Research Systems at TWA. Prior to his 13-year stint with industry, Darrell worked for the US Army TRADOC Analysis Command first as an officer and then as a contractor, developing cost-tradeoff and military war games (Vector-in-Commander (VIC) and TacWar). Mr. Hamilton has a Master of Science in Operations Research from the Naval Post Graduate School and a Bachelor of Science in Chemistry from Georgia Tech. He is a graduate of the US Army Command and General Staff College and served 11 years on active duty as an officer in the US Army Chemical Corps.