# Gödel's Impact on Hilbert's Problems for Cost Estimating

## Cost Consistency and Completeness as an Impossible Exercise

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# Inspiration

- Previous papers by Peter Braxton and Richard Coleman on Hilbert's Problems for Cost Estimating
- Degree in Theoretical Mathematics
- Previous Reading

# Purpose

- Praise Braxton-Coleman Problems
- Look at Hilbert's Problems
- Understand Impact of Gödel's Theorems of Undecidability/Incompleteness
- Examine Ramifications, if any, Regarding Cost Estimating
- Encourage Discussion, Vice Debate Mathematical Precision

# Hilbert's Problems

Circa 1900 Drove 20th Century Math Research

1. "Cantor's problem of the cardinal number of the continuum."
2. "The compatibility of the arithmetical axioms."
3. Give two tetrahedra that cannot be decomposed into congruent tetrahedra directly or by adjoining congruent tetrahedra.
4. Find geometries whose axioms are closest to those of Euclidean geometry if the ordering and incidence axioms are retained, the congruence axioms weakened, and the equivalent of the parallel postulate omitted.
5. Can the assumption of differentiability for functions defining a continuous transformation group be avoided?
6. Can physics be axiomatized?
7. Let!=1!=0 be algebraic and beta irrational. Is alpha^beta then transcalphaendental?
8. Prove the Riemann hypothesis.
9. Construct generalizations of the reciprocity theorem of number theory.
10. Does there exist a universal algorithm for solving Diophantine equations?
11. Extend the results obtained for quadratic fields to arbitrary integer algebraic fields.
12. Extend a theorem of Kronecker to arbitrary algebraic fields by explicitly constructing Hilbert class fields using special values.
13. Show the impossibility of solving the general seventh degree equation by functions of two variables.
14. Show the finiteness of systems of relatively integral functions.
15. Justify Schubert's enumerative geometry.
16. Study the topology of real algebraic curves and surfaces.
17. Find a representation of definite form by squares.
18. Build spaces with congruent polyhedra.
19. Analyze the analytic character of solutions to variational problems.
20. Solve general boundary value problems.
21. Solve differential equations given a monodromy group.
22. Uniformization.
23. Extend the methods of calculus of variations.
24. Criteria of simplicity, or proof of the greatest simplicity of certain proofs.

# Gödel's First Incompleteness Theorem

Assume F is a formalized system which contains Robinson arithmetic Q. Then a sentence GF of the language of F can be mechanically constructed from F such that:

i. If F is consistent, then F ⊬ GF.

ii. If F is 1-consistent, then F ⊬ ¬GF

Such an independent, or “undecidable” (that is, neither provable nor refutable in F) statement GF in F is often called “the Gödel sentence” of F.

# Gödel's Second Incompleteness Theorem

Assume F is a consistent formalized system which contains elementary arithmetic. Then F ⊬ Cons(F).

As it stands, Gödel's second incompleteness theorem only establishes the unprovability of *one* sentence, *Cons*(*F*).

Giving a rigorous proof of the second theorem in a more general form that covers all such sentences, however, has turned out to be very complicated.

# Gödel's Theorems in English

First Theorem:

- If axiomatic set theory is consistent, there exist theorems that can neither be proved or disproved.
  (If the system is consistent, it cannot be complete.)

Second Theorem:

- There is no constructive procedure that will prove axiomatic theory to be consistent.
  ( the consistency of the axioms cannot be proven within the system.)

# Braxton-Coleman Program

- Emulates Hilbert's Problems, as Sequel Set for Cost Estimating
  - Four Categories:
    - Professional Identity;
    - Analytical Techniques; (subsequently split into two)
      - Cost Estimating Techniques and
      - Cost and Schedule Risk Analysis
    - Cost Estimating Implementation; and
    - Integration with Other Disciplines
- Not Meant to be a Formal Axiomatic System, like Hilbert's vision
  - Rather a Program for Investigating the Broader Issues

# Implications for Cost Estimating

- Cost isn't a formal arithmetic system
- Does have a lot of Math to Inform
- Cost is a social science
- Cost is "As Much Art as it is Science"

# Recap

- Hilbert
- Gödel
- Braxton-Coleman
- Where Cost is Today, and Where We Can Take It

***QED***
***quod erat demonstrandum***

**or**

**as Mr. O'Neil used to say…**
**quit, enough done**