Beyond Anderlohr: An Alternative Approach to Estimating Production Gaps

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Abstract: It is widely agreed that gaps in production interrupt the continuous flow of manufacturing and the cost improvement that comes from accumulated experience. Estimating the cost impact of these production interruptions remains a challenge. The estimating community has largely coalesced around the production gap methodology defined by George Anderlohr. However, the Anderlohr methodology requires a significant amount of subjective judgment and can produce substantially different answers given the same set of facts. Recent research on "organizational forgetting", on the other hand, opens the possibility of an empirically-based approach to estimating production breaks and is worthy of further investigation.

Introduction

It is widely agreed that gaps in production interrupt the continuous flow of manufacturing and the cost improvement that comes from accumulated experience. Estimating the cost impact of these production interruptions remains a challenge 80 years after T. P. Wright formulated his theory of the learning curve. Since the 1970s, the estimating community has largely coalesced around the production gap methodology defined by George Anderlohr. However, the Anderlohr methodology requires a significant amount of subjective judgment and can produce substantially different answers given the same set of facts. Recent research on "organizational forgetting", on the other hand, opens the possibility of an empirically-based approach to estimating production breaks and is worthy of further investigation.

The Anderlohr Methodology

In 1969 George Anderlohr released his paper on "What Production Breaks Cost" (Anderlohr, 1969). Since then, Anderlohr's methodology has become the standard mode of analysis for both the government and contractor estimating community. There have been dissenters (most prominently, DCAA), but the Anderlohr methodology is widely accepted as a "best practice" in the estimating community (GAO, 2009; Mislick, 2015). Consequently, the Anderlohr method has become the preferred method of estimating the cost impacts of a production line interruption.

The Anderlohr methodology identifies five elements of learning potentially affected by production gaps: (1) production personnel learning, (2) supervisory learning, (3) continuity of production, (4) improvement of special tooling and (5) improvement of manufacturing methods. Anderlohr defined each of these categories as follows:

Personnel learning deals with the learning lost when shop employees move to other programs or leave the company entirely. Their replacements often have little or no experience with the physical product when production is restarted. In addition, those workers who return to the line upon restart have lost some degree of their physical dexterity ("muscle memory") and their familiarity with the product itself.

Supervisory learning is similar: some supervisors will move to other programs or leave the company. Returning supervisors will be less familiar with their old jobs, and they will have no knowledge about the personalities and capabilities of new hires.

Continuity of production covers the physical location and position of the production line – the relation of one work station to another, the location of tools, bins, parts, et al. – as well as the production line balance. Anderlohr argued this area suffers the greatest initial loss since the initial discontinuities in parts production will create line imbalances and non-optimal work flows.

Methods covers production planning and the associated knowledge of how to build parts and assemblies. Anderlohr argued this area is least affected by a break: "As long as method sheets are kept on file," he wrote, "learning can never be completely lost." However, restarts usually require the incorporation of engineering changes since previous production, which in turn create re-learning.

Tooling refers to the loss, damage or intentional destruction of tools during the production break. In addition, during production breaks the type of tooling sometimes transitions from temporary "soft" tools to more permanent "hard" tooling, forcing a period of relearning and a proofing out of the new tools.

Anderlohr suggested each element of learning be assigned a theoretical value, the five elements adding

in turn to 100%. Then the analyst must estimate the percent learning impact against each element.

Exhibit 1 demonstrates an example calculation.

Exhibit 1. Example of Calculation of Learning Loss (Anderlohr Methodology)

Element of Learning	Element Weighting	x	Percent Available	x	Percent Learning Retained	=	Weighted Learning Retained
Production Personnel Learning	20%		75%		67%		10%
Supervisor Personnel	20%		75%		67%		10%
Continuity of Production	20%		0%				0%
Methods Improvement	20%		90%				18%
Special Tooling	20%		60%				12%
Grand Totals	100%						50%
Learning Lost = 1 - % Learning Retained							50%

The end result of the Anderlohr calculation is the percent of learning achieved to date, which is lost due

to the production interruption. This is then applied to the observed learning curve to calculate the

setback position on the learning curve, as seen in Exhibit 2.

Exhibit 2. Application of Calculated Learning Loss
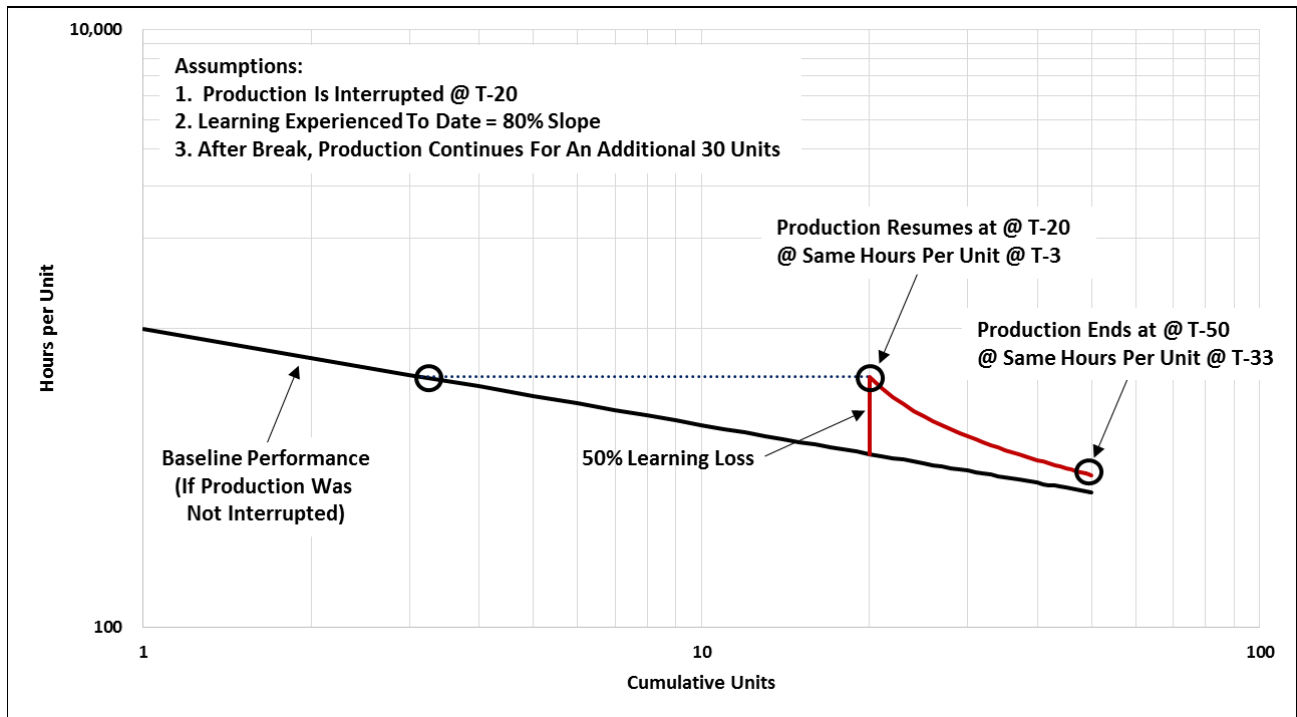
Criticism of Anderlohr has centered on the subjective judgments required to calculate it, judgments sometimes difficult to substantiate to an outside audience. First, the contribution of each element to total learning is entirely subjective – it is difficult to justify why production personnel learning should require a weighting of 30% and tooling improvements only 10% or vice versa. Anderlohr himself seemed to recognize this difficulty, suggesting that each element be weighted equally as a default. This decision too seems arbitrary. Second, it is necessary to estimate how much improvement will be lost under each element. While it might be possible to estimate the number of production personnel or supervisors who will return after an interruption, or the quantity of tools or methods sheets available, the amount of learning lost by mechanics or supervisors in the interim seems little more than a guess. In Exhibit 1, it is estimated that a third of the learning will be lost – it would be just as easy to substitute 50% or 10%, there is little empirical basis for choosing one number or another. This becomes even more difficult when the estimators are asked about production gaps of different lengths – say, a six month gap versus

a one year gap. It is almost a certainty that two estimators independently assigned a given cost analysis and using the Anderlohr method will arrive at different answers.

For this reason, anyone who has ever tried to use Anderlohr during contract negotiations knows that it is difficult to achieve consensus. The government team evaluating the proposal will inevitably arrive at different values than the contractor. First, the government team will question the weighting of the individual elements; there will be no answer to this, since the contractor will have developed the weighting based on estimating judgment or he will have defaulted to each element being equally weighted. Second, the government team will question the contractor's assumptions on the degree of learning lost during the break. "It is only a nine month gap," one might say. "Why do you assume 50% of the learning will be lost in this element?" The conversation will quickly turn to the number of personnel to be laid off or reassigned, the number which can be expected to be recalled, how much learning we can reasonably expect to retain – none of which can be predicted with much certainty before the fact.

Equally unsatisfying to the estimator will be his conversation with his own program manager, who is evaluating schedule alternatives. What is the cost impact of a three month gap, as opposed to a six month gap or a nine-month gap? Is it a linear relationship? At what point would we expect little to no impact from the gap? The estimator must now take his subjective factors and relate them to time. At best, he has one or two examples from prior history to use to guide him – and they will almost certainly not fit the scenarios he has been asked to examine. So he will try to relate his expectation of a 50% loss of learning and relate it to different periods of time (three month gap, six month gap, nine month gap) with little to guide him. He will come up with a number – estimators always do – but he will not have a good feeling that the answer he has derived has much validity. Nor will he be well-prepared if the program manager starts asking hard questions.

6

Organizational Learning and Forgetting

Is there an alternate approach to Anderlohr, one based on empirical data and which can be more rationally applied and defended? One possible source is the theory of organizational learning and forgetting, which made its first appearance in 1990 in a paper by Linda Argote and others (Argote, 1990). Of particular interest to this paper is Lanier Benkard's study of the Lockheed L-1011 commercial jetliner.

Benkard's paper proposes the phenomenon of "organizational forgetting," where production experience depreciates over time. He writes:

 "Organizational forgetting is the hypothesis that a firm's stock of production experience depreciates over time. Since an aircraft firm's experience is embodied in its workers, it seems likely that turnover and layoffs may lead to losses of experience. The traditional learning hypothesis does not allow for this possibility." (Benkard, 2000, pg. 1036.)

The nature of this lost experience can be seen in Anderlohr's five categories of learning. As personnel move in and out of a program, as the production line is altered and the continuity of the production line ebbs and flows, as tools and methods change over time, the accumulated learning over time is depreciated. This forgetting occurs all the time but, provided a constant design configuration and production rate is maintained, it is disguised by the traditional learning curve. Benkard writes: "The firm's experience is not being fully retained over time, which only becomes apparent when production rates are uneven and new models are introduced." (Benkard, 2000, pg. 1046.)

Mathematically, traditional learning is defined as:

$$(1) \qquad \ln L_i = \ln A + \theta \ln E_i$$

where L_i is labor hours per unit, A is the theoretical first unit cost, E is cumulative quantities produced to date and θ is the coefficient of learning.

7

In the traditional learning curve model, variable E can be understood as:

$$(2) \qquad E_i = E_{i-1} + 1$$

In order to incorporate organizational forgetting, Benkard proposes that variable E can be redefined as

$$(3) \qquad E_t = \delta E_{t-1} + q_{t-1}$$

where δ is the coefficient associated with learning retention over time. By definition, $0 \leq \delta \leq 1$. If $\delta = 1$, there is no organizational forgetting (Benkard, 2000, pgs.1041-1042).

Stop and consider our equation (3). The variable δ is based on *time*. Conventional learning theory holds each unit built in the past contributes knowledge toward building the product and that that knowledge is not discounted over time. The first aircraft built, say, ten years ago counts just the same in the count of cumulative units built as the aircraft delivered yesterday. Benkard's theory holds the opposite – knowledge attained through prior build *is* degraded over time. For example, if $\delta = 0.95$, then:

$$E_t = 0.95^0 = 1$$
$$E_{t-1} = 0.95^1 = 0.95$$
$$E_{t-2} = 0.95^2 = 0.90$$
$$E_{t-3} = 0.95^3 = 0.73$$
$$.$$
$$.$$
$$.$$
$$E_{t-12} = 0.95^{12} = 0.54$$

So that a unit built one year ago only contributes 54% of a shipset of knowledge toward today's build.

In Benkard's words:

An implication of the organizational forgetting hypothesis is that recent production is more important than more-distant past production in determining a firm's current efficiency. This prediction is perhaps more intuitive than that of the learning hypothesis, which treats all production equally no matter how old. In a practical example, it is hard to imagine that Boeing's rapid production of 747s in the early 1970s is as important to current unit costs as production in the early 1990s, particularly because it is unlikely that many of Boeing's workers from that period remain with the company today." (Benkard, 2000, pg. 1037).

The Production Gap as a Special Case

A production gap emerges as a special case in Benkard's theory when traditional learning ceases as production rates fall to zero. According to Benkard, experience depreciates at a constant rate, which he estimates with the following coefficient:

$$(4) \qquad y = \delta^m$$

where y = percent of learning retained and m = number of months that have passed. Based on the L-1011 data, Benkard estimates δ = 0.96. This implies that 61% (0.96^{12}) of a firm's experience existing at the beginning of a year survives to the end of the year (Benkard, 2000, pg. 1049).

Graphically, this can be seen as a curve (Exhibit 3) which asymptotically approaches zero learning retention as we move further away from the last production delivery. Also plotted is its inverse – the percent of learning lost over time:

Exhibit 3. Profile of Learning Retention and Loss Over Time
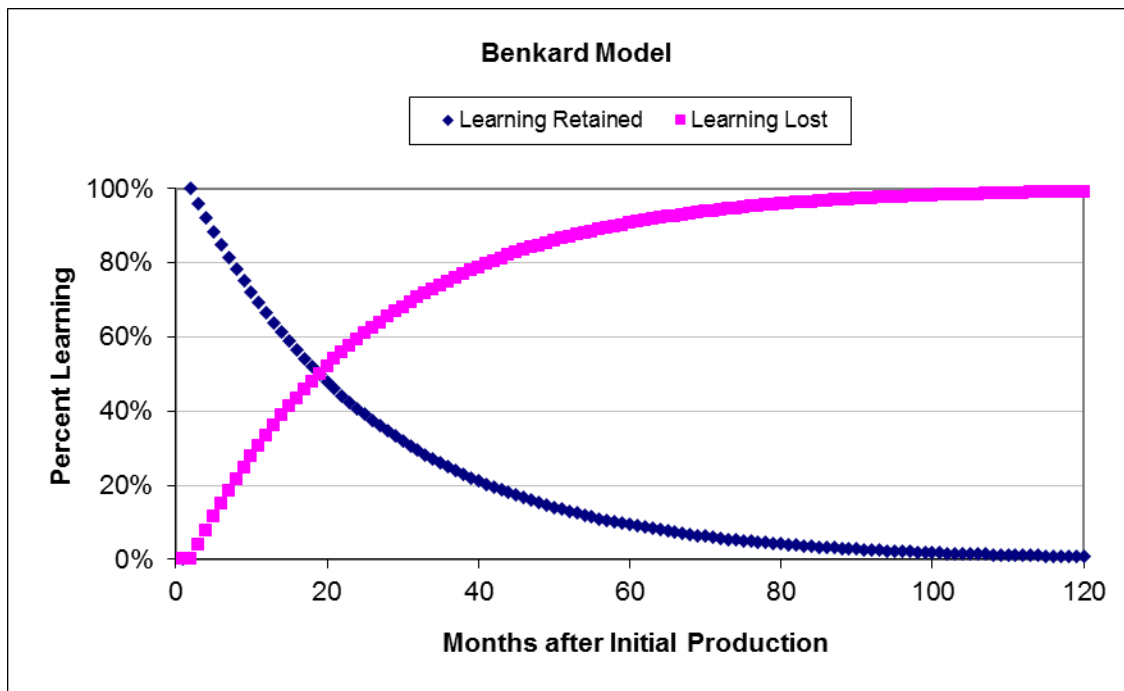
Intuitively, such a profile satisfies a number of conditions. First, it is logical that the degree of lost learning is directly related to the passage of time. Second, it ties to our estimating experience that that relationship is not linear – that certain elements of learning (methods improvement, tooling) are not lost as quickly as operator skill and proficiency – and that, therefore, while most of the lost learning occurs relatively early, after two years there is still a substantial degree of learning retention. In short, the asymptotic shape of the curve seems to tie back to our understanding of the elements that constitute cost improvement. Note too that it is only after many years pass that the learning retention eventually degrades to zero.

One caveat remains. The particular value of the coefficient δ matters a great deal. It can be seen that the lower the value, the more quickly learning is lost during a production gap scenario. As shown in Exhibit 4, a value of $\delta = 0.80$ implies that after a year, only 7% of the prior learning is retained – a value that seems much too fast for most long-cycle manufacturing operations with a normal employee turnover.

Exhibit 4. Comparison of δ Values

δ	% Learning Retained After One Year (δ^{12})	% Learning Lost After One Year ($1 - \delta^{12}$)
0.80	7%	93%
0.85	14%	86%
0.90	28%	72%
0.95	54%	46%
0.99	89%	11%

Theoretically, δ must be between $0 < \delta < 1$. More probably, it needs to lie between $0.90 < \delta < 1$. It is difficult to believe under a normal rate of employee turnover that more than 75% of learning is lost in a single calendar year. Such a low rate of learning retention does not correspond to most cost estimators' experience. So we turn our attention to establishing the value of δ.

Three academic studies performed since 1990 have estimated the value of δ, as shown in Exhibit 5:

Exhibit 5. Published Studies and Estimates of Learning Retention

Author	Value of δ	Source Data
Argote, et al. (1990)	0.75	World War II Liberty ship build, monthly data
Benkard (2000)	0.96	L-1011 aircraft build, unit data
Thompson (2006)	0.943 – 0.964	World War II Liberty ship build, unit data

These published values are limited in number and show a wide variation. Interestingly, Argote (δ = 0.75) and Thompson (δ = 0.94-0.96) worked from the same dataset – hours for the build of Liberty ships, albeit at different levels of data aggregation – and arrived at substantially different answers. Given the spread of the published values of δ, we may wish to establish its value on our own. There are two means to do so. First, we can regress historical cost data to determine its value. Second, we can fit δ based on an observed interruption. Both methods will be shown.

Calculation of δ – Regression of Historical Data

To test the validity of the Benkard approach, a historical dataset from a military aircraft program was developed. The ideal dataset to develop a value for δ would seem to be one with a lengthy production run (> 150 units) with at least one model change and demonstrating fluctuations in production rates. Fortunately such a program is available. Due to the proprietary nature of this data, this program is not specifically identified in this paper. The aircraft is of conventional aluminum manufacture; the hours per unit represent unit values for the development aircraft and lot averages for the production units. The variable t (time) is measured by months from the first aircraft delivery, where 1 = first month of aircraft deliveries.

A significant model upgrade occurred approximately three-quarters through the time line under study. The upgraded model had significant structural revisions and enhanced mission systems capabilities requiring electrical and hydraulic changes throughout the aircraft. The original model will be referred to here as Model A; the upgraded version as Model B. In order to properly account for the potential learning impacts associated with the model upgrade, consistent with Benkard's modeling for the L-1011, an additional variable λ has been introduced to account for experience that "spills over" from model A to model B. If λ = 1, there is complete spillover, i.e., 100% cost commonality between models A and B. There were, in fact, additional model upgrades which occurred after the time period in question, but these units were excluded from the analysis to avoid confusion.

Therefore, the definition of experienced units E with models A and B are defined as follows:

$$(5) \quad E_i = E_{A,t} \quad : \text{if i is model A}$$
$$= E_{B,t} \quad : \text{if i is model B}$$
$$(6) \quad E_{A,t} = \delta E_{A,t-1} + q_{A,t-1} + \lambda q_{B,t-1} \quad \text{and } E_{A,1} = 1$$
$$(7) \quad E_{B,t} = \delta E_{B,t-1} + q_{B,t-1} + \lambda q_{A,t-1} \quad \text{and } E_{B,1} = 1$$

Note that if δ =1 and λ = 1, then our regression equation collapses back to our standard learning equation (1), allowing us to use ordinary least squares (OLS) regression. But if δ ≠1 and λ ≠ 1, then we are now dealing with a nonlinear regression equation. We will need to estimate the values of δ and λ consistent with equations (6) and (7) while at the same time optimizing equation (1) with the goal of minimizing the sum of squared residuals (SSR).

Nonlinear parametric estimation can be performed with a number of statistical software packages such as SAS, R, et al. But estimators are not econometricians, and many of these high-powered tools are not easily available to an estimator. However, the good news is that we can achieve the same result using

Microsoft Excel Solver. While Microsoft Excel Solver is perhaps not the most optimal tool from the statistician's standpoint, it can be adapted for our purposes to estimate the values of δ and λ that minimize SSR. It also has the advantages that it is universally available and that virtually all cost estimators are familiar with it.

In order to use Solver, we need to set up our dataset similar to Exhibit 6:

Exhibit 6. Dataset before Optimization

|		Intercept	Masked
		Seq (E)	Masked
Optimize these values in order to...	→	DELTA	1.00
	→	LAMBDA	Masked

INTERCEPT(J13:J48,K14:K48)

SLOPE(J13:J48,K14:K48)

... minimize this → SSR 1.71

Model	Hrs (K)	Months Since First Delivery (1st Unit =1)		Time-Adjusted Sequence (E)		ln (y)	ln (E)	Predicted	Residual	SSR
		A	B	A	B					
A		1	0	1.0	-		-		0.14	0.02
A		2	0	4.0	-		1.39		(0.05)	0.00
A		4	0	6.0	-		1.79		(0.07)	0.01
A		7	0	8.0	-		2.08		(0.07)	0.01
A		10	0	9.0	-		2.20		(0.10)	0.01
⋮	Masked	⋮	⋮	⋮	⋮	Masked	⋮	Masked	⋮	⋮
B		89	1	901.0	1.0		6.80		(0.79)	0.63
B		94	5	945.0	5.0		6.86		(0.53)	0.28
B		97	8	973.0	25.0		6.91		(0.22)	0.05
B		101	12	983.0	60.0		6.95		(0.09)	0.01
B		105	16	997.0	105.0		7.00		(0.07)	0.00
B		109	20	1,003.0	165.0		7.06		(0.12)	0.01
B		114	25	1,003.0	231.0		7.12		(0.21)	0.04

If we set δ and λ to 1, Microsoft Excel will return the coefficients for A and θ consistent with ordinary least squares by using the Excel functions INTERCEPT and SLOPE. We can also calculate the SSR associated with this equation.

The next step is to allow δ and λ to vary in order to determine which values will minimize the SSR. Solver (reference Exhibit 7) allows us to minimize the SSR by changing δ and λ subject to the constraints that $0 < δ < 1$ and $0 < λ < 1$.

Exhibit 7. Example of Microsoft Solver
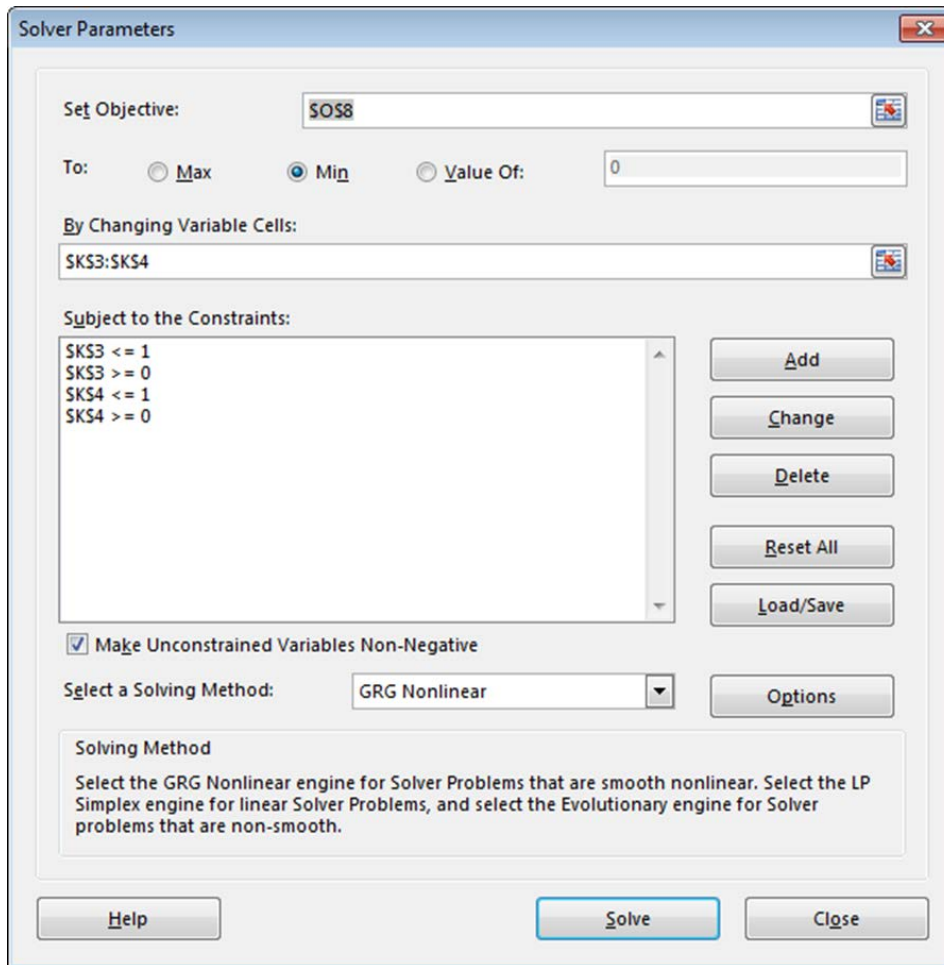
While Solver determines the optimal values for δ and λ, the Excel functions INTERCEPT and SLOPE are

simultaneously calculating the best fit coefficients for A and θ. When we do this we get:

Exhibit 8. Dataset after Optimization

Intercept	Masked	←
Seq (E)	Masked	←
DELTA	0.93	←
LAMBDA	Masked	

SOLVER determined these values...

These values also recalculated by Excel as a result of different values for DELTA and LAMBDA

...will minimize this → SSR 0.52

Model	Hrs (K)	Months Since First Delivery (1st Unit =1) A	B	Time-Adjusted Sequence (E) A	B	ln (y)	ln (E)	Predicted	Residual	SSR
A		1	0	1.0	-		-		0.30	0.09
A		2	0	3.9	-		1.37		(0.03)	0.00
A		4	0	5.4	-		1.69		(0.06)	0.00
A		7	0	6.4	-		1.85		(0.04)	0.00
A		10	0	6.2	-		1.82		(0.02)	0.00
⋮	Masked	⋮	⋮	⋮	⋮	Masked	⋮	Masked	⋮	⋮
B		89	1	230.8	1.0		4.18		(0.33)	0.11
B		94	5	206.4	4.8		4.13		(0.03)	0.00
B		97	8	195.2	23.9		4.36		0.21	0.04
B		101	12	157.3	53.0		4.57		0.26	0.07
B		105	16	132.8	85.0		4.80		0.21	0.05
B		109	20	106.2	124.2		5.04		0.09	0.01
B		114	25	74.7	153.4		5.16		(0.03)	0.00

If we compare our results to both the traditional (cumulative experience) learning model and the initial

organizational forgetting model with δ and λ to 1, we can see:

Exhibit 9. Comparison of Traditional Learning & Organizational Forgetting (O.F.) Models

	Traditional Learning	O.F. (δ, λ = 1)	O.F. (δ, λ ≠ 1)
N	36	36	36
Intercept	Masked	Masked	Masked
Slope	Masked	Masked	Masked
δ	N/A	1.00	0.93
λ	N/A	1.00	Masked
R^2	86%	87%	96%
SEE	23%	22%	12%
F-Statistic	211.6	221.8	809.6
SSR	1.78	1.71	0.52

It can be seen that the organizational forgetting model where δ ≠1 and λ ≠ 1has a better set of fit statistics than the traditional learning model. Interestingly, the derived value δ = 0.93 is slightly below the range previously reported by Benkard and Thompson, but within our proposed range of reasonableness (δ > 0.90).

Calculation of δ – Fitting an Observed Interruption

Alternatively, we could attempt to fit δ based on a previously observed production interruption.

To illustrate this approach, we will take a hypothetical case not based on actual experience. For our case, we will assume a 24-month production gap between Engineering and Manufacturing Development (EMD) to the Low Rate Initial Production (LRIP). The gap is measured by observing a given work station and determining the period of time it is idle during the interruption. We will also assume a cost increase in the first production aircraft measured against the last EMD aircraft that equates to a 50% loss of learning.

We can estimate the coefficient value required to produce 50% learning loss after a 24 month gap, as follows:

$$\text{50\% learning loss = 50\% learning retention } (1 - 0.50 = 0.50)$$

$$\delta^{24} = 0.50$$

Solving for δ, we take the natural logarithm of both sides:

$$\ln(\delta^{24}) = \ln(0.50)$$

$$24 \ln \delta = \ln(0.50)$$

$$\ln \delta = \ln(0.50) / 24$$

$$\ln \delta = -0.69315 / 24$$

$$\ln \delta = -0.02888$$

$$\delta = 0.97$$

The advantage of this approach is it allows us to use historical experience from prior production interruptions and scale that historical experience to fit production gaps of a greater or lesser length of time. If several examples of production breaks can be constructed, it might also help us to empirically determine the impact of programmatic events that may have impacted the degree of learning lost, i.e., a transition from soft to hard tooling or a relocation of the physical production line.

Application to Estimating Production Gaps

Whether δ is calculated based on a historical regression or fitting it to an observed interruption, once its value has been established, a table can be built showing the relationship between the percent of learning retained and the length of the production gap in months. Once this is done, the estimator need only refer to the table to establish the percent of learning loss (or retention) that correlates to the length of the production interruption. For example, if $\delta = 0.96$, then:

Exhibit 10. Table of Learning Loss (Organizational Forgetting Methodology)

$\delta = 0.96$	Learning	
Months	Retained	Lost
1	96%	4%
2	92%	8%
3	88%	12%
4	85%	15%
5	82%	18%
6	78%	22%
12	61%	39%
24	38%	62%
36	23%	77%
48	14%	86%

The end result of this is a calculation of learning loss similar to the Anderlohr methodology, and its use in calculation of the estimate would be similar to Exhibit 2 – we would calculate a new position on the

learning curve set back from the break-in point of the production interruption and then proceed down the curve in an asymptotic recovery to the baseline.

Conclusions and Further Research

This methodology, however, arrives at the estimation of the learning loss by a different means than Anderlohr. It presents us with several potential advantages.

First, it is empirically based and eliminates the use of estimating judgment inherent in the Anderlohr methodology. Therefore, it is more defensible to a hostile audience and more likely to produce an answer agreeable to both parties in a negotiation. Second, it allows us to logically correlate learning loss to the length of the interruption, making it easier to determine the cost impacts of different production schedule scenarios.

In addition, the organizational forgetting model could have uses beyond the production interruption. It could potentially provide insight into cost impacts related to production rate increases or decreases, as well as cost impacts due to aircraft model changes or major engineering changes.

In order to be accepted in the cost community, however, further research is required. In particular, additional estimates of the learning retention variable δ are needed to determine if it is sufficiently stable to warrant use to estimate production gap impacts. For example, further research into additional weapons systems would demonstrate the robustness of δ across programs. In addition, the estimates of δ could be applied retrospectively to historical production gap estimates to validate that it produces reasonable estimates for production breaks. Finally, it should be determined if δ is more reliably determined by the regression of historical actuals or by calculating it based on previously observed production interruptions.

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Biography

Brent Johnstone is a production air vehicle cost estimator at Lockheed Martin Aeronautics Company in Fort Worth, Texas. He has 28 years' experience in the military aircraft industry, including 25 years as a cost estimator. He has worked on the F-16 program, and since 1997 has been the lead Production Operations cost estimator for the F-35 program. He has a Master of Science from Texas A&M University and a Bachelor of Arts from the University of Texas at Austin.