# Improving the Accuracy of Cost Estimating Relationships (CERs) for NSS Software Systems

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## Agenda

*Estimating the Uncertainty in Cost Estimating Relationships (CERs)*

- Statement of the Problem
- Introduction
- Basic Concepts and Terminology Used in Parametric Modeling
- Empirical Analysis Results
- Statistical Characterization of Normalized ESLOC
- Proposed Solution
- Empirical Results from Applying Modified PI Equation
- S-Curve Generation
- Application to Cost Prediction
- Summary

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## Statement of the Problem

- Current Department of Defense acquisition policy guidance mandates funding at a set percentile of confidence level
  - The confidence level percentile estimate is typically derived from Cost Estimating Relationship (CERs), the CER prediction interval (PI), and associated S-curve
- Numerous studies by GAO and others have shown there is significant cost growth in many National Security Space (NSS) acquisition programs
  - The results from these studies suggest that the CERs and associated S-curves may be underestimating the true cost
- A more accurate and robust CER would allow decision-makers to be better informed on how much money is needed to fund a particular NSS acquisition program
- Our analysis results suggest the conventional Prediction Interval equation may be too optimistic
- We show in this presentation a practical method for improving the accuracy of the prediction interval estimate, thereby improving the accuracy of the resulting S-curve

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## Introduction

*Software Permeates All Elements of National Security Space (NSS) Systems [Eslinger, 2010]*

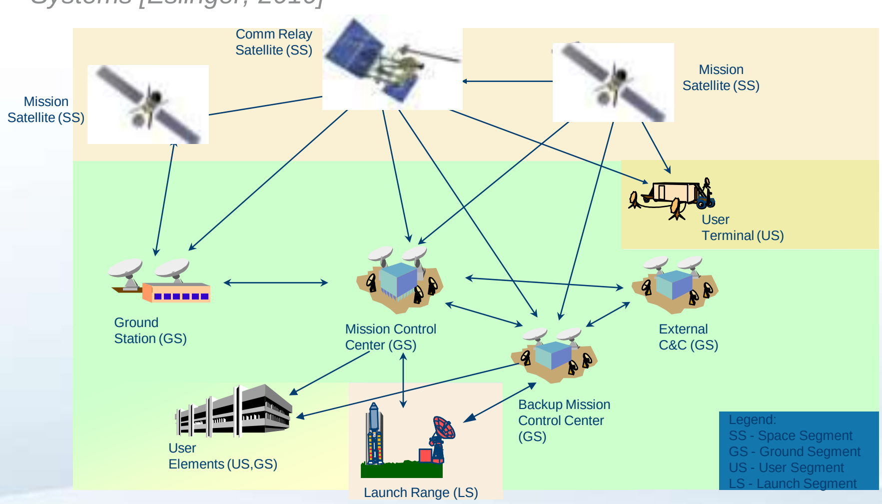


**Figure 1**

***In order to develop useful and predictive CER for NSS systems, it is necessary to develop a predictive CER for NSS software systems***

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# Basic Concepts and Terminology Used in Parametric Modeling

## Introduction to CERs

- CERs express cost as a function of one or more independent cost drivers

  Y = f(x, β);

  *where*

  x is a vector representing the cost driver variables

  β is a vector of coefficients to be estimated by the regression analysis of the sample cost data points

- Below are examples of the common parametric cost model equations for hardware or software systems:
  - *Linear:* $Y = \sum_{1}^{N} \beta_i \cdot x_i$
  - *Non-Linear:* $Y = A \cdot X^B$

    where A and B are constants derived from the regression

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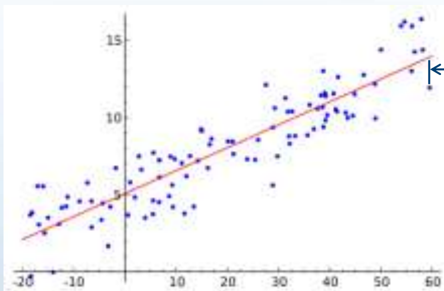
## Linear CER Model

- Linear CER Model: $Y = A + B \cdot X$
- CER residual error, $\varepsilon_i$, is represented as additive errors: $Y = A + B \cdot x_i + \varepsilon_i$
- Problem: Find A and B such that the Sum of the Squared Error ( $SSE = \sum_i^N \varepsilon_i^2$ ) is minimized




$\varepsilon_i$ : delta between the $i^{th}$ data point and the linear regression line

Linear CER assumption:
(1) An error around the regression line, ε, is distributed normally, and is symmetric; or
(2) The number of an observation, N, is sufficiently large so that Central Limit Theorem is applicable

**Figure 2** Linear CER Model

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## Common Measures of CER Uncertainty

- The Standard Error of the Estimate (SEE) is the standard deviation of the cost estimates from a CER
  - SEE is <u>not</u> the CER regression error
- The Confidence Interval (CI) is expressed as $(1 - \alpha) \cdot 100\%$ confident that the true mean value is contained within the calculated range; where α is the probability that the population mean for a parameter lies outside of the CI; ( $0 \le \alpha \le 1$ )
  - e.g., An α of 0.20 represents a confidence level of 80% (i.e., there is 80% certainty that the true value of the mean lies within the CI)
- The Prediction Interval (PI) measures the range of uncertainty around the cost estimates from a CER

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## Prediction Interval Equation

*For single variate linear CER*

$$\hat{Y} \pm t_{\alpha/2,df} \times \text{SEE} \sqrt{\frac{n+1}{n} + \frac{(X-\bar{X})^2}{\sum X^2 - n\bar{X}^2}} \qquad \text{(Eqn 1)}$$

Where

$\hat{Y}$ is the CER prediction

$t_{\alpha/2,df}$ is the upper α/2 cut-off point of the student's t distribution (for the simple linear regression, df = n-2)

n is the number of observations

SEE is the Standard Error of the Estimate

X is the value of the independent variable used in calculating the estimate

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## Basic Parametric Software Cost Model Equation

- Basic Parametric Software Cost Model: $Cost = A \cdot ESLOC^B$ (Eqn 2)

  where

  Cost is the Development Effort in Person-months

  A is the proportionality constant calculated from the cost driver parameters

  ESLOC is the Equivalent Software Lines of Code which normalizes the amount of new, modified, and re-used code applied to calculate the effort to produce the total software product

  B is an exponent (depends on the specific software cost model used, but always > 1)

- Translate into linear CER by transforming into the natural log domain
  - Ln (Cost) = ln (A) + B * ln (ESLOC)

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# Empirical Analysis Results

## Linear Regression Model

*(Data Samples from NSS Software Systems)*

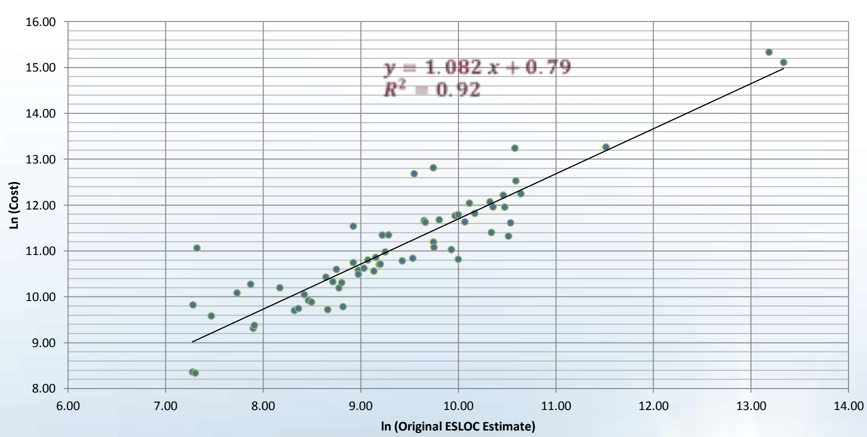


Figure 3

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## Applying 95% Confidence Bands from PI Equation to Historical Data

*Data Collection Samples from NSS Software Systems*

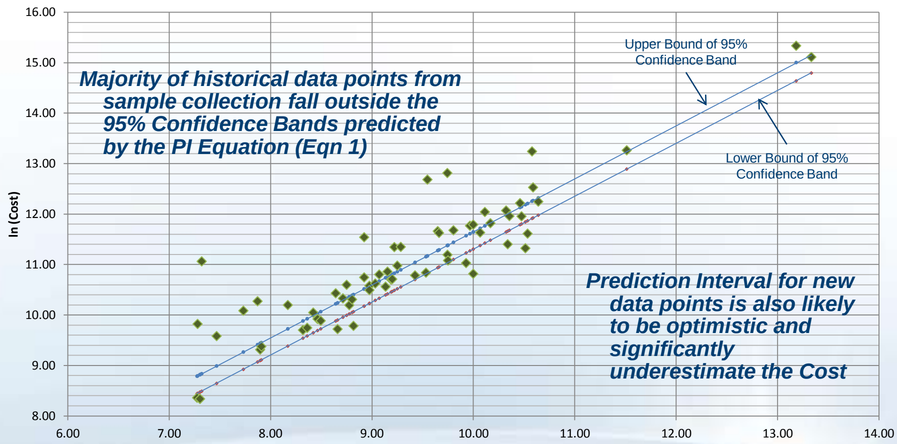


Figure 4

***There are significant statistical variation in addition to the regression errors that the PI Equation is not accounting for***

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## Observation

- There are significant statistical variations in addition to the regression errors that the PI Equation is modeling
  - *PI equation estimates the prediction interval based on the second-order statistics of the CER cost estimates,*
    - $Cost = A * ESLOC^{B} * \varepsilon$
      - *Where ε is the CER regression error*
      - *A and B are constants*
    - SEE (the standard error of the cost estimate) is a function of ε and ESLOC
  - *The independent driver variable (ESLOC) is typically assumed to have insignificant variations relative to the regression errors*
  - *If ESLOC varies significantly, then the SEE term in the PI Equation will significantly underestimate the true prediction interval*
- Question:
  - *How much does ESLOC vary?*

***Empirical and historical data for ESLOC growth provides a definitive answer!!***

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## Empirical Data on Uncertainties of ESLOC Estimates

*High degree of uncertainty and variation in ESLOC estimates!*

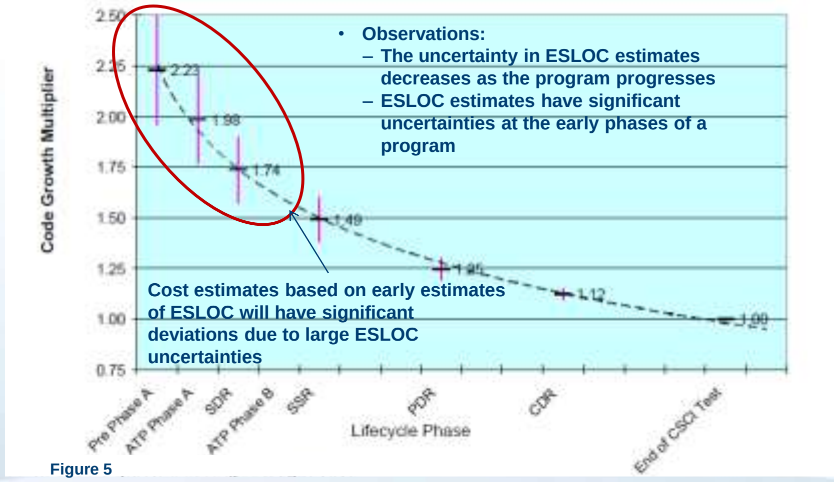


Figure 5

**Based on Barry Holchin's code growth algorithm for medium-to-high complexity software from: [Holchin, 2003]**

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## Historical Data on ESLOC Growth

*Significant growth between initial ESLOC estimate and actual ESLOC*

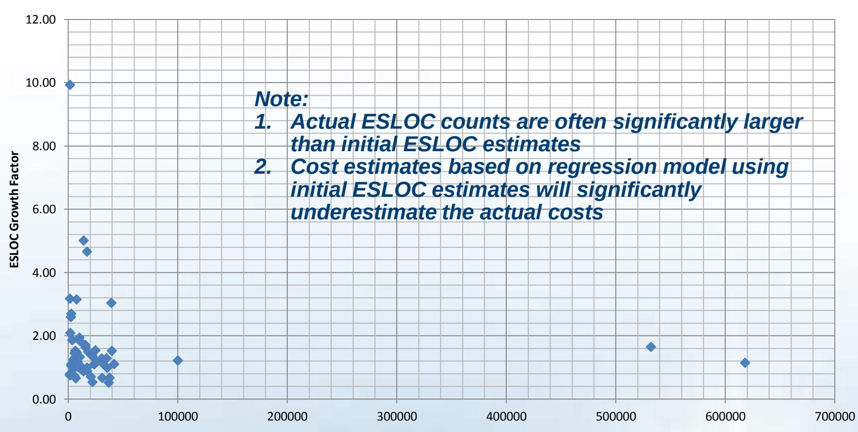


Figure 6

***Improving the prediction interval estimate requires a better statistical characterization of ESLOC growth***

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# Statistical Characterization of Normalized ESLOC

## Basic Software Schedule and Software Cost Models

*Cost* is the development effort (Person-months)
$T$ is the development Time or Duration (Months)
$C$ is a proportionality Constant
D is an exponent
(where $C$ and D depend on the specific software cost model used)

**BASIC PARAMETRIC SOFTWARE SCHEDULE MODEL EQUATION**

$$Cost = A \bullet ESLOC^{B}$$

*Cost* is the development effort (Person-months)
$A$ is a proportionality constant calculated from cost driver parameters
$ESLOC$ is the Effective number of SLOC
$B$ is an exponent (depends on specific software cost model used, but always >1)

**BASIC PARAMETRIC SOFTWARE COST MODEL EQUATION**

$$T = C_1 \bullet ESLOC^{B_1}$$

$T$ is the development Time or Duration (Months)
$C_1$ is the proportionality Constant (i.e., $C \bullet A^D$)
$B_1$ is an exponent (i.e., B • D)

**Alternate Formulation of Parametric Software Schedule Model Equation**

- ESLOC is the independent variable
- ESLOC is assumed to be a known value

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## Known Results from Prior Studies

- Schedule delays exhibit fat-tail behaviors [Wang, 2013], [Smart, 2013], [Wang, 2012]
  - Schedule delays extreme statistics can be approximated by Extreme Value distribution or Log Normal distribution
- Cost growths exhibit fat-tail behaviors [Smart, 2013], [Smart, 2011]
  - Cost growth extreme statistics can be approximated by Log Normal distribution

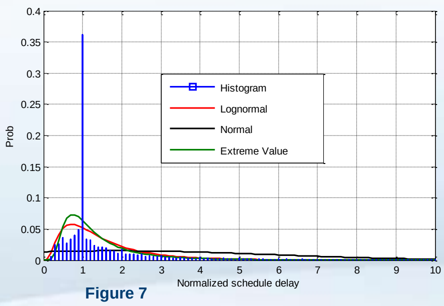
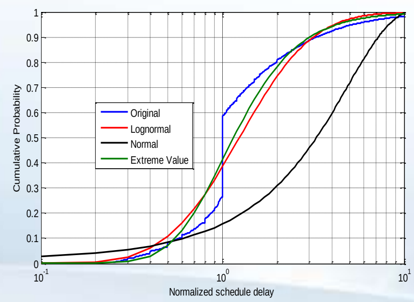



Figure 7

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## Fundamental Theorem

*[Papoulis] "Probability, Random Variables, and Stochastic Processes"*

- For $Y = A * X^B$, (where A and B are real number, and B > 1)

If Y is (Extreme Value , or Log Normal, or Normal) distributed, then X is also (Extreme Value , or Log Normal, or Normal) distributed

If X is (Extreme Value , or Log Normal, or Normal) distributed, then Y is also (Extreme Value , or Log Normal, or Normal) distributed

***Numerical Simulation Results***

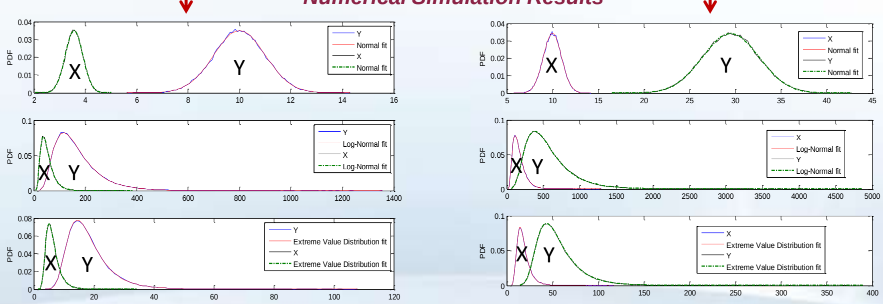


Figure 8

***Conclusion: Normalized ESLOC is characterized by Extreme Value distribution or Log Normal distribution***

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## Empirical Data on Normalized ESLOC Statistics

*Empirical Data confirmed that Normalized ESLOC statistics are approximated by Extreme Value distribution or Log Normal distribution*

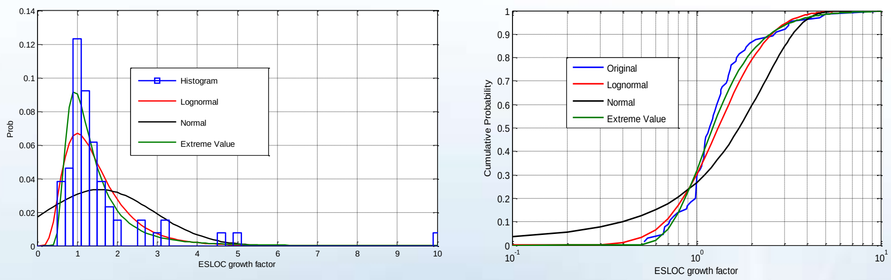


**Figure 9**

***Large Variability of ESLOC violates key assumption in PI Equation***

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## Effect of Normalized ESLOC Statistical Analysis Results

- The PI equation (Eqn 1) will significantly underestimate the prediction interval range for a given α, and thus overestimate the confidence level of a cost estimate or schedule estimate, because:
  - Empirical and historical data show clearly that the key assumption of a regression model's SEE is not applicable for NSS software systems
    - Normalized ESLOC (i.e., ESLOC Growth) can be approximated by fat-tail distributions (e.g., Extreme Value distribution or Log Normal distribution)
    - the variation of Normalized ESLOC is significantly larger relative to the regression error ε
- Adjustment to the Prediction Interval equation is needed to account for the large variability of Normalized ESLOC

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# The Proposed Solution

## Proposed Adjustment to the PI Equation

- Recall from PI Equation (Eqn. 1)

$$\hat{Y} \pm t_{\alpha/2,df} \times \text{SEE}\sqrt{\frac{n+1}{n} + \frac{(X-\bar{X})^2}{\sum X^2 - n\bar{X}^2}}$$

  - *Where*
    - X is the independent cost driver, i.e., ESLOC
    - SEE is a function of $X_\alpha$ and α: SEE($X_\alpha$, α)
    - $X_\alpha$ is the value of $X$ that corresponds to a confidence of $(1 - \alpha/2)$
      - *As the $(1 - \alpha/2)$ percentile increases, the corresponding value for SEE(X, α) will also increase*
  - *As the value for SEE($X_\alpha$, α) increases, the prediction interval will increase accordingly*

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## Empirical Results from Applying Modified PI Equation

## Modified PI Equation Confidence Level Bands

*(in natural log space)*

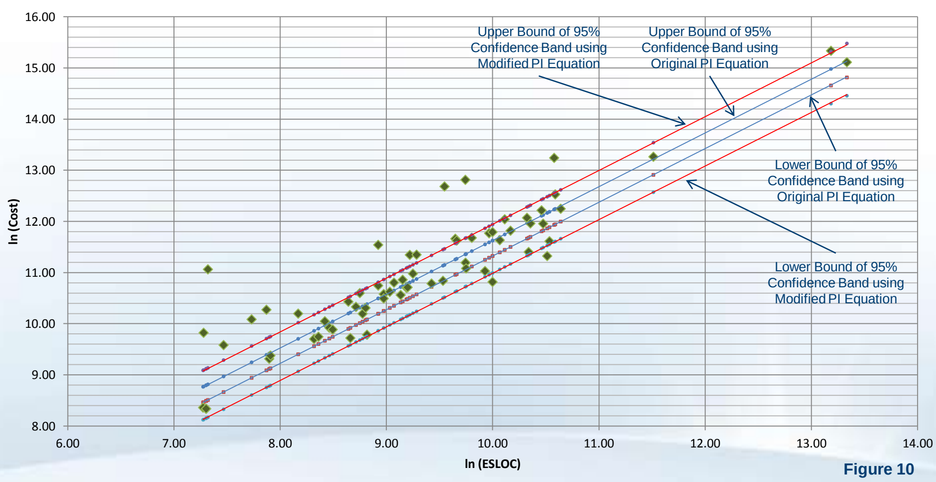


**Figure 10**

***Empirical results show clearly that the modified PI Equation Confidence Level Bands are more accurate than the original PI Equation's***

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## Modified PI Equation Confidence Level Bands

*(in ESLOC, Cost space)*

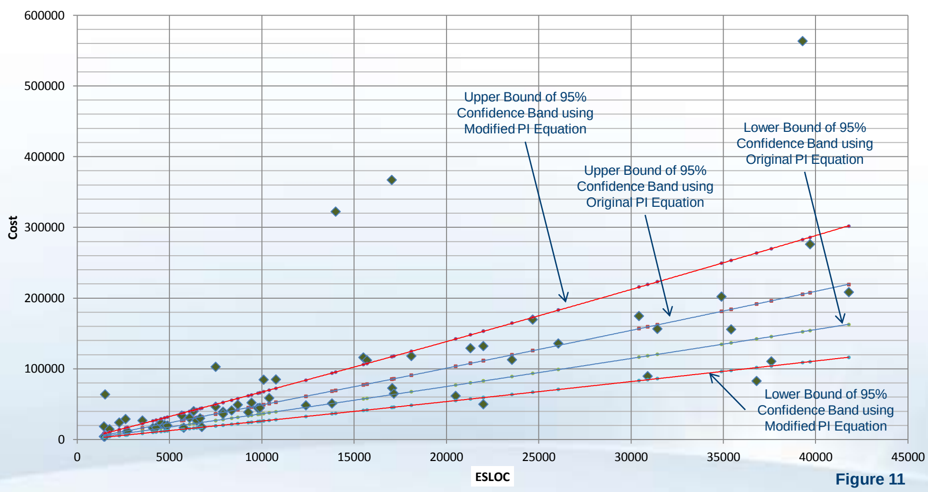


**Figure 11**

***Empirical results show clearly that the modified PI Equation Confidence Level Bands are more accurate than the original PI Equation's***

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## S-Curve Generation

## Generating an S-Curve from a Set of PI Curves

*Notional Example*

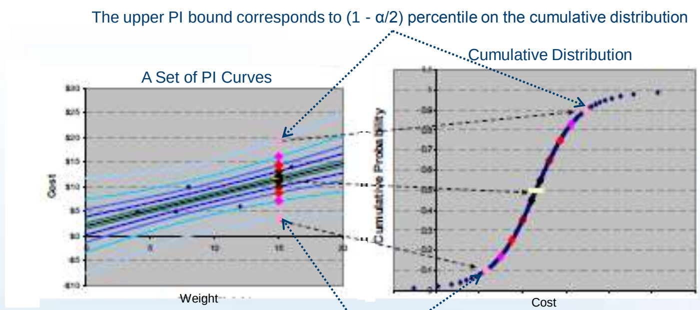


Figure 12

- Generating an S-Curve by varying α from 0 to 1

***Note: the S-Curve will overestimate the cumulative probability, if the Prediction Interval is underestimating the true variation of CER prediction***

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## Empirical S-Curves vs S-Curves from CER Predictions

*Notional Example*

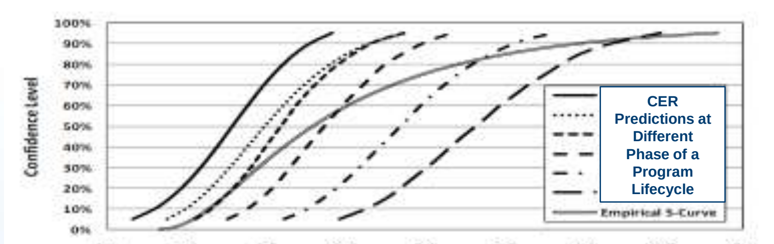


Figure 13

- S-curves from the CER predictions shift to the right as the program progresses
- Empirical S-curves show a more accurate description of the actual cost behaviors

***Conjecture: If we improve the accuracy of the Prediction Interval, then the resulting S-curve should better approximate the actual cost behavior***

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## S-Curves for NSS Software Systems

*The modified PI equation results in S-Curves that approximate the actual cost behavior*

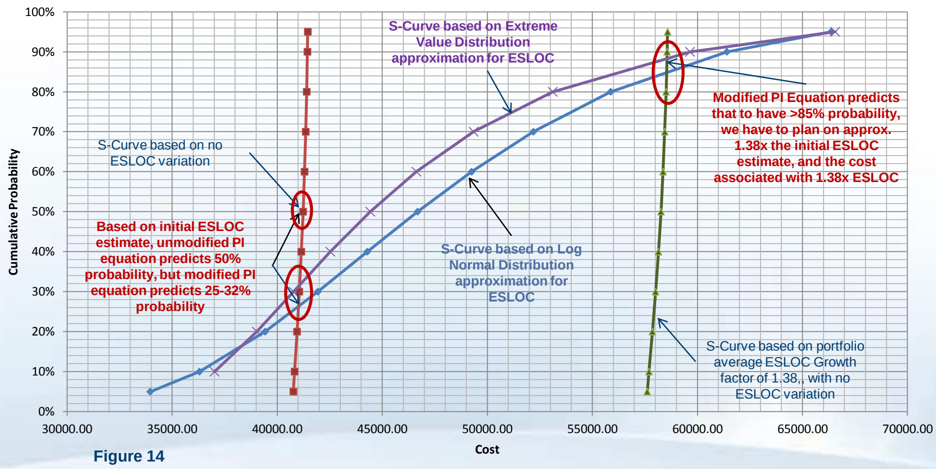


Figure 14

***Most recent actual program experience confirms the prediction from the modified PI equation***

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## Application to Cost Prediction

## Cost Prediction Example

- S-Curves based on modified PI Equation predicts there will be 38% ESLOC Growth on average for a Cumulative Probability of 85-90%
  - Actual program data: 39% ESLOC Growth
- Apply regression model derived from historical data, CER (with modified PI eqn) predicts a 43% Cost Growth
  - Detailed SEER-SEM model with 39% ESLOC Growth predicts a 47% Cost Growth

***The Modified PI Equation produces a more realistic forecast of ESLOC Growth and Cost Growth than unmodified PI Equation***

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## Summary

- In this presentation, we presented analytical analysis as well as empirical data that the existing well-known PI equation consistently underestimates the prediction interval.
  - This underestimation of the prediction interval results in an inflated S-curve confidence level.
- We presented results that show the cause of the PI equation underestimating the prediction interval.
- We presented a proposed modification to the PI equation to account for the variability of the independent cost driver, ESLOC.
- We applied the proposed modification to the PI equation, and showed that the prediction of the modified PI equation is more accurate.
- We generated S-Curves based on the modified PI equation.
  - Our S-Curves better approximate the S-Curve derived from empirical data.
  - Our S-Curve prediction was confirmed by actual program experience.
- Cost Prediction based on our modified PI equation is a close approximation of the Cost Prediction using a detailed SEER-SEM model.

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## References


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# Backup